How students learn from multiple contexts and definitions: Proper time as a coordination class

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(Received 21 October 2007; published 23 April 2008)

This article provides an empirical analysis of a single classroom episode in which students reveal difficulties with the concept of proper time in special relativity but slowly make progress in improving their understanding. The theoretical framework used is "coordination class theory," which is an evolving model of concepts and conceptual change. The paper will focus on showing to what extent and in what sense most of the conditions and events in the data corpus seem understandable from the point of view of coordination class theory. In addition, however, some extensions of the theory are implicated, although we argue that they are "natural" extensions, improvements that extend, but do not threaten, the core theory. In particular, we observe students articulately aligning different ways of determining proper time, and we conjecture, more generally, that such a process is strongly consistent with coordination class theory and likely to be productive in other cases of conceptual change. The empirical analysis is explicitly connected to the general issue of theories and theory development in studies of conceptual change.

DOI: 10.1103/PhysRevSTPER.4.010107 PACS number(s): 01.40.Fk, 01.40.Ha, 01.40.ek

I. INTRODUCTION

A. Developing "humble" theories

This article applies and extends an existing model of concepts and conceptual change to an episode of problematic learning. A class of high school students struggles with the difficult idea of "proper time" in special relativity. The data reveal some particular aspects of difficulty and, in the end, show that much progress has been made and also how it was made. Both (a) the nature of the problems encountered and (b) how one can understand progress in general terms are important empirical and theoretical foci here.

Although this article has a strong empirical orientation, its main importance is theoretical. Its aim is to advance the theory of conceptual change in learning science. The main part of this introduction describes in a general way the kind of theory we are aiming to develop. It also anticipates what we can expect from applying it to a context and, thus, frames the main empirical results of the article. A substantial part of the reason that we include this metatheoretical overlay is that we strongly believe that physics education research could benefit a great deal from more active consideration of its theories and the nature of theory building.

In "applying a theory or model"[1] to a situation, one expects a good model to invoke insights about an empirical case in question, explaining in some measure how and why things work in the way they do. Good science also does not presume its models but tests them in application to cases. Beyond possible rejection, testing in cases often results in refining the model, either substantially (introducing new entities and relations) or analytically (changing or refining definitions and meanings).

The theory in question here is an example of a "humble theory,"[2] which is a deliberate attempt to step back from grand theories to more local ones that are, at once, more specific (thus more easily applied to cases) and self-consciously incomplete. The rationale for humble theories is that we are still early in the learning sciences, and a premature jump to theories intended to cover all circumstances is likely to create theories that are vague and distanced from particular cases, and thus both very difficult to apply and also even to test. Constructivism, in its broadest sense—the view that students learn on the basis of prior conceptualizations—may be so general that it is essentially impossible to refute, and its particular implications in a case may be highly ambiguous.

The theory applied here is a *coordination class* view of conceptual change[3] (see Ref. 4 for an updated review). Briefly, a "coordination class" is a particular type of concept. The theory specifies the organization of knowledge in a well-developed coordination class and specifies several hurdles that must be overcome in achieving "well-developed" status. As such, the theory also (partially) specifies the processes of acquisition of a coordination class. Coordination class theory is humble in two senses: First, it is not intended to cover all cases of conceptual change; some concepts are simply not coordination classes.[5] Second, coordination class theory provides only a sketch of the related conceptual change process. Still, coordination class theory is much more detailed than most theories of conceptual change. For a concept to be a coordination class, several theoretical categories must be cogently identified in empirical data, with particular relationships among them, and a set of consequences is entailed. The particulars will be treated later in the article.

In working with humble theory, nuanced consequences of studies of cases of application are generally more important than more categorical consequences.[2] Of course, humble theories may be categorically refuted and also one expects insights in applying them. However, because humble theories are not intended to be universal, even their applicability is at issue. We are not guaranteed that a given model will apply to particular circumstances. In a case where we thought it might apply, but it does not, we might improve the model by better laying out and understanding its conditions for applicability. More generally, humble theories are designed to be revised, improved, and extended, typically in iterative attempts to consequentially apply them.

To sum up, in "applying our model" to the data presented here, we aim to do the following.

(1) Check the *applicability* of the model and possibly refine our understanding of when and how it applies. In this, one also determines particular cases of the general theoretical categories, which heighten our understanding of what is general and what is specific in various cases of conceptual change.

(2) If the model applies, *test* it in a more or less conventional sense. Are predictions and expectations violated or ratified?

(3) Check the *insightfulness* of the model. Does it show why some things happened, why others did not? Is it explanatory?

(4) *Refine*, as necessary. In addition to refining applicability conditions [(1) above], definitions of terms and detail in relations in the model may be improved by further specification or modification.

(4) *Extend*, as necessary. While humble theories are not expected to cover all aspects of a scientific phenomenon (e.g., conceptual change), more aspects may be brought under the aegis of the theory with extensions and also, details may be filled in.

In considering modifications and extensions, an important distinction is between natural and unnatural refinements and extensions. For example, a model with explicit built-in limits may be extended to go beyond those limits; Einstein's original formulation of general relativity was extended with its later tensor-based mathematical formulation. An empirical phenomenon related to those covered by the theory, but one that initially appeared to be distinct, might be brought under the aegis of the theory by an extension; quantum mechanics gradually grew to cover a much broader range of phenomena than that which originally motivated it. Similarly, one might, after the fact, realize that processes and cases that *should* be covered were not originally. Extending the model in these ways likely constitutes a natural extension.

In contrast, unnatural modifications may be *ad hoc* and disconnected from the model. In the extreme, continually changing core terms, suppositions, and claims or adding more and more ill-understood parameters or conditions on the theory provides evidence that the model might better be abandoned than "tinkered with."

The central aim of this study, then, is to make the case that the relevant model applies to the present empirical case and that it is insightful. The data also suggest a number of refinements and extensions, which we will argue are natural, and, thus, they suggest the continued productiveness of the relevant humble theory.

B. Process orientation

We make a final general point about the theorizing and data analysis here. Part of the relevant theory concerns processes of change. In that respect, it is somewhat unusual. Many ideas about conceptual change center on before and after "snapshots," for example, that students start with a naive theory and move to a normative one. A great proportion of conceptual change research follows this model (see Ref. 6 for a review). Another tradition, expert/novice studies, also focuses on a snapshot view; novices work in one way and experts work in a quite distinct way (for two classic references, see Refs. 7,8). In a similar manner, theories may specify *conditions for change* without specifying detailed processes of change (e.g., Ref. 9). Here, while the core of coordination class theory concerns the structure of a well-formed[10] concept, there are significant process implications. Consequently, process data are more relevant than in most studies of conceptual change. While coordination class theory has always included process considerations, few prior empirical analyses have focused substantially or entirely on process data. Parnafes[11] and Wagner[12] are exceptions in that they do focus on process and change. This is the only article using the idea of coordination classes of which we are aware that focuses on the cumulative details of conceptual change in one extended but continuous episode. In addition, we feel that it is important that this is the first coordination class analysis that deals with change in the course of classroom interaction.

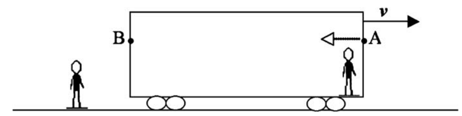


FIG. 1. The exercise (following Halliday *et al.*, Ref. 14).

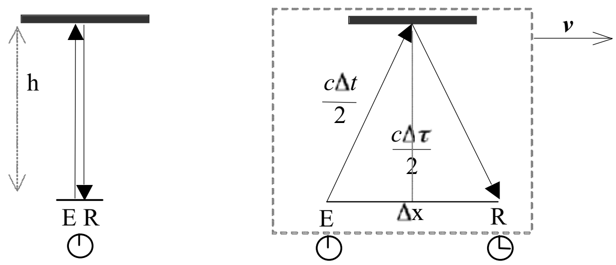
II. EMPIRICAL AND INSTRUCTIONAL SETTING: INITIAL RESEARCH QUESTIONS

The study consists of an analysis of students' difficulties in understanding the concept of proper time. The context is a practice period[13] involving 2 future teachers (T1 and T2) with 12 18–19 year old high school students. The data come from a discussion of about 90 min length, mainly directed by T1 (Bonazzi), about an exercise on proper time (see Fig. 1). The question is whether an observer on a train measures the proper time in finding the time difference between emission at A and reception at B of a light ray. The critical point is that almost all the students initially gave the *wrong* answer: "The experimenter on the train measures *a* proper time because he is in the same frame of reference as the light signal."

The few students who gave the correct answer (the experimenter does not measure proper time) were rapidly convinced by their classmates that they were wrong.

The students had previously studied special relativity following a path designed by the classroom teacher. Proper time was introduced through the light clock experiment (Fig. 2): A laser ray is emitted at the base of "the clock" (event E). The ray goes up, is reflected at the top of the clock (point S), comes down to the base of the clock, and is "received" (event R). The "tick" of this ideal clock is the time interval between event E and event R. The clock is used to argue that the clock's tick depends on the frame of reference with respect to which the clock is observed. In the frame of reference at rest with respect to the clock, the time interval is $\Delta\tau = 2h/c$, where c is the speed of light. In a frame of reference with respect to which the clock is moving with velocity

From the light clock thought experiment:



$$\Delta\tau = \sqrt{\Delta t^2 - \frac{\Delta x^2}{c^2}} = \Delta t\sqrt{1-\frac{v^2}{c^2}} = \frac{\Delta t}{\gamma}$$

where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

Calculating proper time by applying the Pythagorean theorem.

FIG. 2. The light clock experiment.

v (perpendicular to the direction of the light ray), the time interval between the two events (the tick) is longer: The light ray has a longer path to travel, while, as postulated by relativity, the speed of light is the same in all the (inertial) frames of reference. The algebraic relation between the two time intervals can be easily calculated by applying the Pythagorean theorem: $\Delta\tau = \Delta t\sqrt{1-v^2/c^2}$. $\Delta\tau$ is called *proper time*.

As will become clear in the following data, the teacher had emphasized the invariance of proper time in prior instruction.

We began with the following questions.

(1) Why did students give the wrong answer?[15]

(2) More importantly, why were they not willing to accept the correct answer provided by the teacher during an engaging and lively discussion? For future reference, we note, in particular, that students explicitly rejected the teacher's counterargument to their claim. The teacher stated that the two events, "emission" and "reception," of the light ray do not occur in the same spatial position; hence, the measured time is not proper time.

We now turn to describe the coordination class model, which frames our analysis of the data.

III. COORDINATION CLASS MODEL

A. Preliminaries

Coordination class theory was designed to directly deal with some questions often taken for granted among researchers in science education.

(1) What does it mean "to have a concept"? How can we take this vague term and make it more precise and useful in educational research?

(2) What does "conceptual change" mean? That is, what changes in students' minds, and how, in the process of conceptual change?[3]

As already mentioned, a coordination class is a model of a particular kind of concept. Not all concepts are coordination classes, but (for reasons that will become apparent) physical quantities, such as force or proper time, are especially good candidates for coordination class status. The theory centers on general properties of a well-formed concept that must be developed during learning; hence, "observing" how these properties develop is an important focus.

Coordination class theory is framed in a broader view called the "complex knowledge system" perspective. In this view, concepts are large and intricately organized systems, which effectively coordinate the activation and use of many specific elements according to context. Learning a concept is seen as a process of recruiting and "coordinating a large number of elements in many ways." Some or even most of the elements that are coordinated come from prior competence of the learner,[2] so the perspective is deeply constructivist. Because concepts are systems, only part of which shows in any given context, assessment becomes more difficult. However, we view this difficulty as a "fact of life" with which we need to deal rather than something that can be easily overcome or something that is a problem with the theory. One of the compensating advantages of a complex systems view is that learning can be tracked at a small grain size and, thus, it is quite synergistic with using process data, as opposed to before and after snapshot data.

Other theories of concepts are also systemic and relational, like the coordination class theory. However, the most familiar of these see concepts as given their meaning in a web of relations with other concepts (e.g., Refs. 16,17). In contrast, coordination class theory has the following properties.

(1) It is specifically oriented toward *entering* the structure of each concept (rather than viewing it only in relation to other concepts) and analyzing it as a coordination of the various pieces of knowledge involved. Concept maps and semantic networks, for example, show the *external* relations of concepts with other concepts, but coordination classes aim to show the *inner* relations of parts of a concept that make it well formed and powerful.

(2) By considering the internal structure of concepts and their gradual construction, coordination class theory provides operational tools for interpreting learning difficulties and what, in general, happens during the processes of concept learning.

Coordination class theory has a notion of "the family of relations in which a concept participates" (see discussion of the "inferential net" below), which roughly corresponds to the semantic field or semantic net that is given priority in other views of concepts. However, the emphasis in coordination class theory is on *the processes by which such inferential relations are assembled and used in specific situations* to do the conceptual work characteristic of the concept at issue.

B. Defining structure and function of coordination classes

The central function that ties together all the various pieces of a coordination class is to *allow people to read one* *particular class of information out of the huge variety of situations in which the concept is useful in the real world* (Ref. 4, p. 131). It is, in fact, that diversity of contexts that ensures concepts must include many context-specific elements, as will be elaborated later. The *relevant information* that defines a coordination class might be, for example, (a) the point of application, magnitude, and direction of *a force* or (b) the number associated with a pair of points in space-time (which satisfy the special relation called "timelike") that we call proper time. Succinctly, having a concept, according to coordination class theory, is in essence being able to "see" the information that defines the concept in an appropriate range of relevant situations.

We briefly sketch the rest of the coordination class model in the following categories:[4] (a) the architecture (organizational structure of the internal elements) of a coordination class, (b) the processes that build a coordination class, and (c) the characteristic difficulties students encounter in building a coordination class.

(a) *Architecture*: In general, people do not directly and transparently see the relevant information of a coordination class. Instead, typically they *read out* some related information and then *infer* the coordination-characteristic information. For example, in order to see the magnitude of a force, one may "observe" mass and acceleration and then "infer" force by multiplying mass times acceleration. Noting these two steps motivates splitting the architecture of a well-formed coordination class into two parts.

(1) *Readout strategies*: "the ways in which people focus their attention and read out *any* related information from the real world" (where "related" can be exemplified by the fact that mass and acceleration information are related to force).

(2) The *causal net (sometimes called the inferential net)*: "the total set of inferences one can use to turn related information readouts into the *particular* information at issue."

(b) While coordination class theory accepts that entirely new elements may be created in constructing a new concept, an overriding concern is understanding how *prior knowledge* contributes to or detracts from the construction of a coordination class. This emphasis represents the claim that coordination class theory provides a model for *conceptual change*, not just (blank slate) learning. With respect to prior knowledge, the process of building a coordination class involves the two very generic processes.

(1) *Incorporation*: recruiting elements of prior conceptualization into partial encoding of the new concept. Typically, "partial encoding" means that those elements will be used in some circumstances but not in others.

(2) *Displacement*: "dismissing" elements of prior conceptualization that may initially and inappropriately "take over" the function of the coordination class in certain circumstances. For example, novices often determine the existence and magnitude of a force by using the inference "if there is motion, there must be a force." That, of course, is incorrect, and the inferential relation must be displaced from at least the context of "steadily moving things."[18]

(c) Coordination class theory hypothesizes two particular and characteristic difficulties that students have in creating new coordination classes. These have to do with the ability to the coordination class properly "work" across a wide range of situations in which it is useful. The theory presumes that "working the concept" may use different knowledge in different situations. The particular knowledge used in specific applications of the concept is called a *concept projection*.

(1) The problem of *span*: having adequate conceptual resources to operate the concept across a wide range of contexts in which it is applicable.[19] For example, can one determine forces in the situation of a tossed object and also in the situation of a book resting on a table? Span amounts to having or being able to produce concept projections in an appropriately broad range of circumstances.

(2) The problem of *alignment*: being able to determine the *same* concept-characteristic information across diverse circumstances. This is a well-formedness or coherence principle that "what you see" does not vary when you use different methods of seeing it. Said in terms of concept projections, alignment means that each concept projection gets the same results. Students often feel that they can determine forces in different situations, however, for situation-specific reasons, they may fail. A student may see forces adequately in situations of movement but may not see them properly in static situations. For example, a student may reason in the particular case of air pressure that it acts equally in all directions, so its net value is always zero. In the special case of air pressure, such a student has heard that pressure equally acts in all directions and concludes (incorrectly) that air pressure must always be zero; in other cases, the student might determine forces more normatively.

The process of extending the span of a concept may seem analogous to the idea of transfer, coming to be able to use the same idea in new contexts. However, coordination class theory takes the position that knowing a concept is being able to use it in a broad range of circumstances. Specifically, being able to use it in one context is too soft a test to allow the judgment that one has the concept (and all that remains is extending it to other contexts). In terms of learning, coordination class theory is a bootstrapping theory, where students begin to use the concept in some contexts but may not be able to make it work properly in others. In contrast to a view that assumes situation specifics fall away in finding universal characteristics of concepts that work in all situations (see discussion in Ref. 12), coordination class theory assumes that many context-specific aspects of the concept remain in different concept projections, which *in toto* result in competence.

Specific contexts are important to coordination class theory in several ways. First, they are loci in which important work is done in bootstrapping a concept to have more span and better invariance. However, contexts are also the loci of different projections, which remain as part of the concept into expertise. In what follows, we trace both these perspectives on student thinking in particular contexts.

C. Anticipating empirical analyses

Cognitive theories and models are complicated. They have many elements and many general implications, and also specific implications for particular cases. In order to prepare readers for the empirical observations, below, this section synthesizes aspects of coordination class theory that relate fairly directly to following analyses. We anticipate some specifics of later analysis in general terms. In a rough sense, this section contains the *predictions* that we aim to validate in empirical analysis.

From a coordination class point of view, one expects that learning is a long process, and one expects to track progress, bit by bit, in terms of gradual construction of a robust well-formed concept. It is nearly certain that for an extended time, students will appear competent in some contexts but will show weaknesses in other contexts. The data below clearly illustrate this as students begin with a certain competence but progress incrementally, in several stages, toward a broader more systematic competence.

Learning should be expected to be relatively context specific. As students work from one context to another, they should improve both span and alignment. Span will be improved, prototypically, by adding knowledge that allows their concept to work in contexts where it could not work before. In the data below, new definitions help students see proper time in new contexts. Alignment will be specifically improved by correcting inferences or readouts ("observations") that cause them to see things that are *not* the concept-defining information in certain circumstances. In the data below, students "observe" things (one of them being "happenings") that they falsely take to be relevant to (that is, are inferentially related to) determining proper time.

In the best of circumstances, coordination class theory looks to identify particular knowledge that is invoked and used (*incorporated*) in learning. Intuitive elements are often used productively, even if their productive use must gradually become limited to particular contexts (displaced from others). Here, students' use of the idea of happenings creates an intuitive plausibility for early understanding of proper time in the case of Einstein's light clock, but that idea needs refining and displacing from other contexts. They also use the idea of "sitting on an object" as a proxy for what becomes a better-justified more general strategy for "finding the right frame of reference."

Some parts of our analysis, below, do not follow these general patterns. However, we will argue that most or all of those exceptions require only natural extensions and refinements to coordination class theory.

IV. EMPIRICAL ANALYSIS

A. Strategy

The analysis that follows highlights how students "coordinate"[20] proper time, how their coordination changes, and what relationships exist between students' coordination and teaching choices. Our aim is to see certain teaching choices as inviting or sustaining coordination class characteristic learning problems and other choices as designed to obviate these.

The discussion can be roughly schematized in three stages.[21]

(1) The first stage: How do students coordinate proper time at the beginning of the discussion? What definition did they give for proper time?[22] It turns out that their coordination, at this stage, closely reflects the classroom teacher's introduction to proper time.

(2) An intermediate stage: How do students start to incorporate some ideas introduced by the student-teacher—a new and more carefully phrased definition of proper time and a new context of use? We might expect *a priori* that a second definition raises issues of alignment. Will students use both definitions "properly"? If one is used systematically improperly, or improperly in some contexts, or if the definition is incomplete or incorrect in circumstances, issues of alignment arise. Furthermore, we shall see, the fact of two definitions raises a metaconceptual issue: Do students see them as "the same" or related in comprehensible ways?

(3) The third stage: How do students coordinate proper time at the end of the discussion after the student-teacher invited them to situate the discussion within the debate between Einstein and Minkowski?

B. Results

1. First Stage: How do students coordinate proper time at the beginning of the lesson?

(a) Excerpt 1a. The students' definition of proper time and its ambiguities: "being at rest with respect to something:" What is that something?

T1: The first question one should ask is "what is proper time"? Why is it called *proper* time?… What is proper time for you?

Marco: Proper time of something is that time measured in the same frame of reference or in a frame of reference at rest with respect to that something.

T1: What about this "something"? Can you give us an example?

Marco: For example, this laser ray.

T1: So, can you repeat your definition?

Marco: Proper time is the time calculated in the same frame of reference as the object, of the laser ray… of what one wants to calculate proper time… that is, the event. […]

Elis: We defined proper time starting from a moving light clock. And we defined proper time… we found a relation that was $\Delta\tau = \Delta t\sqrt{(1-\beta^2)}$ [where β is used, as by convention, for v/c]. According to the way in which we found proper time, we had a light ray going up and down. And we found proper time because one was in the frame of reference of the clock.

Comments: The students seem to have the necessary readout strategies for finding proper time in the light clock experiment and for formalizing its relationship with time in another frame of reference. They start from a definition for proper time (Marco says, "Proper time of something is that time measured…in a frame of reference at rest with respect to that something."), and they rightly recognize that such a "definition" works without problem to provide a method to "see" proper time when applied in the context at issue here, the light clock experiment. To paraphrase: "To coordinate proper time, find the right frame, then measure time in that frame." The general phenomenon of a definition "providing" a projection (or a class of projections, some of which may be specific to particular contexts) will be important for later discussion.

"Finding the right frame" is specified in their definition as "being at rest with respect to…." In this case, "being at rest with respect to" can be easily referred to the light clock, as Elis explicitly does in the final contribution, above.[23] In the horizontal light ray exercise, however, they feel confused by the fact that the readout strategies provided by the definition are not transparent for individuating the appropriate frame of reference; the teacher is telling them that they have the wrong frame.

In theoretical terms, alignment is threatened since a critical part of the process specified by the definition (finding the right frame of reference) is uncertain. The ambiguity and lack of clarity is evident in their indecision in describing the "something" with respect to which the frame of reference can be said at rest. Marco introduces several partial and ambiguous descriptions. He says: "in the same frame of reference as the *object*…" [Emphasis added.] This description is ambiguous since it does not specify which object, and it seems to presume "the object" will be obvious. A way of describing what they are doing wrong in the exercise is picking an obvious object, which, however, is not "the correct object." Indeed, this "object" description, in principle, has limited span since proper time exists and can be measured or computed where no object exists, between two "arbitrary" timelike points in space-time. If "object" means "physical object" (as opposed to light ray), then there is simply no object that defines the frame of reference in the horizontal light ray exercise.

Marco's next try is in the frame "of the laser ray." This is also a vague description since the beam of light in the light clock travels up and down, and it is evident in the other things that he says (and what others also say) that no vertically moving frame is seen (yet) as relevant to this problem. Finally, Marco specifies the frame of reference as that "of what one wants to calculate proper time…that is, *the event*." [Emphasis added.] Marco's use of "event" is telling and ironic. It is telling because he uses this word in a commonsense way that we would gloss as "a happening" or (the students' words) "a phenomenon." This implicates a classical ontology, "happenings have a characteristic place, and they also have a characteristic duration." While completely commonsensical, such "events" either have no status at all or they have a secondary status in special relativity. The use of the word "event" is ironic since, as we will explain later, seeing and using events in the proper sense of special relativity is, we propose, an important "fix" for the problems students have using the commonsense ontology event.

Elis points out that the reference frame is obvious in the light clock experiment, that of "the clock" (as a physical structure). That is the correct frame, but Elis does not mention (and we doubt he could produce) a justification for that judgment.

(b) Excerpt 1b. Proper time and its equation: Proper time of the moving ray light must exist.

Lorenzo: But if $\Delta\tau=\gamma\Delta t$, in this case does $\Delta\tau$ exist? [According to the usual convention, Lorenzo is using a wrong relation (it should be $\Delta\tau=\Delta t/\gamma$), but it is possible that he intends γ to mean $\sqrt{1-v^2/c^2}$.]

T1: Does it exist in this case? Let's see if it can exist.

Elis: I thought that proper time exists inasmuch as a time exists.

Lorenzo: If $\Delta\tau$ is always equal to $\gamma\Delta t$, it must exist.

Comments: There are two arguments for the existence of a proper time here. The easier one is implied by Lorenzo; if there is an equation for the quantity, we can determine it (and, thus, it must exist). We pointed out earlier, with the example of F being inferred from m and a, that equations can form obvious parts of the causal net.

Elis's argument is more subtle. Our interpretation of his assertion that "proper time exists inasmuch as time exists" is ontological in the sense we introduced above. That is, any extended happening (event in the students' parlance) must have a duration associated with it, between its start and end. Unfortunately, this particular inference in the naive causal net concerning durations must be displaced in the case where the "beginning" and "end" of the happening are in a lightlike relation. The emission and reception of a light ray terminate the travel of the ray; how could there not be a time (and hence, by the equation, also a proper time) associated with that happening? In other cases, the inference is unproblematic; most happenings have a (nonzero) proper time, as given by the equation. This necessary but undone displacement (of "happenings take time") is a typical coordination class learning event: An inference that is sometimes appropriate must be restricted to a particular subset of circumstances.

(c) Excerpt 1c. Proper time and its invariance.

Lorenzo: We said that proper time is a feature of a given phenomenon. We… in the sense that… in class we defined proper time as *an invariant feature of a determinate object*. [Emphasis added.]

T1: OK, you have introduced the word invariant… What does invariant mean? What does invariant mean to you?

Lorenzo: Something that it does not change in that frame of reference.

T1: What do you mean?

Lorenzo: It is in that way in that frame of reference. [Lorenzo is saying it has a particular value in the specified frame of reference. A more normative description of Lorenzo's portrayal here is "uniqueness"—a quantity (in a particular frame) has a unique value—not invariance.]

Marco: In my opinion, invariant is something that does not change even when you change frame of reference.

Lorenzo: Yes, but proper time is a characteristic just of *that* object in *that* frame of reference.

Marco: Yes, OK, but the proper time you have in a frame of reference, you have it also in another frame of reference. You can find it, it is not that…

T1: Sorry, what…

Marco: If you change frame of reference, proper time of the other frame of reference still stays constant. I mean, if you are in a frame of reference A and calculate proper time and then you go in a frame of reference B, proper time of A is still that. It is this that, in my opinion, it [invariance] means.

T1: And how do you do to calculate proper time if you aren't in the rest frame of reference?

Marco: Beh, there are also the formulas!

Comments: In his opening remark, Lorenzo says "proper time is a feature of a given phenomenon," which seems to be saying "phenomena have durations, and that's that." We believe that he is interpreting the definition of proper time as invariant to be sanctioning, in this case (proper time), the use of the classical ontological inference that durations simply exist as unique quantities. What relativity may have added, for him, is that one must be careful to use only one special frame to determine time duration. Notice that Lorenzo protests when Marco introduces the possibility of proper time seen in a different frame of reference.

Marco, on the other hand, has the essence of invariance correct. A quantity can be measured or computed in multiple frames of reference, and one must get the same number. What he is adding to the conversation is the recognition that "determining" may look very different in different frames of reference. In one case (the "proper" frame), the determining is simply the measurement of a single clock. [Marco does not recount that possibility here, but he makes it clear on other occasions that he knows that this is possible. In Marco's first comment in segment 1a, he speaks of measurement (as in reading off a clock) rather than "calculating."]. In another frame, the determination involves not only time difference but also spatial differences, and there is a complicated formula one must use to "see" the quantity [$\Delta\tau = \sqrt{(\Delta t)^2 - (\Delta x)^2/c^2}$]. Notice that Marco talks of "finding it" in another frame of reference (which may well implicate something more indirect than measuring), and he introduces the idea of calculating (in his second to last turn). Coming to see proper time as an invariant means coming to accept that two very different looking projections of determining time can actually be the same thing. Generalizing this case, we offer as a conjecture that the perception of unity of diverse projections is an essential metaconceptual step in learning a new concept.

(d) Students' coordination of proper time in the beginning. In the beginning, we see three conceptual elements that play a clear role in determining proper time.

(a) The determining of proper time (the generic "determining concept-characteristic information" in coordination class theory) involves an implicit and ambiguous process by which the relevant reference frame is to be determined. A good guess is that students simply expect to see an object (e.g., the light clock) that determines the relevant frame or, slightly more complexly, the frame is determined by the (potentially moving) "location of the phenomenon" whose duration is to be measured. This implicates the persistence of "classical ontological inferences" that take for granted the existence of happenings (phenomena) as unproblematic things that have a place and a duration. Later, we will see how their perceptions shift when an object moves, in place of the light ray.

(b) The equation for proper time justifies its existence and, possibly, its uniqueness.

(c) The invariance property is interpreted sometimes (Lorenzo) to justify the naive ontological inference that phenomena simply have durations.

The light clock experiment plays a special role in developing or sustaining at least the first two of these ideas. First, since it "puts forward" an obvious frame of reference, that of the clock itself, or of the phenomenon at issue, it subverts the need for more refined, stable (perhaps explicit), and reliable (aligned) readouts and inferences. Second—perhaps less interestingly—it is the context in which the equation they know and use for its inferential properties (not its quantitative properties) is derived. We are not suggesting that considering the light clock is necessarily a bad pedagogical move. However, we are saying that the unreflective intuitive processes that make the relevant frame of reference (the locus of a phenomenon) obvious, if not augmented by more careful consideration of how, in other contexts, one constructs or notices "the right" frame of reference, seeds the strong potential for nonalignment via taking an obvious but incorrect frame as the relevant one.

The parts of the *causal net* built or sustained ("sustained" means that displacement of naive ontological inferences was not forced) through the light clock experiment prevents them from recognizing any difference between the light clock and the exercise situation; they are *the same* context in the students' view.

In particular, the students see the following:

(1) the same way of determining the relevant "frame of reference" (at rest with respect to the object or phenomenon);

(2) the same phenomenon (the propagation of a light ray);

(3) the same "invariant" (intrinsic) property ("the proper duration") is requested.

So, naturally, their projections of proper time used in the light clock and in the exercise do not differ; they conclude, as with the light clock, that proper time in the exercise is the time measured in the "obvious" frame of reference. Anticipating later discussion, we believe that the main lack here is that they need to have a focus of attention for readouts that are more explicitly considered and consistent with special relativity. Instead of "phenomena" with intrinsic locations and durations, they need to conceptualize and use events construed as special-relativistic things. Events in the proper relativistic sense will provide a stable focus of attention in the determining process. We return to this point later in the article.

Although vague (risking misalignment) and retaining classical inferences (proper time exists as an invariant—invariant here means "intrinsic" rather than properly Lorentz invariant), the causal net built through the light clock experiment appears very sensible to them. Consequently, the new problem and the teacher's initial explanation that they are doing something wrong are insufficient to really disturb their ways of thinking about proper time.

In summary, the light clock experiment really provided a "short circuit" in letting students adequately perform in conceptualizing it, without (1) really challenging some central naive inferences and without (2) introducing enough refinement in their conceptualization of proper time to see that a new projection or at least some new elements of contexuality and inferencing, discussed below, are necessary in the new exercise. With respect to (1), the idea that duration is just a unique property of a happening seems to persist. With regard to (2), the students' process of determining proper time relied on a vague description of the relevant frame of reference (the obvious frame, centering on a classical object, the phenomenon at issue, or on a typical frame-determining thing such as a boxcar).

2. Second stage: Increasing the span of students' proper time strategies and the role of awareness in conceptual change; characterizing the change in coordinating proper time in view of teacher interventions

The second stage concerns students' reactions to the teacher's introduction of two new elements.

(1) A "more precise" definition of proper time: "proper time as the time interval measured between two events occurring at the same position." As intimated above, definitions often, if not always, imply or provide a way of determining concept-characteristic information, so they are strongly associated with a class of determining procedures (a class of projections). For example, people perceptually determine speed with a wide range of strategies. (They use, e.g., "blurriness," gestalt judgments, patterns of overtaking, etc., see Ref. 11.) However, the definition of speed as the limit of the ratio of distance traveled divided by time interval implies a specific procedure for determining speed. The formal definition thus introduces the possibility of increased span in that the definition may be easy to apply in situations (e.g., using a table of distances and times) in which the older determining strategies are difficult to apply.[24] However, this also introduces the possibility of misalignment or, at least, the perception of *possible* misalignment. Does the definition determine the same information as prior strategies or definitions? We introduced earlier the potential importance of the perception by students that different projections determine "the same thing."

(2) A new context: a ball moving horizontally on the train (from the front of the wagon, A, to the back, B). Once again, new contexts are critical to coordination class theory. Generally, a new context provides an opportunity to expand span, test it, and also test alignment (adding or modifying projections).

(a) Excerpt 2a: Students' difficulties in integrating the teacher's definition in the exercise context.

Lorenzo: There *[in the light clock experiment]* proper time can be also defined as the time between two phenomena occurring at the same spatial coordinate. [Notice, for future reference, that this sentence is literally consistent with interpreting "phenomenon" as "space-time event," and it is correct in that construal. The two events are the emission and reception of the light ray from a single place, where a clock is imagined to reside.] There, the two phenomena are going back and going forth. [However, this makes it clear that Lorenzo means "phenomenon" in the sense of a happening, presumably having a duration, and he is focusing on "location" not as a point, but as a "place" (the light clock).] Here [in the horizontal light ray exercise] the light ray goes only forth. It does not go back.

[…]

Lorenzo: Yes, but if it is not a proper time [the time measured by the observer on the train] and if proper time is $\Delta\tau = \gamma\Delta t$ [sic], what do you do to find the proper time from A to B?

T1: Can you find a time between A and B?

Lorenzo: You find Δt.

T1: Yes, Δt, that is a time…

Lorenzo: But if $\Delta\tau = \gamma\Delta t$ [sic], in this case does $\Delta\tau$ exist?

T1: Does it exist in this case? Let's see if it can exist.

Comments: The teacher's definition of proper time does not make sense to students since it seems to imply that proper time does not exist in the case of the light ray moving horizontally. That is, in the clock case, the light ray returns to the (critical) "same place," while with the new problem, the ray only goes "forth;" it does not go "back" to the definition-critical same place. The problem in Lorenzo's first take is, in our theoretical terms, that he does not have a means (cannot construct a relevant projection) for implementing the definition as a determining process in this particular case. We see a lack of span in the definition precisely caused by the fact that Lorenzo cannot see the relevant pair of events as happening at the same place. He is missing a specific construction (finding a frame in which end points of a duration occur at the same place).[25]

Lorenzo's second assertion is that the formula also does not obviously apply. How can one determine γ and, thus, $\Delta\tau$ from Δt? Although he does not say exactly what the problem is, we believe that it is likely that, as he does not see a second frame of reference, he is wondering where the relative velocity involved in γ is. Without another frame of reference how can γ be determined? In our theoretical terms, this context also tests the span of the formula for determining proper time and shows a specific lack of the ability to construct the situation as involving two frames of reference, which determines γ.

(b) Excerpt 2b. The new context of the ball moving on the train allows the teacher's definition to be partially incorporated into their conceptual schemes for interpreting proper time. However, the methods (projections) of determining proper time suggested by the different definitions are not yet aligned.

T1: Now, let's suppose that, instead of having a light ray, we have a particle moving slower than light with respect to this frame of reference [the train] and in this direction [from the front to the back]. So, how can I calculate proper time from A to B?

Marco: I should be on the particle.

T1: Why?

Marco: Spatial coordination is fixed on the particle. [This is *a* way of saying that the two space-time events are at the same place, consistent with the definition provided by the teacher.[26]]

T1: And then?

Marco: In order to measure a proper time, we have said that the spatial coordinate must be fixed… and then the fixed spatial coordinate we have here is the particle in this case… then… measuring the time staying on the particle.

T1: Did you understand what he said? If I am on the ball, I must measure the time interval passing from the moment in which the ball is at A—that is, from the moment at which the point A coincides with the ball—to the moment at which point B coincides with the ball point. Do you agree?

Lorenzo: But it will move… [Our interpretation: Lorenzo is persistent in seeing the boxcar frame as the relevant one, hence A and B are not at the same point; the ball is moving.]

T1: Yes… the train will move under the ball… if I stay in the frame of reference of the ball… [The instructor emphasizes that the train, not the ball, is moving when the reference frame is the ball.]

Comments: The new context allows the students to make sense of the teacher's definition of proper time; the definition "works" (generates a projection) for the horizontally moving ball. They do this by translating the condition "occurring at the same position" into the more intuitive idea of "being on the ball." Marco says, "I should be on the particle." Notice that this coincides with our interpretation of how the relevant frame of reference was described by students earlier. "Things" (a ball or the locus of a phenomenon) easily seed the consideration of a frame of reference. Marco, at least, has refined this condition to be a special case of the requirement to measure in a frame where the end points of the time interval occur at the same spatial location.

(c) Excerpt 2c. The new context allows the exercise context to be recognized as a limit case in which proper time is equal to zero.

[after the discussion about the ball]

T1: In this case [the exercise with the horizontal light ray] may I do it? Here, in the exercise, can I sit on the laser ray and calculate proper time?

T2: Does someone remember the idea that came to Einstein?

Simone: of riding a light ray and…

T2: And what did he deduce?

Elis: That it couldn't be.

T1: Yes, that no one could ride a light ray. Why?

[confused discussion]

T1: The speed of light is a limit for all the velocities and no object with mass can arrive at that velocity. Thus no observer can ride a light ray and measure a proper time in this case. Nevertheless you have seen also another definition of proper time… [The teacher's explanation is not completely adequate, in our view. In any case, however, the idea of "the frame of reference moving with a light ray" is problematic in at least some respects.]

[At this point the students move from the discussion of proper time as invariant (excerpt 1c) and a digression on muons occurs. After that, Lorenzo picks up the thread.]

Lorenzo: So, how can I, in this case [the exercise with the horizontal light ray] be in a frame of reference in which I measure a proper time?

Luca: It does not exist.

Lorenzo: Then this is why proper time does not exist… But if proper time is $\gamma\Delta t$ [sic]…

Marco: It becomes zero, in this case, since $v=c$! [This appears to confirm that the students are interpreting γ to be $\sqrt{1-\beta^2}$, which goes to zero as v approaches c. Thus, they are giving the wrong name to the correct functional relationship.]

Lorenzo: Then proper time is equal to zero, it isn't that it does not exist! [While proper time exists, even if it has the intuition-defying value of zero, the limit frame of reference, according to what they have been told, does not exist.]

Comments: The ball context allows students to recognize that the exercise problem is just like the ball case, except that the speed of the moving object is at the limit, at the speed of light. Following this observation, in coordination class terms, students notice that the class of projections seeded by the "same spatial coordinate" definition of proper time fails in this limit ("how can I be in a frame of reference in which I should measure proper time?"). There is no such frame of reference. However, they then notice that another definition (technically, we would describe this as a class of projections seeded by the definition)—to wit, the $\Delta\tau=\gamma\Delta t$ formula—does, in fact, work, leading to a "zero" result for proper time. In the last two contributions, Marco makes the critical invocation of the formula for proper time, and Lorenzo ratifies Marco's result with the observation that zero is different from not existing.

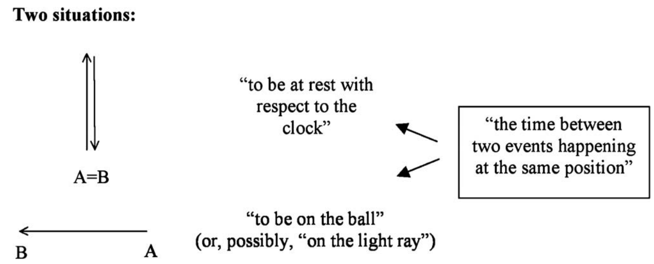


FIG. 3. Students' coordination at the second stage. (The light ray version of the exercise, being "on the light ray," must be understood only as a limiting case.)

(*d*) *Students' coordination of proper time at this second stage*.

At the end of this intermediate step, students can distinguish three different situations (Fig. 3): (1) the light clock experiment, in which the ray goes back and forth; (2) the "ball situation," in which the object goes only in one direction; (3) the light ray situation of the exercise, which is operationally different from the ball situation (no observer can make the relevant measurements). However, considering the equation, the light ray result can be seen as a limit case of the ball result.

They are able to conclude that the definition of proper time given by the student-teacher, "the time between two events happening in the same spatial position," works in the first two situations: it is consistent both with the students' original definition "being at rest with respect to…" (when applied to clock in the light clock experiment) and with the other more intuitive one "being on the ball." Moreover, the teacher's definition allows the exercise situations to be aligned as a limit case. That definition seems to have aligned the first two cases (and also the third, but only partially) not only in the standard coordination class sense of correctly determining proper time in the different cases, but in the stronger sense that students perceive the process of determining to be "the same;" it is (and is seen as) the same in the different cases. What is happening here is a particularly strong form of alignment that has not been previously identified in coordination class theory, that is, alignment not only in *having* different concept projections that determine the same information but consciously and articulately putting those determining processes in relationship with each other, noticing *that and how* they produce the same result. Here, the key observation is that "at the same spatial coordinate" leads to the same thing as taking what is sometimes "the obvious" frame of reference, but sometimes (the horizontal light ray) the relevant frame is not so obvious.

This accomplishment by the students represents what we take to be a transparently natural extension to coordination class theory. In its original formulation, the distinction between conscious and unconscious processing is simply ignored. Furthermore, the theory offered no high-level categorization scheme for ways in which alignment is achieved. Here, a deliberate process of articulating determination processes, their relations, and results produces a likely to be stable (because it is conscious, articulate, and rationalized) alignment. Furthermore, we can see why this attractive process of creating alignment could work well here. Different projections—identified by the class members and sanctioned by the teacher as "different definitions" of the same thing—make the fundamental "goal" of alignment obvious and something consciously to aim toward. The class explicitly works on the problem of alignment, even if they do not know coordination class theory.[27] The simple and obvious idea that different definitions of, or ways of determining, the same thing should lead to the same determination of that thing is a metaconceptual driver of alignment.

At this point, everything seems in order. Students have the key idea for giving the correct answer to the exercise and a "consistent" criterion that aligns the different determinations of proper time in the light clock and in the exercise: by singling out the appropriate frame of reference in different contexts.

Nevertheless, as we will see below, since attention has been explicitly drawn to the different determination processes, nonparallelisms are troubling to the students; the proposed alignment is not fully sensible to them. Once again, the construction of a complete well-formed coordination class requires extended consideration and learning, involving experimenting with various ideas across various contexts.

One particular issue still makes such an alignment fragile: "Why didn't we ride the light ray in the light clock experiment?"

Danilo: But in the light clock I saw a ray leaving, coming back… but I did not move myself, I was not on the ray… nevertheless that was a proper time…

[confused discussion]

Danilo: The clock wasn't on the ray.

T1: There *[in the light clock experiment]*, I had to calculate the time occurring in the same position. Here I sit on the particle only because, in that way, the events A and B in the frame of reference of the particle occur in the same position. So, also in this case I can use only one clock linked with the particle. Right?

Students: Mmm [confused discussion]

[The succeeding discussion did not resolve the problem]

Comments: The students remain troubled by an apparent nonalignment. "Riding the light ray" worked in the exercise; why can that not be taken back to the original light clock experiment as a method of proper time determination? Alignment always seems to be an extended process.

3. Characterization of students' conceptual state: Framing a conceptual problem in coordination class terms and formulating a possible solution

So far, students have made significant progress. Initially, they all gave the wrong answer to the horizontal light ray problem. They thought they could determine proper time, but they actually did not (a case of misalignment). The incorrect determination was mediated by the critical question of selecting the frame of reference in which to measure. Ambiguous specifications of the frame as "that of the object, or of the phenomenon" supported choosing the wrong frame. Incorrect elements in the causal net included the critical idea that a phenomenon has a locus, and that locus (read as a frame of reference) is the "home" for determining proper time. We can imagine that to be an unintended consequence of teaching "sitting on the clock/particle" as a heuristic for finding the proper frame, but we also see that the naive ontology of phenomenon or happening contributes, where students (uncritically) see a phenomenon as happening in a place that is characteristic of it (the train car is the place for both the light clock and the horizontal light ray problems).

A new context (a ball in place of the horizontal light ray) and a new teacher-provided definition for proper time (the time separation between two events measured in the inertial frame where the "begin" and "end" events happen in the same place) move the conceptual change process along. The newly provided definition, as might be expected of a new projection, provides a greater span for determining proper time. Speaking more precisely (since students thought that the new situation was within the span of their ways of thinking about the light clock), it provided a new projection (the limiting process) on the horizontal light ray problem that was properly aligned. Furthermore, students saw the light ray case as (the limiting case of) the same process of analysis as the light clock (or, what we did not discuss, time dilation for a fast-moving muon). That is, alignment in this case is more than seeing the correct proper time in the horizontal light ray (alignment in the technical sense of coordination class theory) but seeing *how the processes of determining it may be viewed as the same as that used in the light clock* (alignment as a conscious and articulated "seeing as the same").

Still, with the issue of alignment on the table, students persisted. Now that the option of (virtually) riding a light ray is in the picture, why should not one ride the light ray in the light clock in order to determine proper time?

One might propose three potential "solutions" to the problem.

(1) You need to choose one inertial frame of reference, not two (or more). The outgoing and returning light rays determine two different frames of reference. This seems, essentially, the proposal of the teacher, T1 (where he used a "clock" as a proxy for one frame of reference; see above).

(2) One can resort to the technicality that the frame of reference riding on the light ray does not exist, even if this idea provides a measure of the proper time of a lightlike path as zero in a limit of faster moving objects.

(3) One can make salient to the students that the light ray path for the light clock is a different space-time path between the events occurring in A and B, and its length is not necessarily the same as the direct path from the two events. In the same way, the twin paradox demonstrates two paths with the same beginning and end points that have different space-time lengths associated with them.

What we want to propose is an encompassing and "deeper" solution (thus, it may in practice be harder to implement). Since it is motivated fairly directly by coordination class theory, our solution also has the advantage of being less *ad hoc*; it is situated in a fairly well-elaborated and empirically supported theory of conceptual change. We propose to shift the fundamental view of the universe as a place in which there are objects and phenomena, which can all move around, to the universe as an ensemble of events. In coordination class terms, we want to introduce a preferred focus of attention for all relevant determinations. That locus is space-time events. Thus, objects relate to space-time measurements only and precisely for the family of events that the object "lays down" in space time (all events that correspond to the particle's position at all times). This proposal is closest to the third, above, but has the advantage that it applies to all coordinations in space-time, not just proper time, and not just in this family of examples.

A major advantage of this formulation is that it can be used to directly problematize phenomena as things that have a location and a duration. As we argued, these ontological assumptions are persistent and problematic. What locus of space-time events corresponds to a phenomenon? In the light clock, there are two possibilities: the set of space-time events directly between emission and reception and the family of events determined by the trajectory of the light ray. Which set is relevant? The observer sitting on the clock precisely "experiences" the former set; he does not travel with the light ray. That is the time we want to relate to that observed by another observer, not what might be associated with the space-time events making up the path of the light ray.

We are proposing a "factoring" of the space of possible projections of all coordinations in special relativity. There are those that first identify the set of relevant events, and then proceed from there (determining other things on the basis of properties of the relevant events), and there are others, such as "sitting on the relevant object or phenomenon" or better (but not ideal) "finding the right frame of reference to make our coordination easier." We are proposing to give the first class of coordination general priority over the others.

This structuring of the coordination class into projections that preferentially use a particular class of readouts, focused specifically on space-time events, is, of course, compatible with coordination class theory as "classically" presented. However, it is a structuring having to do with one particular conceptual space, special relativity; thus, it is a natural extension specialized to the conceptual landscape of one domain. At this stage, we present no arguments that such factoring is possible in other conceptual domains. However, if possible, it seems likely to be powerful in organizing much of students' conceptual change work.

This developmental path—focusing on space-time events as the first stage of coordination—was not taken in the classes studied here. It will remain, in this paper, motivated by theory and by particular empirical observations, but not empirically validated.

The third stage analysis, below, builds on what is probably a more general and more widely applicable instructional suggestion that we see embedded in the above data: Learning may often be aided by explicitly delineating different projections, or classes of projections, and considering their unity or subtle contextual differences. This idea directly follows from coordination class as a model of concepts together with the idea that it is often useful for students explicitly to consider the nature of the conceptual landscape that they need to engage in learning. In this case, this principle is less to have students consider directly their naive ideas (as typical in "misconception" research) but more to work to expose the structural characteristics of the conceptual change that they are experiencing.

C. Historical, practical, and theoretical perspectives on teaching special relativity

The point to which we have come—putting forward a high-level instructional strategy suggested by coordination class theory (explicit consideration of multiple projections and their relations)—is opportune for connecting what we have said with historical traditions for teaching special relativity. This will anticipate the final data presentation and analysis, and it will also explain some important elements of the instructional strategy as it was implemented, *before* the idea of coordination classes was applied to these data.

Most secondary textbooks teach special relativity following paths similar to Einstein's original 1905 presentation.[28] They introduce the historical context by presenting the inconsistencies between classical mechanics and electromagnetism, they focus on Lorentz transformations, and they use algebraic representations predominantly.

However, unlike Einstein's papers, they pay little attention to what it means to measure space and time, taking into account the new constraints of the theory: the unsurpassable and constant speed of light. In other words, despite following Einstein's reasoning, they do not attach relevance to his original operational perspective, which we view as consistent with the idea that *space and time are special names we give to ways of relating events by measurement*. We have seen that the operationalization of "observing" plays a special role in coordination class theory, so this "slight" of Einstein's orientation is substantial, and it is more general than just "a way to teach relativity." In particular, the arguments and empirical results of this study affirm a contention of other research (e.g., see Refs. 29,30): the crucial necessity of introducing students to special relativity by operatively constructing, for *a single frame of reference*, the time between events (colloquially, the duration of an event) and the procedures for defining the time of a distant event. In this manner, one arrives at the analysis of the Einstein's thought experiments, concerning the consequences of the speed of light invariance with respect to different frames of reference, with a structured and operational concept of frame of reference as a lattice of rules and synchronized clocks[31] or as Scherr *et al.* say, as "a system of intelligent observers."[29,30]

Some innovative proposals (for example, the proposal by Taylor and Wheeler[31]), in contrast to an Einsteinian approach, rely heavily on a geometrical (Minkowskian) formulation of special relativity. This potentially offers an elegant and conceptually transparent representation of the theory. One of its main advantages is paving the way to contemporary physics, including particle physics and general relativity. According to such an approach, the concepts of event, of space-time interval, and of its invariance are strongly emphasized as well as proper time as the space-time interval between two events occurring in the same position and, then, measurable by what they call the "wristwatch." In order to stress this point, Taylor and Wheeler suggest calling proper time as "wristwatch time."

Nevertheless, Taylor and Wheeler's proposal has some problems of implementation at the secondary school level. The main sources of the problems are the "length" of the outlined path (with respect to the time that can be scheduled for relativity within the whole physics curriculum) and its being so unfamiliar that teachers may find it difficult to compare it with the paths that they followed as students or that they find in textbooks.[32] As a result, such an approach tends to be oversimplified when implemented. In our judgment, this is what happened in the class on which this study is based. The teacher who taught this class up to the point where we began analysis had decided, indeed, to minimize the role of Lorentz's transformations and to precisely focus on the effectiveness of the geometrical approach for representing relativistic effects (the relativity of simultaneity, the length contraction, and the time dilation). Also, her decision to stress the invariance of proper time was made to align with the geometrical approach proposed by Taylor and Wheeler, and this emphasis constituted the main difference between her class and more traditional approaches. However, unlike Taylor and Wheeler, she de-emphasized what we consider here the main point: Minkowskian geometry is an extremely powerful tool to enforce events as foci of attention and supporting geometry as a way to coordinate them.

Our analysis so far reinforces the good properties of the geometrical approach for specifically pedagogical reasons (not on the relevant, but different criteria of "elegance" and "power"). In addition, the analysis shows why and when it is a good approach, not just *that* it is a good approach. In particular, the teacher's oversimplication invited Newtonian ways of viewing the subject, preventing a deeper transformation.[33] That is, we argued that students were "invited" to use their classical concept of a phenomenon with characteristic duration in the light clock case. That use promoted the understandability of the example in students' eyes but did not displace the "lurking" classical dispositions. Students' incorporating prior ideas that need eventual displacement is a canonical coordination class "danger" to be attended to in instruction.

In contrast, we argue that changing students' perspectives from "looking in terms of phenomena" to "looking in terms of events" is an educational goal that, if accomplished, substantially promotes effective conceptual competence, whatever particular approach is followed. As mentioned, that instructional strategy, emphasizing events in contrast to all other ways of coordinating, was not directly implemented. However, the more general idea of explicitly treating multiple projections was implemented. At the time of implementation, this strategy was not conceived in coordination class terms but as a way to bring the complexities of historically different approaches to special relativity into the classroom. In the following section, below, (a) we show a little of that instructional strategy in action, (b) we argue, briefly, that, in fact, it explicitly highlighted and considered different projections, and (c) we show that what data we have suggest that it was effective. In passing, we note that framing the consideration of multiple projections as historical perspectives, by itself, seems a useful strategy. Historical perspectives are potentially interesting and important in their own right, they may have independent good pedagogical properties, and they constitute an easier "handle" on diverse projections for students who, of course, have no reference in coordination class theory.

Third stage: The debate between Einstein and Minkowski as a way of considering and managing different kinds of projections

During the lesson, the student-teachers now and then recalled the debate between Einstein and Minkowski. The ground here had been reasonably well prepared: The teachers had discussed this debate with the students in the previous lesson. (The lesson on the debate was based on the analysis of Minkowski's papers reported in Ref. 34). In particular, the teachers thought that the comparison could be a way to support different ways of creating relations between events: *measuring*, according to the Einstein's operational perspective, and *calculating*, according to the Minkowski's geometrical perspective based on the space-time interval invariance. We interpret these as ways to support reflection on different projections (or classes of projections). Thus, the effect might be to further promote systematic and articulate consideration of paths of determining, leading to an elaborate version of articulate alignment of which we spoke earlier. Here are some snippets of the discussion.

T1: Let's try, however, to give a definition more operational a la Einstein… proper time as the result of a measurement… […]

T1: If you want to properly measure a time, you have to use…

Simone: You have to use a chronometer. [and, we would add, a chronometer only]

Notice that T1 starts by asking for a definition of a certain type (Einsteinian), presumably in contrast to definitions of a different type. Furthermore, the definition is explicitly framed as a coordination in our sense, proper time as the result of a way of determining it. T1 says (roughly) "proper time as the result of a measurement."

T2: Do you remember Minkowski? "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only… [The sentence comes from Minkowski's paper "space and time,"[35] which the student teachers had discussed with the students in the previous lesson. The sentence goes on: "only a kind of union of the two will preserve an independent reality." The union of which Minkowski speaks is prototypically exemplified by what is now called "4-interval" or space-time interval.]

T1: At this point I ask you a question. Let's suppose we are in a frame of reference where two events occur in the same position and I want to calculate this quantity that we are calling "4-interval." To what is this equal?

[silence]

Δx in this case is zero and, then, the 4-interval coincides with the time. And, in this case, I can call it proper time. Then, proper time coincides with this expression when it is true that I am in a frame of reference where the two events occur in the same positions and Δx vanishes.

The teacher is articulately aligning two means of determining, showing that one is the same as the other when one arranges for the spatial component to disappear. The sparseness of student contributions is unfortunate from the point of view of tracking student thinking. However, what the teacher is intending is relatively clear, and students sometimes made appropriate contributions (such as Simone, above). In general, the teacher is pointing out different determination methods (e.g., a "more Einsteinian" approach) and aligning them by emphasizing their relations (T1's final contribution, above).

At the end of the whole lesson, the student teacher used the comparison between Einstein and Minkowski to go back to the exercise. The excerpt reported below is the last part of the lesson. At this point, the frame of mind of the students seemed to be confident, and they appeared to be able to manage conceptualizing proper time in all of the situations well, explicitly considering different classes of projections (which they know as "perspectives" on special relativity):

T1: Then [coming back to the exercise] in your opinion, what would Einstein have answered? How much is proper time?

All (chorus): It does not exist! It cannot be measured!

T1: And Minkowski?

Luca: Zero! He would have taken the mathematical concept as the starting point for his deductions… as foundation….

Again, although the process data during this last stage are meagre, we were impressed with the confidence and precision of conceptualization with which the students seemed to wind up. They appeared quite clear on subtle distinctions in the same "concept" (proper time), which are, in fact, differences of coordination in special boundary cases (the speed of light).

Follow up on this instructional event is somewhat encouraging in its outcome. Four successive courses on special relativity were organized around this key idea of explicitly considering classes of projections (interpreted, at the time, as historical perspectives). Instruction was organized around (a) operational (Einsteinian) view, (b) algebraic (Lorentz equations) view, and (c) geometric (Minkowskian) view. However, that was before the present analysis and interpretation, and our (Levrini and the Italian research group's) argument, at the time, was that the richness and diversity of the thinking about the historical context was the "secret ingredient" in achieving greater conceptual competence. While we found in summative assessment that students, by and large, did not have the difficulties exposed here (with the horizontal light ray), we have little process data to analyze the details. In particular, we do not have an inclination to reject that the historical context *per se* had influence in the effectiveness of the instruction. Instead, we now believe—following our analyses above—that a coordination class perspective explains some of that success. Explicitly exposing, managing, and relating multiple classes of projections seems to be a good theory-motivated instructional technique to work around documented difficulties in conceptual change in special relativity.

V. CONCLUSION

This section concludes the article in three ways. First, we review some of the important results of our analysis seeking to make clear how coordination class theory illuminated the case of learning investigated here while at the same time showing aspects of the analysis that are not entailed by coordination class theory *per se*. In preparation, the reader may choose to review Sec. III C. Next, we reflect on the consequences of this analysis of a case, using coordination class theory, in terms of theory validation, refinement, and extension. Finally, we provide a brief comparison and contrast with some other views of conceptual change.

A. Review of results

1. Contexts and classes of projections

Different contexts are a critical focus in coordination class theory, and many general expectations of coordination class theory concerning contexts are borne out in these data. First, we saw what we described as a short circuit with regard to the light clock. Initially, a commonsense intuitive scheme—which we proposed to be the idea that happenings (or phenomena) have characteristic locations and times—took the place of more refined special-relativistic ways of thinking in this context. When students looked at the light clock, they saw a phenomenon with a location (corresponding roughly to the clock itself) and a duration, the latter being the happening's proper time. While this idea actually promoted the students' sense of understanding, it seeded a misalignment when a new context was explored, the horizontal light ray. This new problem appeared identical in terms of their naive schemata, and they concluded that the situation was the same: The location of the phenomenon (emission and reception of the light ray) was the train car and that should be the relevant frame of reference for determining proper time. However, of course, this was an inappropriate conclusion that led to misreading proper time in this context (misalignment).

Naturally, some aspects of this learning situation are not generic to coordination class theory. The fact that determining a particular frame of reference served to mediate information determination is special to special relativity, if not more locally, to a few concepts like proper time and length.[36]

How did students make progress out of this difficulty? Two contributing factors seemed salient. First, the teachers introduced a new class of projections by fixing the definition of proper time. In view of general coordination class considerations, seeding a new class of projections (from the newly provided definition) should potentially increase span but at the cost of potential misalignment. Both those expectations were borne out. Students (eventually) managed to expand proper alignment to both old and new contexts. However, reaching alignment was an extended process, which one should always expect from a coordination class theory point of view. Recall that, even after students appeared to have solved the alignment problem, new questions arose: If we move with the object in the train car to measure its proper time, why do we not also do that in the case of the light clock?

Emphasizing the role of multiple contexts in developing a well-formed coordination class, a third context played a critical role, where a horizontally moving object replaced the horizontally moving light ray. Our interpretation is that the role of this context is only partly generic in coordination class terms. What is generic is that some particular contexts productively "harness" particular intuitive schemes. In this case, the intuitive idea of "riding an object or phenomenon" was used, but in such a way that it seemed little more than a way of speaking. Students became clear that "riding an object" was justified precisely by the more technical requirement that start and termination events be viewed in a frame that sees them happening "at the same place." Recall that Marco initially spoke merely about being on the object, but then made clear that the justification for this was that riding an object made the terminal events happen in the same place. This is the kind of special-relativistic justification that, for example, Lorenzo did not give (and, we believe, could not give) for the analysis of the light clock.

What was clearly not generic coordination class phenomena was that the horizontal light ray was aligned as a limiting case of the coordination that worked for the horizontal-moving object. There are—and we have always posited—particular characteristics of concepts, contexts, and intuitive resources that must be added to coordination class theory to complete a view of conceptual change. For example, which intuitive conceptualizations are productive in which contexts cannot be given *a priori* in general coordination class terms. This is a limitation of coordination class theory that we have not transcended here and, in fact, we do not expect to transcend within the framework of coordination class theory alone. It has to rely on knowledge of students' available and developmentally contingent supply of resources and on the relation of those resources to the particular cognitive demands of the new concepts being acquired.

2. Articulate alignment

The way that alignment was accomplished in these data constitutes a natural extension of coordination class theory. First, we see an attractive and stronger form of alignment than has been discussed in the past. Students not only learn to correctly determine the relevant information (proper time) in different circumstances, but they come to explicitly and articulately view the relation between those projections: They come to see them as "the same" or as "related in an understandable particular way." In this case, for example, one projection was seen as a limiting case of another. Our interpretation also involved a natural extension of coordination class theory by noting the after-the-fact obvious idea that definitions can seed different classes of projections (by using the definition in various contexts). Definitions thus "parse" the space of projections and, furthermore, they do it in an explicit way: Students find the task of seeing that the definitions "get the same answer" (are aligned) completely obvious. Moreover, they furthermore find articulate alignment to be a natural goal: Not only should projections (definitions) get the same answer, but we should be able to figure out why and how they do.

Later data suggest (although we do not take it to be proven) that elaborating students' consideration of the relations among definitions in the context of differing historical perspectives made a positive contribution to an integrated, flexible, and robust understanding. At a minimum, students clearly felt much more confident about the particular set of contexts considered here and about the relations among different ways of viewing them in order to coordinate proper time, after drawing out implications of different "historical perspectives" (viewed here as proxies for different classes of projections). Recall the confident and nearly unanimous "chorusing" in reviewing how proper time could be viewed (determined) differently through different perspectives.

Beyond this generic factoring that multiple definitions would seem always to seed, we conjectured one substantial factoring: focusing coordination on space-time events rather than (just) on frames of reference, objects, or happenings. In fact, this conjecture was driven by trying to understand how, in general, one might provide for systematic displacement of intuitive schemes such as "happenings have characteristic places and durations" when they are inappropriate. This, of course, is very unlikely to be a generic coordination class phenomenon and, is, instead, special to special relativity. Whether there are similar primary factorings for other concepts, at this point, is speculative.

B. Review of the "fate" of coordination class theory in its application to proper time

(1) *Applicability.* We feel that coordination class theory fares well in its application to this empirical context. Proper time seems to have all the earmarks of a coordination class. We see examples of many coordination class categories, such as information determining processes, focus of attention during determining (*readout*), and inferences in that process (*causal net*). It is clear that the development of *alignment of projections* across different contexts takes time and consideration. Lack of *span and alignment* characterize particular student difficulties. When we can "see" the coordination process, we can sometimes see intuitive schemes apt for certain contexts "taking the place" of more appropriate ideas. That makes those intuitive ideas in need of *displacement* (the happening story).

(2) *Test*. Many of the core general expectations seemed borne out in this analysis, including loci of difficulty and the function of learning in different contexts (*recruitment* and *displacement*).

(3) *Insightfulness*. We have advanced the case that coordination class theory has been insightful about both the problems but also the successes of these students' conceptual change. The advantages and disadvantages of various pedagogical strategies were interpreted in coordination class terms. The light clock, by itself, is prone to invoke but not displace or refine classical intuitions. Focusing explicitly on multiple determination strategies and focusing space-time event as a general foundation for "seeing" relativistic quantities seem likely to be generally productive.

(4) *Refinements and extensions*. At least two important but, we argue, completely natural refinements and extensions

to coordination class theory were proposed. First, we make the after-the-fact obvious observation that definitions can seed classes of projections; we might say that they factor the space of projections. Thus, multiple definitions provide a good proxy for the general fact of life, in the view of coordination class theory, that a variety of different projections are always involved in a well-formed concept. The second extension builds on this idea. Students can be reflectively engaged in the goal-directed process of *articulate alignment*. Students can, under certain circumstances, consciously consider the issue of alignment (in the case that definitions or historical perspectives serve as graspable foci for determination, that is, for classes of projections). As a result, students may achieve what apparently happened here, articulate alignment as an end state (rather than as a process)—where projections not only have become aligned (in part) in a conscious attempt to do so, but the resulting alignment is more than "getting the same answer in different circumstances or by different methods." Instead, the state of articulate alignment means the ability to make well-reasoned comparisons and to understand the important relationships among different projections, for example, as "being the same"[37] or "being a limiting case of another."

C. Brief comparison to other perspectives

We believe that a comparison with other points of view that have been prominent in the history of conceptual change research will help highlight the distinctive features of coordination class theory. Readers who are not familiar with that history may consult reviews (e.g., Ref. 6) or the particular literature cited. In the following, we do not aim to provide a convincing competitive argumentation that coordination class theory is right or that other views are wrong. However, we aim to suggest how they are different and, obviously, to point out features of the present empirical analysis that favor coordination class theory.

1. Categorical views of conceptual change

Conceptual change, outside physics education research,[38] is dominated by "snapshot" views of the process.[6] That is, naive ideas are contrasted categorically with professional ones. The discontinuity is often emphasized in the extreme. For example, Carey[39] picked up Kuhn's idea of incommensurability of paradigms. She made the idea that prior and postconceptual change concepts cannot even be translated into each others' terms the center of her view on what constituted the meaning and difficulty of conceptual change. Others characterize naive "theories" and distinct stages of advancement[40] not only as landmarks, but essentially the entirety of the their theory of change, in contrast to "change" as gradual structural reorganization, as it is viewed here.

Such categorical views are systematically weak in dealing with processes and intermediate states, as we anticipated in the Introduction. Here, we began the analysis at a point where students have made significant progress. They understand the light clock and its relation to proper time to a level that is fairly impressive. (They understand, for example, that time durations may differ in different frames of reference.) In fact, they have made the light clock analysis compatible with the classical ontology of happenings that have characteristic places and durations. In that context, their performance might be all but unobservably different from an expert performance. Recall that several critical remarks that the students made were essentially correct, if event (or phenomenon) were interpreted in the special space-time sense rather than as happenings. The teachers of this class initially felt that their students had a firm grasp of the ideas but only discovered their limitations in a different context, where misalignment became clear, and also the reason for the misalignment was suggested. This is a very particular state of partial construction, and we saw many particulars, at least some of them generic in the sense of being expectable from a coordination class point of view. We saw this stage as on the way to a more widely spanning, aligned, and even articulately aligned concept of proper time.

2. Rational views of conceptual change

In the Introduction, we also cited views of conceptual change that purport to set the conditions for change but stop short of the kind of structural analysis that coordination class theory provides in considering multiple projections, multiple contexts, and the role of displacing particular intuitive ideas. Posner *et al.*[9] proposed, for example, that students need to experience systematic difficulties in their own conceptions, they need to find the new conceptions both understandable and plausible, and, finally, they need to anticipate productivity of the new concepts in a "progressing paradigm," to use Lakatos's words.[41]

Long complex paths of conceptual change, as portrayed here, make rational choice points by students unlikely, whatever the criteria of choice. If it takes a substantial amount of time and reorganization to create a robust concept, choice does not even make sense until the concept has been substantially constructed. To understand that progress, one needs, we claim, some articulated view of the specifics of construction, which coordination class theory provides. Along the way, of course, implausibilities and perhaps even local "contests" between old and new ideas are inevitable. However, in the structural view, these issues are resolved piecemeal, where old distributed elements are displaced and new projections are constructed to cover cases of lack of span and lack of alignment. In this view, then, it seems immensely plausible that commitment to the new ideas gradually grows with the construction of the concept and, importantly, along with its gradual construction *so as to be sensible at each stage*. By the time there could be a choice point, the issue is moot. All the little battles have accumulated in the construction of the concept and, simultaneously, in belief and confidence in it.[42]

Students did not and should not have made a choice between classical ideas and special relativity due to bad properties of their old ideas and good properties of the new ones. Instead, a local problem (lack of alignment in a new context) provided an occasion to extend further the meaning and power of the new ideas. Indeed, students were simply unaware—much less capable of articulating—that they had or were in need of "giving up" what is, after all, only a tiny piece of their intuitive conceptions of space and time: happenings as entities with characteristic location and duration. They already had, in some contexts, substantial parts of the new conceptual structure that they would eventually have more completely. Understanding the light clock and the implications of the formula it derived (including the idea of different durations for the same happening viewed from different frames of reference) must have seemed to them new and a substantial accomplishment that they did not necessarily see as countering some "abandonable" prior concept (at least, no data presented here suggested this possibility). Certainly, teachers regarded students' proper time concept as fairly well constructed, before some weaknesses were observed. So, a good deal of construction probably went on with no particular global conflict or choice point, and the "abandoning" happened invisibly, in pieces, as plausible alternatives to these old ways of thinking were constructed in the context of local failures.

3. Ontological views of conceptual change

We introduced "ontological" as a description that we feel is apt for the naive idea that happenings exist and have the intrinsic properties of place and duration. However, the history of conceptual change research includes the use of ontological in a different sense and within a radically different view of conceptual change. Chi, for example, sees the fact of systematic difficulties in learning in specific ontologies that experts have, and novices simply do not have. Novices, according to Chi,[43,44] have problems because they use and abuse one particular ontology, "matter," and they simply do not have more sophisticated ontologies, such as "constraint-based event" that experts supposedly use.

Our ontological interpretation is distinct from Chi's in several ways. First, the problem ontology is "small" and local. It is not the (one) ontology that students use that keeps them from conceiving in the least degree the explanatory concepts of experts. Students have many ways of thinking about space and time that they can well adapt to learn special relativity, enough so that they can learn a good approximation to the critical ideas illustrated and developed in the light clock, for example. Their naive ontology even served a productive role in allowing them to "find the right frame of reference" or, at least, to judge that it seemed right, after the fact, in the light clock case. The limits in their understanding only became evident to students and teachers alike in a new context. Similar to the difficulties we attributed to categorical views of conceptual change, novices do not systematically have only one way of viewing the world that prevents substantial construction in particular contexts. Neither does one nameable ontology, such as constraint-based event,[43] covers all of expert conceptualization of special relativity (although we cannot take the time to specifically argue this point).

More specifically, "matter ontology" is simply not the problem that students have with special relativity, at least not in this case. The problem ontology, as we think we see in these data, is event (happening).[45] Furthermore, we pointed out that event does not need to be abandoned, but, in fact, a refinement of the naive "event," namely, the space-time event, is precisely what we expect to be a key professional-grade concept that will supersede the intuitive version. Recall again, that some important statements that students made about events could be interpreted as absolutely correct, if they meant space-time event rather than commonsense event.

Finally, whatever ontological constructions or reconstructions are needed are not a matter of "starting from scratch" and simply telling students about the new ontology (see Ref. 43 and subsequent empirical work). In our view, those accomplishments must be gradual, piece-by-piece constructions in no obvious way different from the gradual conceptual constructions that have been the focus of coordination class theory.

ACKNOWLEDGMENTS

The authors wish to thank readers of previous drafts of this paper for comments that improved the content and presentation. This included the Boxer Research Group at the University of California, Berkeley (Jeanne Bamberger, Lauren Barth-Cohen, Janet Casperson, Karen Chang, and Colleen Lewis), the Research Group in Physics Education at the Physics Department of the University of Bologna (Nella Grimellini Tomasini, Marta Gagliardi, Barbara Pecori, and Eugenio Bertozzi), Anna De Ambrosis, Melinda diSessa, Paola Fantini, Orit Parnafes, and Joseph Wagner. The authors moreover wish to thank the teachers, Fabio Bonazzi, Stefano Cristoni, Laura del Pennino, Bruna Fergnani, and Marina Marani, who collaborated on the experiment. Manuscript reviewers for the *journal* helped refine and clarify arguments. A grant from the Spencer Foundation to A.A.dS. (No. 200500036) facilitated discussion and the development of ideas. We are grateful for the support. The study is also carried out within the Italian project PRIN F21 coordinated by Paolo Guidoni.

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[1] We use the terms "theory" and "model" roughly synonymously in this paper. However, "model" emphasizes the expectation and perhaps deliberate articulation of limits to validity and coverage, a point that is important here.

[2] A. A. diSessa and P. Cobb, Ontological innovation and the role of theory in design experiments, J. Learn. Sci. **13**, 77 (2004).

[3] A. A. diSessa and B. Sherin, What changes in conceptual change?, Int. J. Sci. Educ. **20**, 1155 (1998).

[4] A. A. diSessa and J. F. Wagner, in *Transfer of Learning From a Modern Multidisciplinary Perspective*, edited by J. Mestre (Information Age, Greenwich, CT, 2005), pp. 121–154.

[5] A coordination class, as we define it in the text, involves coming to be able to see *one particular thing* (e.g., force). Artificial concepts, such as those used by psychologists in early "concept learning" experiments ("grall" means an object that is both green and tall), are not coordination classes because people

learning the concept can already "see" the elements (green and tall). Complex "concepts," such as Newton's second law entail several "subconcepts" (e.g., mass and force), which are at least partially independently learned and are individually candidates for coordination classes. So, such compound entities are not, themselves, coordination classes. See the discussion of different kinds of concepts in Ref. 4. Finally, no concept is assumed to be a coordination class. It must empirically reveal the structure and learning properties of a coordination class.

[6] A. A. diSessa, in *Cambridge Handbook of the Learning Sciences*, edited by K. Sawyer (Cambridge University Press, Cambridge, 2006), pp. 265–281.

[7] M. T. H. Chi, P. Feltovich, and R. Glaser, Categorization and representation of physics problems by experts and novices, Cogn. Sci. **5**, 121 (1981).

[8] J. Larkin, J. McDermott, D. P. Simon, and H. A. Simon, Expert and novice performance in solving physics problems, Science **208**, 1335 (1980).

[9] G. J. Posner, K. A. Strike, P. W. Hewson, and W. A. Gertzog, Accommodation of a scientific conception: Toward a theory of conceptual change, Sci. Educ. **66**, 211 (1982).

[10] "Well formed" does not mean "correct." Instead, it refers to several specific "organizational" properties, as will be explained shortly. Thus, "prior conceptions" might very well be coordination classes. In the subject at issue here, the classical concept of time may very well be a coordination class. (Other "prior conceptions," such as "force," are unlikely to constitute coordination classes for reasons we do not discuss here.) In net, the goal of teaching a coordination class includes both that it is well formed and that it is normative.

[11] O. Parnafes, What does ′′fast′′ mean? Understanding the physical world through computational representations, J. Learn. Sci. **16**, 415 (2007).

[12] J. F. Wagner, Transfer in pieces, Cogn. Instruct. **24**, 1 (2006).

[13] In Italy, credentialing as a secondary school teacher involves a two-year postgraduate program, culminating in a teaching project that involves teaching in a "real" classroom under the supervision of an experienced teacher. In Italian, the classroom sessions are called *tirocini*. The experiment described in the article took place at Liceo Scientifico ''N. Coperico,'' Bologna. Italy.

[14] D. Halliday, R. Resnick, and J. Walker, *Fundamentals of Physics* (Wiley, New York, 1997).

[15] Members of the research team that originally organized and studied this instruction felt that the roots of the difficulty were in the classroom teachers' failure to emphasize the definition of proper time as the time between two events measured in a frame of reference in which the two events occur at the same spatial location. Here, in following the implications of a coordination class perspective, we come to a distinct though related conclusion.

[16] P. Thagard, *Coherence in Thought and Action* (MIT, Cambridge, MA, 2000).

[17] M. R. Quillian, in *Semantic Information Processing*, edited by M. Minsky (MIT, Cambridge, MA, 1969), pp. 227–270.

[18] One must keep in mind that displacement is intended to be a subconceptual move, operating on pieces of a concept. Thus, the "displacement of one concept by another" is a move at an entirely different level of aggregation. Coordination class theory is an attempt to replace monolithic treatments of displacement or replacement of concepts with a theory of the fine-grained elements and processes that go into such macrochanges. So, historically, older accounts of biology took life to be an irreducible quality of things. Modern biology sees life to be a consequence of a large number of elements and processes. This observation notwithstanding, when many concepts are at issue in a conceptual system, it may be useful to speak synoptically of the construction of some of them, which are accomplished prior to others, as events rather than as extended processes.

[19] Experts will have a pretty secure sense of the full range of contexts in which a coordination class should apply. For pedagogical purposes, one might make the practical decision that "a set of 'typical' end-of-chapter problems" provides a practical test of adequate span.

[20] While "seeing" the class-relevant information is a productive metaphor, we use the term "coordinate" to denote seeing in the general coordination class sense of determining via readouts and inferences.

[21] The stages are not strictly temporal. The discussion is quite complicated and some of the excerpts that were put in stage 1—for example, the "invariance" 1c excerpt—occurred in the middle of the discussion. However, our interpretation is that these excerpts belonged together logically, for example, as concerning preinstruction knowledge; the particular times at which they emerged in the discussion were a matter of contingency.

[22] Whether the definition is precise and technically correct is not a relevant criterion for "being a definition." Instead, we want to describe how *the students* defined and used the concept. The foreground in an analysis like this is always how students think.

[23] Psychologically, how do students actually pick out "the thing we want to be at rest with respect to"? It is possible that finding the relevant frame is guided by literal linguistic memory, something like "the frame of reference of phenomenon," whether this was ever uttered by the teacher or found in the text. Alternatively, perhaps they have come to expect that there will be a single obvious frame of reference and let their instincts find that frame. Marco's initial formulation is ambiguous, "time of something" and "frame of reference at rest with respect to that something." As he initially applies it to the light clock, the "something" is described as "the light ray," although he probably means the physical clock structure, which is for him "the location of the phenomenon." Later, he identifies the relevant frame as that of "the event." However, it is effectively encoded, linguistically or simply expecting the relevant frame to be obvious, the point is that the frame is not unambiguously and securely defined; its ambiguity can result in misalignment via choosing the wrong frame in which to find the proper time.

[24] Relying exclusively on a new definition is quite likely to reduce span. Students may not be able to produce a projection consistent with that definition in contexts where an older definition or older strategies smoothly work.

[25] Anticipating later discussion, most physicists would say that the frame actually does not exist; there is no (proper) frame of reference traveling at the speed of light with respect to any physical frame of reference. So, Marco's inability to construct a frame of reference having A and B at the same place turns out to be a principled impossibility.

[26] An alternate interpretation is that Marco here is using the more diffuse interpretation that we claimed was consistent with what students got from the light clock: "Proper time is what you mea-

sure 'sitting on' the phenomenon." However, directly below, he clarifies the meaning of his statement consistent with the teacher's definition, which involves finding a frame in which two events happen at the same place. Marco has learned.

[27] It seems plausible to us that it would help both teachers and students to know some version of coordination class theory. However, this is a conjecture for future work.

[28] A. Einstein, Zur Elektrodynamik bewegter Körper, Ann. Phys., **17**, 891 (1905).

[29] R. E. Scherr, P. S. Shaffer, and S. Vokos, Students understanding of time in special relativity: Simultaneity and reference frames, Am. J. Phys. **69**, S24 (2001).

[30] R. E. Scherr, P. S. Shaffer, and S. Vokos, The challenge of changing deeply held student beliefs about the relativity of simultaneity, Am. J. Phys. **70**, 1238 (2002); R. E. Scherr, Modeling student thinking: An example from special relativity, Am. J. Phys. **75**, 272 (2007).

[31] E. F. Taylor and J. A. Wheeler, *Spacetime Physics*, 2nd ed. (W. H. Freeman, New York, 1992).

[32] A. De Ambrosis and O. Levrini, Insegnare relatività ristretta a scuola: esigenze degli insegnanti e proposte innovative, G. Fis. **XLVIII**, 255 (2007).

[33] The session analyzed here, in fact, followed on the discovery that students had some significant remaining difficulties with special relativity. The sessions were initiated in order to discover the reasons for and, if possible, remedy the students' problems.

[34] O. Levrini, The substantivalist view of spacetime proposed by Minkowski and its educational implications, Sci. & Educ. **11**, 601 (2002).

[35] H. Minkowski, Raum und Zeit, Phys. Z. **10**, 104 (1909).

[36] On the other hand, similar phenomena might well apply in Galilean relativity and, perhaps, more generally concerning choices of coordinate systems in both mathematics and physics.

[37] "Being the same," here, implicates a complex articulate construction that takes time and effort to lay out. It is not a presumption or a mere observation.

[38] Snapshot views of conceptual change are not endemic in physics education research. However, it is still worth noting the pervasiveness of such views in the broader field of conceptual change, and physics educators may still benefit from explicit consideration of the limits of categorical views of conceptual change.

[39] S. Carey, in *Conceptual Development: Piaget's Legacy*, edited by E. Scholnick, K. Nelson, S. Gelman, and P. Miller (Lawrence Erlbaum Associates, Mahwah, NJ, 1999), pp. 293–326.

[40] C. L. Smith, M. Wiser, C. W. Anderson, and J. Krajcik, Implications of research on children's learning for standards and assessment: A proposed learning progression for matter and the atomic molecular theory, Meas. Interdiscip. Res. Perspect. **4**, 1 (2006).

[41] I. Lakatos, in *Criticism and the Growth of Knowledge*, edited by I. Lakatos and A. Musgrave (Cambridge University Press, London, 1970), pp. 91–196.

[42] It is possible to attempt to rescue rational views by asserting that rational criteria apply to the small and time-local learning events that are described in coordination class theory. However, criteria such as recognition that prior ideas are beset by "a sea of anomalies" are implausible in a 20 min episode. Furthermore, estimating the future productivity of a new idea in "a progressing paradigm" seems just as implausible at the idea's "birth." Finally, we know of no empirical analysis of conceptual change where individual learning events are held accountable to seeing these criteria enacted in empirical data, as opposed to the time-localized analyses we present here using coordination class theory.

[43] M. T. H. Chi, in *Cognitive Models of Science: Minnesota Studies in the Philosophy of Science*, edited by F. Giere (University of Minnesota Press, Minneapolis, 1992), pp. 129–160.

[44] M. T. H. Chi, Commonsense conceptions of emergent processes: Why some misconceptions are robust, J. Learn. Sci. **14**, 161 (2005).

[45] This particular point and several others in this section were made at greater length in Ref. 46.

[46] A. A. diSessa, Toward an epistemology of physics, Cogn. Instruct. **10**, 272 (1993).