Semileptonic decays of D and D_s mesons in the relativistic quark model

R. N. Faustov^{*} and V. O. Galkin[†]

Institute of Cybernetics and Informatics in Education, FRC CSC RAS, Vavilov Street 40, 119333 Moscow, Russia

Xian-Wei Kang[‡]

College of Nuclear Science and Technology, Beijing Normal University, Beijing 100875, China

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The form factors parametrizing the weak D and D_s transitions to light pseudoscalar and vector mesons are calculated in the framework of the relativistic quark model based on the quasipotential approach. The special attention is paid to the systematic account of the relativistic effects including transformation of the meson wave function from the rest to moving reference frame and contributions of the intermediate negative-energy states. The form factors are expressed through the overlap integrals of the meson wave functions, which are taken from previous studies of meson spectroscopy. They are calculated in the whole range of the transferred momentum q^2 . Convenient parametrization of the form factors which accurately reproduces numerical results is given. The obtained values of the form factors and their ratios at $q^2 = 0$ agree well with the ones extracted form the experimental data. On the basis of these form factors and helicity formalism, differential and total semileptonic decay rates of D and D_s mesons as well as different asymmetries and polarization parameters are calculated. The detailed comparison of the obtained results with other theoretical calculations and experimental data is given.

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I. INTRODUCTION

Semileptonic decays of heavy mesons provide an important information on the values of the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements V_{Qq} (with Q denoting the heavy quark and q the light one), which are essential ingredients of the standard model. Experimentally such decays can be measured more accurately than pure leptonic ones since there is no helicity suppression for them. Theoretically semileptonic decays are significantly less complicated than hadronic ones as they contain one meson and a lepton pair in the final state. The lepton part is easily calculated using standard methods, while the hadronic part factorizes thus reducing theoretical uncertainties. The hadronic matrix element is usually parametrized by the set of invariant form factors, which are calculated using nonperturbative approaches based on quantum chromodynamics (QCD), such as lattice QCD, QCD sum rules, potential quark models.

Recently significant experimental progress has been achieved in studying semileptonic decays of the open charm mesons [1]. More precise and detailed measurements of the absolute and differential branching fractions and form factors for D and D_s decays to pseudoscalar and vector mesons became available due to high statistics accumulated at BES III [2–10]. Various CKM- favored and suppressed decay modes both with positron and muon were investigated. This allows one to check the lepton universality in D meson decays. Note that possible hints of its violation were recently found in B decays [11]. More precise and comprehensive data are expected form BES III and Belle II [12] in near future.

In this paper we calculate the matrix elements of the flavor changing charged weak current between initial D or D_s mesons and final light pseudoscalar or vector mesons in the framework of the relativistic quark model based on the quasipotential approach. This model was successfully applied for the calculations of the hadron spectroscopy [13–16] and weak decays [17–21]. It was found that relativistic effects play a very important role both for light and heavy hadrons. Thus the form factors are calculated with the consistent account of the relativistic quark dynamics. They are expressed through the overlap integrals of the meson wave functions which are known from the study of

^{*}faustov@ccas.ru

galkin@ccas.ru

[‡]11112018023@bnu.edu.cn

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their spectroscopy. The momentum transfer q^2 dependence of form factors is explicitly determined in the whole kinematical range without additional assumptions and extrapolations.¹ Then we use these form factors and the helicity formalism for the calculation of the differential and total branching fractions as well as polarization and asymmetry parameters. We also compare our results with available experimental data and previous predictions.

The paper is organized as follows. In Sec. II we briefly describe our relativistic quark model with special emphasis on calculation of the weak decay matrix elements between meson states with the account of relativistic effects. This model is applied in Sec. III to the consideration of semileptonic decay form factors of open charm mesons. We give the analytic expressions for the form factors which accurately reproduce the numerical results for the momentum transfer q^2 dependence of the form factors in the whole accessible kinematical range and compare them with available data. Then in Sec. IV we use these form factors to calculate the differential and total D and D_s meson semileptonic decay rates and different asymmetries and polarization parameters. Decays both with positrons and muons are considered. This allows us to give predictions for the ratios of the corresponding decay rates which can be used for the test of the lepton universality in charm meson decays. Finally, Sec. VI contains our conclusions.

II. RELATIVISTIC QUARK MODEL

For the calculation of meson properties we employ the relativistic quark model based on the quasipotential approach. In this model a meson with the mass M is described by the wave function $\Psi_M(\mathbf{p})$ of the quark-antiquark bound state which satisfies the Schrödinger-like quasipotential equation $[13,23]^2$

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right) \Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M) \Psi_M(\mathbf{q}), \quad (1)$$

where \mathbf{p} is the relative quark momentum. The relative momentum squared in the center of mass system on the mass shell is given by

$$b^{2}(M) = \frac{[M^{2} - (m_{1} + m_{2})^{2}][M^{2} - (m_{1} - m_{2})^{2}]}{4M^{2}}, \quad (2)$$

and the relativistic reduced mass is defined by

$$\mu_R = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3},\tag{3}$$

here $m_{1,2}$ are the quark masses. The kernel of this equation $V(\mathbf{p}, \mathbf{q}; M)$ is the QCD-motivated quark-antiquark potential which is constructed by the off-mass-shell scattering amplitude projected on the positive energy states. We assume [13] that it consists from the one-gluon exchange term which dominates at small distances and a mixture of the scalar and vector linear confining interactions which dominate at large distances. Note that the similar Lorentz structure for the covariant interaction kernel was also assumed in Ref. [24]. Moreover, we assume that the long-range vertex of the confining vector interaction contains additional Pauli term. Then the quasipotential is given by

$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p},\mathbf{q};M)u_1(q)u_2(-q), \quad (4)$$

with

$$\mathcal{V}(\mathbf{p}, \mathbf{q}; M) = \frac{4}{3} \alpha_s D_{\mu\nu}(\mathbf{k}) \gamma_1^{\mu} \gamma_2^{\nu} + V_{\text{conf}}^V(\mathbf{k}) \Gamma_1^{\mu}(\mathbf{k}) \Gamma_{2;\mu}(\mathbf{k}) + V_{\text{conf}}^S(\mathbf{k}), \qquad \mathbf{k} = \mathbf{p} - \mathbf{q},$$

where α_s is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge, and γ_{μ} and u(p) are the Dirac matrices and spinors, respectively. The long-range vector vertex has the form

$$\Gamma_{\mu}(\mathbf{k}) = \gamma_{\mu} + \frac{i\kappa}{2m} \sigma_{\mu\nu} k^{\nu}, \qquad (5)$$

where κ is the long-range anomalous chromomagnetic quark moment. In the nonrelativistic limit confining vector and scalar potentials reduce to

$$V_{\rm conf}^V(r) = (1 - \varepsilon)(Ar + B), \quad V_{\rm conf}^S(r) = \varepsilon(Ar + B), \quad (6)$$

and in the sum they reproduce the linear rising potential

$$V_{\rm conf}(r) = V_{\rm conf}^{S}(r) + V_{\rm conf}^{V}(r) = Ar + B, \qquad (7)$$

where ε is the mixing coefficient. Thus this quasipotential can be viewed as the relativistic generalization of the nonrelativistic Cornell potential

$$V_{\rm NR}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + Ar + B. \tag{8}$$

Our quasipotential contains both spin-independent and spindependent relativistic contributions.

All parameters of the model were fixed from the previous consideration of hadron spectroscopy and decays 13]]. Thus the values of constituent quark masses are $m_b = 4.88$ GeV, $m_c = 1.55$ GeV, $m_s = 0.5$ GeV, $m_{u,d} = 0.33$ GeV; the parameters of the linear potential are

¹Note that this property is shared by other approaches working in Minkowski space, such as the ones of Ref. [22]. The advantage of our treatment is the knowledge of the meson wave functions as the numerical solutions of the corresponding quasipotential equation [13–16].

²This equation was obtained from the usual quasipotential equation with the square root form of the off-shell energies by the "rationalization" procedure in terms of covariant center of mass system quantities [23].

 $A = 0.18 \text{ GeV}^2$ and B = -0.30 GeV; the mixing parameter of the vector and scalar confining potential is $\varepsilon = -1$, while the anomalous chromomagnetic quark moment $\kappa = -1$. We take the running QCD coupling constant with infrared freezing

$$\alpha_{s}(\mu) = \frac{4\pi}{\beta_{0} \ln \frac{\mu^{2} + M_{0}^{2}}{\Lambda^{2}}},$$
(9)

where $\beta_0 = 11 - \frac{2}{3}n_f$, n_f is the number of flavors, $\Lambda = 413$ MeV, $M_0 = 2.24\sqrt{A} = 0.95$ GeV and the scale μ is set to $\frac{2m_1m_2}{m_1+m_2}$.

The spectroscopy of heavy-light and light mesons was discussed in detail in Refs. [14,15]. The calculated masses for both of the ground and excited states were found in agreement with available experimental data and exhibit linear Regge trajectories. The meson wave functions were also calculated and can be used for the evaluation of the meson decays.

For the consideration of the *D* meson semileptonic decays it is necessary to calculate the hadronic matrix element of the local current governing the $c \rightarrow f$ (f = s, d) weak transition. In the quasipotential approach the matrix element of this weak current $J^W_\mu = \bar{f}\gamma_\mu(1-\gamma_5)c$ between the initial $D_{(s)}$ meson with four-momentum $p_{D_{(s)}}$ and final meson *F* with four-momentum p_F is given by [17]

$$\langle F(p_F) | J^W_{\mu} | D_{(s)}(p_{D_{(s)}}) \rangle$$

$$= \int \frac{d^3 p d^3 q}{(2\pi)^6} \bar{\Psi}_{F\mathbf{p}_F}(\mathbf{p}) \Gamma_{\mu}(\mathbf{p}, \mathbf{q}) \Psi_{D_{(s)}\mathbf{p}_{D_{(s)}}}(\mathbf{q}), \quad (10)$$

where $\Psi_{M\mathbf{p}_M}$ are the initial and final meson wave functions projected on the positive energy states and boosted to the moving reference frame with the three-momentum \mathbf{p}_M . The vertex function

$$\Gamma = \Gamma^{(1)} + \Gamma^{(2)},\tag{11}$$

where $\Gamma^{(1)}$ is the leading-order vertex function which corresponds to the impulse approximation

$$\Gamma_{\mu}^{(1)}(\mathbf{p},\mathbf{q}) = \bar{u}_f(p_f)\gamma_{\mu}(1-\gamma^5)u_c(q_c)(2\pi)^3\delta(\mathbf{p}_q-\mathbf{q}_q) \quad (12)$$

and contains the δ function responsible for the momentum conservation on the spectator q antiquark line. The vertex function $\Gamma^{(2)}$ takes into account interaction of the active quarks (c, f) with the spectator antiquark (q) and includes the negative-energy part of the active quark propagator. It is the consequence of the projection on the positive energy states and has the form

$$\begin{split} \Gamma^{(2)}_{\mu}(\mathbf{p},\mathbf{q}) &= \bar{u}_f(p_f)\bar{u}_q(p_q) \\ &\times \left\{ \mathcal{V}(\mathbf{p}_q - \mathbf{q}_q) \frac{\Lambda^{(-)}_f(k')}{\epsilon_f(k') + \epsilon_f(q_c)} \gamma^0_1 \gamma_{1\mu} (1 - \gamma^5_1) \right. \\ &+ \gamma_{1\mu} (1 - \gamma^5_1) \frac{\Lambda^{(-)}_c(k)}{\epsilon_c(k) + \epsilon_c(p_f)} \gamma^0_1 \mathcal{V}(\mathbf{p}_q - \mathbf{q}_q) \right\} \\ &\times u_c(q_c) u_q(q_q), \end{split}$$

where $\mathbf{k} = \mathbf{p}_f - \Delta$; $\mathbf{k}' = \mathbf{q}_c + \Delta$; $\Delta = \mathbf{p}_F - \mathbf{p}_D$; $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$; and the projection operator on the negativeenergy states

$$\Lambda^{(-)}(p) = rac{\epsilon(p) - (m\gamma^0 + \gamma^0(\mathbf{\gamma p}))}{2\epsilon(p)}.$$

Note that the δ function in the vertex function $\Gamma^{(1)}$ [Eq. (12)] allows us to take off one of the integrals in the expression for the matrix element Eq. (10). As the result the usual expression for the matrix element as the overlap integral of the meson wave functions is obtained. The contribution $\Gamma^{(2)}$ [Eq. (13)] is significantly more complicated and contains the quasipotential of the quark-antiquark interaction \mathcal{V} [Eq. (4)] which has nontrivial Lorentz-structure. However, it is possible to use the quasipotential equation (1) to get rid of one of the integrations in Eq. (10) and thus get again the usual structure of the matrix element as the overlap integral of meson wave functions (for details see Refs. [17,18]).

Calculations of hadron decays are usually done in the rest frame of the decaying hadron, the $D_{(s)}$ meson in the considered case, where the decaying meson momentum $\mathbf{p}_D = 0$. Then the final meson F is moving with the recoil momentum $\mathbf{\Delta} = \mathbf{p}_F$ and its wave function should be boosted to the moving reference frame. The wave function of the moving meson $\Psi_{F\mathbf{\Delta}}$ is connected with the wave function in the rest frame $\Psi_{F\mathbf{0}}$ by the transformation [17]

$$\Psi_{F\Delta}(\mathbf{p}) = D_f^{1/2}(R_{L_{\Delta}}^W) D_q^{1/2}(R_{L_{\Delta}}^W) \Psi_{F0}(\mathbf{p}), \qquad (14)$$

where R^W is the Wigner rotation, L_{Δ} is the canonical Lorentz boost from the meson rest frame to a moving one and $D^{1/2}(R)$ is the spin rotation matrix.

III. WEAK DECAY FORM FACTORS

In the standard model the semileptonic D and D_s meson decays to a pseudoscalar (P) or a vector (V) mesons are governed by the flavor-changing $c \to q\ell\nu_\ell$ (q = s, d) current. The corresponding matrix element \mathcal{M} between meson states factorizes in the product of the leptonic (L_μ) current and the matrix element of the hadronic (H^μ) current with the corresponding CKM matrix element V_{cq} and the Fermi constant G_F

$$\mathcal{M}(D_{(s)} \to P(V)\ell\nu_{\ell}) = \frac{G_F}{\sqrt{2}}V_{cq}H^{\mu}L_{\mu}, \qquad (15)$$

where $L_{\mu} = \bar{\nu}_{\ell} \gamma_{\mu} (1-\gamma_5) \ell$ and $H^{\mu} = \langle P(V) | \bar{q} \gamma_{\mu} (1-\gamma_5) c | D_{(s)} \rangle$. The leptonic part is easily calculated using the lepton spinors and has a simple structure, while the hadronic part is significantly more complicated and requires nonperturbative treatment within QCD.

The hadronic matrix element of weak current J^W between meson states is usually parametrized by the following set of the invariant form factors.

(i) For $D_{(s)}$ transitions to pseudoscalar $P(\pi, K, \eta, \eta')$ mesons

$$\langle P(p_P) | \bar{q} \gamma^{\mu} c | D_{(s)}(p_{D_{(s)}}) \rangle$$

$$= f_+(q^2) \left[p_{D_{(s)}}^{\mu} + p_P^{\mu} - \frac{M_{D_{(s)}}^2 - M_P^2}{q^2} q^{\mu} \right]$$

$$+ f_0(q^2) \frac{M_{D_{(s)}}^2 - M_P^2}{q^2} q^{\mu},$$

$$\langle P(p_P) | \bar{q} \gamma^{\mu} \gamma_5 c | D_{(s)}(p_{D_{(s)}}) \rangle = 0,$$
(16)

(ii) For $D_{(s)}$ transitions to vector $V(\rho, \omega, K^*, \phi)$ mesons

$$\langle V(p_{V}) | \bar{q} \gamma^{\mu} c | D_{(s)}(p_{D_{(s)}}) \rangle$$

$$= \frac{2iV(q^{2})}{M_{D_{(s)}} + M_{V}} \epsilon^{\mu\nu\rho\sigma} \epsilon^{*}_{\nu} p_{D_{(s)}\rho} p_{V\sigma},$$

$$\langle V(p_{V}) | \bar{q} \gamma^{\mu} \gamma_{5} c | D_{(s)}(p_{D_{(s)}}) \rangle$$

$$= 2M_{V} A_{0}(q^{2}) \frac{\epsilon^{*} \cdot q}{q^{2}} q^{\mu} + (M_{D_{(s)}} + M_{V}) A_{1}(q^{2})$$

$$\times \left(\epsilon^{*\mu} - \frac{\epsilon^{*} \cdot q}{q^{2}} q^{\mu} \right) - A_{2}(q^{2}) \frac{\epsilon^{*} \cdot q}{M_{D_{(s)}} + M_{V}}$$

$$\times \left[p_{D_{(s)}}^{\mu} + p_{V}^{\mu} - \frac{M_{D_{(s)}}^{2} - M_{V}^{2}}{q^{2}} q^{\mu} \right].$$

$$(17)$$

At the maximum recoil point $(q^2 = 0)$ these form factors satisfy the following conditions:

$$f_{+}(0) = f_{0}(0),$$

$$A_{0}(0) = \frac{M_{D_{(s)}} + M_{V}}{2M_{V}} A_{1}(0) - \frac{M_{D_{(s)}} - M_{V}}{2M_{V}} A_{2}(0).$$

We use the quasipotential approach and the relativistic quark model discussed in Sec. II for the calculation of the weak decay matrix elements and transition form factors. We substitute the leading $\Gamma^{(1)}$ [Eq. (12)] and subleading $\Gamma^{(2)}$ [Eq. (13)] vertex functions in the expression for the matrix element of the weak current between meson states (10). This matrix element is considered in the rest frame of the decaying $D_{(s)}$ meson, then the boost of the final meson wave function Ψ_F from the rest to moving reference frame

TABLE I. Form factors of the weak D meson transitions.

Decay	Form factor	F(0)	$F(q_{\max}^2)$	σ_1	σ_2
$D \to K$	f_+	0.716	1.538	0.902	1.07
	f_0	0.716	1.086	0.360	1.657
$D \to K^*$	V	0.927	1.305	0.356	-0.490
	A_0	0.655	1.048	0.432	-0.840
	A_1	0.608	0.660	0.410	0.166
	A_2	0.520	0.623	0.582	-0.917
$D \to \pi$	f_+	0.640	2.336	0.332	0.557
	f_0	0.640	1.318	-0.345	1.133
$D \rightarrow \rho$	V	0.979	1.884	0.264	-2.001
	A_0	0.712	1.377	0.282	-0.826
	A_1	0.682	0.782	0.567	0.352
	A_2	0.640	0.815	0.964	0.645
$D \rightarrow \eta$	f_+	0.547	1.228	1.153	1.519
	f_0	0.547	0.683	0.408	3.147
$D \rightarrow \eta'$	f_+	0.538	0.804	-0.203	-4.686
	f_0	0.538	0.547	-0.950	1.038
$D \to \omega$	V	0.871	1.709	0.146	-2.775
	A_0	0.647	1.178	0.224	-0.759
	A_1	0.674	0.765	0.542	0.350
	A_2	0.713	0.802	0.997	2.176

with the recoil momentum $\Delta = \mathbf{p}_F$ should be considered. It is given by Eq. (14). Thus we take into account all relativistic effects including the relativistic contributions of intermediate negative-energy states and relativistic transformations of the meson wave functions. The resulting expressions for the decay form factors have the form of the overlap integrals of initial and final meson wave functions. They are rather cumbersome and are given in Refs. [17,18]. For the numerical evaluation of the decay form factors we use the meson wave functions obtained in calculating their mass spectra [14,16]. This is a significant advantage of our approach since in most of the previous model calculations some phenomenological wave functions (such as Gaussian) were used. Moreover, our relativistic approach allows us to

TABLE II. Form factors of the weak D_s meson transitions.

Decay	Form factor	F(0)	$F(q_{\max}^2)$	σ_1	σ_2
$\overline{D_s \to \eta}$	f_+	0.443	1.554	0.675	-0.856
	f_0	0.443	0.550	-0.302	1.634
$D_s \rightarrow \eta'$	f_+	0.559	1.001	0.719	-2.123
	f_0	0.559	0.654	-0.499	-0.124
$D_s \rightarrow \phi$	V	0.999	1.687	0.467	-4.020
	A_0	0.713	0.988	0.412	0.903
	A_1	0.643	0.746	0.621	-0.317
	A_2	0.492	0.645	0.447	-3.622
$D_s \to K$	f_{+}	0.674	2.451	1.255	-0.935
-	f_0	0.674	1.174	0.216	1.241
$D \to K^*$	V	0.959	1.966	0.425	-2.444
	A_0	0.629	1.103	0.281	-0.435
	A_1	0.596	0.733	0.835	0.423
	A_2	0.540	0.702	1.266	1.425



FIG. 1. Form factors of the weak D meson transitions.

determine the form factor dependence on the transferred momentum q^2 in the whole accessible kinematical range without additional approximations and extrapolations.

We find that the numerical results for these decay form factors can be approximated with high accuracy by the following expressions: (a) $f_+(q^2)$, $V(q^2)$, $A_0(q^2)$

$$F(q^2) = \frac{F(0)}{\left(1 - \frac{q^2}{M^2}\right) \left(1 - \sigma_1 \frac{q^2}{M_{D^*_{(s)}}^2} + \sigma_2 \frac{q^4}{M_{D^*_{(s)}}^4}\right)},$$
 (18)

(b) $f_0(q^2), A_1(q^2), A_2(q^2)$

$$F(q^2) = \frac{F(0)}{\left(1 - \sigma_1 \frac{q^2}{M_{D^*_{(s)}}^2} + \sigma_2 \frac{q^4}{M_{D^*_{(s)}}^4}\right)}, \qquad (19)$$

where for the decays governed by the CKM favored $c \rightarrow s$ transitions masses of the intermediate D_s mesons are used: $M = M_{D_s^*} = 2.112$ GeV for the form factors $f_+(q^2)$, $V(q^2)$ and $M = M_{D_s} = 1.968$ GeV for the form factor $A_0(q^2)$. While for the decays governed by



FIG. 2. Form factors of the weak D_s meson transitions.



FIG. 3. Comparison of our predictions for the product $f_+(q^2)|V_{cq}|$ (solid curve) and $f_0(q^2)|V_{cq}|$ (dashed curve) with experimental data for $f_+(q^2)|V_{cq}|$ form Belle [25] (orange dots with error bars) and *BABAR* [26] (red dots with error bars) and lattice results [27] for $f_+(q^2)|V_{cq}|$ (green dots with error bars) and $f_0(q^2)|V_{cq}|$ (magenta dots with error bars) of the weak $D \to K$ (q = s) and $D \to \pi$ (q = d) transitions.

the CKM suppressed $(c \rightarrow d)$ transitions masses of the intermediate *D* mesons are taken as follows: $M = M_{D^*} = 2.010$ GeV for the form factors $f_+(q^2)$, $V(q^2)$ and $M = M_D = 1.870$ GeV for the form factor $A_0(q^2)$. The values of form factors F(0), $F(q_{\text{max}}^2)$ and fitted parameters $\sigma_{1,2}$ are given in Tables I and II. We estimate the uncertainties of the calculated form factors to be less

than 5%. The form factors are plotted in Figs. 1 and 2. In Fig. 3 we compare our predictions for the product $f_+(q^2)|V_{cq}|$ with experimental data from Belle [25] and *BABAR* [26] and lattice results [27,28] for the weak $D \rightarrow K$ and $D \rightarrow \pi$ transitions. On the same plots we also show our results for $f_0(q^2)|V_{cq}|$ in comparison with lattice [27] data. We find the agreement within error bars with experimental values in the whole kinematical range for both transitions. There is also a nice accord with lattice results for the form factors of the $D \rightarrow \pi$ transition (there is a small difference only near q_{max}^2), while for the $D \rightarrow K$ transition our form factors have systematically somewhat larger values for $q^2 > 0.7 \text{ GeV}^2$. Note that in general our form factors better agree with data in the accessible kinematical range than lattice ones.

In Table III we compare theoretical predictions for the form factors of the weak D and D_s meson transitions to pseudoscalar mesons at $q^2 = 0$ with available experimental data. The authors of Ref. [29] calculated form factors in the framework of the covariant confining quark model. The covariant light-front quark model was employed in Refs. [30-32], while in Ref. [33] calculations were done using light-cone sum rules in the framework of heavy quark effective field theory. Lattice QCD simulations in Ref. [27] were carried out with $N_f = 2 + 1 + 1$ dynamical quarks. Experimental values were taken from very recent report on world averages of measurements of hadron properties obtained by the Heavy Flavor Averaging Group [34]. Good agreement of our predictions with data is found. Only for the $D \rightarrow \eta$ transition $f_{+}(0)$ is somewhat larger than experimental value.

For the weak D^+ and D_s meson transitions to vector mesons only the ratios of the form factors at

TABLE III. Comparison of various theoretical predictions for the form factors $f_+(0)$ of the weak D and D_s meson transitions to pseudoscalar mesons with available experimental data.

Decay	Our	[29]	[30–32]	[33]	Lattice [27]	Experiment [7,8,34]
$D \rightarrow K$	0.716	0.77	0.79(1)	$0.661^{(67)}_{(66)}$	0.765(31)	0.7361(34)
$D \rightarrow \pi$	0.640	0.63	0.66(1)	$0.635^{(60)}_{(57)}$	0.612(35)	0.6351(81)
$D \rightarrow \eta$	0.547	0.36	0.55(1)	$0.556^{(56)}_{(53)}$		0.38(3)
$D \rightarrow \eta'$	0.538	0.36	0.45(1)	(55)		
$D_s \rightarrow \eta$	0.443	0.49	0.48(3)	$0.611^{(62)}_{(54)}$		0.4576(70)
$D_s \rightarrow \eta'$	0.559	0.59	0.59(3)	(54)		0.490(51)
$D_s \to K$	0.674	0.60	0.66	$0.820_{(71)}^{(80)}$		0.720(85)

	Ratio		TI	heory		Ex	Experiment	
Decay		Our	[29]	[30–32]	[33]	PDG [1]	BES III [2-10]	
$D \to K^*$	r_V	1.53	1.22(24)	1.36(2)	$1.39^{(9)}_{(10)}$	1.49(5)	1.406(62)	
	r_2	0.85	0.92(18)	0.83(3)	$0.60^{(9)}_{(8)}$	0.802(21)	0.784(48)	
$D \rightarrow \rho$	r_V	1.44	1.26(25)	1.46(3)	$1.34_{(13)}^{(0)}$	1.48(16)	1.695(98)	
	r_2	0.94	0.93(19)	0.78(2)	$0.62^{(8)}_{(8)}$	0.83(12)	0.845(68)	
$D \rightarrow \omega$	r_V	1.29	1.24(25)	1.47(4)	$1.33_{(13)}^{(6)}$	1.24(11)		
	r_2	1.05	0.95(19)	0.84(2)	$0.60^{(9)}_{(0)}$	1.06(16)		
$D_s \rightarrow \phi$	r_V	1.56	1.34(27)	1.42(2)	$1.37^{(24)}_{(21)}$	1.80(8)		
	r_2	0.77	0.99(20)	0.86(1)	$0.53^{(10)}_{(6)}$	0.84(11)		
$D_s \to K^*$	r_V	1.61	1.40(28)	1.55(5)	$1.31_{(16)}^{(0)}$		1.67(38)	
	r_2	0.90	0.99(20)	0.82(2)	$0.53^{(10)}_{(6)}$		0.77(29)	

TABLE IV. Comparison of various theoretical predictions for the ratios of the form factors $r_V = V(0)/A_1(0)$ and $r_2 = A_2(0)/A_1(0)$ of the weak D^+ and D_s meson transitions to vector mesons with available experimental data.

maximum recoil of the final meson $(q^2 = 0)$ are obtained experimentally

$$r_V = \frac{V(0)}{A_1(0)}, \qquad r_2 = \frac{A_2(0)}{A_1(0)}.$$
 (20)

There have been many measurements and calculations of these ratios. In Table IV we compare some of theoretical

predictions with averaged experimental data from PDG [1] and recent BES III [2–10] measurements. Once again our results agree well with data.

IV. SEMILEPTONIC DECAYS

The differential decay rate of the semileptonic $D_{(s)}$ decays can be expressed in the following form [29]

$$\frac{d\Gamma(D_{(s)} \to F\ell^+ \nu_{\ell})}{dq^2 d(\cos\theta)} = \frac{G_F^2}{(2\pi)^3} |V_{cq}|^2 \frac{\lambda^{1/2} (q^2 - m_{\ell}^2)^2}{64M_{D_{(s)}}^3 q^2} \Big[(1 + \cos^2\theta)\mathcal{H}_U + 2\sin^2\theta\mathcal{H}_L + 2\cos\theta\mathcal{H}_P + \frac{m_{\ell}^2}{q^2} (\sin^2\theta\mathcal{H}_U + 2\cos^2\theta\mathcal{H}_L + 2\mathcal{H}_S - 4\cos\theta\mathcal{H}_{SL}) \Big],$$
(21)

where $\lambda \equiv \lambda(M_{D_{(s)}}^2, M_F^2, q^2) = M_{D_{(s)}}^4 + M_F^4 + q^4 - 2(M_{D_{(s)}}^2 M_F^2 + M_F^2 q^2 + M_{D_{(s)}}^2 q^2)$, m_ℓ is the lepton mass, and the polar angle θ is the angle between the momentum of the charged lepton in the rest frame of the intermediate *W*-boson and the direction opposite to the final *F* meson momentum in the rest frame of $D_{(s)}$. The bilinear combinations \mathcal{H}_I of the helicity components of the hadronic tensor are defined by [29]

$$\mathcal{H}_{U} = |H_{+}|^{2} + |H_{-}|^{2}, \qquad \mathcal{H}_{L} = |H_{0}|^{2}, \qquad \mathcal{H}_{P} = |H_{+}|^{2} - |H_{-}|^{2}, \qquad \mathcal{H}_{S} = |H_{t}|^{2}, \qquad \mathcal{H}_{SL} = \Re(H_{0}H_{t}^{\dagger}), \qquad (22)$$

and the helicity amplitudes are expressed through invariant form factors.

(i) For $D_{(s)} \rightarrow P$ transitions

$$H_{\pm} = 0, \qquad H_0 = \frac{\lambda^{1/2}}{\sqrt{q^2}} f_+(q^2), \qquad H_t = \frac{1}{\sqrt{q^2}} (M_{D_{(s)}}^2 - M_P^2) f_0(q^2). \tag{23}$$

(ii) For $D_{(s)} \rightarrow V$ transitions

$$\begin{split} H_{\pm}(q^2) &= \frac{\lambda^{1/2}}{M_{D_{(s)}} + M_V} \bigg[V(q^2) \mp \frac{(M_{D_{(s)}} + M_V)^2}{\lambda^{1/2}} A_1(q^2) \bigg], \\ H_0(q^2) &= \frac{1}{2M_V \sqrt{q^2}} \bigg[(M_{D_{(s)}} + M_V) (M_{D_{(s)}}^2 - M_V^2 - q^2) A_1(q^2) - \frac{\lambda}{M_{D_{(s)}} + M_V} A_2(q^2) \bigg], \\ H_t &= \frac{\lambda^{1/2}}{\sqrt{q^2}} A_0(q^2). \end{split}$$
(24)



FIG. 4. Comparison of our predictions for the weak $D \rightarrow Ke\nu_e$ and $D \rightarrow \pi e\nu_e$ differential decay rates with experimental data form *BABAR* [26,37] (blue dots with error bars), CLEO [35] (orange dots with error bars), and BES III [2,36] for neutral D^0 (red dots with error bars) and charged D^+ with the account of isospin factor (green dots with error bars).

The expression (21) normalized by the decay rate $d\Gamma/dq^2$, which is obtained by the integration of (21) over $\cos \theta$, can be rewritten as

$$\frac{1}{d\Gamma/dq^2} \frac{d\Gamma(D_{(s)} \to F\ell^+ \nu_\ell)}{dq^2 d(\cos\theta)} = \frac{1}{2} \left[1 - \frac{1}{3} C_F^\ell(q^2) \right] + A_{FB}(q^2) \cos\theta + \frac{1}{2} C_F^\ell(q^2) \cos^2\theta,$$
(25)

where the forward-backward asymmetry is defined by

$$A_{FB}(q^{2}) = \frac{\int_{0}^{1} d(\cos\theta) d\Gamma/d(\cos\theta) - \int_{-1}^{0} d(\cos\theta) d\Gamma/d(\cos\theta)}{\int_{0}^{1} d(\cos\theta) d\Gamma/d(\cos\theta) + \int_{-1}^{0} d(\cos\theta) d\Gamma/d(\cos\theta)} = \frac{3}{4} \frac{\mathcal{H}_{P} - 2\frac{m_{\ell}^{2}}{q^{2}} \mathcal{H}_{SL}}{\mathcal{H}_{\text{total}}},$$
(26)

and lepton-side convexity parameter, which is the second derivative of the distribution (25) over $\cos \theta$, is given by

$$C_F^{\ell}(q^2) = \frac{3}{4} \left(1 - \frac{m_{\ell}^2}{q^2} \right) \frac{\mathcal{H}_U - 2\mathcal{H}_L}{\mathcal{H}_{\text{total}}}.$$
 (27)

Here the total helicity structure

$$\mathcal{H}_{\text{total}} = (\mathcal{H}_U + \mathcal{H}_L) \left(1 + \frac{m_{\ell}^2}{2q^2} \right) + \frac{3m_{\ell}^2}{2q^2} \mathcal{H}_S \quad (28)$$

enters the differential decay distribution (21) integrated over $\cos \theta$

$$\frac{d\Gamma(D_{(s)} \to F\ell^{+}\nu_{\ell})}{dq^{2}} = \frac{G_{F}^{2}}{(2\pi)^{3}} |V_{cq}|^{2} \frac{\lambda^{1/2}(q^{2} - m_{\ell}^{2})^{2}}{24M_{D_{(s)}}^{3}q^{2}} \mathcal{H}_{\text{total}}.$$
(29)

Other useful observables are the longitudinal polarization of the final charged lepton ℓ defined by [29]

$$P_L^{\ell}(q^2) = \frac{(\mathcal{H}_U + \mathcal{H}_L)(1 - \frac{m_{\ell}^2}{2q^2}) - \frac{3m_{\ell}^2}{2q^2}\mathcal{H}_S}{\mathcal{H}_{\text{total}}},\qquad(30)$$

and its transverse polarization [29]

$$P_T^{\ell}(q^2) = -\frac{3\pi m_{\ell}}{8\sqrt{q^2}} \frac{\mathcal{H}_P + 2\mathcal{H}_{SL}}{\mathcal{H}_{\text{total}}}.$$
 (31)

For the decays $D_{(s)} \rightarrow V$ the longitudinal polarization fraction of the final vector meson is given by [29]

$$F_L(q^2) = \frac{\mathcal{H}_L(1 + \frac{m_\ell^2}{2q^2}) + \frac{3m_\ell^2}{2q^2}\mathcal{H}_S}{\mathcal{H}_{\text{total}}},$$
(32)

then its transverse polarization fraction $F_T(q^2) = 1 - F_L(q^2)$.

V. RESULTS AND DISCUSSION

Now we substitute the form factors calculated in Sec. III in the expressions for helicity amplitudes, Eq. (23) and Eq. (24), and then evaluate differential and total decay rates of semileptonic $D_{(s)}$ decays. In Fig. 4 we confront our results with experimental data from *BABAR* [26], CLEO [35] and BESIII [2,36] Collaborations for $D \rightarrow Ke\nu_e$ and $D \rightarrow \pi e\nu_e$ differential decay rates. Good agreement in the whole accessible kinematical range is observed. In Tables V–VII we compare our and previous theoretical predictions [29,30,33] with experimental data from

			Theory	Ex	Experiment	
Decay	Our	[29]	[30]	[33]	PDG [1]	BES III [2,6,9]
$D^+ \to \bar{K}^0 e^+ \nu_e$	9.02	9.28	10.32(93)	$8.12^{(1.19)}_{(1.08)}$	8.73(10)	8.60(16)
$D^+ o ar{K}^0 \mu^+ u_\mu$	8.85	9.02	10.07(91)	$7.98^{(1.16)}_{(1.06)}$	8.76(19)	
$D^0 \to K^- e^+ \nu_e$	3.56	3.63	4.1(4)	$3.20^{(47)}_{(43)}$	3.542(35)	3.505(36)
$D^0 o K^- \mu^+ u_\mu$	3.49	3.53	4.2(4)	$3.10^{(45)}_{(42)}$	3.41(4)	3.413(39)
$D^+ \to \bar{K}^{*0} e^+ \nu_e$	4.87	7.61	7.5(7)	$5.37^{(24)}_{(23)}$	5.40(10)	
$D^+ ightarrow ar{K}^{*0} \mu^+ u_\mu$	4.62	7.21	7.0(7)	$5.10^{(23)}_{(21)}$	5.27(15)	
$D^0 \to K^{*-} e^+ \nu_e$	1.92	2.96	3.0(3)	$2.12^{(9)}_{(9)}$	2.15(16)	2.033(66)
$D^0 \to K^{*-} \mu^+ \nu_\mu$	1.82	2.80	2.8(3)	$2.01^{(9)}_{(8)}$	1.89(24)	

TABLE V. Comparison of various theoretical predictions for the branching ratios (in %) of the CKM-favored $D \rightarrow K^{(*)} \ell \nu_{\ell}$ semileptonic decays with available experimental data.

PDG [1] and recent data from BES III [2–10] Collaboration. We roughly estimate the uncertainties of our calculations to be within 10%. For all decays we find agreement with experimental data within error bars. Only for the $D \rightarrow K^* \ell \nu_{\ell}$ decay branching fractions we obtain somewhat lower central values than the data [1], while Refs. [29,30] give significantly larger values. Thus the precise measurement of these branching fractions is the important test of the models. Recently possible hints of the violation of the lepton universality were found in B decays where deviations from the standard model predictions for the ratios of the semileptonic decay branching fractions involving muon and electron were observed. In Table VIII we give our results for the corresponding ratios of D decays

$$R_F = \frac{\Gamma(D_{(s)} \to F\mu^+\nu_{\mu})}{\Gamma(D_{(s)} \to Fe^+\nu_{e})}$$
(33)

TABLE VI. Comparison of various theoretical predictions for the branching ratios (in 10^{-3}) of the CKM-suppressed D meson semileptonic decays with available experimental data.

		Т	heory		E	xperiment
Decay	Our	[29]	[30]	[33]	PDG [1]	BES III [3,5,10]
$D^+ o \pi^0 e^+ \nu_e$	3.53	2.9	4.1(3)	$3.52_{(38)}^{(45)}$	3.72(17)	3.63(9)
$D^+ o \pi^0 \mu^+ u_\mu$	3.47	2.8	4.1(3)	$3.49^{(45)}_{(38)}$	3.50(15)	3.50(15)
$D^0 \to \pi^- e^+ \nu_e$	2.78	2.2	3.2(3)	$2.78^{(35)}_{(30)}$	2.91(4)	2.95(5)
$D^0 o \pi^- \mu^+ u_\mu$	2.74	2.2	3.2(3)	$2.75^{(35)}_{(30)}$	2.67(12)	2.72(10)
$D^+ \to \rho^0 e^+ \nu_e$	2.49	2.09	2.3(2)	$2.29^{(23)}_{(16)}$	$2.18^{(17)}_{(25)}$	1.860(93)
$D^+ o ho^0 \mu^+ u_\mu$	2.39	2.01	2.2(2)	$2.20^{(21)}_{(16)}$	2.4(4)	
$D^0 \to \rho^- e^+ \nu_e$	1.96	1.62	1.8(2)	$1.81_{(13)}^{(18)}$	1.77(16)	1.445(70)
$D^0 \to \rho^- \mu^+ \nu_\mu$	1.88	1.55	1.7(2)	$1.73^{(17)}_{(13)}$	1.89(24)	
$D^+ \to \eta e^+ \nu_e$	1.24	0.938	1.2(1)	$0.86^{(16)}_{(15)}$	1.11(7)	1.074(98)
$D^+ o \eta \mu^+ u_\mu$	1.21	0.912	1.2(1)	$0.84^{(16)}_{(14)}$		
$D^+ \to \eta' e^+ \nu_e$	0.225	0.200	0.18(2)	()	0.20(4)	0.191(53)
$D^+ o \eta' \mu^+ u_\mu$	0.211	0.190	0.17(2)			
$D^+ \rightarrow \omega e^+ \nu_e$	2.17	1.85	2.1(2)	$1.93^{(20)}_{(14)}$	1.69(11)	2.05(72)
$D^+ ightarrow \omega \mu^+ u_\mu$	2.08	1.78	2.0(2)	$1.85^{(19)}_{(13)}$		

			Theory		Experiment		
Decay	Our	[29]	[30]	[33]	PDG [1]	BES III [4,7,8]	
$\overline{D_s \to \eta e^+ \nu_e}$	2.37	2.24	2.26(21)	$1.27^{(26)}_{(20)}$	2.29(19)	2.323(89)	
$D_s o \eta \mu^+ u_\mu$	2.32	2.18	2.22(20)	$1.25^{(25)}_{(20)}$	2.4(5)	2.42(47)	
$D_s \rightarrow \eta' e^+ \nu_e$	0.87	0.83	0.89(9)	~ /	0.74(14)	0.824(78)	
$D_s o \eta' \mu^+ u_\mu$	0.83	0.79	0.85(8)		1.1(5)	1.06(54)	
$D_s \rightarrow \phi e^+ \nu_e$	2.69	3.01	3.1(3)	$2.53^{(37)}_{(40)}$	2.39(16)	2.26(46)	
$D_s o \eta \mu^+ u_\mu$	2.54	2.85	2.9(3)	$2.40^{(35)}_{(37)}$	1.9(5)	1.94(54)	
$D_s \to K^0 e^+ \nu_e$	0.40	0.20	0.27(2)	$0.390^{(74)}_{(57)}$	0.39(9)	0.325(4)	
$D_s ightarrow K^0 \mu^+ u_\mu$	0.39	0.20	0.26(2)	$0.383^{(72)}_{(56)}$			
$D_s \to K^{*0} e^+ \nu_e$	0.21	0.18	0.19(2)	$0.233^{(29)}_{(30)}$	0.18(4)	0.237(33)	
$D_s \to K^{*0} \mu^+ \nu_\mu$	0.20	0.17	0.19(2)	$0.224^{(27)}_{(29)}$			

TABLE VII. Comparison of various theoretical predictions for the branching ratios (in %) of the D_s meson semileptonic decays with available experimental data.

in comparison with previous predictions [29,30], lattice [27], and experimental data form the BES III Collaboration [3,4,6,34]. We see that the standard model predictions are consistent with current experimental data.

We also calculate the forward-backward asymmetry $A_{FB}(q^2)$ [Eq. (26)], the lepton-side convexity parameter $C_F^{\ell}(q^2)$ [Eq. (27)], the longitudinal $P_L^{\ell}(q^2)$ [Eq. (30)] and transverse $P_T^{\ell}(q^2)$ [Eq. (31)] polarization of the final charged lepton, and longitudinal polarization $F_L(q^2)$ of the final vector meson, Eq. (32). As an example in Figs. 5 and 6 we plot these asymmetries and polarization parameters for $D^+ \to \pi^0 \ell^+ \nu_{\ell}$ and $D^+ \to \bar{K}^{*0} \ell^+ \nu_{\ell}$ decays.

In Table IX we present our predictions for the mean values of the polarization and asymmetry parameters for the semileptonic D and D_s decays. These values were obtained by separately integrating corresponding partial differential decay rates in numerators and the total decay rates in denominators. Since we neglect the small positron mass, for all decays $D^+_{(s)} \rightarrow Ve^+\nu_e$, $\langle P^e_L \rangle = 1$ and $\langle P^e_T \rangle = 0$, while for decays $D^+_{(s)} \rightarrow Pe^+\nu_e$, $\langle A_{FB} \rangle = 0$ and $\langle C^e_F \rangle = -1.5$. Note that in Ref. [29] close values of these parameters were found. Experimentally only the ratios of the partial decay rates of the final vector meson states with longitudinal and

TABLE VIII. Test of the $e - \mu$ lepton flavor universality. Comparison of theoretical predictions for the ratios *R* of the weak *D* and *D*_s meson semileptonic decays with available experimental data.

Decay	Our	[29]	[30]	Lattice [28]	Experiment [3,4,6,34]
$D \to K$	0.980	0.97	0.976	0.975(1)	$\begin{cases} 0.974(14) & K^- \\ 1.00(3) & \bar{K}^0 \end{cases}$
$D \to \pi$	0.985	0.98	1.00	0.985(2)	$\begin{cases} 0.964(40) & \pi^0 \\ 0.922(40) & \pi^- \end{cases}$
$D \to K^*$	0.946	0.95	0.933		
$D \rightarrow \rho$	0.959	0.96	0.957		
$D \rightarrow \eta$	0.976	0.97	1.00		
$D ightarrow \eta'$	0.937	0.95	0.944		
$D \rightarrow \omega$	0.959	0.96	0.952		
$D_s \rightarrow \eta$	0.977	0.97	0.982		1.05(24)
$D_s \to \eta'$	0.952	0.95	0.956		1.14(68)
$D_s \rightarrow \phi$	0.944	0.95	0.936		0.86(29)
$D_s \to K$	0.984	1.00	0.963		
$D_s \to K^*$	0.958	0.95	1.00		







FIG. 6. Polarization and asymmetry parameters for the semileptonic $D^+ \to \bar{K}^{*0} \ell^+ \nu_{\ell}$ decays.

TABLE IX. Predictions for the polarization and asymmetry parameters for the semileptonic D and D_s decays.

Decay	$\langle A_{FB} \rangle$	$\langle C_F^\ell angle$	$\langle P_L^{\ell} \rangle$	$\langle P_T^{\ell} \rangle$	$\langle F_L \rangle$
$\overline{D^+_{(s)} \to Pe^+\nu_e}$	0	-1.5	1	0	
$D^+ \to \bar{K} \mu^+ \nu_\mu$	-0.053	-1.34	0.85	-0.42	
$D^+ \rightarrow \pi^0 \mu^+ \nu_\mu$	-0.040	-1.38	0.89	-0.36	
$D^+ ightarrow \eta \mu^+ u_\mu$	-0.052	-1.34	0.85	-0.40	
$D^+ o \eta' \mu^+ \nu_\mu$	-0.097	-1.20	0.72	-0.56	
$D^+ \rightarrow \bar{K}^{*0} e^+ \nu_e$	-0.22	-0.47	1	0	0.54
$D^+ \rightarrow \bar{K}^{*0} \mu^+ \nu_\mu$	-0.25	-0.37	0.90	-0.15	0.54
$D^+ \rightarrow \rho^0 e^+ \nu_e$	-0.26	-0.42	1	0	0.52
$D^+ o ho^0 \mu^+ u_\mu$	-0.28	-0.34	0.92	-0.12	0.52
$D^+ \rightarrow \omega e^+ \nu_e$	-0.25	-0.39	1	0	0.51
$D^+ \to \omega \mu^+ \nu_\mu$	-0.27	-0.32	0.93	-0.11	0.50
$D_s \rightarrow \bar{K}^0 \mu^+ \nu_\mu$	-0.038	-1.38	0.89	-0.34	
$D_s^+ \rightarrow \eta \mu^+ \nu_\mu$	-0.043	-1.37	0.88	-0.35	
$D_s^+ o \eta' \mu^+ u_\mu$	-0.080	-1.26	0.77	-0.51	
$D_s \rightarrow \bar{K}^{*0} e^+ \nu_e$	-0.26	-0.41	1	0	0.52
$D_s \rightarrow \bar{K}^{*0} \mu^+ \nu_\mu$	-0.29	-0.33	0.92	-0.11	0.51
$D_s \rightarrow \phi e^+ \nu_e$	-0.21	-0.49	1	0	0.54
$D_s ightarrow \phi \mu^+ u_\mu$	-0.24	-0.35	0.90	-0.15	0.54

transverse polarization $\Gamma_L/\Gamma_T = \langle F_L \rangle/(1 - \langle F_L \rangle)$ have been measured for $D^+ \rightarrow \bar{K}^{*0}\ell^+\nu_\ell$ and $D_s \rightarrow \phi \ell^+\nu_\ell$ decays. The experimental values for these ratios are 1.13 ± 0.08 and 0.72 ± 0.18 [1], respectively. The first value is in agreement with our prediction $\Gamma_L/\Gamma_T = 1.16$, while the second one is somewhat smaller than predicted $\Gamma_L/\Gamma_T =$ 1.19. Values of these ratios, close to ours, were obtained in Ref. [29].

VI. CONCLUSIONS

In the framework of the relativistic quark model based on the quasipotential approach we calculated the form factors of the semileptonic D and D_s meson transitions. The relativistic effects including wave function transformations from the rest to moving reference frame and contributions of the intermediate negative energy states were consistently taken into account. This allowed us to reliably calculate the form factors in the whole accessible kinematical range without additional approximations and/or extrapolations. The form factors were expressed through the overlap integrals of the meson wave functions. These wave functions were obtained in our previous study of the heavy-light and light meson spectroscopy. This fact significantly increases self-consistency and reliability of our approach, since in most of the previous quark model studies of semileptonic decays some *ad hoc* form of the wave function (mostly Gaussian) had been used. It was found that our numerical results for the form factors and their q^2 dependence can be well approximated by Eqs. (18) and (19). The parameters of the fit are collected in Tables I, II. The calculated values of the form factors $f_+(q^2)$ for the decays to pseudoscalar mesons and the ratios of the form factors r_2 and r_V for the decays to vector mesons at $q^2 = 0$ agree within errors with the experimental data (see Tables III, IV). The form factors $f_+(q^2)$ of the weak *D* transitions to pseudoscalar *K* and π mesons agree well with data form Belle [25] and *BABAR* [26] Collaborations in the whole q^2 range.

These form factors were applied for the calculation of differential and total decay rates of semileptonic decays of D and D_s using the helicity formalism. The obtained differential decay distributions (29) for D decays to pseudoscalar mesons, plotted in Fig. 4, agree with experimental data from the BABAR [26] and CLEO [35] Collaborations. The calculated total branching fractions are also in good agreement with averaged experimental values from PDG [1] and recent data from the BES III Collaboration [2-10]. To test lepton universality in the semileptonic $D_{(s)}$ meson decays we calculated the ratios of the branching fractions of decays involving muons to the ones involving positrons. These ratios are collected in Table VIII in comparison with other theoretical predictions and available experimental data. Within current experimental accuracy no deviations of data from the standard model predictions are found. We also calculated the forward-backward asymmetries, the lepton and vector meson longitudinal and transverse polarization parameters which can be measured in future experiments. Their mean values are collected in Table IX.

We presented the detailed comparison of our results with other theoretical predictions. In most cases better agreement of our values with data is found. The further increase of the experimental accuracy and new measurements can help to better understand quark dynamics in mesons. This could be realized in the future super tau-charm facility [38–40] which is hotly discussed to construct.

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