Probing the single-particle behavior above ¹³²Sn via electromagnetic moments of ^{133,134}Sb and N = 82 isotones

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Magnetic and quadrupole moments of the $7/2^+$ ground state in ¹³³Sb and the (7⁻) isomer in ¹³⁴Sb have been measured by collinear laser spectroscopy to investigate the single-particle behavior above the doubly magic nucleus ¹³²Sn. The comparison of experimental data of the $7/2^+$ states in ¹³³Sb and neighboring N = 82 isotones to shell-model calculations reveals the sensitivity of magnetic moments to the splitting of the spin-orbit partners $\pi 0g_{9/2}$ and $\pi 0g_{7/2}$ across the proton shell closure at Z = 50. In contrast, quadrupole moments of the N = 82isotones are insensitive to cross-shell excitations, but require the full proton model space from Z = 50 to 82 for their accurate description. In fact, the linear trend of the quadrupole moment follows approximately the expectation of the seniority scheme when filling the $\pi 0g_{7/2}$ orbital. As far as the isomer in ¹³⁴Sb is concerned, its electromagnetic moments can be perfectly described by the additivity rule employing the moments of ¹³³Sb and ¹³³Sn, respectively. These findings agree with shell-model calculations and thus confirm the weak coupling between the valence proton and neutron in ¹³⁴Sb.

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I. INTRODUCTION

Out of the over 3000 atomic nuclei discovered so far [1], only about ten represent nuclides with closed nuclear shells for both protons and neutrons. Such rare exemplars, found at the traditional shell closures Z, N = 2, 8, 20, 28, 50, 82, or 126, are called doubly magic nuclei. Studies of electromagnetic moments in the region of doubly magic species contribute to our understanding of the single-particle behavior predicted by the spherical shell model [2]. In particular, magnetic dipole moments of systems with one proton on top of a doubly magic nucleus are sensitive to meson-exchange currents and core-polarization effects [3,4]. Moreover, deviations from the Schmidt moment give insights on the purity of the nuclear configuration, while electric quadrupole moments allow the investigation of second order core-polarization effects. Examples for this can be found in 41,49 Sc [4] (40,48 Ca

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core + proton), 57,69 Cu [5] (56,68 Ni core + proton), and 209 Bi [2] (208 Pb core + proton).

Another candidate is ¹³³Sb with one proton in the $0g_{7/2}$ orbital on top of a doubly magic ¹³²Sn core. Despite its large N/Z ratio, the magicity of ¹³²Sn was already firmly established [6,7]. The latest evidence was added by laser spectroscopy of ¹³³Sn [8], where it was shown that the electromagnetic moments of ¹³³Sn can be well described in a single-particle picture with the valence neutron in the $1f_{7/2}$ orbital on top of a ¹³²Sn core. A similar behavior would be expected for ¹³³Sb. However, in Ref. [3] a rather large deviation of the magnetic moment to the Schmidt value was found. This deviation was mainly attributed to meson-exchange currents and first-order core-polarization effects. The latter is driven by the interplay of the spin-orbit partners $0g_{9/2}$ and $0g_{7/2}$ $(j = l \pm s)$, which belong to different shells. An analogous behavior was observed in the equivalent magnetic moment of ²⁰⁹Bi with a $0h_{9/2}$ proton above a doubly magic ²⁰⁸Pb core [2].

As a missing piece in this discussion, the quadrupole moment of ¹³³Sb has not been measured so far. Furthermore, electromagnetic moments of 134 Sb (132 Sn + one proton + one neutron) would allow for further investigations of singleparticle behavior above ¹³²Sn, but are unknown. The objective of this research was therefore to measure the electromagnetic moments of ^{133,134}Sb with high-resolution laser spectroscopy in order to establish a better understanding of nuclear structure above the doubly magic nucleus ¹³²Sn. To extend the perspective, electromagnetic moments of N = 82 isotones above ¹³³Sb with protons filling the $0g_{7/2}$ orbital are included in the discussion. Simple trends in this isotonic chain can be qualitatively explained by using the seniority scheme for a single-orbital configuration. Additionally, state-of-the-art shell-model calculations are employed for a more quantitative understanding. This theoretical work illustrates the importance of proton excitations across the Z = 50 shell for the description of magnetic moments in N = 82 isotones, while they do not affect quadrupole moments. On the other hand, the moments of the (7^-) isomer in ¹³⁴Sb are well described by the weak coupling of the valence proton of ¹³³Sb and the valence neutron of ¹³³Sn, providing a textbook example for the additivity rule of electromagnetic moments.

II. EXPERIMENT

The experiment was carried out at the radioactive ion-beam facility ISOLDE-CERN [9], where a large variety of nuclides can be synthesized by high-energy protons impinging on a thick target. Neutron-rich antimony (Sb) isotopes were formed by directing the protons onto a neutron converter to enable neutron-induced fission in a UCx target. Subsequently, atomic Sb was selectively laser ionized [10], electrostatically accelerated to 50 keV, magnetically mass separated, and accumulated in a buffer-gas filled radio-frequency quadrupole cooler and buncher [11]. Cooled ion bunches with a temporal width of around 6 μ s were reaccelerated to 50 keV and sent to the COLLAPS beam line [12]. There, the ions were neutralized in a potassium-filled charge-exchange cell [13,14] as depicted in Fig. 1. The atom bunches were then collinearly overlapped with a narrow-band continuous-wave laser beam. Photons,



FIG. 1. Overview of the COLLAPS setup. The laser system consisted of a Ti:Sa laser, pumped by a frequency-doubled Nd:YAG laser and producing infrared light at 870 nm. The fundamental light was frequency doubled twice to obtain a laser beam at 217 nm. Ion bunches (red) from ISOLDE were guided by a 10° electrostatic bender onto the axis of the experiment, where they were collinearly overlapped with the laser beam. In the reacceleration region, consisting of four ring electrodes and the charge-exchange cell (CEC), the final beam energy was defined. Ions were neutralized in the CEC by collisions with potassium. The atom bunches (blue) then entered the optical detection region. Photons, emitted after resonant excitation of the Sb atoms, were collected by pairs of aspheric lenses and detected by four photomultiplier tubes (PMTs).

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In order to obtain hyperfine spectra, the Doppler-shifted laser frequency observed by the fast moving atoms was scanned by varying the floating voltage of the chargeexchange cell, which resulted in a change of the longitudinal atom velocity. Hence, the laser frequency in the laboratory frame was kept fixed, which enabled faster measurements together with an increased laser stability. The large beam energy E, compared to its energy spread $\delta E \approx 3$ eV, allowed high-resolution measurements of the hyperfine structure with linewidths of about 80 MHz by minimizing the Doppler broadening according to $\delta v_D \propto \delta E / \sqrt{E}$ [15]. Furthermore, the bunched structure of the beam allowed a gate to be placed on the photon counting for the time when the atoms passed the photon detectors, vastly improving the signal-to-background ratio compared to continuous beams [16,17]. More details on collinear laser spectroscopy as performed at COLLAPS can be found in the review of Neugart et al. [12].

The atomic transition $5s^25p^3 \, {}^4S_{3/2} \rightarrow 5s^25p^26s \, {}^4P_{3/2}$ was probed by laser light at a wavelength of 217 nm. This transition was chosen due to its sensitivity to magnetic and quadrupole moments. The laser beam was produced by frequency quadrupling the fundamental of a Ti:Sa laser (Matisse 2TS) as depicted in Fig. 1. The stability of the laser frequency was ensured by locking the fundamental of the Ti:Sa laser to a wavelength meter (HighFinesse WSU10). The wavelength meter itself was calibrated once per minute to a diode laser (Toptica DLPRO780), which was locked to the $F = 2 \rightarrow$ F = 3 hyperfine transition of the D1 line in ⁸⁷Rb. Hyperfine spectra of ^{133,134}Sb have been recorded for the

Hyperfine spectra of ^{133,134}Sb have been recorded for the first time, in addition to those of the two stable Sb isotopes, ^{121,123}Sb. The resulting data were analyzed with the SATLAS package [18], using χ^2 minimization fitting routines. The line

shape is described by a Voigt profile and two side peaks. These account for the asymmetric peak shape caused by energy losses during the charge-exchange process [19,20]. As fitting function, Eqs. (20) and (21) from Ref. [18] were employed, which yielded the hyperfine parameters *A* and *B* for the lower (1) and upper (u) level of the transition. Since B_1 is too small to be properly resolved, the ratio of B_u/B_1 was determined for the two stable isotopes ¹²¹Sb and ¹²³Sb by varying B_1 within the range of the high-precision values obtained from atomic-beam measurements [21]. This yielded a *B* ratio of 128.6(6), which was set as a constraint for the analysis of ¹³³Sb and ¹³⁴Sb. The hyperfine peak intensities were free parameters in the fit.

The magnetic moment μ and the quadrupole moment Q were derived from the hyperfine parameters with respect to the reference isotope ¹²³Sb:

$$\mu = \mu_{\rm ref} \frac{AI}{A_{\rm ref} I_{\rm ref}} \tag{1}$$

and

$$Q = Q_{\rm ref} \frac{B}{B_{\rm ref}}.$$
 (2)

This approach is valid as long as the ratios between the hyperfine values from lower and upper levels are constant. This holds true for *B*, but is not fully correct for magnetic moments. Variations of the magnetic hyperfine field over the nuclear volume can cause a hyperfine anomaly (hfa) Δ , which can have a sizable influence on the magnetic moment. Thus, Eq. (1) has to be adapted to

$$\mu = \mu_{\rm ref} \frac{AI}{A_{\rm ref} I_{\rm ref}} (1 + \Delta). \tag{3}$$

For the atomic ground state of antimony the hfa was measured to be ${}^{121}\Delta^{123} = -0.317(3)\%$ [21] and, therefore, cannot be neglected. To evaluate the approximate size of the hfa for ¹³³Sb and ¹³⁴Sb with respect to ¹²³Sb, simple calculations using a single-particle model [22,23] were carried out. These calculations showed that the hfa is negligible between ¹²³Sb and ¹³³Sb, as expected since both isotopes are in the same nuclear configuration. On the other hand, the calculations yield $^{134}\Delta^{123}\approx 0.5\%$, which has to be taken into account. Since more reliable calculations are difficult and not available at the moment, the hfa from the single-particle model is presently the best estimate. In order to account for uncertainties in this single-particle model, the derived hfa value is multiplied by a factor of 2 (thus, $^{134}\Delta^{123} \approx 1\%$) and added as a systematic uncertainty to the magnetic moment of ¹³⁴Sb, which is calculated via Eq. (1). These calculations are only valid for the lower atomic level. Therefore, only A_1 is used for the determination of the magnetic moments except for ¹³³Sb, where we expect a negligible hfa with respect to ¹²³Sb in the upper atomic level as well. Details about the hfa calculations can be found in Ref. [24].

III. RESULTS

The spectra of ¹³³Sb and ¹³⁴Sb are shown in Fig. 2 together with their hyperfine level scheme. For ¹³³Sb, the spin of the ground state was already firmly assigned to I = 7/2



FIG. 2. Hyperfine spectra of (a) the $7/2^+$ ground state of ¹³³Sb and (b) the (7^-) isomeric state of ¹³⁴Sb in the $5s^25p^3$ ${}^{4}S_{3/2} \rightarrow 5s^25p^26s$ ${}^{4}P_{3/2}$ transition. The frequency is given as an offset from the transition frequency [45 945.340(5) cm⁻¹ [25]]. A scheme of the hyperfine splitting is shown, where each arrow is connected to one hyperfine peak. While the spectrum of ¹³³Sb was obtained in around 10 min, the one for ¹³⁴Sb took more than 7 h due to large isobaric contamination.

[26] and is a consequence of the valence proton in the $0g_{7/2}$ orbital. This assignment is confirmed by this paper as shown in Fig. 3(a). For ¹³⁴Sb, only the isomer was observed in the hyperfine spectrum [see Fig. 2(b)]. This isomer is tentatively assigned with a spin/parity of $I^{\pi} = (7^{-})$ and has an excitation energy of 0.279(1) MeV [27]. In the present analysis it was not possible to make a distinction between I = 7, 8, and 9 [see Fig. 3(b)], while all other spins can be excluded. However, in order to have I = 8, the isomer of ¹³⁴Sb would need to be in a $\pi 0g_{7/2} \times \nu 0h_{9/2}$ configuration. This seems improbable since



FIG. 3. Spin assignment of (a) ¹³³Sb and (b) ¹³⁴Sb with the χ^2 from the data fit as a function of the nuclear spin. In the case of ¹³³Sb, only I = 7/2 gives a reasonable result. On the other hand, ¹³⁴Sb has a minimum at I = 8, but the difference in χ^2 to I = 7 and I = 9 is too small for a clear spin assignment. However, other possibilities can be excluded. See text for details.

in ¹³³Sn the $\nu 0h_{9/2}$ state is around 1.5 MeV above the $\nu 1f_{7/2}$ ground state. Similar arguments can be made to exclude I = 9. Therefore, I = 7 is used in the following as also suggested in Ref. [27] and supported by shell-model calculations in this paper, which place the 7⁻ and 8⁻ states at excitation energies of 0.408 and 1.67 MeV, respectively.

Table I gives an overview of the measured hyperfine parameters in comparison to known values in literature of the stable isotopes ¹²¹Sb and ¹²³Sb. With the exception of A_1 in ¹²³Sb, excellent agreement with literature is obtained. A difference of close to 2σ in $A_1(^{123}Sb)$ is seen. We found no obvious reason to doubt our result or the literature value. In the end, the 2σ level does not exclude compatibility of the two results. Due to its higher precision, A_1 from Ref. [21] is used as reference value to derive the magnetic moments.

From the values of the hyperfine parameters in Table I, together with reference values from ¹²³Sb of $A_1 = -162.451(3)$ MHz [21] and $B_u = -602.9(7)$ MHz (from this paper) the magnetic and quadrupole moments of ¹²¹Sb, ¹³³Sb, and ¹³⁴Sb given in Table II were calculated via Eqs. (1) and (2), respectively. In the case of ¹³³Sb, the magnetic moment was additionally derived for the upper atomic level and the weighted average from both levels is given in Table II. Correlations between A_1 and A_u of ¹³³Sb, which would slightly reduce the

uncertainty, are not included in our final value. The very good agreement with literature in stable ¹²¹Sb gives confidence in our results for the radioactive isotopes. Note that the uncertainty on Q_{ref} , taken from Ref. [30], limits the precision of the new quadrupole moments in this paper.

There is a small discrepancy between the magnetic moment of ¹³³Sb from this paper and from Ref. [3], in which Stone *et al.* employed the technique of nuclear magnetic resonance on oriented nuclei, which relies on the magnetic hyperfine field $B_{\rm hf}$ of Sb in iron. They used $B_{\rm hf}$ from Ref. [33], while applying $B_{\rm hf}$ from Ref. [34] would yield good agreement with our value. Hence, the deviation arises most probably from the choice of $B_{\rm hf}$.

IV. THEORY

A. Single-particle considerations

For a single-orbital configuration with n protons, the magnetic moment of seniority-1 states can be written as [2,35]

$$\mu(j^{n}) = \mu_{\rm SP} = \begin{cases} (j - \frac{1}{2})g_{l} + \frac{g_{s}}{2}, & \text{for } j = l + \frac{1}{2}, \\ \frac{j}{j+1} [(j + \frac{3}{2})g_{l} - \frac{g_{s}}{2}], & \text{for } j = l - \frac{1}{2}, \end{cases}$$
(4)

where *j* and *l* are the total and orbital angular momenta characterizing the orbital and μ_{sp} stands for the single-particle moment, also called the Schmidt value. The quantities g_l and g_s are the orbital and spin *g* factors for the proton, the free values of which are $g_l = 1$ and $g_s = 5.585\,694\,689\,3(16)$ [36,37], respectively. However, effective *g* factors are usually introduced in single-*j* as well as in multi-*j* models to account for the neglected degrees of freedom, namely, the remaining nucleons as well as the truncation of the Hilbert space.

In the case of odd-odd nuclei with a rather pure configuration (e.g., 134 Sb), the magnetic moment of a state with spin *I* can be estimated from a composition of the valence proton and neutron via an additivity rule [2]:

$$\mu(I) = \frac{I}{2} \left[\frac{\mu(I_{\pi})}{I_{\pi}} + \frac{\mu(I_{\nu})}{I_{\nu}} + \left(\frac{\mu(I_{\pi})}{I_{\pi}} - \frac{\mu(I_{\nu})}{I_{\nu}} \right) \frac{I_{\pi}(I_{\pi} + 1) - I_{\nu}(I_{\nu} + 1)}{I(I + 1)} \right].$$
(5)

TABLE I. Hyperfine parameters of 121,123,133,134 Sb in comparison to literature. Note that B_1 of 121,123 Sb was fixed to the given literature values, while B_1 of 133,134 Sb was derived from B_u using the *B* ratio $B_u/B_1 = 128.6(6)$, which was fixed in the fit of the hyperfine spectra. See text for details.

	A ₁ (MHz)		A _u (MHz)		B_1 (MHz)		B _u (MHz)		
	I^{π}	Exp.	Lit.	Exp.	Lit.	Exp.	Lit.	Exp.	Lit.
¹²¹ Sb	$5/2^{+}$	-299.0(3)	-299.034(4) [21]	523.8(5)	519(6) [28]		-3.68(2) [21]	-471.1(17)	-480(15) [28]
¹²³ Sb	$7/2^+$	-162.59(8)	-162.451(3) [21]	285.20(8)	282(4) [<mark>29</mark>]		-4.67(3) [21]	-602.9(7)	
¹³³ Sb	$7/2^+$	-196.1(2)		343.7(2)		-2.06(2)		-265(2)	
¹³⁴ Sb	(7-)	-55.7(3)		97.7(3)		-3.24(5)		-416(7)	

TABLE II. Magnetic and quadrupole moments of ^{121,123,133,134}Sb in comparison to literature where ¹²³Sb was used as reference to derive the moments of the other isotopes. Literature magnetic moments given here are obtained from a reevaluation in Ref. [31], while the second citation indicates the original measurement.

	I^{π}	$\mu_{\mathrm{exp}}~(\mu_{\mathrm{N}})$	$\mu_{ m lit}~(\mu_{ m N})$	$Q_{\exp}(b)$	$Q_{ m lit}(b)$
¹²¹ Sb	$5/2^{+}$	3.357(4) ^a	3.3580(16) [31,32]	-0.541(11)	-0.543(11) [30]
¹²³ Sb	$7/2^+$		2.5457(12) [31,32]		-0.692(14) [30]
¹³³ Sb	$7/2^+$	3.070(2)	3.00(4) [3,31]	-0.304(7)	
¹³⁴ Sb	(7-)	1.745(8){17} ^b		-0.477(12)	

^aA hyperfine anomaly of -0.317(3)% from Ref. [21] is included in this value.

^bThe calculated hyperfine anomaly is denoted in curly brackets as a systematic uncertainty.

For ¹³⁴Sb as an example, the proton and neutron moments are taken from ¹³³Sb and ¹³³Sn, respectively.

Similar considerations can be made for quadrupole moments. For a single-j orbital containing n particles, the quadrupole moment of seniority-1 states can be expressed as [35,38]

$$Q(j^n) = Q_{\rm SP} \frac{2j+1-2n}{2j-1},$$
(6)

with the single-particle moment Q_{sp} . Hence, in the extreme single-*j* configuration picture a linear trend in quadrupole moments is expected as a function of *n*. It reverses its sign at the middle of the orbital and for n = 2j has the opposite sign, but the same absolute value as for n = 1. This behavior differs from that of the magnetic dipole moment which is independent of *n* in absolute magnitude as well as in sign. Analogously to the magnetic moments, an additivity rule can be applied for odd-odd nuclei in the weak coupling regime [2]:

$$Q(I) = \begin{pmatrix} I & 2 & I \\ -I & 0 & I \end{pmatrix} (-1)^{I_{\pi}+I_{\nu}+I} (2I+1) \\ \times \left[\begin{cases} I_{\pi} & I & I_{\nu} \\ I & I_{\pi} & 2 \end{cases} \frac{Q(I_{\pi})}{\begin{pmatrix} I_{\pi} & 2 & I_{\pi} \\ -I_{\pi} & 0 & I_{\pi} \end{pmatrix}} + \begin{cases} I_{\nu} & I & I_{\pi} \\ I & I_{\nu} & 2 \end{cases} \frac{Q(I_{\nu})}{\begin{pmatrix} I_{\nu} & 2 & I_{\nu} \\ -I_{\nu} & 0 & I_{\nu} \end{pmatrix}} \right],$$
(7)

involving Wigner-3*j* and Wigner-6*j* symbols.

B. Large-scale shell-model calculations

For a more detailed theoretical understanding we have performed realistic shell-model calculations using the shell-model code KSHELL [39].

As shown in Fig. 4 in magenta, we consider ¹³²Sn as a closed core and let the valence protons occupy the singleparticle orbitals of the Z = 50-82 shell $(0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, \text{ and } 0h_{11/2})$ and let the valence neutrons occupy the orbitals of the N = 82-126 shell $(0h_{9/2}, 1f_{7/2}, 1f_{5/2}, 2p_{3/2}, 2p_{1/2}, \text{ and } 0i_{13/2})$. The starting point to construct the effective shell-model Hamiltonian is the high-precision CD-Bonn nucleon-nucleon potential [40] for which the short-range repulsive components are renormalized by the so-called Vlow-k approach [41]. This method provides a smooth potential that preserves the low-energy physics properties of the original potential and can be employed directly in the many-body perturbation theory. The low-momentum potential, with the addition of the Coulomb force for protons, is then used to derive the two-body matrix elements for the effective Hamiltonian within the framework of the \hat{Q} -box-plus-foldeddiagram method [42,43], where the \hat{Q} box is expressed as a perturbative diagrammatic expansion. The one-body matrix elements of the Hamiltonian, instead, have been taken, where possible, from experiment (see Ref. [8] for details). This effective Hamiltonian was already applied in several other studies of neutron-rich nuclei beyond ¹³²Sn [44].

For the calculation of the electromagnetic moments, we have adopted microscopic as well as empirical effective operators. The microscopic *M*1 and *E*2 effective operators are constructed consistently with the derivation of the effective Hamiltonian, by resorting to the Suzuki-Okamoto formalism [45,46], an extension of the \hat{Q} -box-plus-folded-diagram approach for transition operators. For the empirical magnetic dipole operator, we have used $g_l^{\text{eff}}(p) = 1.18$, $g_l^{\text{eff}}(n) = 0$, and $g_s^{\text{eff}}(p, n) = 0.7g_s(p, n)$, where the effective neutron g factors are taken from Ref. [8] while the proton ones are fixed so as to reproduce the magnetic moment of the $7/2^+$ state in ¹³³Sb. For the electric quadrupole operator, the same effective charges of Ref. [8], $e_p^{\text{eff}} = 1.7e$ and $e_n^{\text{eff}} = 0.7e$, have been adopted.



FIG. 4. Relevant orbitals for the shell-model calculations. The model space for calculations with a ¹³²Sn core is indicated by magenta rectangles, while the one for a ⁸⁸Sr core is defined by green rectangles.

In order to check the dependence of the theoretical moments of N = 82 isotones with respect to proton core excitations, we have also performed calculations by assuming $^{88}_{38}$ Sr as core, with a valence space spanned by the proton orbitals $1p_{1/2}$, $0g_{9/2}$, $0g_{7/2}$, and $1d_{5/2}$ and the neutron orbitals $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, and $0h_{11/2}$ (see Fig. 4). In this case, the effective Hamiltonian is taken from Ref. [47], where all details about its derivation can be found. Here, we only mention that it is obtained through the double-step procedure outlined in Ref. [47]. Namely, by following the procedure described above, we have derived the effective Hamiltonian in a larger model space including the $1d_{3/2}$, $2s_{1/2}$, and $0h_{11/2}$ proton orbitals. Then, by applying a unitary transformation we have computed the Hamiltonian in the truncated space. In the calculations with ⁸⁸Sr as a core, we have used only empirical effective operators. The adopted values for the proton g factors are $g_l^{\text{eff}}(p) = 1.065$ and $g_s^{\text{eff}}(p) = 0.7g_s(p)$ and for the proton effective charge $e_p^{\text{eff}} = 1.6e$, neutrons being ineffective since the corresponding model space is completely filled.

V. DISCUSSION

A. N = 82 isotones

Figure 5 shows magnetic and quadrupole moments of the $7/2^+$ state in ¹³³Sb and higher mass isotones. As already discussed in Ref. [3], there is a large offset of the magnetic moment of ¹³³Sb to the Schmidt moment as shown in Fig. 5(a). This discrepancy was attributed to meson-exchange currents and first-order core polarization. By including higher mass isotones in the discussion, a trend towards the Schmidt value with increasing Z up to La is observed. A similar trend is known to exist for N = 126 isotones above Pb with protons occupying the $0h_{9/2}$ orbital [2]. There, the spin-orbit partners $0h_{9/2}$ and $0h_{11/2}$ are also stretched across the shell closure, analogously to the $0g_{7/2}$ and $0g_{9/2}$ orbitals in the N = 82isotones discussed here (see Fig. 4). The decrease in magnetic moments towards the Schmidt moment with increasing Z could be qualitatively explained in part by Pauli blocking of core polarization. An increased occupancy of the $0g_{7/2}$ orbital reduces the probability of core excitations from $0g_{9/2}$ to $0g_{7/2}$ and thus limits the effect from core polarization. ¹⁴¹Pr does not seem to follow the general trend, but its experimental uncertainty is too large for firm conclusions.

From the comparison between the experimental magnetic moments and the shell-model results based on a ¹³²Sn core, two observations can be made. First, the microscopic *M*1 operator yields results closer to the Schmidt line deviating by a factor of about 1.5 to experimental values. This might be a consequence of the strong renormalization of the *M*1 operator induced by the missing spin-orbit partner of the $0g_{7/2}$ orbital, which is only partially taken into account by our perturbative calculation. This hypothesis is supported by the magnetic moment of the $I^{\pi} = 5/2^+$ ground state in ¹⁴¹Pr in Fig. 5(c), the structure of which is mostly unrelated to a $j = l \pm s$ configuration across a closed shell. Here, the calculation with the microscopic *M*1 operator agrees with experiment within 5%. Moreover, the magnetic moment of ¹³³Sn was also accurately reproduced with the same calculation method [8].



FIG. 5. (a) Magnetic and (b) quadrupole moments of ¹³³Sb and its higher mass isotones in the $7/2^+$ state compared to shell-model calculations using once a ¹³²Sn core with microscopic (¹³²Sn_{micro}) and empirical (¹³²Sn_{emp}) effective *M*1 and *E*2 operators and once a ⁸⁸Sr core with empirical operators (⁸⁸Sr_{emp}). The solid line indicates the Schmidt moment of the (a) $0g_{7/2}$ and (c) $d_{5/2}$ orbital. (b) The linear fit (solid gray line) was obtained solely from experimental data. (c, d) Electromagnetic moments of the $5/2^+$ ground state of ¹⁴¹Pr. Note the different scaling on the *y* axis here. Literature values taken from Refs. [3,31,48–57].

The second observation from Fig. 5(a) is that the theoretical results follow approximately a horizontal line over proton number, which does not reflect the experimental trend. In order to understand if this discrepancy arises from proton excitation across the Z = 50 shell closure, additional calculations with ⁸⁸Sr as a core were carried out, which include the $\pi 0g_{9/2}$ orbital (see again Fig. 4). As shown in Fig. 5(a), the obtained magnetic moments follow closely the experimental trend, indicating that contributions arising from the $0g_{9/2}$ orbital are essential to reproduce the experimental behavior up to La. The number of proton holes in the $0g_{9/2}$ orbital decreases slightly from Sb to Pr as visualized in Fig. 6. Note that holes in the $0g_{9/2}$ orbital are only shown indirectly by a proton occupation above the nominal occupation number. Despite their very small share of the proton occupation, these holes can have a big impact on magnetic moments due to the large value of the $\langle 0g_{9/2}|M1|0g_{7/2}\rangle$ matrix element. However, calculations with ⁸⁸Sr as a core do not reproduce the rise of



FIG. 6. Proton occupation of the orbitals in the Z = 50-82 shell for the yrast $7/2^+$ state in N = 82 isotones, obtained from ¹³²Sncore and ⁸⁸Sr-core calculations. Values above the nominal proton occupation (1, 3, 5, 7, and 9) for the ⁸⁸Sr core arise due to holes in the $0g_{9/2}$ orbital. Note that the model space of the ⁸⁸Sr calculations does not include orbitals above $1d_{5/2}$. Contributions from the $2s_{1/2}$ and $1p_{1/2}$ orbitals are negligible for ¹³²Sn and ⁸⁸Sr as core, respectively, and are thus not shown in the figure. In the employed model spaces the neutron orbitals are empty (full) and can be therefore neglected.

the magnetic moment in Pr. This is most likely due the omission of the $0h_{11/2}$ orbital, which gives a positive contribution as observed in calculations with ¹³²Sn as core (see Figs. 5 and 6).

The experimental results for the quadrupole moments of the N = 82 isotones are shown in Fig. 5(b). They follow a linearly increasing trend over proton number, which can be easily explained by the seniority scheme [see Eq. (6)]. Around closed shells, an orbital occupied by a single particle is expected to result in a negative quadrupole moment and thus an oblate shape. This is indeed the case for ¹³³Sb with a single proton in the $0g_{7/2}$ orbital, while the neutron shell is closed. By filling up the $0g_{7/2}$ orbital, the nucleus should transform to a prolate shape with positive quadrupole moment and cross Q = 0 (i.e., spherical shape) at a half-filled orbital. Hence, the trend seen in Fig. 5(b) can be described qualitatively in such a simplistic picture, indicated by a linear fit. Similar trends have been observed for instance along the $v0f_{7/2}$ and $v1p_{3/2}$ orbitals of Ca [58] as well as the $v0h_{11/2}$ and $v1d_{3/2}$ orbitals in Cd [59] and Sn [60], respectively.

The quadrupole moments calculated within the shell model using a ¹³²Sn core are in very good agreement with the experiment, where microscopic and empirical operators deliver equally good results. This shows that, in contrast to the microscopic *M*1 operator, the *E*2 operator seems to be insensitive to the explicit presence of the $0g_{9/2}$ orbital in the model space. The fact that the linear fit in Fig. 5(b) does not cross Q = 0in the middle of the $0g_{7/2}$ can be attributed to contributions from higher-lying orbitals. These may be seen by looking at the proton occupation as shown in Fig. 6. The $0g_{7/2}$ orbital includes the major share of the proton configuration, but, with increasing *Z*, contributions from $1d_{5/2}$ and $0h_{11/2}$ also rise.





FIG. 7. Magnetic and quadrupole moments of ¹³⁴Sb compared to shell-model calculations with ¹³²Sn as core and the additivity rule [Eq. (5)]. For the additivity rule, the experimental moments from ¹³³Sb and ¹³³Sn [8] were taken for the proton and neutron contribution, respectively.

The occupation of the $0g_{7/2}$ orbital increases only slightly in ^{141m}Pr compared to ¹³⁹La while higher-lying orbitals receive the largest portion of the additional two protons. Therefore, the quadrupole moment of the $I^{\pi} = 7/2^+$ isomer in ¹⁴¹Pr is quite close to that of the ground state in ¹³⁹La, which is dominated by a one-hole configuration in the $0g_{7/2}$ orbital.

Unlike the magnetic moments, the ⁸⁸Sr-core calculations lead to larger discrepancies between theoretical and experimental quadrupole moments as shown in Fig. 5(b). In particular, they do not reproduce the correct sign for heavier isotones. From the analysis of the wave functions we have found that the missing contributions from $1d_{3/2}$, $2s_{1/2}$, and $0h_{11/2}$ orbitals, which are outside the model space of our ⁸⁸Sr calculations, are responsible for such discrepancies. Actually, a more complete description of the nuclei under investigation would require the inclusion of these proton orbitals in addition to the ones below the Z = 50 shell closure. This would lead to quite a large model space, which is very demanding from a computational point of view.

B. ¹³⁴Sb

In a simplistic view, ¹³⁴Sb can be understood as the sum of the valence proton of ¹³³Sb in the $0g_{7/2}$ orbital and the valence neutron of ¹³³Sn in the $1f_{7/2}$ orbital. Indeed, as shown in Fig. 7, the additivity rules for the electromagnetic moments [see Eqs. (5) and (7)] provide a very good agreement with the new experimental data for the $I = (7^-)$ isomer in ¹³⁴Sb when the measured moments of ¹³³Sb and ¹³³Sn are used.

Evidence of the validity of the additivity rule emerges also from shell-model calculations (performed with a ¹³²Sn core). These predict for the 7⁻ state the dominant $\pi 0g_{7/2} \times \nu 1f_{7/2}$ maximally aligned configuration, which accounts for almost 100% of the wave function. Consequently, the theoretical dipole and quadrupole moments of this state, obtained with both empirical and microscopic operators, are very close to those computed by summing the theoretical moments of the $7/2^+$ state in ¹³³Sb and of the $7/2^-$ state in ¹³³Sn. As shown in Fig. 7, shell-model calculations reproduce very well the experimental magnetic moment of the (7⁻) isomeric state in ¹³⁴Sb once the empirical *g* factors are adopted, while a good agreement with the experiment is obtained for the quadrupole moments by using both empirical and microscopic approaches, deviating by 15% at most.

Applying the additivity rule to the calculated magnetic moments from the microscopic approach of ¹³³Sb and ¹³³Sn yields almost the same value for ¹³⁴Sb as the shell-model calculations. Hence, the large underestimation of the experimental magnetic moment by the microscopic approach with a factor >2 is solely caused by the disagreement of the calculated moment of ¹³³Sb [see Fig. 5(a)].

VI. CONCLUSION

Electromagnetic moments of the $7/2^+$ ground state in ¹³³Sb and the (7⁻) isomer ¹³⁴Sb were determined by collinear laser spectroscopy. These observables provide new insights into single-particle behavior of nuclides above the doubly magic nucleus ¹³²Sn.

Experimental quadrupole moments of N = 82 isotones with a valence proton in the $0g_{7/2}$ orbital follow a linear trend with respect to the proton number up to ¹³⁹La. Therefore, they represent a textbook example for the application of the seniority scheme to the filling of the $0g_{7/2}$ orbital. In contrast, experimental magnetic moments, in addition to being rather far from the Schmidt values, deviate from the constant behavior predicted by the seniority scheme, thus indicating the sensitivity to more complex phenomena.

In order to interpret the data, we have performed shellmodel calculations with ¹³²Sn as a closed core and employed microscopic and empirical operators for computing electromagnetic properties. The theoretical results with both operators are in very good agreement with the experimental data for the quadrupole moments, while they fail in reproducing the experimental behavior of the magnetic moments. Moreover, the use of a microscopic *M*1 operator yields magnetic moments that are very close to the Schmidt line, and which largely underestimate the experimental value. These deviations may be related to *M*1 core-polarization excitations, also observed above the doubly magic ²⁰⁸Pb, and especially to the lack of the *M*1 spin-flip $0g_{9/2} \rightarrow 0g_{7/2}$ transition in our model space. In light of these results, we have carried out shell-model calculations by considering a ⁸⁸Sr core, including the $0g_{9/2}$ and $1p_{1/2}$ proton orbitals below Z = 50. Indeed, these calculations reproduce the experimental magnetic moments very well. However, the ⁸⁸Sr model space misses important contributions from $1d_{3/2}$, $2s_{1/2}$, and $0h_{11/2}$ proton orbitals, leading to a disagreement in the quadrupole moments. An ideal model space in this region of the nuclear chart, that reproduced both magnetic and quadrupole moments equally well, would require the inclusion of the proton $0g_{9/2}$ orbital as well as of the whole Z = 50–82 shell. This is however rather cumbersome from a computational point of view.

On the other hand, ¹³⁴Sb is easier to interpret and constitutes another textbook example. In this case, the additivity rule for odd-odd nuclei gives excellent agreement with the experiment. These results are supported by shell-model calculations revealing a pure configuration of the proton in the $0g_{7/2}$ and the neutron in the $1f_{7/2}$ orbital above a ¹³²Sn core.

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