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PROPOSAL FOR A CYCLOTRON RESONANCE MASER IN InSb

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Abstract

A scheme for amplifying cyclotron resonance radiation in InSb is outlined. To obtain amplification two requirements must be satisfied. The first is that the electron velocity distribution be monoenergetic. Such distributions can be produced by photoexciting electrons in cold p-type InSb. Detailed calculations show that, with a reasonable electron density, the distribution remains monoenergetic throughout the electron lifetime. The second requirement for amplification is that stimulated absorption of cyclotron radiation be inhibited as compared to stimulated emission. This criterion is met by producing the photoelectrons at an energy just below that of the optical phonon in InSb. As a consequence, the stimulated absorption line is broad compared to that for emission, so the latter predominates. Numerical estimates indicate that there is a reasonable chance of producing a far infrared amplifier based upon this technique.

1. Introduction

A NUMBER of authors [1] have discussed the possibility of amplifying cyclotron resonance radiation in plasmas containing free electrons. In general, maser action is difficult to attain because the electronic energy levels are equally spaced and form a ladder (the Landau ladder) which extends to infinite energy. As a consequence, it is not possible to achieve a negative electron temperature in such systems. Nevertheless, the authors cited have demonstrated that negative *conductivities* can occur. To obtain them two requirements must be met. The first is that the electron velocity distribution be decidedly non-Maxwellian — preferably mono-energetic. The second is that, in some manner, stimulated absorption from a given level be inhibited, as compared to stimulated emission. This condition can be satisfied in a variety of ways: by relativistic effects; with an effective mass that is a function of velocity; or with a collision rate that is a sufficiently rapidly increasing function of energy.

That these ideas are not theoretical fictions has been shown by the actual construction [2] of a cyclotron resonance amplifier based on the use of relativistic electrons. There is also experimental evidence [3] that amplification is possible in a system in which the electron collision frequency changes rapidly with velocity. However, both of these experiments were performed on free electrons for which the cyclotron frequency is limited (by available magnetic fields) to the microwave range. In solids, on the other hand, effective masses can be quite small compared to that of the free electron and it is then possible to push the frequency into the near infrared. An obvious candidate for such an application is the crystal InSb, in which the electron effective mass is about one hundredth of that of a free electron. Lax [4] has suggested that the non-parabolicity of the conduction band in InSb (which causes unequal

Landau level spacings) might make possible the construction of a cyclotron resonance maser in this material. To date, however, such a device has not been made to operate. Even if it can be constructed, it will require large fields and pumping powers, and be restricted to the near infra-red range. The purpose of this paper is to analyze a different arrangement which might make possible the amplification of far infrared cyclotron resonance radiation in InSb. The method does not require unequal Landau level spacings, but instead relies upon an electron collision rate which is a rapidly increasing function of velocity to inhibit stimulated absorptions.

The scheme to be considered was suggested by recent experiments [5,6] on the intrinsic photoconductivity of InSb. In this work it is observed that, at low temperatures, the photoconductivity is a strongly oscillating function of photon energy for energies in the range 0.24-0.50 ev. The explanation offered for this phenomenon involves the optical phonons in InSb, and can be most easily understood by reference to Figure 1, which shows the energy band structure of the crystal. Here we see that, because of the $\Delta k = 0$ optical selection rule, the kinetic energy of the photoelectrons is determined by the photon energy.

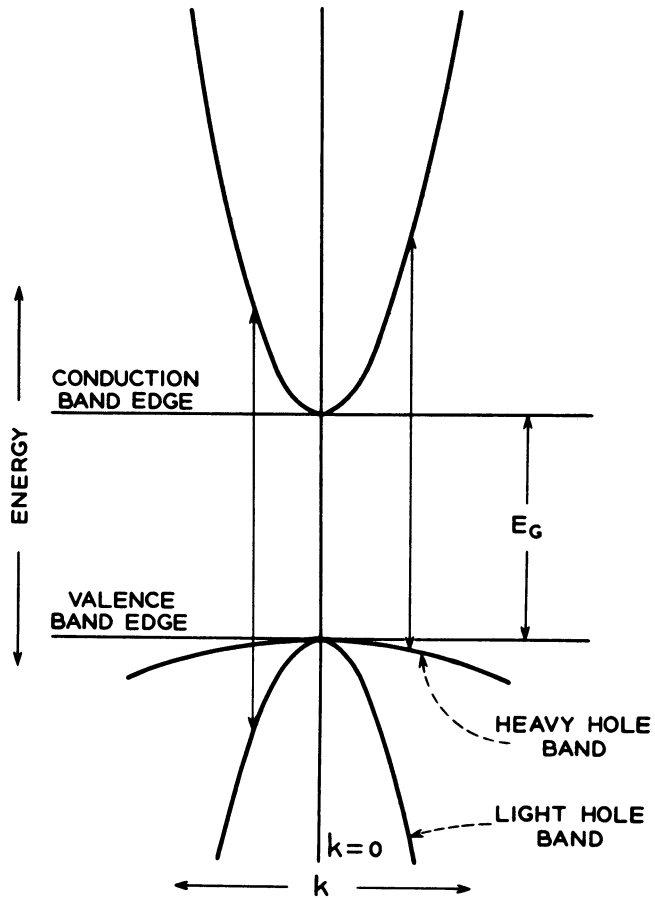

FIGURE 1
Energy bands of InSb

There are, of course, two groups of photoelectrons — those arising from the heavy hole band, and those from the light hole band. For a given photon energy, the kinetic energy of the former is very nearly twice that of the latter — the fractional deviations from this ratio being of order m^*/M, where m^* is the electron mass, $M \sim 30m^*$ that of the heavy hole. For the time being let us focus our attention on electrons that are excited from the heavy hole band. They are more numerous — because of a higher joint density of

states — than those excited from the light hole band, and seem to be primarily responsible for the oscillations. Upon comparing Figure 1 with the experimental data, one finds that the photoconductivity minima occur at precisely those photon energies for which the electrons have a kinetic energy that is an integral multiple of the optical phonon energy, $\hbar\omega_0 \simeq 0.024\ eV$. This is the clue which enables one to understand the photoconductivity results. For, consider the behavior of these electrons when the crystal is irradiated with an arbitrary photon frequency. The electron kinetic energy may then be written as

$$E = n\hbar\omega_0 + \Delta E \tag{1}$$

where n is an integer, and $0 \leq \Delta E < \hbar\omega_0$. Since the electron is very strongly coupled to the optical phonons, it loses the energy $n\hbar\omega_0$ almost instantaneously (in a time of the order of 10^{-13} sec [6]) and is then left in a state of energy ΔE in which it can no longer emit optical phonons. At this point, the only way its energy can relax is via acoustic phonon interactions. However, the time constant for this process is known [7] to be quite long ($\simeq 10^{-7}$ sec) when the InSb lattice is at liquid helium temperatures. Thus, for all practical purposes the photoelectron is isolated from the lattice and remains at energy ΔE until it recombines (in about 10^{-10} sec [5]). Since the mobility, and also possibly the recombination rate, are rapidly varying functions of ΔE, it is easy to see how the photoconductivity oscillations arise.

The observation of conductivity oscillations emphasizes an important feature of the photoconduction process in InSb; namely, that it quite naturally produces a monoenergetic supply of conduction electrons which preserve their energy for a long time, provided that it is less than $\hbar\omega_0$. This statement does not, unfortunately, imply that the velocity distribution remains monoenergetic, for electron-electron interactions may convert it to a Maxwellian without changing the average energy. Moreover, the statement only applies to those electrons which are produced from a single one of the hole bands, say the heavy hole band. There will always be, in addition, another group of photoelectrons, with half as much energy, that come from the light hole band. Nevertheless, the experiment suggests that monoenergetic distributions are, at least, a possibility. We shall see presently that, with reasonable electron densities, the electron-electron interactions are not, in fact, sufficiently rapid to cause conversion. Furthermore, the apparent difficult due to the existence of two separate velocity distributions is probably not serious. Theory [8] indicates that the average matrix elements for the two types of transitions should be the same. However, at a given photon energy, the joint density of states for transitions from the heavy hole band is about three times greater than that from the light hole band. Thus, on this basis alone, the photoelectrons from the heavy hole band should be three times more numerous. In addition, electrons from the light hole band are produced at lower energies, which means a smaller matrix element for the cyclotron resonance transition, and a less effective contribution to amplification (or absorption). Finally, because they have small energies, the electrons from the light hole band may recombine more rapidly. The combination of the three effects makes it very likely that these electrons can be ignored — we will generally do so in the following. However, it should be mentioned that, if this is not the case, there is a scheme (albeit, a fairly complicated one) for actually bringing the two distributions to the same energy, and thus producing a truly monochromatic distribution. It will be described briefly in a later section. In any case, the important point is that it *does* seem possible to produce a monoenergetic distribution in InSb. This, as was indicated earlier, is the primary requirement for obtaining cyclotron resonance amplification.

The second condition for amplification also follows, in a very natural way, from the photoproduction process. For, consider the collision rate of electrons whose kinetic energy is near that of the optical phonon. One expects, from mobility and cyclotron resonance measurements, that electrons with energy less than the optical phonon energy have a relatively small collision rate. On the other hand, the photoconductivity results imply that electrons with energy greater than $\hbar\omega_0$ scatter almost instantaneously. It follows that the scattering rate (or, equivalently, the line width of the cyclotron resonance transition) is an exceedingly rapidly increasing function of energy for energies near $\hbar\omega_0$. This is one way of inhibiting stimulated absorptions, thereby satisfying the second amplification condition.

We see, therefore, that both of the requirements for cyclotron resonance amplification are, at least potentially, met in InSb. In the remainder of the paper we will analyze these possibilities more closely, and suggest arrangements for actually achieving amplification.

2. The Amplification Process

In this section we wish to consider the formula which relates the rate of photon generation (or absorption) to the electron velocity distribution. Discussion of the factors which determine the velocity distribution, itself, will be postponed to the next section. We begin by considering the energy levels of electrons in the conduction band of InSb in a constant magnetic field, B_0. To simplify matters the electron effective mass will be assumed constant and spin splitting (which is a third of the cyclotron frequency) will be ignored. The energy levels are then given by the expression

$$E(n,k_z) = \left(n + \frac{1}{2}\right)\hbar\omega_c + \frac{\hbar^2 k_z{}^2}{2m^*} \quad , \tag{2}$$

where $n = 0,1,2,\ldots$, $\omega_c = \dfrac{eB_0}{m^*c}$ is the cyclotron frequency, and $\hbar k_z$ is the electron momentum in the direction of the magnetic field. The distribution function for electrons among these levels will be denoted by $F(n,k_z)$. Since elastic scattering processes are generally quite rapid in InSb ($\tau_{elastic} \simeq 10^{-11}$-$10^{-12}$ sec), the function F will usually be a function of $E(n,k_z)$ alone.

Now let us imagine that an electromagnetic wave, of frequency ω, propagates through the InSb crystal in the direction of the applied magnetic field. This wave will stimulate transitions between the energy levels of equation (2). The resulting rate of change of photon number (neglecting spontaneous emission) is given by the equation

$$\frac{dN}{dt} = \frac{2\pi}{\hbar^2}\left(\frac{e\mathbf{A}}{m^*c}\right)^2 \sum_{n,k_z} \left\{ | < n,k_z \mid \pi^+ \mid n\text{-}1,\ k_z > |^2 \times [F(n,k_z) - F(n\text{-}1,k_z)]\ g[\omega; n,n\text{-}1,k_z] \right\} . \tag{3}$$

Here $\mathbf{A}$ is the vector potential of the electromagnetic wave which is assumed to be circularly polarized, $\pi^+ = \left(\dfrac{\pi_x + i\pi_y}{2}\right)$ where the operator $\pi = (\mathbf{p} - \dfrac{e}{c}\mathbf{A}_0)$, $\mathbf{A}_0$ is the vector potential of the d.c. magnetic field, and $g(\omega; n,n\text{-}1,k_z)$ is the line-shape function for optically induced transitions between the energy levels $E(n,k_z)$ and $E(n\text{-}1,k_z)$. Equation (3) incorporates the standard dipole selection rules; $\Delta n = \pm 1$, $\Delta k_z \simeq 0$. The extra factor of $1/\hbar$ that appears in its coefficient arises because the line-shape function, g, is written in terms of frequency, rather than energy, variables. We will assume that g has the Lorentzian form

$$g(\omega; n,n\text{-}1,k_z) = \frac{1}{\pi}\left[\frac{\gamma(n,n\text{-}1,k_z)}{(\omega-\omega_c)^2 + \gamma^2(n,n\text{-}1,k_z)}\right] \quad , \tag{4}$$

where $\gamma(n,n\text{-}1,k_z)$ is the line width for the transition in question. For our purposes, the important feature of the function $\gamma(n,n\text{-}1,k_z)$ is that its value changes very abruptly at that point at which $E(n,k_z) = \hbar\omega_0$. When $E(n,k_z) < \hbar\omega_0$ optical phonon emission is not possible and we may expect the line width to be of the order of the elastic collision rate — i.e. $\gamma \simeq \dfrac{1}{\tau_{elastic}} \cong 10^{11} - 10^{12}$ sec^{-1} for $E(nk_z) < \hbar\omega_0$. For convenience, we will also assume that it has a constant value, γ_0, in this range. On the other hand, for electrons with $E(n,k_z) \geq \hbar\omega_0$ the line width is at least an order of magnitude greater since the lifetime for optical phonon emission is about 10^{-13} sec. Here the line broadening is extreme. Therefore, at resonance ($\omega=\omega_c$), transitions involving states with $E(n,k_z) \geq \hbar\omega_0$ are severely inhibited. This is particularly important for the absorption $(n\text{-}1,k_z) \rightarrow (n,k_z)$, in which $E(n\text{-}1,k_z) < \hbar\omega_0 < E(n,k_z)$. Transitions in which both the initial and final states have energy greater than $\hbar\omega_0$ will contribute very little to equation (3), in any case, since we expect the distribution function $F(n,k_z)$ to be small for $E(n,k_z) > \hbar\omega_0$. Thus, to a fairly good approximation, we may say that at resonance no states with energy greater than $\hbar\omega_0$ contribute to equation (3), either as initial *or final* states. The latter is, of course, the vital point.

Setting $\omega = \omega_c$ and $\gamma(n,n-1,k_z) = \gamma_0$ for $E(n,k_z) < \hbar\omega_0$, we then find that equation (3) takes the form

$$\frac{dN}{dt} = \frac{2}{\hbar^2\gamma_0}\left(\frac{eA}{m^*c}\right)^2 \sum_{\substack{n,k_z \\ (E(n,k_z) < \hbar\omega_0)}} \left\{ | < n,k_z \mid \pi^+ \mid n-1,k_z > |^2 \, [F(n,k_z) - F(n-1,k_z)] \right\} \tag{5}$$

The important feature of this formula is the fact that the summation over states is now restricted to the finite range, $E(n,k_z) < \hbar\omega_0$. As a consequence, we may obtain a positive value of $\frac{dN}{dt}$ (implying amplification) if the requirement

$$F(n,k_z) - F(n-1,k_z) > 0 \tag{6}$$

for $E(n,k_z) < \hbar\omega_0$ is satisfied. Physically, one may say that the extreme broadening of those levels with $E(n,k_z) > \hbar\omega_0$ truncates the Landau ladder at $E(n,k_z) = \hbar\omega_0$. Thus, we have only a finite series of energy levels to deal with, and something akin to a negative temperature is possible within this restricted manifold. The crucial question then becomes that of satisfying the criterion of equation (6). In general, it is exceedingly difficult in solids to maintain such highly non-Maxwellian velocity distributions. However, the case of InSb is somewhat special, and the remarkable photonconductivity results give reason to hope that it may be possible to do so. This is the problem to which we turn our attention in the next section.

3. Thermalization Rates for the Electron Distribution

In the following we will imagine that electrons are produced photoelectrically in p-type InSb (cooled to liquid helium temperatures) by a monochromatic light source. Our aim is to produce electrons whose kinetic energy is just below that of the optical phonon. If, as seems quite probable (see Section 4), the effects of electrons excited from the light hole band can be ignored, this goal can be achieved relatively easily. One merely chooses the appropriate frequency and band width for the light, in such a way that photoelectrons are produced with kinetic energies in the range $\hbar(\omega_0 - \omega_c) < E < \hbar\omega_0$. (We assume $\omega_c < \omega_0$.)*

On the other hand, if electrons excited from the light hole band cannot be ignored, a somewhat more involved pumping scheme is necessary. The trick in this case is to use light which excites electrons from the heavy hole band to kinetic energies very slightly less than that of *two* optical phonons, say with energy $E_1 = 2\hbar\omega_0 - \delta$. Such electrons will immediately radiate an optical phonon and end up with energy $E_1 = \hbar\omega_0 - \delta$. The same light produces electrons from the light hole band with kinetic energy $E = \hbar\omega_0 - \delta/2$ (neglecting corrections of order m^*/M. Thus, if δ is small, the two distributions are superposed to produce a single monoenergetic one with energy just below $\hbar\omega_0$.

In any case, we will assume that by some means a distribution satisfying the condition $\hbar(\omega_0 - \omega_c) < E < \hbar\omega_0$ has been created. Such a distribution has the property that $F(n-1,k_z) = 0$ for $E(n,k_z) < \hbar\omega_0$. Thus, equation (6) is satisfied, at least initially. The important question is how long such a monoenergetic distribution can persist. In particular, we want *all* thermalization times for the distribution to be long compared to the recombination time. If this condition is met, the criterion of equation (6) will be maintained. Thus, we are faced with the problem of examining various thermalization mechanisms, and comparing their rate with the recombination rate. In the experiments of ref. [5] the recombination time, τ_r, was about 10^{-10} sec. This turns out to be a convenient value so, henceforth, we will assume that it is typical of the samples we consider. Any thermalization process which takes appreciably longer than 10^{-10} sec will not then perturb the energy distribution.

There appear to be four mechanisms by which the initially monoenergetic electron distribution can relax to a Maxwellian. They are electron-phonon interactions, electron-electron interactions,

*This condition restricts the device to the wave length range $\lambda \gtrsim 50\,\mu$.

electron-hole interactions, and, finally, the possibility that the velocity distribution may break up by the spontaneous generation of plasma oscillations. We will consider these processes in turn.

Electron-phonon interactions can be of two sorts — those with optical phonons, and those with acoustic phonons. However, we have deliberately chosen the initial distribution in such a way that the electrons have insufficient energy to generate optical phonons. We are left, therefore, with the electron — acoustic phonon interaction — which is known to be quite weak when the InSb lattice is at 4°K. Peskett and Rollin [7] have measured a thermalization time of 3×10^{-7} sec under these circumstances, and this number is in fairly good agreement with theoretical estimates [9]. We conclude that the electron-acoustic phonon interaction is about three orders of magnitude too small to perturb the velocity distribution. The observation of photoconductivity oscillations also forces one to a similar conclusion.

Another important thermalization mechanism is electron-electron scattering. This process differs from the phonon interaction in that it does not change the total energy of the electrons, but merely converts a monochromatic distribution into a Maxwellian with the same average energy. From the point of view of maser operation, however, the two processes are equally undesirable. No measurements of electron-electron relaxation rates have been made, but there are a number of theoretical estimates in the literature [10]. In particular, MacDonald, Rosenbluth and Chuck have done a calculation, using the Fokker-Planck equation, which closely approximates the situation we wish to investigate. They start a group of electrons in a nearly monoenergetic distribution, and determine the time at which it becomes Maxwellian. This time is given by $\tau_{e\text{-}e} = 0.73\, t_c$ where t_c is the "self-collision time" defined by Spitzer [11]:

$$t_c = \frac{(m^*)^{1/2}(3kT)^{3/2}\epsilon^2}{(8\times 0.714)\pi\rho e^4 \ln(\Lambda)} \tag{7}$$

where

$$\Lambda = \frac{3}{2}\sqrt{\frac{(kT)^3\epsilon^3}{\pi\rho e^6}}\ , \tag{8}$$

$\epsilon = 16$ is the dielectric constant of the InSb lattice, ρ the electron density, and $(3kT)/2$ the average electron energy. For the concentrations of interest to us the value of $\ln(\Lambda)$ is about 10, and $\tau_{e\text{-}e}$ is approximately given by

$$\frac{1}{\tau_{e\text{-}e}} \simeq 10^{-3}\rho \text{ sec}^{-1}\ . \tag{9}$$

This thermalization time is to be compared with the recombination time, $\tau_r \simeq 10^{-10}$ sec. The two become equal when $\rho = 10^{13}$ cm^{-3}. To be on the safe side we will restrict ρ to be less than 10^{12} cm^{-3}. This insures that the electron-electron relaxation time is about an order of magnitude longer than τ_r. Thus, this process should not appreciably modify the velocity distribution. An electron density of 10^{12} carrier/cm^3 appears to be quite small, but we will see that, because of the large matrix element for the cyclotron transition, it is adequate to produce amplification.

The third thermalization process to be considered is the interaction of the photoelectrons with holes in the p-type InSb. At low temperatures these are bound to acceptor centers in the lattice. Here we can arrange things, by proper choice of the acceptor, so that the electrons do not have sufficient energy to excite the bound holes, and thus cannot lose energy to them. In particular, it has been found [12] that the acceptors Ag and Au in InSb have binding energies

$$E_i = 0.023 - 0.028 \text{ eV} \quad \text{for Ag}$$

$$E_i = 0.032 - 0.043 \text{ eV} \quad \text{for Au}\ .$$

The two values in each case are obtained from different types of measurements, with the upper probably being most reliable. We see that either of these acceptors has an ionization energy which is sufficiently large to prevent electrons with energy less than the optical phonon energy ($\simeq$ 0.024 eV) from losing energy to it. Again, this conclusion is consistent with the observation of photoconductivity oscillations — no oscillations would be observed if there were appreciable energy interchange between the electrons and holes. In this regard it is significant that in the one experiment [6] in which the dopant was specified, it was the acceptor Ag.

Finally, we must consider the possible break-up of the velocity distribution by generation of plasma oscillations. It is known that in sufficiently non-equilibrium distributions plasma oscillations are spontaneously generated and cause extremely rapid thermalization. This problem has been treated by Jackson [13] who gives a simple criterion for the stability of a velocity distribution. A distribution is stable towards the generation of a plasma oscillation travelling in the direction $\vec{n}$ if the one-dimensional velocity distribution, defined by

$$\tilde{F}(v_1) = \int F(v)\, \delta(\mathbf{n}\cdot\mathbf{v} - v_1)\, d^3v\,, \tag{10}$$

satisfies the condition

$$v_1 \tilde{F}'(v_1) \leq 0 \tag{11}$$

for all v_1. A straightforward calculation shows that this criterion *is* met by the monoenergetic distribution $F(v) \propto \delta\left(\frac{m^*v^2}{2} - E_0\right)$, which is then stable in the sense discussed by Jackson. Similar conclusions are also obtained by Penrose [14].

To summarize the preceding conclusions we may say that, by working in Ag or Au doped InSb at low temperatures, and restricting photoelectron densities to the range $\rho \leq 10^{12}$ cm^{-3}, it seems possible to insure that an initially monoenergetic electron distribution will remain so throughout a recombination time, $\tau_r \simeq 10^{-10}$ sec. This, as we have seen, is the crucial requirement for obtaining amplification of cyclotron resonance radiation. Assuming that such a monoenergetic distribution can be maintained, we now turn to the problem of how much amplification one might expect to obtain in a practical situation.

4. Calculation of Gain

In the preceding section we have shown that it is probable that a non-equilibrium velocity distribution, of the type required for cyclotron resonance amplification, can be maintained in p-type InSb doped with Ag or Au. Henceforth, we will assume that this criterion (see equation 6) is fulfilled, and investigate the sort of amplification rates one might expect with reasonable values of the parameters. To be specific, we will consider a situation such as that shown in Figure 2, in which there are several Landau sub-bands below the energy $\hbar\omega_0$. The light which produces the photoelectrons has a frequency such that these electrons are produced in the kinetic energy interval $(\hbar\omega_0 - \hbar\omega_c) - \hbar\omega_0$ (the banded region in the figure). If these electrons absorb a photon of frequency ω_c they arrive in the energy region in which optical phonon emission is possible. Such absorption processes have a very broad line and contribute relatively little to $(dN)/(dt)$ (equation 3). They are ignored in the approximation which leads from equation (3) to equation (5). The important processes are the stimulated emissions which have a narrow line — and can produce amplification.

With the sort of excitation described above, the distribution function $F(n-1,k_z) = 0$ whenever $E(n,k_z) \leq \hbar\omega_0$. Thus, the second term of equation (5) does not contribute, and the formula simplifies to

$$\frac{dN}{dt} = \frac{2}{\gamma_0 \hbar^2}\left(\frac{eA}{m^*c}\right)^2 \sum_{\substack{n,k_z \\ (E(n,k_z) \leq \hbar\omega_0)}} \left\{ |< n,k_z \mid \pi^+ \mid n-1,k_z >|^2 F(n,k_z)\right\} \tag{12}$$

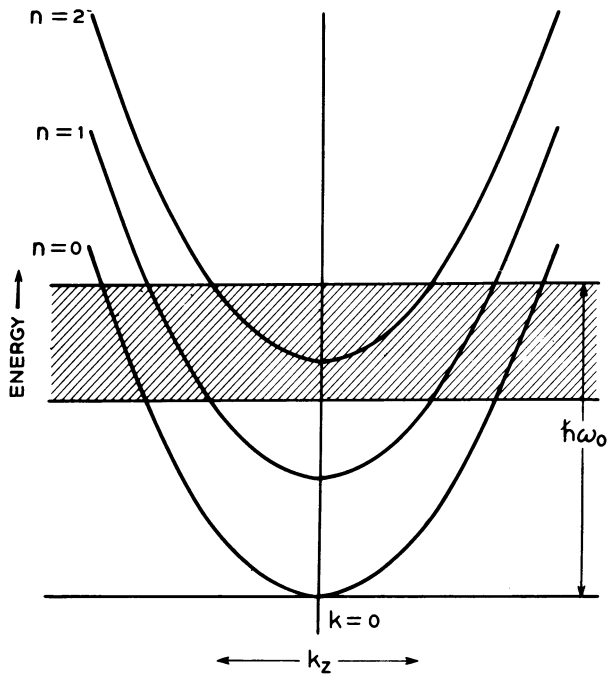

FIGURE 2
Landau sub-bands in the conduction band of InSb

The matrix element of π^+ may be evaluated using the well-known harmonic oscillator wave functions, giving the result

$$\frac{dN}{dt} = \frac{e^2A^2\rho}{\hbar\gamma_0 m^* c^2}\bar{n}\omega_c \quad , \tag{13}$$

where

$$\bar{n} = \left(\frac{1}{\rho}\right)\sum_{n,k_z}[nF(n,k_z)] \tag{14}$$

is the average value of the Landau level quantum number. Finally, we may relate the value of the vector potential to the photon density through the standard formula for the energy density of the electromagnetic field. The result is

$$\frac{1}{N}\frac{dN}{dt} = \frac{4\pi\rho e^2}{m^*\epsilon}\left(\frac{\bar{n}}{\gamma_0}\right) \tag{15}$$

or, as a gain per unit length,

$$G = \frac{d(\ln N)}{dx} = \frac{4\pi\rho e^2}{m^*}\left(\frac{n}{\gamma_0 c\sqrt{\epsilon}}\right) \quad . \tag{16}$$

The authors of ref. [5] make estimates of electron mobilities in their InSb samples, from which one may deduce an electron collision rate. For electrons with energy equal to that of the optical phonon one finds $\gamma_0 \simeq 2 \times 10^{11}$ sec^{-1}. For ρ we use the previously estimated upper limit of 10^{12} cm^{-3}, and assume $\bar{n} = 1$. The value of G then turns out to be 10 cm^{-1}, which is fairly substantial. As usual in maser

processes, the value of G is quite sensitive to the line width (γ_0) for the transition in question. The value given above is based upon the assumption of ionized impurity scattering, but in the sort of samples we are considering, with deep-lying acceptor levels, most of the centers would actually be neutral at liquid helium temperatures. Under these circumstances it might be possible to achieve smaller values of γ_0 than that quoted above. This is a point of vital importance to the operation of the device, and one on which it would be helpful to have detailed experimental information.

Until now we have ignored the effect of electrons excited from the light hole band. It is fairly easy to see, however, that they do not materially alter the gain estimates made above. To investigate this point we return to equation (5) and include in the velocity distribution a component describing electrons excited from the light hole band. These electrons are produced at an energy well below $\hbar\omega_0$ — in a region where the line width is slowly varying. As a consequence, both emission and absorption processes play a role in determining their contribution to equation (16). The calculation is straightforward and exactly parallels that for the conductivity of an electron gas. One finds that equation (16) should be corrected by a term $-\frac{4\pi\rho' e^2}{m^*}\frac{1}{\gamma_0 c\sqrt{\epsilon}}$, where $\rho' \simeq 1/3\rho$ is the density of electrons excited from the light hole band. Notice that the factor $\bar{n}$ does not appear in this expression. It is absent because the stimulated emission rate nearly cancels that due to stimulated absorption. Numerically, this correction is considerably smaller than the G of equation (16), and would be even less important if one took account of any velocity variation in γ_0.

Finally, it is important to consider whether or not the photoelectron densities we have imagined ($\rho \approx 10^{12}$ cm^{-3}) can be achieved with reasonable pumping powers. At helium temperatures it is possible to dissipate of the order of 1 W/cm^2 of sample surface. This is also about the power radiated by a black body source at a few thousand degrees in an energy interval of 0.0025 eV at 0.25 eV. With photons of this energy, and an electron lifetime of 10^{-10} sec one finds that the steady state number of electrons in the sample (with a surface of 1 cm^2) is 2×10^9. Thus, one achieves the required electron density if the sample thickness is 2×10^{-3} cm. Since this is also about the diffusion length in material with such a low lifetime, the numbers work out in quite a convenient way. A two-way pass through such a sample gives a gain of 4 per cent. This is sizable, and should be detectable in an infrared cyclotron resonance experiment. However, one can do even better with a multi-pass geometry such as that shown in Figure 3.* Here

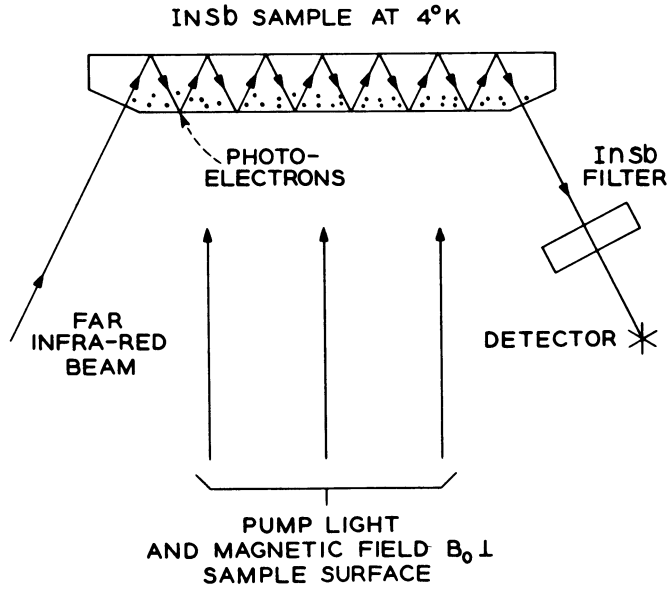

FIGURE 3

Schematic diagram of experiment to observe gain

*I am grateful to L. C. Hebel for suggesting this arrangement.

the pumping light is incident normal to the surface of the sample. The infrared signal enters the InSb wafer at its end, makes several passes through the active region, and emerges to the detector at the opposite side. A convenient wave length for such an experiment would be 100 μ. Conventional infrared methods work fairly well in this range and a magnetic field of about 15 kg would be required.

5. Conclusion

In the preceding sections it has been demonstrated that there is a reasonable chance of amplifying cyclotron resonance radiation in InSb. The scheme envisaged uses the extreme line widths of Landau levels with energy greater than that of the optical phonon to inhibit induced absorption. Throughout the calculations it has been assumed, for simplicity, that m^* is constant. Any variation of m^* with energy can always be used to assist the amplification process.

The amplification technique depends upon the attainment of a nearly monoenergetic distribution of photoelectrons, with energy just below that of the optical phonon. Such distributions are naturally produced in the optical pumping process. Various relaxation mechanisms for such a distribution are considered and, with conservative values of the parameters, it is concluded that a distribution containing 10^{12} electrons/cc remains monoenergetic over an electron lifetime of 10^{-10} sec. Such a distribution, in a sample 2×10^{-3} cm thick, produces a gain of 2 per cent per pass. However, it should be realized that this figure is quite sensitive to the collision rate of the photoelectrons and that a considerable improvement might be achieved in a suitable sample.

Finally, a few words should be said about the pump. Powers of 1 W/cm^2 in the narrow frequency range required may be obtainable from conventional sources with appropriate filtering. In the long run, however, the elegant way to do the job is with a laser pump — preferably one of InSb itself. Unfortunately, the InSb junction laser [15] has somewhat too low a frequency for this purpose. However, it might be possible to make such a laser in a mixed crystal having a slightly larger band gap (GaSb, for example). If it could be made, such a pump would have the twofold advantage of giving high power in a narrow line, and being magnetically tunable.

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