Possible partner state of the $Y(2175)$

Hua-Xing Chen,[1,*] Cheng-Ping Shen,[1,†] and Shi-Lin Zhu[2,3,4,‡]

[1] *School of Physics and Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 100191, China*

[2] *School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*

[3] *Collaborative Innovation Center of Quantum Matter, Beijing 100871, China*

[4] *Center of High Energy Physics, Peking University, Beijing 100871, China*

(Received 5 June 2018; published 9 July 2018)

We study the $Y(2175)$ using the method of QCD sum rules. There are two independent $ss\bar{s}\bar{s}$ interpolating currents with $J^{PC} = 1^{--}$, and we calculate both their diagonal and their off-diagonal correlation functions. We obtain two new currents that do not strongly correlate to each other, so they may couple to two different physical states: one of them couples to the $Y(2175)$, while the other may couple to another state whose mass is evaluated to be 2.41 ± 0.25 GeV. Evidence of the latter state can be found in the *BABAR* [B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **76**, 012008 (2007)], BESII [M. Ablikim *et al.* (BES Collaboration), Phys. Rev. Lett. **100**, 102003 (2008)], Belle [C. P. Shen *et al.* (Belle Collaboration), Phys. Rev. D **80**, 031101 (2009)], and BESIII [M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **91**, 052017 (2015)] experiments.

DOI: 10.1103/PhysRevD.98.014011

I. INTRODUCTION

In recent years there have been lots of exotic hadrons observed in hadron experiments [1], which cannot be explained in the traditional quark model and are of particular importance to understand the low energy behaviors of quantum chromodynamics (QCD) [2–8]. Most of them contain heavy quarks, such as the charmoniumlike states $X(3872)$ [9], $Y(4220)$ [10,11], and $Z_c(3900)$ [12,13]. However, there are not so many exotic hadrons in the light sector containing only light $u/d/s$ quarks. The $Y(2175)$ is one of them, which is often taken as the strange analogue of the $Y(4220)$ [10,11].

The $Y(2175)$ was first observed in 2006 by the *BABAR* Collaboration in the $\phi f_0(980)$ invariant mass spectrum [14–17], and later confirmed in the BESII [18], Belle [19], and BESIII [20,21] experiments. Its mass and width were measured to be $M = 2188 \pm 10$ MeV and $\Gamma = 83 \pm 12$ MeV, respectively, and its spin-parity quantum number is $J^{PC} = 1^{--}$ [1]. We list some of these experiments in Fig. 1, including the following:

Figure 1(a): the *BABAR* experiment [14] discovering the $Y(2175)$ in the $e^+e^- \to \phi f_0(980)$ cross section in 2006.
Figure 1(b): the *BABAR* experiment [15] in 2007.
Figure 1(c): the Belle experiment [19] in 2009.
Figure 1(d): a combined fit to the BaBar [14, 15] and Belle [19] measurements of the $e^+e^- \to \phi f_0(980)$ cross sections, performed by Shen and Yuan in Ref. [22].
Figure 1(e): the BESII experiment [18] in 2007.
Figure 1(f): the BESIII experiment [20] in 2014.

Besides the $Y(2175)$, there might be another structure in the $\phi f_0(980)$ invariant mass spectrum at around 2.4 GeV, whose evidence can be found in the *BABAR* [15] [Fig. 1(b) around 2.4 GeV], Belle [19] [Fig. 1(c) around 2.40 GeV], BESII [18] [Fig. 1(e) around 2.46 GeV], and BESIII [20] [Fig. 1(f) around 2.35 GeV] experiments. The *BABAR* experiment [15] determined its mass and width to be 2.47 ± 0.07 GeV and 77 ± 65 MeV, respectively. Shen and Yuan [22] also used the *BABAR* [14,15] and Belle [19] data to fit its mass and width to be 2436 ± 34 MeV and 99 ± 105 MeV, respectively. However, its statistical significance is smaller than 3.0σ. In this paper we shall study this structure as well as the $Y(2175)$ simultaneously using the method of QCD sum rules.

Since its discovery, the $Y(2175)$ has attracted much attention from the hadron physics community, and many theoretical methods and models were applied to study it. By using both the chiral unitary model [23,24] and the Faddeev

*hxchen@buaa.edu.cn
†shencp@ihep.ac.cn
‡zhusl@pku.edu.cn

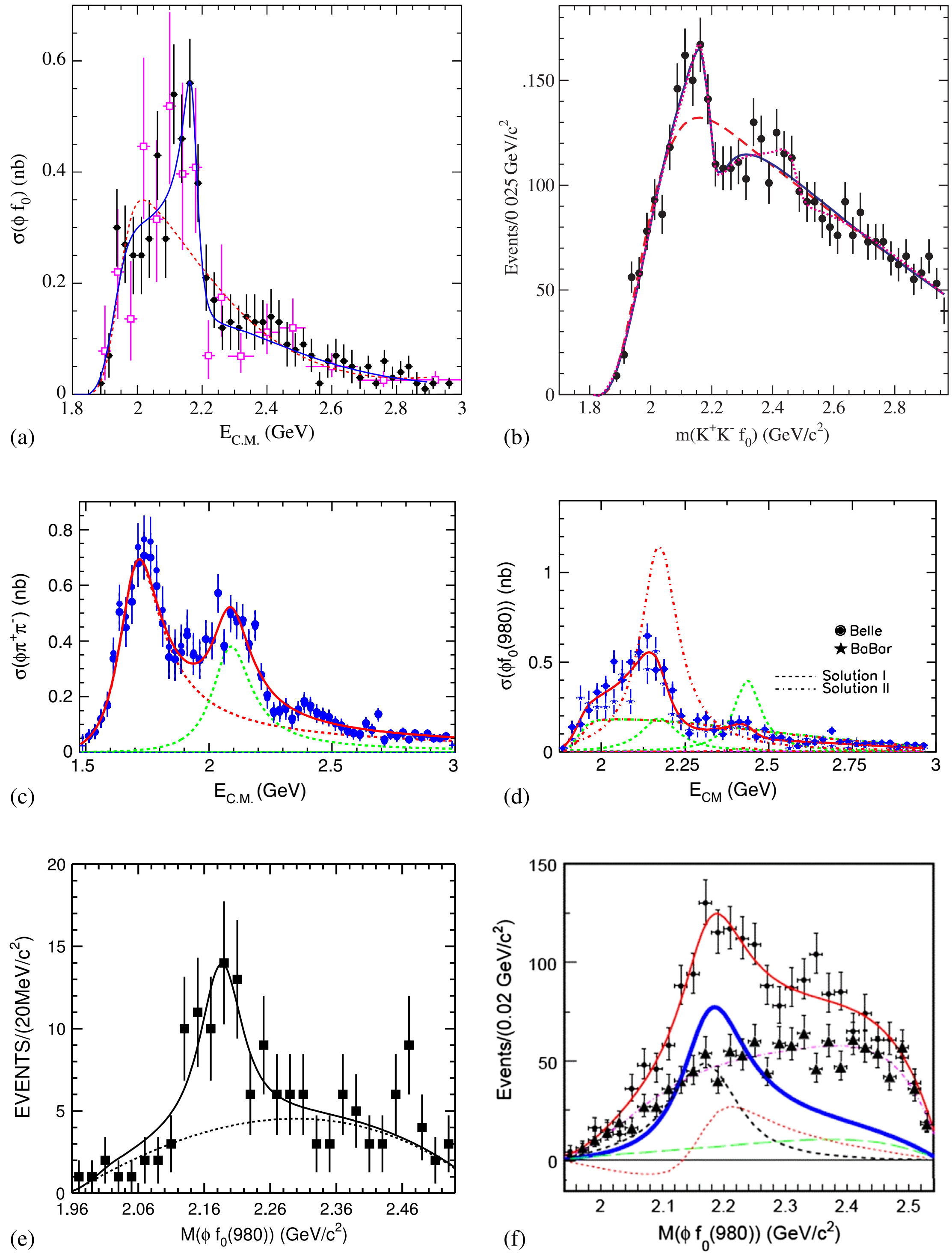

FIG. 1. The *BABAR* [14,15], Belle [19], BESII [18], and BESIII [20] experiments observing the $Y(2175)$ as well as the fit performed in Ref. [22]. (a) The $e^+e^- \to \phi f_0(980)$ cross section. Taken from *BABAR* [14]. (b) The $K^+K^-\pi^+\pi^-$ invariant mass distribution in the $K^+K^-f_0(980)$ threshold region. The fits are done by including no (dashed line), one (solid line) and two (dotted line) resonances. Taken from *BABAR* [15]. (c) The $e^+e^- \to \phi\pi^+\pi^-$ cross section with two incoherent Breit-Wigner functions, the $\phi(1680)$ and the $Y(2175)$. Taken from Belle [19]. (d) Fits to the *BABAR* [14, 15] and Belle [19] measurements of the $e^+e^- \to \phi f_0(980)$ cross sections with two coherent Breit-Wigner functions, performed by Shen and Yuan and taken from Ref. [22]. (e) The $\phi f_0(980)$ invariant mass spectrum. Taken from BESII [18]. (f) The $\phi f_0(980)$ invariant mass spectrum. Taken from BESIII [20].

equations [25], the authors interpreted the $Y(2175)$ as a dynamically generated state in the $\phi K\bar{K}$ and $\phi\pi\pi$ systems, and more states were predicted in the $\phi\pi^0\eta$ [26] and $NK\bar{K}$ [27,28] systems. By using similar approaches, the $Y(2175)$ was interpreted as a dynamically generated resonance by the self-interactions between the ϕ and $f_0(980)$ resonances [29], while the resonance spectrum expansion formalism by including the $f_0(980)$ as a resonance in the coupled $\pi\pi$-KK system is also able to generate the $Y(2175)$ in the $\phi f_0(980)$ channel [30].

Besides the dynamically generated resonance, there are many other interpretations to explain this structure. In Ref. [31] the authors interpreted the $Y(2175)$ as a 2^3D_1 $s\bar{s}$ meson, and calculated its decay modes using both the $^3P_0$ model and the flux-tube model. In Ref. [32] the authors used a constituent quark model to interpret the $Y(2175)$ as a hidden-strangeness baryon-antibaryon state ($qqs\bar{q}\,\bar{q}\,\bar{s}$) strongly coupling to the $\Lambda\bar{\Lambda}$ channel. Later in Ref. [33] the authors applied the one-boson-exchange model to interpret the $Y(2175)$ and $\eta(2225)$ as the bound states of $\Lambda\bar{\Lambda}(^3S_1)$ and $\Lambda\bar{\Lambda}(^1S_0)$, respectively. In Ref. [34] the authors interpreted the $Y(2175)$ as a strangeonium hybrid state and used the flux-tube model to study its decay properties. However, this interpretation is not supported by the nonperturbative lattice QCD calculations [35]. Productions of the $Y(2175)$ were studied in Refs. [36,37] by using the Nambu–Jona-Lasinio model and the Drell-Yan mechanism, while its decay properties were studied in Refs. [38,39] via the initial single pion emission mechanism.

The method of QCD sum rules was also applied to study the $Y(2175)$ [40,41]. Especially, in Ref. [41] we have systematically constructed the $ss\bar{s}\,\bar{s}$ interpolating currents and found that there are only two independent ones. We have separately used them to perform QCD sum rule analyses, both of which can be used to explain the $Y(2175)$. However, in Ref. [41] we only calculated the diagonal terms of these two currents, and in this work we shall further calculate their off-diagonal term to study their correlation. This can significantly improve our understanding on the relations between interpolating currents and physical states (see Refs. [42–45] for more relevant discussions).

This paper is organized as follows. In Sec. II, we list the two independent $ss\bar{s}\,\bar{s}$ interpolating currents with $J^{PC}=1^{--}$ and discuss how to diagonalize them. In Sec. III, we use two diquark-antidiquark $(ss)(\bar{s}\,\bar{s})$ interpolating currents to perform QCD sum rule analyses, and we obtain two new currents that do not strongly correlate to each other. In Sec. IV, we use these two new currents to calculate mass spectra, and Sec. V is a summary.

II. INTERPOLATING CURRENTS AND THEIR RELATIONS TO POSSIBLE PHYSICAL STATES

The interpolating currents having the quark content $ss\bar{s}\,\bar{s}$ and with the quantum number $J^{PC}=1^{--}$ have been systematically constructed in Ref. [41], where we found that there are two nonvanishing diquark-antidiquark $(ss)(\bar{s}\,\bar{s})$ interpolating currents with $J^{PC}=1^{--}$:

$$\eta_{1\mu}=(s_a^TC\gamma_5 s_b)(\bar{s}_a\gamma_\mu\gamma_5 C\bar{s}_b^T)-(s_a^TC\gamma_\mu\gamma_5 s_b)(\bar{s}_a\gamma_5 C\bar{s}_b^T), \tag{1}$$

$$\eta_{2\mu}=(s_a^TC\gamma^\nu s_b)(\bar{s}_a\sigma_{\mu\nu}C\bar{s}_b^T)-(s_a^TC\sigma_{\mu\nu}s_b)(\bar{s}_a\gamma^\nu C\bar{s}_b^T). \tag{2}$$

Here a and b are color indices, $C=i\gamma_2\gamma_0$ is the charge conjugation operator, and the superscript T represents the transpose of Dirac indices. These two currents are independent of each other.

In Ref. [41] we have separately used $\eta_{1\mu}$ and $\eta_{2\mu}$ to perform QCD sum rule analyses; i.e., we have calculated the diagonal terms

$$\langle 0|T\eta_{1\mu}(x)\eta_{1\nu}^\dagger(0)|0\rangle \quad \text{and} \quad \langle 0|T\eta_{2\mu}(x)\eta_{2\nu}^\dagger(0)|0\rangle. \tag{3}$$

However, although $\eta_{1\mu}$ and $\eta_{2\mu}$ are independent of each other, they can be correlated to each other; i.e., the off-diagonal term can be nonzero,

$$\langle 0|T\eta_{1\mu}(x)\eta_{2\nu}^\dagger(0)|0\rangle \neq 0, \tag{4}$$

suggesting that $\eta_{1\mu}$ and $\eta_{2\mu}$ may couple to the same physical state. In this paper we shall evaluate this off-diagonal term in order to find two noncorrelated currents,

$$\begin{aligned} J_{1\mu}&=\cos\theta\eta_{1\mu}+\sin\theta i\eta_{2\mu},\\ J_{2\mu}&=\sin\theta\eta_{1\mu}+\cos\theta i\eta_{2\mu}, \end{aligned} \tag{5}$$

satisfying

$$\langle 0|TJ_{1\mu}(x)J_{2\nu}^\dagger(0)|0\rangle=0, \\ \text{or}\begin{cases} \ll \langle 0|TJ_{1\mu}(x)J_{1\nu}^\dagger(0)|0\rangle \\ \ll \langle 0|TJ_{2\mu}(x)J_{2\nu}^\dagger(0)|0\rangle \end{cases}. \tag{6}$$

Then we shall use $J_{1\mu}$ and $J_{2\mu}$ to perform QCD sum rule analyses. Because of the above Eq. (6), $J_{1\mu}$ and $J_{2\mu}$ should not strongly couple to the same physical state, so we assume

$$\langle 0|J_{1\mu}|Y_1\rangle=f_1\epsilon_\mu, \tag{7}$$

$$\langle 0|J_{2\mu}|Y_2\rangle=f_2\epsilon_\mu, \tag{8}$$

where Y_1 and Y_2 are two different states with $J^{PC}=1^{--}$, f_1 and f_2 are decay constants, and ϵ_μ is the polarization vector. Especially, we shall evaluate the mass splitting between these two states/currents.

III. QCD SUM RULE ANALYSIS

The method of QCD sum rules is a powerful and successful nonperturbative method [46,47]. In this method, we calculate the two-point correlation function

$$\Pi_{\mu\nu}(q^2) \equiv i \int d^4x e^{iqx} \langle 0|T\eta_\mu(x)\eta_\nu^\dagger(0)|0\rangle \tag{9}$$

at both the hadron and the quark-gluon levels.

At the hadron level we simplify its Lorentz structure to be

$$\Pi_{\mu\nu}(q^2) = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\Pi(q^2) + \frac{q_\mu q_\nu}{q^2}\Pi^{(0)}(q^2) \tag{10}$$

and express $\Pi(q^2)$ in the form of the dispersion relation

$$\Pi(q^2) = \int_{16m_s^2}^\infty \frac{\rho(s)}{s - q^2 - i\varepsilon} ds. \tag{11}$$

Here $\rho(s)$ is the spectral density, for which we adopt a parametrization of one pole dominance for the ground state Y and a continuum contribution:

$$\begin{aligned}\rho(s) &\equiv \sum_n \delta(s - M_n^2)\langle 0|\eta|n\rangle\langle n|\eta^\dagger|0\rangle \\ &= f_Y^2\delta(s - M_Y^2) + \text{continuum}.\end{aligned} \tag{12}$$

At the quark-gluon level, we insert $J_{1\mu}$ and $J_{2\mu}$ into Eq. (9) and calculate the correlation function using the method of operator product expansion (OPE). After performing the Borel transformation at both the hadron and the quark-gluon levels, we obtain

$$\Pi^{(all)}(M_B^2) \equiv \mathcal{B}_{M_B^2}\Pi(p^2) = \int_{16m_s^2}^\infty e^{-s/M_B^2}\rho(s)ds. \tag{13}$$

After approximating the continuum using the spectral density of OPE above a threshold value s_0, we obtain the sum rule equation

$$\Pi(M_B^2) \equiv f_Y^2 e^{-M_Y^2/M_B^2} = \int_{16m_s^2}^{s_0} e^{-s/M_B^2}\rho(s)ds. \tag{14}$$

We can use this equation to calculate M_Y through

$$M_Y^2 = \frac{\frac{\partial}{\partial(-1/M_B^2)}\Pi(M_B^2)}{\Pi(M_B^2)} = \frac{\int_{16m_s^2}^{s_0} e^{-s/M_B^2} s\rho(s)ds}{\int_{16m_s^2}^{s_0} e^{-s/M_B^2}\rho(s)ds}. \tag{15}$$

The sum rules for the currents $\eta_{1\mu}$ and $\eta_{2\mu}$ have been separately calculated and given in Eqs. (13) and (14) of Ref. [41]. In this paper we revise these calculations by adding the diagram shown in Fig. 2. We write them as $\Pi_{\eta_1\eta_1}(q^2)$ and $\Pi_{\eta_2\eta_2}(q^2)$ in the present study, which are transformed to be $\Pi_{\eta_1\eta_1}(M_B^2)$ and $\Pi_{\eta_2\eta_2}(M_B^2)$ after the Borel transformation. The results are shown in Eqs. (21) and (22), which do not change significantly compared to Ref. [41].

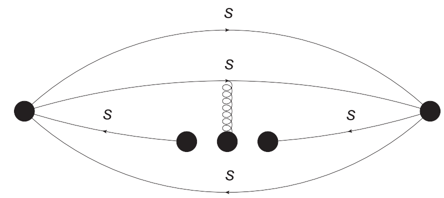

FIG. 2. Feynman diagram related to the quark-gluon mixed condensate $\langle g_s\bar{s}\sigma Gs\rangle$.

In Eqs. (21) and (22) we have calculated the OPE up to 12 dimensions, including the strange quark mass, the perturbative term, the quark condensate $\langle\bar{s}s\rangle$, the gluon condensate $\langle g_s^2GG\rangle$, the quark-gluon mixed condensate $\langle g_s\bar{s}\sigma Gs\rangle$, and their combinations $\langle\bar{s}s\rangle^2$, $\langle\bar{s}s\rangle^3$, $\langle\bar{s}s\rangle^4$, $\langle g_s\bar{s}\sigma Gs\rangle^2$, $\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle$, $\langle\bar{s}s\rangle^2\langle g_s\bar{s}\sigma Gs\rangle$, $\langle g_s^2GG\rangle\langle\bar{s}s\rangle$, $\langle g_s^2GG\rangle\langle\bar{s}s\rangle^2$, $\langle g_s^2GG\rangle\langle g_s\bar{s}\sigma Gs\rangle$, and $\langle g_s^2GG\rangle\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle$. These parameters take the following values [1,48–54]:

$$\begin{aligned}\langle\bar{q}q\rangle &= -(0.24 \pm 0.01\ \text{GeV})^3, \\ \langle\bar{s}s\rangle &= -(0.8 \pm 0.1) \times (0.240\ \text{GeV})^3, \\ \langle g_s^2GG\rangle &= (0.48 \pm 0.14)\ \text{GeV}^4, \\ \langle g_s\bar{q}\sigma Gq\rangle &= -M_0^2 \times \langle\bar{q}q\rangle, \\ M_0^2 &= (0.8 \pm 0.2)\ \text{GeV}^2, \\ m_s(2\ \text{GeV}) &= 96^{+8}_{-4}\ \text{MeV}, \\ \alpha_s(1.7\ \text{GeV}) &= 0.328 \pm 0.03 \pm 0.025.\end{aligned} \tag{16}$$

Beside the diagonal terms $\Pi_{\eta_1\eta_1}(q^2)$ and $\Pi_{\eta_2\eta_2}(q^2)$, in the present study we also calculate the sum rules for the off-diagonal term:

$$\begin{aligned}\Pi_{\mu\nu}^{\eta_1\eta_2}(q^2) &= i\int d^4x e^{iqx}\langle 0|T\eta_{1\mu}(x)\eta_{2\nu}^\dagger(0)|0\rangle \\ &= \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\Pi_{\eta_1\eta_2}(q^2) + \frac{q_\mu q_\nu}{q^2}\Pi_{\eta_1\eta_2}^{(0)}(q^2).\end{aligned} \tag{17}$$

After performing the Borel transformation to $\Pi_{\eta_1\eta_2}(q^2)$, we obtain $\Pi_{\eta_1\eta_2}(M_B^2)$ as shown in Eq. (23).

After fixing $s_0 = 6.0\ \text{GeV}^2$, we show $\Pi_{\eta_1\eta_2}(M_B^2)$ as a function of the Borel mass M_B in the left panel of Fig. 3, compared with $\Pi_{\eta_1\eta_1}(M_B^2)$ and $\Pi_{\eta_2\eta_2}(M_B^2)$. Especially, we have

$$\begin{aligned}\left|\frac{\Pi_{\eta_1\eta_2}(3.0\ \text{GeV}^2)}{\Pi_{\eta_1\eta_1}(3.0\ \text{GeV}^2)}\right| &= 0.20, \\ \left|\frac{\Pi_{\eta_1\eta_2}(3.0\ \text{GeV}^2)}{\Pi_{\eta_2\eta_2}(3.0\ \text{GeV}^2)}\right| &= 0.12.\end{aligned} \tag{18}$$

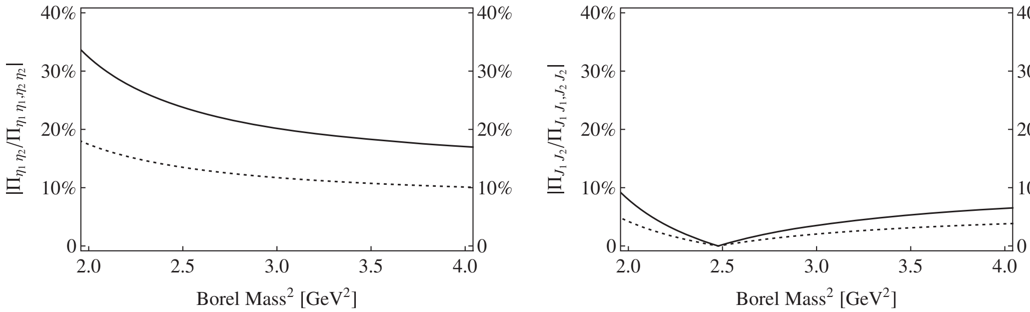

FIG. 3. Left: $\left|\frac{\Pi_{\eta_1\eta_2}(M_B^2)}{\Pi_{\eta_1\eta_1}(M_B^2)}\right|$ (solid line) and $\left|\frac{\Pi_{\eta_1\eta_2}(M_B^2)}{\Pi_{\eta_2\eta_2}(M_B^2)}\right|$ (dotted line), as functions of the Borel mass M_B, when taking $s_0 = 6.0$ GeV^2. Right: $\left|\frac{\Pi_{J_1J_2}(M_B^2)}{\Pi_{J_1J_1}(M_B^2)}\right|$ (solid line) and $\left|\frac{\Pi_{J_1J_2}(M_B^2)}{\Pi_{J_2J_2}(M_B^2)}\right|$ (dotted line), as functions of the Borel mass M_B, when taking $s_0 = 6.0$ GeV^2.

These values suggest that the off-diagonal term is non-ignorable. By diagonalizing the following matrix at around $s_0 = 6.0$ GeV^2 and $M_B^2 = 2.5$ GeV^2:

$$\begin{pmatrix} \Pi_{\eta_1\eta_1} & \Pi_{\eta_1\eta_2} \\ \Pi^\dagger_{\eta_1\eta_2} & \Pi_{\eta_2\eta_2} \end{pmatrix}, \tag{19}$$

we obtain two new currents $J_{1\mu}$ and $J_{2\mu}$ with the mixing angle $\theta = -5.0°$, which do not strongly correlate to each other. Again we fix $s_0 = 6.0$ GeV^2, and show $\Pi_{J_1J_2}(M_B^2)$ as a function of the Borel mass M_B in the right panel of Fig. 3, compared with $\Pi_{J_1J_1}(M_B^2)$ and $\Pi_{J_2J_2}(M_B^2)$. Especially, we have

$$\left|\frac{\Pi_{J_1J_2}(3.0\text{ GeV}^2)}{\Pi_{J_1J_1}(3.0\text{ GeV}^2)}\right| = 0.04,$$
$$\left|\frac{\Pi_{J_1J_2}(3.0\text{ GeV}^2)}{\Pi_{J_2J_2}(3.0\text{ GeV}^2)}\right| = 0.02. \tag{20}$$

$$\begin{aligned}
\Pi_{\eta_1\eta_1} = {} & \int_{16m_s^2}^{s_0}\Bigg[\frac{s^4}{18432\pi^6} - \frac{m_s^2 s^3}{256\pi^6} + \left(-\frac{\langle g^2GG\rangle}{18432\pi^6} + \frac{m_s\langle\bar{s}s\rangle}{48\pi^4}\right)s^2 \\
& + \left(\frac{\langle\bar{s}s\rangle^2}{18\pi^2} - \frac{m_s\langle g\bar{s}\sigma Gs\rangle}{32\pi^4} + \frac{17m_s^2\langle g^2GG\rangle}{9216\pi^6}\right)s + \left(\frac{\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{8\pi^2} - \frac{m_s\langle g^2GG\rangle\langle\bar{s}s\rangle}{128\pi^4} - \frac{29m_s^2\langle\bar{s}s\rangle^2}{12\pi^2}\right)\Bigg]e^{-s/M_B^2}ds \\
& + \left(\frac{5\langle g^2GG\rangle\langle\bar{s}s\rangle^2}{864\pi^2} + \frac{\langle g\bar{s}\sigma Gs\rangle^2}{24\pi^2} + \frac{20m_s\langle\bar{s}s\rangle^3}{9} - \frac{5m_s\langle g^2GG\rangle\langle g\bar{s}\sigma Gs\rangle}{2304\pi^4} - \frac{13m_s^2\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{8\pi^2}\right) \\
& + \frac{1}{M_B^2}\left(-\frac{32g^2\langle\bar{s}s\rangle^4}{81} - \frac{\langle g^2GG\rangle\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{576\pi^2} - \frac{19m_s\langle\bar{s}s\rangle^2\langle g\bar{s}\sigma Gs\rangle}{18} + \frac{m_s^2\langle g^2GG\rangle\langle\bar{s}s\rangle^2}{576\pi^2} + \frac{m_s^2\langle g\bar{s}\sigma Gs\rangle^2}{16\pi^2}\right),
\end{aligned} \tag{21}$$

$$\begin{aligned}
\Pi_{\eta_2\eta_2} = {} & \int_{16m_s^2}^{s_0}\Bigg[\frac{s^4}{12288\pi^6} - \frac{3m_s^2 s^3}{512\pi^6} + \left(\frac{\langle g^2GG\rangle}{18432\pi^6} + \frac{m_s\langle\bar{s}s\rangle}{32\pi^4}\right)s^2 \\
& + \left(\frac{\langle\bar{s}s\rangle^2}{12\pi^2} - \frac{m_s\langle g\bar{s}\sigma Gs\rangle}{24\pi^4} + \frac{35m_s^2\langle g^2GG\rangle}{9216\pi^6}\right)s + \left(\frac{\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{6\pi^2} - \frac{3m_s\langle g^2GG\rangle\langle\bar{s}s\rangle}{128\pi^4} - \frac{29m_s^2\langle\bar{s}s\rangle^2}{8\pi^2}\right)\Bigg]e^{-s/M_B^2}ds \\
& + \left(\frac{5\langle g^2GG\rangle\langle\bar{s}s\rangle^2}{288\pi^2} + \frac{5\langle g\bar{s}\sigma Gs\rangle^2}{96\pi^2} + \frac{10m_s\langle\bar{s}s\rangle^3}{3} - \frac{5m_s\langle g^2GG\rangle\langle g\bar{s}\sigma Gs\rangle}{768\pi^4} - \frac{19m_s^2\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{8\pi^2}\right) \\
& + \frac{1}{M_B^2}\left(-\frac{16g^2\langle\bar{s}s\rangle^4}{27} - \frac{\langle g^2GG\rangle\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{192\pi^2} - \frac{29m_s\langle\bar{s}s\rangle^2\langle g\bar{s}\sigma Gs\rangle}{18} - \frac{m_s^2\langle g^2GG\rangle\langle\bar{s}s\rangle^2}{576\pi^2} + \frac{5m_s^2\langle g\bar{s}\sigma Gs\rangle^2}{48\pi^2}\right),
\end{aligned} \tag{22}$$

$$\begin{aligned}
\Pi_{\eta_1\eta_2} = {} & i\int_{16m_s^2}^{s_0}\left[\frac{\langle g^2GG\rangle}{6144\pi^6}s^2 + \frac{3m_s^2\langle g^2GG\rangle}{1024\pi^6}s - \frac{3m_s\langle g^2GG\rangle\langle\bar{s}s\rangle}{128\pi^4}\right]e^{-s/M_B^2}ds \\
& + i\left(\frac{5\langle g^2GG\rangle\langle\bar{s}s\rangle^2}{288\pi^2} - \frac{5m_s\langle g^2GG\rangle\langle g\bar{s}\sigma Gs\rangle}{768\pi^4}\right) + \frac{i}{M_B^2}\left(-\frac{\langle g^2GG\rangle\langle\bar{s}s\rangle\langle g\bar{s}\sigma Gs\rangle}{192\pi^2} - \frac{m_s^2\langle g^2GG\rangle\langle\bar{s}s\rangle^2}{192\pi^2}\right).
\end{aligned} \tag{23}$$

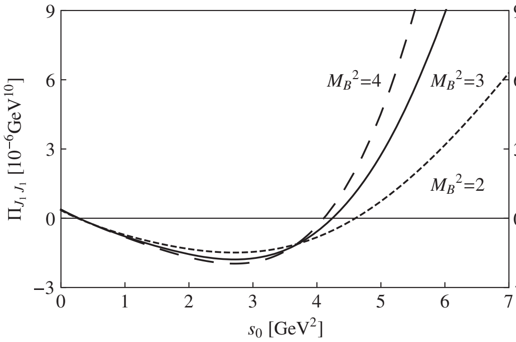

FIG. 4. The correlation function $\Pi_{J_1J_1}(M_B^2)$ as a function of s_0 in units of GeV^{10}. The curves are obtained by taking $M_B^2 = 2.0$ GeV^2 (short-dashed line), 3.0 GeV^2 (solid line), and 4.0 GeV^2 (long-dashed line).

IV. NUMERICAL ANALYSIS

In this section we use the currents $J_{1\mu}$ and $J_{2\mu}$ to perform QCD sum rule analyses. Take $J_{1\mu}$ as an example. First we study the convergence of the operator product expansion, which is the cornerstone of the reliable QCD sum rule analysis. To do this we require that the $D = 10$ and $D = 12$ terms be less than 5%:

$$\mathrm{CVG} \equiv \left|\frac{\Pi^{D=8+10}(M_B^2)}{\Pi(M_B^2)}\right| \le 5\%. \tag{24}$$

After fixing $s_0 = 6.0$ GeV^2, we find that this condition is satisfied when M_B^2 is larger than 2.0 GeV^2.

A common problem, when studying multiquark states using QCD sum rules, is how to differentiate the multiquark state and the relevant threshold, because the interpolating current can couple to both of them. For the case of the $Y(2175)$, its relevant threshold is the $\phi f_0(980)$ around 2.0 GeV, to which $J_{1\mu}$ and $J_{2\mu}$ can both couple. Moreover, the $Y(2175)$ is not the lowest state in the 1^{--} channel containing $s\bar{s}$, and $J_{1\mu}$ and $J_{2\mu}$ may also couple to the $\phi(1680)$ [for example, see the Belle experiment [19] observing the $\phi(1680)$ and $Y(2175)$ at the same time].

If this happens, the resulting correlation function should be positive. Fortunately, we find that the correlation functions $\Pi_{J_1J_1}(M_B^2)$ and $\Pi_{J_2J_2}(M_B^2)$ are negative, and so nonphysical, in the region $s_0 < 4.0$ GeV^2 when taking 2.0 GeV$^2 < M_B^2 < 4.0$ GeV^2. As an illustration, we show the correlation function $\Pi_{J_1J_1}(M_B^2)$ as a function of s_0 in Fig. 4 for $M_B^2 = 2.0/3.0/4.0$ GeV^2. This fact indicates that $J_{1\mu}$ and $J_{2\mu}$ both couple weakly to the lower state $\phi(1680)$ as well as the $\phi f_0(980)$ threshold, so the states they couple to, as if they can couple to some states, should be new and possibly exotic states. However, because of the above negative contributions to the correlation functions, the pole contribution is not large enough. This small pole contribution also suggests that the continuum contribution is important, which demands a careful choice of the parameters of QCD sum rules. Accordingly, in the present study we require that the extracted mass have a dual minimum dependence on both the threshold value s_0 and the Borel mass M_B.

Still using $J_{1\mu}$ as an example, we show the mass obtained using Eq. (15) as a function of the threshold value s_0 and the Borel mass M_B in Fig. 5. We find that there is a mass minimum at around 2.4 GeV when taking s_0 to be around 6.0 GeV^2, and at the same time the Borel mass dependence is weak at around 3.0 GeV^2. Accordingly, we fix s_0 to be around 6.0 GeV^2 and M_B^2 to be around 3.0 GeV^2, and we choose our working regions to be 5.0 GeV$^2 < s_0 <$ 7.0 GeV^2 and 2.0 GeV$^2 < M_B^2 < 4.0$ GeV^2. These regions are moderately large enough for the mass prediction, where the mass is extracted to be

$$M_{Y_1} = 2.41 \pm 0.25 \text{ GeV}. \tag{25}$$

Here the uncertainty is due to the Borel mass M_B, the threshold value s_0, and various condensates [1,48–54].

Similarly, we use $J_{2\mu}$ to perform QCD sum rule analyses. Choosing the same working regions 5.0 GeV$^2 < s_0 <$ 7.0 GeV^2 and 2.0 GeV$^2 < M_B^2 < 4.0$ GeV^2, the mass is extracted to be

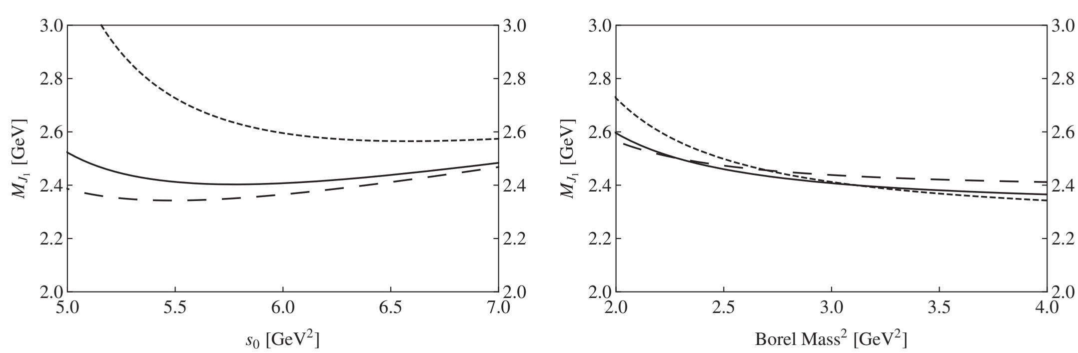

FIG. 5. Mass calculated using the current $J_{1\mu}$, as a function of the threshold value s_0 (left) and the Borel mass M_B (right). In the left panel, the short-dashed/solid/long-dashed curves are obtained by setting $M_B^2 = 2.0/3.0/4.0$ GeV^2, respectively. In the right panel, the short-dashed/solid/long-dashed curves are obtained by setting $s_0 = 5.5/6.0/6.5$ GeV^2, respectively.

$$M_{Y_2} = 2.34 \pm 0.17\ \text{GeV}. \tag{26}$$

As we have discussed in previous sections, $J_{1\mu}$ and $J_{2\mu}$ may couple to two different physical states. Using the same working region, we evaluate the mass splitting between these two states/currents to be

$$\Delta M = 71^{+172}_{-48}\ \text{MeV}. \tag{27}$$

V. SUMMARY AND DISCUSSIONS

In this work we apply the method of QCD sum rules to study the $Y(2175)$ by using local $ss\bar{s}\bar{s}$ interpolating currents with $J^{PC} = 1^{--}$. The relevant diquark-antidiquark $(ss)(\bar{s}\bar{s})$ and meson-meson $(\bar{s}s)(\bar{s}s)$ interpolating currents have been systematically constructed in Ref. [41], where their relations have also been derived. There we found two independent currents, so there are (at least) two different internal structures. In Ref. [41] we have calculated the two diagonal terms using the two diquark-antidiquark $(ss)(\bar{s}\bar{s})$ currents $\eta_{1\mu}$ and $\eta_{2\mu}$, and in this work we further calculate their off-diagonal term

$$\langle 0|T\eta_{1\mu}(x)\eta^{\dagger}_{2\nu}(0)|0\rangle. \tag{28}$$

We find two new currents $J_{1\mu}$ and $J_{2\mu}$ with the mixing angle $\theta = -5.0°$:

$$\begin{aligned} J_{1\mu} &= \cos\theta\eta_{1\mu} + \sin\theta i\eta_{2\mu}, \\ J_{2\mu} &= \sin\theta\eta_{1\mu} + \cos\theta i\eta_{2\mu}. \end{aligned} \tag{29}$$

These two currents do not strongly correlate to each other, suggesting that they may couple to different physical states.

We use $J_{1\mu}$ and $J_{2\mu}$ to perform QCD sum rule analyses. Especially, we find that $J_{1\mu}$ and $J_{2\mu}$ both couple weakly to the lower state $\phi(1680)$ as well as the $\phi f_0(980)$ threshold, so the states they couple to, as if they can couple to some states, should be new and possibly exotic states. Accordingly, we assume $J_{1\mu}$ and $J_{2\mu}$ separately couple to two different states with the same quantum number $J^{PC} = 1^{--}$, whose masses are extracted to be

$$M_{Y_1} = 2.41 \pm 0.25\ \text{GeV}, \tag{30}$$

$$M_{Y_2} = 2.34 \pm 0.17\ \text{GeV}. \tag{31}$$

These results do not change significantly compared with those obtained in Ref. [41]. However, their mass splitting depends significantly on the mixing angle, and we use $J_{1\mu}$ and $J_{2\mu}$ with $\theta = -5.0°$ to evaluate it to be

$$\Delta M = 71^{+172}_{-48}\ \text{MeV}. \tag{32}$$

The mass extracted using $J_{2\mu}$ is consistent with the experimental mass of the $Y(2175)$, suggesting that $J_{2\mu}$ may couple to the $Y(2175)$, while the mass extracted using $J_{1\mu}$ is a bit larger, suggesting that the $Y(2175)$ may have a partner state whose mass is 2.41 ± 0.25 GeV, that is about 71^{+172}_{-48} MeV larger than the $Y(2175)$.

Because $J_{1\mu}$ and $J_{2\mu}$ are two $ss\bar{s}\bar{s}$ interpolating currents with $J^{PC} = 1^{--}$, both the $Y(2175)$ and its possible partner state should be vector mesons containing large strangeness components. Note that our results do not definitely suggest that they are $ss\bar{s}\bar{s}$ tetraquark states, because the interpolating current sees only the quantum numbers of the physical state, that is, $J^{PC} = 1^{--}$. We can further use the Fierz transformation to obtain that the $Y(2175)$ and its possible partner state can both be observed in the $\phi f_0(980)$ channel, while the latter may also be observed in the $\phi f_1(1420)$ channel, as if kinematically allowed.

Experimentally, the $Y(2175)$ has been well established by the *BABAR*, BESII, BESIII, and Belle experiments. Besides it, there might be another structure in the $\phi f_0(980)$ invariant mass spectrum at around 2.4 GeV. This might be the partner state of the $Y(2175)$, which is coupled by the current $J_{1\mu}$. To end this paper, we note that the two mass values we obtained, 2.34 ± 0.17 GeV and 2.41 ± 0.25 GeV, are both around 2.4 GeV, indicating that there might be even more complicated structures in this region, such as two coherent resonances. We also note that there are many charmoniumlike Y states of $J^{PC} = 1^{--}$, so it is natural to think that there can be more than one Y state in the light sector. Accordingly, we propose to carefully study the structure in the $\phi f_0(980)$ invariant mass spectrum at around 2.4 GeV in future experiments.

ACKNOWLEDGMENTS

This project is supported by the National Natural Science Foundation of China under Grants No. 11475015, No. 11575008, No. 11575017, No. 11722540, No. 11261130311, and No. 11761141009, the National Key Basic Research Program of China (2015CB856700), the Fundamental Research Funds for the Central Universities, and the Foundation for Young Talents in College of Anhui Province (Grant No. gxyq2018103).

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