Partial Wave Amplitude Basis and Selection Rules in Effective Field Theories

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We derive the generalized partial wave expansion for $N \to M$ scattering amplitude in terms of spinor helicity variables. The basis amplitudes of the expansion with definite angular momentum j consist of the Poincaré Clebsch-Gordan coefficients. Moreover, we obtain a series of selection rules that restrict the anomalous dimension matrix of effective operators and how effective operators contribute to some $2 \to N$ amplitudes at the loop level.

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Introduction.—Scattering is the most important and fundamental process in particle physics, which is formally a quantum transition between asymptotic states. It is natural to study the selection rules for transitions due to conservation laws, as practiced in theories of molecules, atoms, nuclei, as well as particle decays such as the Landau-Yang theorem [1,2]. The selection rule in particle scattering due to angular momentum conservation is usually achieved by partial wave expansion, formulated for limited cases such as $2 \to 2$ scattering. It is therefore intriguing to apply the selection rule to generic scattering processes.

Such a selection rule turns out to be very powerful. Recent studies [3–7] in the on-shell approach to the standard model effective field theory (SMEFT) have observed nontrivial relations among operators and observables [8–14], shedding light on the precision test of the standard model: a crucial task in particle physics. We show that the selection rule from angular momentum conservation could explain many of these nontrivial relations.

One of the central roles in the explanation is played by the one-to-one correspondence between effective operators and the on-shell amplitude basis, established in [4,15], where it helps construct a complete operator basis without redundancy in the SMEFT. In short, the amplitude basis is a basis in the space of contact amplitudes, with each corresponding to a local operator that exclusively generates it. For example, the partial wave amplitudes for $2 \to 2$ scattering of massless particles are the Wigner d matrices $d^j_{\nu\nu'}$, which precisely form an amplitude basis up to dimensionality. By the operator-amplitude basis correspondence $d^j_{\nu\nu'}$ induce an operator basis, which inherits the label j as the total angular momentum in the particular scattering channel. These operators thus only contribute to processes with selected total angular momentum, which is the key of our selection rule.

To set up the selection rule for generic scattering, we first generalize the partial wave expansion for arbitrary $N \to M$ scattering by utilizing the spinor helicity variables [16]. The amplitude basis with definite total angular momentum is obtained as a result; based on which, we find two types of selection rules: First, we find nontrivial constraints on the anomalous dimension matrix beyond the non-renormalization relations shown in the literature [11,12]; and second, operators are selected in certain cases while contributing to some one-loop diagrams, verifying the discovery of "vanishing rational terms" in [14] and leading to more nontrivial patterns in one-loop amplitudes.

Generalized partial waves.—Multi-particle states in scattering are conventionally tensor products of single-particle asymptotic states $|\Psi_N\rangle_\otimes = \otimes_{i=1}^N |\psi_i\rangle$, with each classified by the Poincaré algebra and labeled by the on-shell four-momentum p_μ and the little group representation [17]—helicity for massless particles $|\psi_i\rangle = |p_i, h_i\rangle$, and spin for massive ones $|\psi_i\rangle = |p_i, s_i, \sigma_i\rangle$. However, due to the angular momentum conservation, it is also convenient to work with basis of definite total angular momentum j, dubbed the *partial wave basis*, for which the S matrix is block diagonal. Instead of computing j from the composition of total spin S

and the orbital angular momentum L, which is mostly useful for two-massive-particle states in the c.m. frame, we propose to treat the eigenstates with definite j as irreducible representations of the Poincaré group, which is now for multiparticle states. Just like how we deal with the rotation group in quantum mechanics, we define the overlap between the tensor representation states and the irreducible representation states as the *Poincaré CG coefficients*:

$$_{\otimes}\langle\Psi_N|\Psi_N\rangle_j \equiv C_N^{j,j_z}(P;\psi_{i=1,\ldots,N})\delta\left(P-\sum p_i\right). \quad (1)$$

The key distinction from the SO(3) CG coefficients is that they already include the partial wave function of the scattering angles via the dependence on p_i—for two-massive-particle states, the Poincaré CG coefficient is the product of the SO(3) CG coefficient and the spherical harmonics $Y_{lm}(\theta,\phi)$.

To evaluate the Poincaré CG coefficients in general, we make use of the helicity spinor variables (λ_α^I, $\tilde{\lambda}_{\dot{\alpha}}^I$) [18], where the superscript I is introduced for little group SU(2) of the massive particles [16]. These spinors not only indicate the momentum $p_\mu\sigma^\mu_{\alpha\dot{\alpha}} = \epsilon_{IJ}\lambda_\alpha^I\tilde{\lambda}_{\dot{\alpha}}^J$ but are also able to represent the spin. For instance, the scattering amplitude involving a massive spin-s particle should have $2s$ totally symmetric little group indices I coming from the spinor variables of this particle, i.e.,

$$\mathcal{A}^{\{I_1\ldots I_{2s}\}} = \mathcal{A}^{\{\alpha_1\ldots\alpha_{2s}\}}\lambda_{\alpha_1}^{I_1}\cdots\lambda_{\alpha_{2s}}^{I_{2s}}, \quad (2)$$

so that it transform correctly under the little group. The factor $\mathcal{A}^{\{\alpha_1\ldots\alpha_{2s}\}}$ is the amputated amplitude whose symmetrized spinor indices come from the spinor variables of the other particles. A simple example would be the $Z\bar{e}e$ coupling $\langle Z^{I_1}e\rangle\langle Z^{I_2}|p_e|\bar{e}]$, where we use the bracket convention for spinor contractions. Now that we introduce the auxiliary spinor variables $(\chi^I,\tilde{\chi}^I)$ for the total momentum P_μ of a multiparticle state, the Poincaré CG coefficient should have the same property as Eq. (2); thus, we define [19]

$$C_N^j(\chi^I\tilde{\chi}_I;\psi_{i=1,\ldots,N}) = f^j(\psi_{i=1,\ldots,N})\cdot(\chi^{I_1}\cdots\chi^{I_{2j}}), \quad (3)$$

where f^j is the multiparticle wave function consisting of spinor variables of all the constituting particles, and the symmetrized spinor contractions are abbreviated by a dot. Note that the projection j_z is represented by the $2j+1$ independent components of the little group tensor.

In this Letter, we will be focusing on the multi-massless-particle states that are relevant for the massless effective field theories (EFTs), although it is straightforward to extend to states with massive particles by decorating the wave function f with more little group SU(2) indices. Let us take the simplest example of a two-massless-particle state with helicity h_1, h_2. The Poincaré CG coefficient can be constructed as amplitudes for two massless particles and one massive spin-j particle, whose general form is shown in [16] as

$$C_{h_1,h_2}^j \sim \frac{[12]^{j+h_1+h_2}}{s^{(3j+h_1+h_2)/2}}(\langle 1\chi\rangle^{j-h_1+h_2}\langle 2\chi\rangle^{j+h_1-h_2}), \quad (4)$$

where the normalization by a power of $s=P^2$ keeps it dimensionless.

With the partial wave basis, the scattering amplitudes can be expanded on a basis with definite angular momentum as

$$\begin{aligned}\mathcal{A}_{N\to M} &\equiv {}_{\otimes}\langle\Psi_M|\mathcal{M}|\Psi_N\rangle_{\otimes}\\ &= \sum_j {}_j\langle\Psi_M|\mathcal{M}|\Psi_N\rangle_j \sum_{j_z} C_M^{j,j_z}(C_N^{j,j_z})^*\\ &\equiv \sum_{j,a}\mathcal{M}^{j,a}(s)\mathcal{B}_{N\to M}^{j,a},\end{aligned} \quad (5)$$

while the coefficient function $\mathcal{M}(s)$ carries the information of the dynamics. The basis $\{\mathcal{B}^{j,a}\}_{N\to M}$ with possible degeneracy labeled by a is therefore defined as the *partial wave amplitude basis*, or *j basis* for short, which is an alternative type of amplitude basis introduced in [4,15]. The sum over j_z turns into the contraction of little group indices in the helicity spinor representation [Eq. (3)]

$$\begin{aligned}\mathcal{B}_{N\to M}^{j,a} &\equiv \sum_{j_z} C_M^{j,j_z}(C_N^{j,j_z})^*\\ &= f^j(\psi_{1,\ldots,M})\cdot(\chi^I\tilde{\chi}_I)^{2j}\cdot f^j(\psi_{1,\ldots,N})^*. \end{aligned} \quad (6)$$

The auxiliary spinors χ can be eliminated via the identity $\chi_\alpha^I\tilde{\chi}_{I\dot{\alpha}} = P_\mu\sigma^\mu_{\alpha\dot{\alpha}}$ such that the amplitude basis $\mathcal{B}^{j,a}$ only depends on the wave functions f^j of the external particles. As a cross-check, one can verify that for $2\to 2$ scattering of massless particles, the j basis derived from the two-particle wave function [Eq. (4)] precisely matches with the classical Wigner d matrices $d^j_{\nu,\nu'}$ in the c.m. frame. The derivation is also provided in the Supplemental Material for readers interested [18]. The advantage of our construction is the application of scattering of an arbitrary number of massive or massless particles.

Poincaré algebra in helicity spinor representation.—In quantum mechanics, we use the nonrelativistic J^2 operator to obtain the angular momentum of a wave function. But in relativistic scenario, we need to use the Pauli-Lubanski operator $W_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu M^{\rho\sigma}$, which induces a Casimir invariant W^2 for the Poincaré group with eigenvalue $-P^2 j(j+1)$, where j is the covariant version of the total angular momentum. In this section, we propose to apply this operator to the multiparticle wave functions f and the amplitude basis $\mathcal{B}$ in order to construct the partial wave (amplitude) basis systematically.

In the helicity spinor representation [20], the Lorentz generators are given as $M_{\mu\nu}\sigma^{\mu}_{\alpha\dot{\alpha}}\sigma^{\nu}_{\beta\dot{\beta}} = \epsilon_{\alpha\beta}\tilde{M}_{\dot{\alpha}\dot{\beta}} + M_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}$ and [21]

$$\begin{aligned} M_{\mathcal{I},\alpha\beta} &= i\sum_{i\in\mathcal{I}}\left(\lambda_{i\alpha}\frac{\partial}{\partial\lambda_i^{\beta}} + \lambda_{i\beta}\frac{\partial}{\partial\lambda_i^{\alpha}}\right), \\ \tilde{M}_{\mathcal{I},\dot{\alpha}\dot{\beta}} &= i\sum_{i\in\mathcal{I}}\left(\tilde{\lambda}_{i\dot{\alpha}}\frac{\partial}{\partial\tilde{\lambda}_i^{\dot{\beta}}} + \tilde{\lambda}_{i\dot{\beta}}\frac{\partial}{\partial\tilde{\lambda}_i^{\dot{\alpha}}}\right). \end{aligned} \tag{7}$$

The sum is taken over a group of particles $\mathcal{I}$ for which we want to compute the angular momentum; hence, for an amplitude, we sum over only the initial or only the final state particles. It defines a scattering channel $\mathcal{I}\to\bar{\mathcal{I}}$ that the angular momentum is associated with ($\bar{\mathcal{I}}$ is the complement of $\mathcal{I}$).

Using Eq. (7) and

$$P_{\mathcal{I}} = \sum_{i\in\mathcal{I}}\lambda_i\tilde{\lambda}_i,$$

the Casimir invariant W^2 takes the following form:

$$\begin{aligned} W_{\mathcal{I}}^2\mathcal{B} = {}& \frac{P_{\mathcal{I}}^2}{8}[\mathrm{Tr}(\tilde{M}_{\mathcal{I}}^2\mathcal{B}) + \mathrm{Tr}(M_{\mathcal{I}}^2\mathcal{B})] \\ & -\frac{1}{4}P_{\mathcal{I}}^{\dot{\alpha}\alpha}P_{\mathcal{I}}^{\dot{\beta}\beta}(M_{\mathcal{I}\alpha\beta}\tilde{M}_{\mathcal{I}\dot{\alpha}\dot{\beta}}\mathcal{B}), \end{aligned} \tag{8}$$

where $M_{\mathcal{I},\alpha\beta}^2\mathcal{B} \equiv M_{\mathcal{I},\alpha}^{\gamma}M_{\mathcal{I},\gamma\beta}\mathcal{B}$. It is tempting to show the conservation of angular momentum defined by this operator. We can easily prove that

$$M_{\mathcal{I}}\mathcal{B} = -M_{\bar{\mathcal{I}}}\mathcal{B}, \qquad \mathrm{Tr}M_{\mathcal{I}}^2\mathcal{B} = \mathrm{Tr}M_{\bar{\mathcal{I}}}^2\mathcal{B}. \tag{9}$$

Together with $P_{\mathcal{I}} = -P_{\bar{\mathcal{I}}}$, we find $W_{\mathcal{I}}^2\mathcal{B} = W_{\bar{\mathcal{I}}}^2\mathcal{B}$, which means that for any channel $\mathcal{I}\to\bar{\mathcal{I}}$ of an amplitude $\mathcal{B}$, the angular momentum defined by the operator $W_{\mathcal{I}}^2$ is the same for the initial and the final states.

We also show that the operator $W_{\mathcal{I}}^2$ has the correct eigenvalues. Let us take a simple example of amplitude $\mathcal{B}_{\psi_1\phi_2\to\bar{\psi}_3\phi_4} = \langle 13\rangle$, which in the c.m. frame has the angular distribution $\langle 13\rangle = \cos(\theta/2) \equiv d^{1/2}_{1/2,-1/2}(\theta)$, implying $j = 1/2$. This is reproduced by the Casimir operator as

$$W_{\{1,2\}}^2\langle 13\rangle = -\frac{3}{4}s\langle 13\rangle. \tag{10}$$

The eigenvalue $-P_{\mathcal{I}}^2 j(j+1) = -\frac{3}{4}s$ agrees on the $j = 1/2$ with the Wigner d matrix.

Selection rules.—In [3,4,15], it is proposed that effective operators subject to the equation of motion and integration by part should one-to-one correspond to the unfactorizable amplitudes they generate, dubbed the "operator-amplitude correspondence." It suggests that the multiparticle state generated by an operator must have the quantum number indicated by the corresponding amplitude: especially the angular momentum that we just explored. With this property, we propose the following selection rules in two types of calculations: operator renormalization and one-loop amplitudes.

Operator renormalization: Since the counterterm of an effective operator $\mathcal{O}_m$ should correspond to the same amplitude basis $\mathcal{B}_m$, it is renormalized at one loop when the UV divergence of the one-loop amplitude contains the amplitude basis

$$16\pi^2\mathcal{A}_m^{1\text{-loop}} \supset -\left(\sum_n \gamma_{mn}C_n\right)\mathcal{B}_m\frac{1}{\epsilon},$$

where the sum is taken over contributions from different operators $\mathcal{O}_n$ with Wilson coefficients C_n, and γ_{mn} is called the anomalous dimension matrix. Suppose $\mathcal{O}_n$ connects multiple external legs in the diagram; our previous claim implies that $\mathcal{B}_n$ and $\mathcal{B}_m$ have the same quantum number (especially j) for this multiparticle state. When the two operators do not match, they should not renormalize each other, and $\gamma_{mn} = 0$.

In Table I, we list all the classes of dimension 6 operators [22] (except for F^3) according to their angular momentum at different channels. For any two operators in the same channel, a diagram specified by the shared particles exists for the renormalization, but only those appearing in the same entry could renormalize each other due to the selection rule. There may be operators of the same class that have different angular momenta in a certain channel. Hence, this class may appear in multiple columns in a row,

TABLE I. Dimension 6 operators classified by their angular momentum in the specified channel. Numbers in brackets are (anti)holomorphic weights $(w,\bar{w})$ so that one can further obtain non-renormalization relations for operators in the same entry in [12].

Channels	$j=0$	$j=1/2$	$j=1$
F^+F^+	$F^2\phi^2(2,6)$		
$F^+\psi^+$		$F\psi^2\phi(2,6)$	
$F^+\phi$			$F\psi^2\phi(2,6)$, $F^2\phi^2(2,6)$
$\psi^+\psi^+$	$\psi^4(2,6)$, $\psi^2\bar{\psi}^2(4,4)$, $\psi^2\phi^3(4,6)$		$\psi^4(2,6)$, $F\psi^2\phi(2,6)$
$\psi^+\psi^-$			$\psi\bar{\psi}\phi^2D(4,4)$, $\psi^2\bar{\psi}^2(4,4)$
$\psi^+\phi$		$\psi^2\phi^3(4,6)$, $F\psi^2\phi(2,6)$, $\psi\bar{\psi}\phi^2D(4,4)$	
$\phi\phi$	$\phi^4D^2(4,4)$, $\psi^2\phi^3(4,6)$, $\phi^6(6,6)$		$\psi\bar{\psi}\phi^2D(4,4)$, $\phi^4D^2(4,4)$

like ψ^4 in Table I, whose $j=0$ and $j=1$ bases could renormalize $\psi^2\phi^3$ and $F\psi^2\phi$, respectively, but. In the Warsaw basis [23], it asserts that among $\mathcal{O}^1_{lequ}$ and $\mathcal{O}^3_{lequ}$, $\mathcal{O}_{eH}$ can only be renormalized by the former, whereas $\mathcal{O}_{eW}$ is only by the latter [9,24,25].

The selection rule can also be applied to gauge quantum numbers, or even the mixture of gauge and j. One example in the SMEFT is the H^4D^2 operator, whose corresponding amplitude basis includes both $j=0$, 1 and $I=0$, 1 components in the $\{H^\dagger, H\}$ channel [26]. We denote the basis amplitudes with definite quantum numbers as the j basis of the scattering channel, and we denote the coefficients as $C^{j,I}$. By expanding the amplitudes generated by the Warsaw basis operators $\mathcal{O}_{HD}$ and $\mathcal{O}_{H\Box}$ on the j basis (shown in the Supplemental Material [18]), we derive

$$C^{0,0} = 3C_{H\Box}, \qquad C^{0,1} = C_{HD} - C_{H\Box},$$
$$C^{1,0} = -C_{H\Box} - C_{HD}, \qquad C^{1,1} = -C_{H\Box}, \tag{11}$$

as the combinations of Wilson coefficients that contribute to certain processes of given quantum numbers. From Table I, the class $\psi\bar{\psi}\phi^2D$ is only renormalized at $j=1$ in the $\phi\phi$ channel. In the Warsaw basis, the operators $\mathcal{O}^1_{Hl}$, $\mathcal{O}_{He}$, $\mathcal{O}^1_{Hq}$, $\mathcal{O}_{Hu}$, and $\mathcal{O}_{Hd}$ of this class have $I=0$ in the $\{H^\dagger, H\}$ channel and are renormalized by exactly the combination $C^{1,0}$; whereas $\mathcal{O}^3_{Hl}$ and $\mathcal{O}^3_{Hq}$ of the same class but $I=1$ are renormalized by the other combination $C^{1,1}$. These are verified by the results in [9,24,25].

One should keep in mind that γ_{mn} is basis dependent. Our selection rules are directly applicable for the j basis, and the conclusions need to be translated to other components before cross-check. Table I shows that the operators $\psi\bar{\psi}\phi^2D$ and ϕ^6 could not renormalize each other. However, for the Warsaw basis, as shown in [25], $\dot{C}_H \supset \lambda g_2^2 C^3_{Hl}$ has a nonvanishing coefficient. The fact is that $\mathcal{O}^3_{Hl}$, with ($j=1$, $I=1$) in the $\{H^\dagger, H\}$ channel, only directly renormalizes the j-basis operator $(H^\dagger i\tau^i \overleftrightarrow{D}_\mu H)^2$, which by the EOM is equivalent to the Warsaw basis combination $(H^\dagger i\tau^i \overleftrightarrow{D}_\mu H)^2 \to \mathcal{O}_H + \frac{8}{3}\lambda\mathcal{O}_{H\Box}$. The ratio between the two terms also agrees with [25].

Our selection rule distinguishes operators with different Lorentz structures in the same type, which differs from the non-renormalization relation proposed in [12]. At higher dimensions, our selection rule becomes more important as more Lorentz structures exist within a type of operators. In this case, we need a systematic way to obtain the j basis, which is to diagonalize the Casimir W^2 in the space of the amplitude basis. A nontrivial example illustrates it in the Supplemental Material [18]. We find a selection rule for the anomalous dimension matrix between the operators of classes $\psi_1\phi_2\phi_3\psi_4\phi_5D^2$ and $\psi_1\phi_2\phi_3\bar{\psi}_4F_5D$ operators: both of which have multiple Lorentz structures.

Vanishing loops: In the above case, the "target operator" to be renormalized fixes the angular momentum at a particular channel so that the contributing operator is selected. In computing the full amplitude, however, the angular momentum is usually unconstrained. We find the following two exceptions where we can still apply selection rules based on angular momentum conservation:

Selection rule A: In the c.m. of a two-particle state, as the orbital angular momentum $L = \mathbf{r}\times\mathbf{p}$ has vanishing projection along $\hat{p}$, we deduce $\sigma \equiv J\cdot\hat{p} = S\cdot\hat{p}$. If they are massless, it is further given by $S\cdot\hat{p} = \Delta h$, which is the difference of their helicities; hence, we must have $j \geq |\sigma| = |\Delta h|$ [27]. This constraint holds for any frame as long as we determine j from the eigenvalue of W^2. Consider a generic $2\to N$ scattering that has a contributing diagram as in Fig. 1. The helicity difference on the lhs selects the $j \geq |\Delta h|$ partial waves for the scattering states on both the lhs and rhs due to the conservation law. Therefore, operators corresponding to the amplitude basis with lower j should not contribute.

In Table II, we list all the $2\to 2$ processes for which the contributions from specific operators to the one-loop amplitude vanish. For dimension 6 cases, this table gives the same results as the "absent rational terms" found in [14], where the underlying mechanism—angular momentum conservation—was not pointed out. For dimension 8 cases, which are not studied in [14], we also find two vanishing contributions, namely, the contribution from $F^2\phi^2D^2$ to $\mathcal{A}(\phi\phi F^+F^-)$ and from $F^2\bar{F}^2$ to $\mathcal{A}(\psi^+\psi^-F^\pm F^\pm)$. For the other dimension 8 classes that are partially constrained, the one-loop amplitude has to receive contributions from a subspace of operators spanned by the $j \geq |\Delta h|$ j basis. For example, the amplitude $\mathcal{A}(B^+B^-H_\alpha H^{\dagger\beta})$ has $j\geq 2$ for the $\{H^\dagger, H\}$ channel, which selects the ($j=2$, $I=0$) j basis among the contributing H^4D^4-type operators. Similar to Eq. (11), we find the contributing combination as (the Wilson coefficients are defined in the Supplemental Material [18])

$$C^{2,0} = \frac{1}{6}\left(C_1^{H^4D^4} + \frac{1}{3}C_2^{H^4D^4} + C_3^{H^4D^4}\right). \tag{12}$$

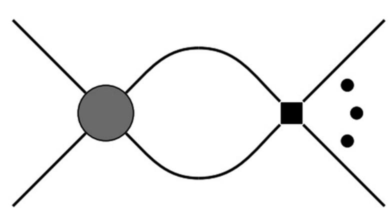

FIG. 1. One-loop diagram for $2\to N$ scattering in EFT. The square on the right-hand side represents a contact interaction; whereas on the left-hand side, the interactions in the shaded region can be arbitrary.

TABLE II. Vanishing one-loop amplitudes from contribution of specific operators. In the first column, we list the two particle states with a minimum angular momentum $j \geq \Delta h$ at the lhs of Fig. 1. In the second column, we list the two-particle external state excited by the operators on the rhs in Fig. 1. The third column contains the operators up to dimension 8 that have vanishing contributions to the one-loop amplitudes. The numbers in parentheses are the possible angular momenta in the given channel; among which, the bold ones have vanishing contributions due to the selection rule.

LHS (Δh)	RHS	Operators at RHS
F^+F^- (2)	$\phi\phi$	$\phi^4(\mathbf{0})$, $\phi^4D^2(\mathbf{0},\mathbf{1})$, $\phi^4D^4(\mathbf{0},\mathbf{1},2)$, $\psi\bar{\psi}\phi^2D(\mathbf{1})$, $\psi\bar{\psi}\phi^2D^3(\mathbf{1},2)$
	$\psi^+\psi^-$	$\psi\bar{\psi}\phi^2D^3(\mathbf{1})$, $\psi\bar{\psi}\phi^2D^3(\mathbf{1},2)$, $\bar{\psi}^2\psi^2(\mathbf{1})$, $\bar{\psi}^2\psi^2D^2(\mathbf{1},2)$
	F^+F^+	$F^2\phi^2(\mathbf{0})$, $F^2\phi^2D^2(\mathbf{0},\mathbf{1})$, $F^2\psi\bar{\psi}D(\mathbf{1})$
$F^+\psi^-$ (3/2)	$\psi^-\phi$	$\psi\bar{\psi}\phi^2D(\mathbf{1/2})$, $\psi\bar{\psi}\phi^2D^3(\mathbf{1/2},3/2)$
	$F^+\psi^+$	$F\psi^2\phi(\mathbf{1/2})$, $F\psi^2\phi D^2(\mathbf{1/2},3/2)$
$F^+\phi$ (1)	$\psi^\pm\psi^\pm$	$\bar{\psi}^2\psi^2(\mathbf{0})$, $\psi^4(\mathbf{0},1)$, $\bar{\psi}^2\psi^2D^2(\mathbf{0},1)$, $\psi^4D^2(\mathbf{0},1,2)$
$\psi^+\psi^-$ (1)	$F^\pm F^\pm$	$F^2\phi^2(\mathbf{0})$, $F^2\phi^2D^2(\mathbf{0},1)$, $F^2\bar{F}^2(\mathbf{0})$, $F^4(\mathbf{0},1,2)$
	$\phi\phi$	$F^2\phi^2(\mathbf{0})$, $F^2\phi^2D^2(\mathbf{0},1)$

Selection rule B: From Eq. (4), we find that if $h_1 = h_2 = h$, the permutation symmetry of the two particles in the CG coefficient is determined by the exponent in $[12]^{j+2h}$. Thus, by spin statistics, once the two particles are identical, the permutation symmetry is allowed only if j is even. If the lhs state in Fig. 1 consists of two identical particles, the $j = 1$ partial wave is forbidden also on the rhs, which selects the operator inserted in this diagram. As an example, consider the one-loop amplitude $\mathcal{A}(\psi^+\psi^-F^\pm F^\pm)$ with operator $\psi\bar{\psi}\phi^2D$ or $\psi^2\bar{\psi}^2$ inserted. Because the two-fermion state created by these effective operators have $j = 1$, their contributions must vanish.

The above discussion can be extended by considering two particles that are not necessarily identical but in the same gauge multiplet so that the spin statistics constrains the combination of permutation symmetries for both the Lorentz structure and the gauge structure [15,28]. For example, the two-W-boson state (in the unbroken phase) can have $j = 1$ as long as the SU(2) indices are taken to be antisymmetric $\epsilon^{ijk}W^jW^k$, namely, the $j = 1$, $I = 1$ state is allowed but the $j = 1$, $I = 0$ state is forbidden. Therefore, we can assert, for instance, that the operator $\mathcal{O}^3_{Hl}$ contributes to the amplitude $\mathcal{A}(l^+l^-W^\pm W^\pm)$ at one loop, whereas $\mathcal{O}^1_{Hl}$ does not.

In this Letter, we construct the partial wave amplitude basis $\mathcal{B}^j$ in terms of spinor helicity variables. The definite angular momentum can be understood via the Poincaré CG coefficients. We further develop the technique to get the complete eigenbasis of angular momentum using the Casimir invariant W^2. Although we mainly present the massless amplitude basis in SMEFT, the technique works for massive ones and other EFTs as well, as shown in the recent paper on monopoles [29], as well as the papers on the QCD chiral Lagrangian [30,31]. In all these scenarios, the partial wave basis allows us to find new selection rules for operator renormalization and loop amplitudes based on angular momentum conservation. When identical particles are present, the additional gauge quantum number and spin statistics have to be taken into account, which we will elaborate on in an upcoming paper [32]. While we are mainly focused on the one-loop cases, the selection rules can be naturally extended to higher loops. Our method can also be applied to calculate the nonvanishing elements of the anomalous dimension matrix with the unitarity method, generalizing the result in [33] to a broader range of loop diagram topologies.

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