García-Aguilar et al. Reply: In their Comment [1], Haas et al. advance two hypotheses on the nature of the shape transformations observed in surfactant-stabilized emulsion droplets [2–9], as well as the theoretical models that us [10] and others [11,12] have introduced to account for these observations. (1) Because of the different surfactants used in Refs. [2–4] and [5–8], the physical mechanisms underpinning the shape transformations may, in fact, differ in spite of the extraordinary resemblance in the experimental output. (2) The theoretical models introduced in Refs. [10] and [11,12] are mathematically equivalent by virtue of the small magnitude of the stretching and gravitational energies considered in Ref. [10]. In this Reply, we argue that neither of these hypotheses is well justified.

To test the first hypothesis, we have performed direct cryo-TEM imaging of Brij 78 nonionic surfactant-stabilized emulsions, as those used in Refs. [5,6]. The interface of a faceted alkane [CH₃(CH₂)₁₄CH₃, denoted as C₁₆] droplet, suspended in a 1.5% (by weight) aqueous Brij solution [inset to Fig. 1(a)], clearly demonstrates the absence of any surface-adjacent structure, such as the rotator crystals hypothesized in Refs. [5,11]. The only detectable feature is a clearly defined interfacial layer, as that observed in C₁₈TAB-stabilized emulsions discussed in Ref. [10]. To extract the interfacial layer's thickness, we fit the intensity profiles across the interfaces with a tilted Gaussian function [Fig. 1(b)]. The full width at half maximum (FWHM) varies with the magnification M, averaging to $t = 2.9 \pm 0.2$ nm at the highest accessible magnification. This t value matches the previously estimated thickness of a monolayer [4]. Furthermore, as in our previous studies [4], we linearly extrapolate the experimental FWHM values to $M^{-1} \rightarrow 0$ [Fig. 1(a)]. The corresponding interfacial thickness of the Brij-stabilized system, 2.3 ± 0.2 nm, perfectly agrees with the 2.2 ± 0.9 nm value previously reported for interfacially frozen C₁₈TAB-stabilized C₁₆ emulsions [4], indicating that only one crystalline monolayer is present at the interface of these faceted droplets, for either of the surfactants. Our analysis is further validated by the much smaller $t = 0.7 \pm 0.1\,$ nm of the C₁₈TAB-stabilized cyclohexane (C₆H₁₂) emulsion, where no interfacial freezing takes place and the droplets are rounded [open circles in Fig. 1(a)].

With respect to the second hypothesis, we notice that the numbers $\Delta \mathcal{E}_S$ and $\Delta \mathcal{E}_G$, used by Haas *et al.* to conclude that stretching and gravity are unimportant, correspond to energy minima and are not representative of the entire energy landscape, hence, cannot be used to support the

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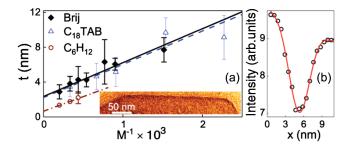


FIG. 1. (a) The cryo-TEM interfacial widths t of the faceted C_{16} emulsion droplets, stabilized with C_{18} TAB [4] or with Brij 78 surfactant (see legend), perfectly coincide. A much thinner interfacial width is obtained for a C_6H_{12} emulsion, where no interfacial freezing takes place. Inset: The cryo-TEM image of the edge of a Brij-stabilized faceted droplet. The "glow" lookup table is employed. (b) Cryo-TEM intensity profile across an interface of a faceted Brij-stabilized droplet. The average fitted FWHM are shown in (a).

authors' conclusions. Specifically, the stretching energy of a spherical crystal depends on the configuration of the topological defects and ranges from the numbers \mathcal{E}_S in Table I of Ref. [10] to infinity (as any two of the twelve seed disclinations approach each other). Thus, stretching cannot be excluded a priori, and its effect is, in fact, pivotal for the emergence of the icosahedral structure, where the twelve seed disclinations are maximally spaced. Similarly, because of the quartic dependence on $r = RH_0$, where R is the droplet radius and H_0 the spontaneous curvature, the gravitational energy \mathcal{E}_G is a rapidly increasing function of R. In the example considered by Haas et al., $R = 10 \mu m$ [1], which, using $H_0^{-1} \approx 60$ nm and $\Pi \approx 10^{-8}$ [10], gives $r \approx 167$ and $\Pi \Delta \mathcal{E}_G r^4 \approx 27$. Thus, $\Pi \Delta \mathcal{E}_G r^4$ is much smaller than the dimensionless bending energy difference $\Delta \mathcal{E}_H r \approx$ 9×10^3 [1]. Yet, it is sufficient to take a droplet of radius $R = 45 \mu \text{m}$ (r = 750), thus, well within the experimental range, i.e., 1–150 μ m [Fig. 1(f) in Ref. [10]], for $\Pi \Delta \mathcal{E}_G r^4$ to have the same order of magnitude of $\Delta \mathcal{E}_H r$. For the largest droplets ($R = 150 \ \mu \text{m}, \ r = 2500$), $\Pi \Delta \mathcal{E}_G r^4$ is 1 order of magnitude larger than $\Delta \mathcal{E}_H r$, emphasizing the importance of gravity in the system.

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