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Windings of twisted strings

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Twistor string models have been known for more than a decade now but have come back under the spotlight recently with the advent of the scattering equation formalism which has greatly generalized the scope of these models. A striking ubiquitous feature of these models has always been that, contrary to usual string theory, they do not admit vibrational modes and thus describe only conventional field theory. In this paper we report on the surprising discovery of a whole new sector of one of these theories which we call "twisted strings," when spacetime has compact directions. We find that the spectrum is enhanced from a finite number of states to an infinite number of interacting higher spin massive states. We describe both bosonic and world sheet supersymmetric models, their spectra and scattering amplitudes. These models have distinctive features of both string and field theory, for example they are invariant under stringy T-duality but have the high energy behavior typical of field theory. Therefore they describe a new kind of field theories in target space, sitting on their own halfway between string and field theory.

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I. INTRODUCTION

String theories based on Penrose's twistor theory have lead to a revolution in our understanding of the S-matrix of quantum field theory [1]. The most remarkable feature of these models is that, contrary to string theory, they describe only field theory (spin ≤ 2 fields), that is, higher spin massive excitations are absent from their spectrum. These models combine the mathematical elegance of string theory with twistor methods to describe the low energy field theories of nature.

In the field of scattering amplitudes, the recent introduction of the "scattering equations formalism" [2–4] lead to the discovery of the ambitwistor string, which in turn lead to advances which were out of reach of the previous twistor methods. Most notably this includes loop-level [5–9] amplitudes and curved space [10,11]. But an important open question remained, the connection between these new models to traditional string theory. The best framework to understand these questions relies on a recently rediscovered quantization ambiguity that leads to closed theories which we call "twisted strings" below. The idea is that the ambitwistor string arises in the *tensionless limit* of the twisted string [12–16].

The main result of this paper is the following: While in flat space the twisted type II string describes gravity only, in spacetimes with different topologies, where the string can wind around compact dimensions, an infinite tower of massive states arises. This twisted string theory remains distinct from usual string theory, as we explain below, and describes the new type of theories halfway between string and field theory.

We study the bosonic and world sheet supersymmetric models. The bosonic twisted string has tachyonic excitations (whose masses squared are bounded by $-4/\alpha'$ where α' is the Regge slope) and ghosts (negative normed states) in the physical spectrum. The Ramond-Neveu-Schwarz (RNS) formulation with $\mathcal{N} = (1,1)$ world sheet supersymmetry has no tachyons but also seems to have nondecoupling ghosts.

Remarkably, we also find that the compactified theory has T-duality: the spectrum is invariant when the radius R of the compact dimension is exchanged with α'/R. At the self-dual radius $R = \sqrt{\alpha'}$, both in the RNS and bosonic models, infinitely many higher spin states become massless, a rather striking fact that will be explored elsewhere.

We compute the scattering amplitude of these twisted theories and find that, in contrast to string theory, these theories do *not* enjoy exponential suppression at high energies [17,18].

The combination of these three elements: stringy T-duality, field-theory power law suppression at high energy and the fact that these theories only describe conventional field theory in flat space give these theories a genuinely novel status somewhere between field and string theory.

We also comment on the connection to the ambitwistor string [19] and scattering equations in the tensionless

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limit [12,13]. We find that the winding modes decouple from the scattering equations, but modify the integrand in a way that we describe. This gives new scattering-equation like formulas for winding states which include higher-spin states.

II. TWISTED STRINGS

A. Review

We call twisted strings the world sheet models described by Siegel in the context in [12,20]. The $\alpha' \to \infty$ limit of these models produces the ambitwistor string [12–14]. We present here a brief description of the model; for more details the reader is referred to [14]. The twisted bosonic string action is the Polyakov action

$$S = \frac{1}{2\pi\alpha'} \int d^2 z \partial X^\mu \bar{\partial} X_\mu, \tag{1}$$

here written in flat space and in conformal gauge. We expand the field $X(z,\bar{z}) = X_L(z) + X_R(\bar{z})$ in the usual way:

$$X_L^\mu(z) = x_L^\mu - i\frac{\alpha'}{2} p_L^\mu \ln(z) + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{\infty} \frac{1}{m}\frac{\alpha_m^\mu}{z^m}$$
$$X_R^\mu(z) = x_R^\mu - i\frac{\alpha'}{2} p_R^\mu \ln(\bar{z}) + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{m=-\infty}^{\infty} \frac{1}{m}\frac{\tilde{\alpha}_m^\mu}{\bar{z}^m} \tag{2}$$

with canonical commutation relations

$$[\alpha_n^\mu, \alpha_m^\nu] = [\tilde{\alpha}_n^\mu, \tilde{\alpha}_m^\nu] = n\delta_{n+m}\eta^{\mu\nu} \tag{3}$$

$$[x_{L/R}^\mu, p_{L/R}^\nu] = i\eta^{\mu\nu}, \qquad [x_{L/R}^\mu, p_{R/L}^\nu] = 0, \tag{4}$$

where $\eta^{\mu\nu} = (-,+,\ldots,+)$.

The difference to conventional strings comes from the choice of vacuum [12–14,21,22],

$$\alpha_n|0\rangle = \tilde{\alpha}_{-n}|0\rangle = 0, \quad \forall\, n > 0, \tag{5}$$

which we call *twisted vacuum*. The negative modes of $\tilde{\alpha}$ annihilate the vacuum in contrast to the conventional string vacuum where the positive modes annihilate the vacuum. This defines the following operator ordering:

$$:\alpha_n^\mu \alpha_{-m}^\nu: = \alpha_{-m}^\nu \alpha_n^\mu, \quad :\tilde{\alpha}_{-m}^\mu \tilde{\alpha}_n^\nu: = \tilde{\alpha}_n^\nu \tilde{\alpha}_{-m}^\mu, \quad \forall\, n, \quad m > 0. \tag{6}$$

The spectrum of these theories can be computed from the Virasoro constraints,

$$(\partial X)^2 = 0, \quad (\bar{\partial} X)^2 = 0. \tag{7}$$

The bosonic theory is found to live in 26 dimensions. In the twisted vacuum the zero modes L_0 and $\bar{L}_0$ of (7) acquire a normal ordering constant [23],

$$L_0 = \frac{\alpha'}{4} p_L^2 + N - 1, \qquad \bar{L}_0 = \frac{\alpha'}{4} p_R^2 - \bar{N} + 1 \tag{8}$$

with $N = \sum_{n=1}^{\infty} :\alpha_{-n}\cdot\alpha_n:$ and $\bar{N} = -\sum_{n=1}^{\infty} :\tilde{\alpha}_{-n}\cdot\tilde{\alpha}_n:$—the minus signs comes from the choice of vacuum. The zero modes of the Virasoro conditions then read

$$(N - \bar{N}) - \alpha' m^2 = 0; \qquad N + \bar{N} - 2 = 0. \tag{9}$$

The normal ordering constant appears in the level matching truncating the spectrum to a finite number of states. These are the massless sector of the bosonic string plus massive spin two states, with $m^2 = \pm 4/\alpha'$.

Adding a pair of fermions gives the RNS model which also has a twisted vacuum [see Eq. (16)] but in this case there is no tachyon. We proved in [13] following on [12] that both the bosonic and the type II models have as the tensionless limit ambitwistor strings.

B. Bosonic model on a circle

Taking one of the target-space dimensions to be a circle of radius R allows winding modes $X^{25}(\sigma + 2\pi) = X^{25}(\sigma) + 2w\pi R$ with $w \in \mathbb{Z}$. In the presence of winding and Kaluza-Klein modes ($n \in \mathbb{Z}$) in the 25th dimension, the momentum zero modes are

$$p_{L/R}^\mu = \left(k^m, \frac{n}{R} \pm \frac{Rw}{\alpha'}\right), \tag{10}$$

where $m = 0, \ldots, 24$. The constraints (9) become

$$m^2 = \frac{n^2}{R^2} + \frac{R^2 w^2}{\alpha'^2} + \frac{2}{\alpha'}(N - \bar{N})$$
$$N + \bar{N} + nw = 2. \tag{11}$$

The compactified model contains many new tachyonic states. To see this take $k = -nw > 0$. The lowest possible mass squared states have $(N, \bar{N}) = (0, k+2)$. The mass-shell constraint then gives

$$m^2 = \frac{n^2}{R^2} + \frac{R^2 w^2}{\alpha'^2} + \frac{2}{\alpha'}(-k-2) = \left(\frac{n}{R} + \frac{wR}{\alpha'}\right)^2 - \frac{4}{\alpha'}. \tag{12}$$

Taking $n > 0$ and $w < 0$, $m^2 < 0$ if

$$n > \frac{|w|R^2}{\alpha'} - 2 \quad \text{and} \quad n < \frac{|w|R^2}{\alpha'} + 2. \tag{13}$$

There are always integer solutions for n, therefore there are infinitely many tachyonic states in the bosonic twisted string, with masses squared bounded from below by $-4/\alpha'$. In addition, many of them have negative norm [14,24], we come back to this point for the RNS model below.

C. RNS model on a circle

The RNS model contains two Majorana-Weyl fermions $\psi(z), \bar{\psi}(\bar{z})$ and has the standard RNS world sheet supersymmetric action. Following conventions of [25] the fermions have mode expansion

$$\psi(z) = \sum_{r\in\mathbb{Z}+\nu} \frac{\psi_r}{z^r}, \qquad \bar{\psi}(\bar{z}) = \sum_{r\in\mathbb{Z}+\tilde{\nu}} \frac{\bar{\psi}_r}{z^r} \tag{14}$$

with $\nu = 1/2$ is the NS sector while $\nu = 0$ is the Ramond sector, and we keep

$$\{\psi^\mu_r, \psi^\nu_s\} = \{\tilde{\psi}^\mu_r, \tilde{\psi}^\nu_s\} = \eta^{\mu\nu}\delta_{r+s} \tag{15}$$

while we define the twisted vacuum

$$\psi^\mu_r|0\rangle = 0, \qquad \tilde{\psi}^\mu_{-r}|0\rangle = 0 \quad \forall\, r > 0 \tag{16}$$

depending on the NS or R vacuum. In a given sector for the closed string, NS-NS, NS-R, R-NS or R-R, the constraints are

$$\begin{aligned} m^2 &= \frac{n^2}{R^2} + \frac{R^2 w^2}{\alpha'^2} + \frac{2}{\alpha'}(N - \bar{N} - a^\nu + a^{\bar{\nu}}) \\ N^{\text{tot}} &+ \bar{N}^{\text{tot}} + nw = a^\nu + a^{\bar{\nu}} \end{aligned} \tag{17}$$

with $a^{NS} = 1/2$, $a^R = 0$. The operators N^{tot} and $\bar{N}^{\text{tot}}$ count the total number of oscillators, ψ's and α's.

These models have a Gliozzi-Scherck-Olive (GSO) projection which eliminates the tachyonic state. The argument is: In the NS-NS sector, the GSO-even states are built out of at least one ψ^μ_r and $\tilde{\psi}^\nu_r$ modes, for example $\psi^\mu_{-1/2}\psi^\nu_{1/2}|0\rangle$. Therefore the number operators on physical states always obey $N^{\text{tot}} \geq \frac{1}{2}$ and $\bar{N}^{\text{tot}} \geq \frac{1}{2}$.

It is easy to see that there are no tachyonic modes in the NS-NS sector; the only nontrivial case has $nw = -k$ with $k > 0$. The twisted level matching constraint implies that the lowest mass in (17) is a state with $\bar{N}^{\text{tot}} = k + \frac{1}{2}$ and $N^{\text{tot}} = 0$:

$$m^2 = \frac{n^2}{R^2} + \frac{R^2 w^2}{\alpha'^2} + \frac{2}{\alpha'}\left(\frac{1}{2} - k - \frac{1}{2}\right) = \left(\frac{n}{R} + \frac{wR}{\alpha'}\right)^2, \tag{18}$$

which is always positive. In the R-R sector both N^{tot} and $\bar{N}^{\text{tot}}$ can be zero but there is no normal ordering constant. The lowest possible value for the mass is given by the state with $(N, \bar{N}) = (0, k)$ for which

$$m^2 = \frac{n^2}{R^2} + \frac{R^2 w^2}{\alpha'^2} + \frac{2}{\alpha'}(-k) = \left(\frac{n}{R} + \frac{wR}{\alpha'}\right)^2 > 0. \tag{19}$$

The mixed NS-R and R-NS sectors are also tachyons-free, the argument follows as above. In conclusion, the compactified twisted model has no tachyons.

D. Negative-norm states

So far we have described the masses of the states in the spectra of these new theories. It turns out that the models possess ghosts in the physical spectrum, which naively spoils the unitarity of the theories. In the flat space case, it was known that the bosonic model had ghosts, while the supersymmetric model was ghost-free. Here this is not the case anymore and we argue now that both models do contain ghosts in the physical spectrum. In light-cone gauge, the GSO admissible supergravity states are of the form $\psi^I_{-1/2}\bar{\psi}^J_{1/2}|0,k\rangle$ with $I, J = 1, \ldots, d-1$ have positive norm if $\langle 0,k|0,k\rangle > 0$. A winding state with Kaluza-Klein (KK) momentum and winding is created by acting with $\exp(ik_L\alpha_0)$ and $\exp(ik_R\tilde{\alpha}_0)$ on $\psi^I_{-1/2}\bar{\psi}^J_{1/2}|0,k\rangle$, adding compensating $\tilde{\alpha}_i$'s and setting the number of α's to zero such that $\bar{N}^{\text{tot}}, n, w$ solve the constraints. These states have negative norms whenever there is an odd number of $\tilde{\alpha}$ [26]. An interesting prospect would be to find a target space with extra symmetries that would remove these states.

E. T-duality

Another fact to add to the list of curious properties of these new theories is that they are invariant under T-duality,

$$(n, w, R) \leftrightarrow \left(w, n, \frac{\sqrt{\alpha'}}{R}\right). \tag{20}$$

This is surprising since in normal string theory, T-duality reflects the existence of a minimum spacetime length and is connected to the UV completeness of string theory. The amplitude analysis below shows that, contrary to string theory amplitudes that are exponential soft at high energies, twisted strings have a power-law falloff and therefore behave like field theories, which do generically suffer from UV divergences.

Since the action is the same for twisted strings and conventional strings we expect that the Buscher rules [27] for T-duality in nontrivial backgrounds are the same in twisted strings as in string theory.

At the self-dual radius $R = \sqrt{\alpha'}$ a surprising effect arises: both the twisted bosonic and RNS strings do have infinitely many extra massless states at the self-dual radius. The Virasoro conditions obeyed by physical states can be written as follows:

$$m^2 = (k_L)^2 + \frac{4}{\alpha'}\left(N^{\text{tot}} - \frac{1}{2}\right) = (k_R)^2 + \frac{4}{\alpha'}\left(\frac{1}{2} - \bar{N}^{\text{tot}}\right). \tag{21}$$

Since N^{tot}, $\bar{N}^{\text{tot}} \geq \frac{1}{2}$ for most values of N^{tot} the first condition has no solution when $m^2 = 0$. However there is an interesting class of solutions with $N^{\text{tot}} = \frac{1}{2}$. The first equation is solved by $w = -n$ and the second by $w = \pm\sqrt{q}$ with $\bar{N}^{\text{tot}} = \frac{1}{2} + q$. For the $\sqrt{q}$ integer, these are

consistent solutions that describe massless spacetime vectors with nonzero KK and winding charge. This suggests an exotic gauge symmetry enhancement [28]. It would be nice to understand this strange sector of the theory. One interesting application would be to take the tensionless limit of an amplitude between these gauge bosons and obtain corresponding Cachazo-He-Yuan (CHY) formulas. Since the heterotic ambitwistor string is inconsistent [19], this may provide an alternative approach to gauge interactions.

III. PROPERTIES OF TWISTED STRINGS

In this section we study various properties of the twisted strings with windings; scattering amplitudes, high-energy behavior and partition functions. We also comment on the form of ambitwistor or CHY integrands with windings.

A. Scattering amplitudes and high-energy behavior

It was shown in [12,14,24,29] that the oscillator flip in scattering amplitudes is implemented by using the twisted world sheet correlators $\langle XX\rangle$ (see also [30])

$$\langle X^\mu(z,\bar{z})X^\nu(w,\bar{w})\rangle = -\frac{\alpha'}{2}\ln\left(\frac{z-w}{\bar{z}-\bar{w}}\right) \tag{22}$$

or, equivalently,

$$\langle X_L^\mu(z)X_L^\nu(w)\rangle = -\eta^{\mu\nu}\frac{\alpha'}{2}\ln(z-w) \tag{23}$$

$$\langle X_R^\mu(\bar{z})X_R^\nu(\bar{w})\rangle = \eta^{\mu\nu}\frac{\alpha'}{2}\ln(\bar{z}-\bar{w}). \tag{24}$$

The part of the string integrands that is affected by the sign flip is the Koba-Nielsen factor, the correlator between the plane-wave part of the vertex operators $\exp(ik\cdot X(z,\bar{z}))\to\exp(i(k_L X(z)+k_R X_R(\bar{z})))$ [31]:

$$\left\langle\prod_{j=1}^{n} e^{i(k_{Lj}X_L(z_j)+k_{Rj}X_R(\bar{z}_j))}\right\rangle = e^{-\sum_{i,j}k_{Li}\cdot k_{Lj}\langle X_L(z_i)X_L(z_j)\rangle}e^{-\sum_{i,j}k_{Ri}\cdot k_{Rj}\langle X_R(\bar{z}_i)X_R(\bar{z}_j)\rangle}. \tag{25}$$

In ordinary string theory, this reduces to the standard expression $\prod_{i<j}|z_{ij}|^{\alpha' k_i\cdot k_j}$ or $\prod_{i<j}(z_{ij})^{\alpha' k_{Li}\cdot k_{Lj}/2}(\bar{z}_{ij})^{\alpha' k_{Ri}\cdot k_{Rj}/2}$ when windings are included.

In the twisted string, we use (24) to obtain

$$\prod_{i<j}((z_i-z_j)^{\frac{1}{2}\alpha' k_{Li}\cdot k_{Lj}}\times(\bar{z}_i-\bar{z}_j)^{-\frac{1}{2}\alpha' k_{Ri}\cdot k_{Rj}}). \tag{26}$$

Given the definitions of the momenta in (10), we have that $\alpha' k_{L1}\cdot k_{L2}=\alpha' k_{R1}\cdot k_{R2}-2n_1w_2-2n_2w_1$. In the presence of windings, Eq. (26) is then rewritten

$$\prod_{i<j}\left(\frac{z_i-z_j}{\bar{z}_i-\bar{z}_j}\right)^{\frac{1}{2}\alpha' q_i\cdot q_j}|z_i-z_j|^{n_iw_j+n_jw_i}, \tag{27}$$

where we used an effective higher-dimensional shorthand notation $q=(k^m,\frac{n}{R},\frac{wR}{\alpha'})$, such that $2q_i\cdot q_j=k_{Li}\cdot k_{Lj}+k_{Ri}\cdot k_{Rj}=2k_i^m k_{jm}+n_in_j/R^2+w_iw_j(R/\alpha')^2$.

To compute these amplitudes we generalize the original observation of [29] which implies a remarkable property: The results of these integrals are rational functions of the kinematic invariants. This can be seen explicitly from the following formula:

$$\int d^2z\left(\frac{z}{\bar{z}}\right)^a|z|^{2n}\left(\frac{1-z}{1-\bar{z}}\right)^b|1-\bar{z}|^{2m} = \frac{(-a-n)^{\underline{2n+1}}(-b-m)^{\underline{2m+1}}}{(-1-a-b-n-m)^{\underline{2n+2m+3}}}, \tag{28}$$

where $(a)^{\underline{n}}=\frac{\Gamma(a+n)}{\Gamma(a)}$ is known as a Pochhammer symbol. It simplifies to a product when n is integer $(a)^{\underline{n}}=a(a+1)\cdots(a+n-1)$.

This rational result is typical of a field theoretic behavior at tree level. By factorization, higher point amplitudes should also be given by rational functions.

This surprising fact which starkly contrasts with string theory calculations, where the oscillator spectrum gives rise to infinitely many poles in the S-matrix, can be understood as follows. The discrete momenta in physical states require compensating oscillator excitations [see again Eq. (11)]. Momentum quantization and conservation in the extra dimensions therefore imply that only finitely many states can be exchanged in a given channel [33].

At loop orders, unitarity cuts predict that the twisted-string integrand in a loop-momentum formulation [37] must also be a rational function. In ordinary string theory, these integrands are typically given by elliptic multiple zeta values [38–41] and the fact that these new integrands should be rational is an interesting mathematical fact deserving further investigation.

B. CHY integrands for higher spins

A prescription for obtaining the scattering equations from the twisted string was given in [12]. We apply it here to obtain scattering equations for states carrying winding modes and corresponding Cachazo-He-Yuan [3] formulas. The integrand for a generic amplitude in a space with one compact dimension has a universal contribution from the twisted Koba-Nielsen factor (26). Following [12], we shift the antiholomorphic coordinate as $\bar{z}\to z-\frac{1}{\beta}\bar{z}$ and take the limit when $\beta\to\infty$ so that $\bar{z}\to z$. Contrary to the uncompactified case, the exponentials in the Koba-Nielsen factor do not cancel completely leaving a $(z_{ij})^{n_iw_j+n_jw_i}$ contribution to the amplitude, as can be seen from Eq. (27). The scattering equations come at the next order by

integrating over $\bar{z}$ to obtain $\bar{\delta}(E_i)$ where $E_i = \sum_{i<k} \frac{q_i \cdot q_j}{z_{ij}} - \frac{2}{\alpha'} \frac{(n_i w_j + n_j w_i)}{z_{ij}} = 0$. In the limit Eq. (27) becomes

$$\int (\cdots) \prod_{i<j} (z_i - z_j)^{n_i w_j + n_j w_i} \bar{\delta}(E_i). \tag{29}$$

The integral supported on the scattering equations, done naively, does not reproduce the one obtained using formulas like (28) (but it still is a rational function). This is similar to the procedure described in [42] where, after changing the integration measure to one containing the scattering equation in a string amplitude, one does not have the same amplitude anymore and a limit must be taken.

In our case here, the proper CHY amplitude is obtained after the $\alpha' \to \infty$ limit is taken. This decouples the windings from the scattering equations but leaves a contribution of $(z_i - z_j)^{n_i w_j + n_j w_i}$ to the integrand. We hope this formula paves the way to describe higher spin (and perhaps massive) amplitudes in the CHY framework.

C. Partition function

The partition function of the bosonic twisted model is readily written in terms of a state sum. The twisted oscillator part was computed in [14] [below Eq. (5.15)]. The only new ingredient here is the (ordinary) lattice sum coming from the left- and right-moving zero modes. As emphasized above, this piece is unaffected by the changes in the oscillator sector so the whole partition function is given by

$$Z(\tau, \tilde{\tau}) = \int d^{26}k e^{i\pi(\tau - \tilde{\tau})\frac{\alpha' k^2}{2}} \sum_{n,m \in \mathbb{Z}^2} \frac{q^{p_L^2/2} \tilde{q}^{p_R^2/2}}{\eta(q)^{24} \eta^{24}(1/\tilde{q})}, \tag{30}$$

where $q = \exp(2i\pi\tau)$, $\tilde{q} = \exp(-2i\pi\tilde{\tau})$ and $\eta(q) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n)$. As observed in [14], the unusual $1/\tilde{q}$ makes a naive interpretation of the integration variable τ impossible, for otherwise $\eta(1/\tilde{q})$ is not defined. The model should be complexified, a matter that will be studied somewhere else and connected with similar problems in the ambitwistor string side [13,43,44].

Physical states are obtained in the state sum by integrating $\int_{-1/2}^{1/2} d(\Re e \tau) Z(\tau, \bar{\tau})$, as is standard and was done in the flat case in [14]. Higher dimensional generalizations are done just like in string theory, and it would of course be interesting to see if some compactification manifolds would produce better behaved theories, without ghosts for instance.

IV. DISCUSSION

In this paper we described a surprising new facet of twisted strings: the winding sector. Its existence calls for a complete reevaluation of the kind of target space theories these models describe and opens the way to many new interesting possibilities. Many baffling aspects of this theory need clarification: for instance, what is the target space theory describing interacting massless higher spins at the self-dual radius. The presence of negative-norm states seems to ruin the unitarity of the theory, but perhaps there is a way to project these states out.

The most urgent question is probably a detailed study of the $\alpha' \to \infty$ limit, which is expected to generalize the ambitwistor string framework. In this limit winding modes are admissible but it is unclear if they contribute or decouple from the spectrum. This question is under investigation and will be discussed elsewhere. Closely related is the possibility of introducing consistent Yang-Mills interactions in the ambitwistor string by taking the tensionless limit from this model at the self-dual radius and a possible inclusion of windings in loop amplitudes in the ambitwistor string [5,7,8].

Another interesting direction is to use the above formalism to obtain CHY formulas for higher spin particles, massive particles and higher dimensional operators generalizing the work of [45].

Finally, the connection with double field theory (DFT) remains to be elucidated. It would be interesting to compare the amplitude calculations of these models with the ones in DFT of [46] for instance. On a more conceptual level, the sign change seems to be connected to the complexification of space-time and exchanging the roles of windings and momenta. It would be interesting to make a connection with [30,47–49]. Some comments on DFT background were also recently made in [50] in relation to Siegel's chiral string (which we call twisted strings) and it would be interesting to try to include compact directions in their analysis.

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