# Erratum: $A_4$ lepton flavor model and modulus stabilization from $S_4$ modular symmetry [Phys. Rev. D 100, 115045 (2019)]

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Equation (24) is incorrect. The correct equation is presented as follows:

$$M_{\nu} = \frac{\langle H_{u} \rangle^{2}}{\Lambda'} \left[ \begin{pmatrix} 2Y_{3} & -Y_{5} & -Y_{4} \\ -Y_{5} & 2Y_{4} & -Y_{3} \\ -Y_{4} & -Y_{3} & 2Y_{5} \end{pmatrix} + aY_{1} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + bY_{2} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \right].$$
(24)

Accordingly, Fig. 1 is replaced in Sec. III, and Figs. 5–10 are replaced in Sec. IV. The numerical results of Sec. V are revised. The conclusion is not changed.

#### V. PHENOMENOLOGICAL ASPECTS OF LEPTONS

#### A. Region A

Let us present numerical results for the modulus  $\tau$  in the region A. The parameter  $\rho$ , which realizes the potential minimum for the superpotential (34), is shown in the Re[ $\rho$ ]-Im[ $\rho$ ] plane in Fig. 5, while Fig. 8 shows  $\rho'$  for the potential minimum for the superpotential (35). In this case, only the NH of neutrino masses is available.

First, we show a correlation between  $\delta_{CP}$  and  $\sin^2 \theta_{23}$  in Fig. 11. The predicted ranges of  $\delta_{CP}$  are  $[-160^\circ, -140^\circ]$  for  $\sin^2 \theta_{23} \le 0.57$  and  $[-140^\circ, -135^\circ]$ ,  $[40^\circ, 45^\circ]$  for  $\sin^2 \theta_{23} \ge 0.57$ . We present the predicted  $\delta_{CP}$  versus the sum of neutrino masses  $\sum m_i$  in Fig. 12, where the cosmological bound  $\sum m_i < 120 \text{ [meV]}$  is imposed. The predicted  $\delta_{CP}$  clearly depends on the sum of neutrino masses. For  $\sum m_i = [78, 88] \text{ [meV]}$ ,  $\delta_{CP}$  is predicted to be in the range  $[-160^\circ, -135^\circ]$ . Near the cosmological bound,  $\sum m_i = [115, 120] \text{ [meV]}$ ,  $\delta_{CP}$  is  $[40^\circ, 45^\circ]$ .



FIG. 1. Allowed regions of  $\tau$  in the Re[ $\tau$ ]-Im[ $\tau$ ] plane. The fundamental domain of  $\Gamma(4)$  is shaded olive green. Cyan and red points denote the cases of NH and IH, respectively.

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FIG. 5. Values of  $\rho$  corresponding to  $\tau$  in the A region for W in Eq. (34).



FIG. 6. Values of  $\rho$  corresponding to  $\tau$  in the B region for W in Eq. (34).



FIG. 7. Values of  $\rho$  corresponding to  $\tau$  in the C region W in Eq. (34).



FIG. 8. Values of  $\rho'$  corresponding to  $\tau$  in the A region for W in Eq. (35).



FIG. 9. Values of  $\rho'$  corresponding to  $\tau$  in the B region for W in Eq. (35).



FIG. 10. Values of  $\rho'$  corresponding to  $\tau$  in the C region W in Eq. (35).

On the other hand, there is no clear neutrino mass dependence for  $\sin^2 \theta_{23}$ , as seen in Fig. 13. Near the upper bound of  $\sum m_i = 120 \text{ [meV]}$ ,  $\theta_{23}$  is predicted in the second octant. The effective mass of the  $0\nu\beta\beta$  decay  $\langle m_{ee} \rangle$  is presented in Fig. 14. The prediction is in the ranges 9.4–12 and 22–24 [meV]. We summarize the predictions for  $\langle m_{ee} \rangle$  and  $\sum m_i$  in Table II.



FIG. 11. The predicted region in the  $\sin^2\theta_{23}$ - $\delta_{CP}$  plane in region A for NH. Vertical red lines denote the  $3\sigma$  bound of observed data.



FIG. 12. The sum of neutrino masses  $\sum m_i$  dependence of  $\delta_{CP}$  in region A for NH. The vertical red line denotes the cosmological bound.



FIG. 13. The sum of neutrino masses  $\sum m_i$  dependence of  $\sin^2 \theta_{23}$  in region A for NH.



FIG. 14. The predicted effective neutrino mass  $\langle m_{ee} \rangle$  versus  $\sum m_i$  in region A for NH.



FIG. 15. The predicted region in the  $\sin^2\theta_{23}$ - $\delta_{CP}$  plane in region B for NH. Vertical red lines denote the  $3\sigma$  bound of observed data.

## **B.** Region B

We discuss the numerical results for the modulus  $\tau$  in the region B. The parameter  $\rho$ , which realizes the potential minimum for the superpotential (34), is shown in the Re[ $\rho$ ]-Im[ $\rho$ ] plane in Fig. 6, while Fig. 9 shows  $\rho'$  for the potential minimum for the superpotential (35). In this case, only the NH of neutrino masses is available.

We show the correlation between  $\delta_{CP}$  and  $\sin^2 \theta_{23}$  in Fig. 15. The predicted ranges of  $\delta_{CP}$  are  $[140^\circ, 160^\circ]$  for  $\sin^2 \theta_{23} \le 0.57$  and  $[135^\circ, 140^\circ]$ ,  $[-45^\circ, -40^\circ]$  for  $\sin^2 \theta_{23} \ge 0.57$ . We present the predicted  $\delta_{CP}$  versus the sum of neutrino masses  $\sum m_i$  in Fig. 16. For  $\sum m_i = [78, 88]$  [meV],  $\delta_{CP}$  is predicted to be in the range  $[135^\circ, 160^\circ]$ . Near the cosmological bound,  $\sum m_i = [115, 120]$  [meV],  $\delta_{CP}$  is  $[-45^\circ, -40^\circ]$ .

The neutrino mass dependence for  $\sin^2 \theta_{23}$  is almost the same as the result for region A, as seen in Fig. 17. The effective mass of the  $0\nu\beta\beta$  decay  $\langle m_{ee} \rangle$  is also the same as the result for region A, as seen in Fig. 18. We summarize the predictions for  $\langle m_{ee} \rangle$  and  $\sum m_i$  in Table II.

	А		В		C	
	NH	IH	NH	IH		NH IH
$\langle m_{ee} \rangle$ [meV]	9–12, 22–24	×	9–12, 22–24	×	×	15-30
$\sum m_i \text{ [meV]}$	77–88, 115–120	×	77–88, 115–120	×	Х	98-110

TABLE II. Predicted  $\langle m_{ee} \rangle$  and  $\sum m_i$  for cases A, B, and C.



FIG. 16. The sum of neutrino masses  $\sum m_i$  dependence of  $\delta_{CP}$  in region B for NH. The vertical red line denotes the cosmological bound.



FIG. 17. The sum of neutrino masses  $\sum m_i$  dependence of  $\sin^2 \theta_{23}$  in region B for NH.

### C. Region C

Finally, discuss the numerical results for the modulus  $\tau$  in the region C. The parameter  $\rho$ , which realizes the potential minimum for the superpotential (34), is shown in the Re[ $\rho$ ]-Im[ $\rho$ ] plane in Fig. 6, while Fig. 9 shows  $\rho'$  for the potential minimum for the superpotential (35). In this case, only the IH of neutrino masses is available.



FIG. 18. The predicted effective neutrino mass  $\langle m_{ee} \rangle$  versus  $\sum m_i$  in region B for NH.



FIG. 19. The predicted region in the  $\sin^2\theta_{23}$ - $\delta_{CP}$  plane in region C for IH. Vertical red lines denote the  $3\sigma$  bound of observed data.



FIG. 20. The sum of neutrino masses  $\sum m_i$  dependence of  $\delta_{CP}$  in region C for IH. The vertical red line denotes the cosmological bound.



FIG. 21. The sum of neutrino masses  $\sum m_i$  dependence of  $\sin^2 \theta_{23}$  in region C for IH.



FIG. 22. The predicted effective neutrino mass  $\langle m_{ee} \rangle$  versus  $\sum m_i$  in region C for IH.

We show the correlation between  $\delta_{CP}$  and  $\sin^2 \theta_{23}$  in Fig. 19. We can see the weak  $\sin^2 \theta_{23}$  dependence for  $\delta_{CP}$ . The predicted  $\delta_{CP}$  is in the ranges  $\pm [40^\circ, 60^\circ]$  and  $\pm [120^\circ, 180^\circ]$ . We show the predicted  $\delta_{CP}$  versus the sum of neutrino masses in Fig. 20. The sum of neutrino masses  $\sum m_i$  is restricted to the narrow range  $\sum m_i = [98, 110]$  [meV].

There is no clear neutrino mass dependence for  $\sin^2 \theta_{23}$ , as seen in Fig. 21. Below  $\sum m_i = 102 \text{ [meV]}$ ,  $\theta_{23}$  is predicted in the second octant, while it is in the first octant for  $\sum m_i \ge 107 \text{ [meV]}$ . The effective mass of the  $0\nu\beta\beta$  decay  $\langle m_{ee}\rangle$  is presented in Fig. 22. The prediction is in the range of 18–34 [meV]. We summarize the predictions for  $\langle m_{ee}\rangle$  and  $\sum m_i$  in Table II.