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**CP violation in the $B_{s}^{0}$ system**
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We review experimental and theoretical studies of CP violation in the $B^0_s$ system. Updated predictions for the mixing parameters of the $B^0_s$ mesons expected in the Standard Model (SM) are given, namely the mass difference $\Delta M_{s}^{\text{SM}} = (18.3 \pm 2.7) \text{ ps}^{-1}$, the decay rate difference $\Delta \Gamma_{s}^{\text{SM}} = (0.085 \pm 0.015) \text{ ps}^{-1}$, and the flavour specific CP asymmetry $a_{s,\text{SM}} = (2.22 \pm 0.27) \times 10^{-5}$ and the equivalent quantities in the $B^0$-sector. Current experimental values of $\Delta M_s$ and $\Delta \Gamma_s$ agree with remarkable precision with theoretical expectations. This agreement supports the applicability of theoretical tools such as the Heavy Quark Expansion (HQE) to these decays. CP violating studies in the $B^0_s$ system provide essential information to test the SM expectations, and to unveil possible contribution of the new physics (NP). NP effects on $\Delta M_s$ of the order of 15% are still possible. The CP phase $\phi_s$ due to CP violation in interference of decays and mixing can accommodate effects of the order of $\mathcal{O}(100\%)$. The semileptonic CP asymmetry $a_{sl}^d$ due to CP violation in mixing could still be a factor of 130 larger than its robust SM expectation and thus provides a very clean observable for NP searches. Theoretical improvements that are necessary to make full use of the experimental precision are discussed.

I. INTRODUCTION

The phenomenon of CP violation, discovered more than 50 years ago (Christenson et al., 1964), is an essential ingredient to explain the apparent imbalance between matter and anti-matter in the Universe (Sakharov, 1967). Consequently, this topic attracts a lot of attention. In the Standard Model (SM) (Glashow, 1961; Salam, 1968; Weinberg, 1967) CP violation arises in the Yukawa-sector...
via quark mixing and it is described by a complex parameter in the Cabibbo-Kobayashi-Maskawa matrix (CKM matrix) (Cabibbo, 1963; Kobayashi and Maskawa, 1973). Intensive studies of CP violation, especially at the $e^+e^-$ $B$ factories (see e.g. (Bevan et al., 2014) for a comprehensive review), provide convincing evidence that the main source of CP violation is the phase in the CKM matrix. More precisely, a vast body of measurements performed in different experimental conditions, such as accelerators, energies of operation, and detectors, confirm the unitarity of the CKM matrix, see (Amhis et al., 2014).

The CKM phase accounts for all the observed CP violating phenomena, but it is too small to account for the abundance of matter in the Universe. Thus additional sources of CP violation must be found. A recent discussion of this problem can be found in (Bambi and Dolgov, 2015). Thus the quest for a broader understanding of CP violation is strongly motivated and may provide hints on the path towards a more complete physics picture of the CP violation in mixing extracted from semileptonic charge asymmetries. Therefore, even a relatively small contribution of new physics effects could be clearly visible in the $B^0$ system, see e.g. (Dunietz et al., 2001). More precisely, the angle $\beta$ describing CP violation in interference of decay and mixing in the $B^0$ system is predicted to be of the order of $22^\circ$. The corresponding angle $\beta_s$ in the $B_s^0$ system is expected to be about $1^\circ$. Thus the sensitivity to new physics is potentially enhanced. Unfortunately, the contribution of the so-called penguin effects to the measured value of $\beta_s$ can also be $\sim1^\circ$. Thus a more precise determination of penguin contributions is mandatory (Aaij et al., 2015e). On the contrary, solid conclusions about the existence of new physics could be drawn by the investigation of CP violation in mixing extracted from semileptonic charge asymmetries. Here, a measured value of about two or three times the value of the SM predictions would be an unambiguous signal for new physics.

The study of $B^0_s$ mesons at $e^+e^-$ $B$-factories is possible only by running at the $\Upsilon(10860)$ center-of-mass energy. The typical center-of-mass energy of both the BaBar and Belle experiments corresponds to the $\Upsilon(4S)$ mass, and is not sufficient to produce $B^0_s\bar{B}^0_s$ pairs. The Belle experiment took some data at the higher energy and obtained several interesting results, notably some branching fractions of $B^0_s$ decays, see e.g. (Olive et al., 2014). However, their statistical accuracy is not sufficient to study CP violation observables. Thus, the main source of information on $B^0_s$ mesons comes from hadron collider experiments at the Tevatron (CDF, D0) and the LHC (ATLAS, CMS, LHCb). In particular LHCb, the first experiment designed to study beauty and charm decays at the LHC, has produced an impressive body of data on CP violation in $B^0_s - \bar{B}^0_s$ mixing and decay.

This paper aims at summarising the current experimental knowledge of CP violation in the $B^0_s$ system as well as the theoretical implication of these data. It is organised as follows. Section II describes the main properties of the $B^0_s$ system, such as its mass and width difference, and the time evolution of the $B^0_s$ system. Section III reports studies CP violation in $B^0_s - \bar{B}^0_s$ mixing. CP violation in interference of $B^0_s$ mixing and decay is discussed in Section IV with a detailed review of penguin contributions. Section V reviews studies of CP violation in $B^0_s$ decays, as well as methods to derive the CKM angle $\gamma$ from $B^0_s$ decays. Section VI examines the data reported in this review in the context of NP searches. Model independent constraints on NP contributions inferred from $B^0_s$ data reported here are presented. Finally, Section VII gives an outlook to future developments. In the appendix details of the numerical updates of the Standard Model predictions for the mixing quantities are listed.

II. THE $B^0_S$ SYSTEM

A. Theory: Basic mixing quantities, time evolution of the $B^0_s$ system and the HQE

1. Mixing observables

The quantum mechanical time evolution of a decaying particle $B$ with mass $m_B$ and lifetime $\tau_B = 1/\Gamma_B$ is given as

$$|B(t)\rangle = e^{-im_Bt-\frac{t}{\tau_B}}|B(0)\rangle,$$

(1)

where $\Gamma_B$ denotes the total decay width of the $B$ particle. We now consider the system of neutral $B^0_s$ mesons, defined by their quark flavour content $|B^0_s\rangle = |(b\bar{s})\rangle$, and their anti-particles, $|\bar{B}^0_s\rangle = |(\bar{b}s)\rangle$. Its time-evolution is described by this simple differential equation for a two-state system:

$$i\frac{d}{dt}\left(\begin{array}{c}
|B^0_s(t)\rangle \\
|\bar{B}^0_s(t)\rangle
\end{array}\right) = \left(\begin{array}{cc}
M_s - \frac{i}{2}\Gamma_s & \Gamma_s \\
\Gamma_s & -M_s
\end{array}\right)\left(\begin{array}{c}
|B^0_s(t)\rangle \\
|\bar{B}^0_s(t)\rangle
\end{array}\right).$$

(2)

Naively one expects the diagonal entries of the $2 \times 2$ matrix $M_s$ to be equal to the mass of the $B^0_s$ meson, $M_{B^0_s}$, the diagonal entries of $\Gamma_s$ to be equal to the decay rate of the $B^0_s$ meson, $\Gamma_s$ and all non-diagonal entries to vanish. However, because of the weak interaction, the flavour eigenstate $B^0_s$ can transform into its anti-particle $\bar{B}^0_s$ and vice versa. This transition is governed by the so-called...
box diagrams, depicted in Fig. 1, and it gives rise to the off-diagonal elements $M_{12}^s$ in $M^s$ and $\Gamma_{12}^s$ in $\Gamma^s$. These box diagrams include contributions from virtual internal particles, denoted by $M_{12}^s$ and contributions from internal on-shell particles, denoted by $\Gamma_{12}^s$. Only internal charm and up quarks are involved in $\Gamma_{12}^s$, while $M_{12}^s$ is sensitive to all possible internal particles, and, in principle, also to heavy new physics particles. Due to the CKM structure both $M_{12}^s$ and $\Gamma_{12}^s$ can be complex.

$$M_{12}^s = |M_{12}^s|e^{i\phi_M}, \quad \Gamma_{12}^s = |\Gamma_{12}^s|e^{i\phi_{\Gamma}}.$$  

The CKM phases $\phi_M$ and $\phi_{\Gamma}$ are not physical, but depend on the phase convention used in the CKM matrix. Later on we will see that

$$e^{i\phi_M} = \frac{V_{ts}^*V_{tb}}{V_{ts}V_{tb}}.$$  

No such simple relation exists for $\phi_{\Gamma}$, because $\Gamma_{12}^s$ depends on three different CKM structures in the Standard Model.

In order to obtain the physical eigenstates of the mesons with a definite mass and decay rate, the matrices $M^s$ and $\Gamma^s$ have to be diagonalised. This gives the meson eigenstates $|B_{s,H}\rangle$ ($H=$heavy) and $|B_{s,L}\rangle$ ($L=$light) as linear combinations of the flavour eigenstates:

$$|B_{s,L}\rangle = p|B_s^0\rangle + q|\bar{B}_s^0\rangle, \quad (6)$$
$$|B_{s,H}\rangle = p|B_s^0\rangle - q|\bar{B}_s^0\rangle, \quad (7)$$

which are in general not orthogonal. The complex coefficients $p$ and $q$ fulfill $|p|^2 + |q|^2 = 1$ and the corresponding masses and decay rates of these states are denoted by $M_L$, $M_H$ and $\Gamma_L$, $\Gamma_H$. The mass eigenstates of the $B_s^0$ mesons are almost CP eigenstates. Using the same conventions as e.g. (Dunietz et al., 2001) for the CP properties and defining

$$CP|B_s^{0}\rangle = -|\bar{B}_s^{0}\rangle, \quad (8)$$

we get for the CP eigenstates of the $B_s^0$ meson

$$|B_s^{even}\rangle = \frac{1}{\sqrt{2}}(|B_s^{0}\rangle - |\bar{B}_s^{0}\rangle), \quad (9)$$
$$|B_s^{odd}\rangle = \frac{1}{\sqrt{2}}(|B_s^{0}\rangle + |\bar{B}_s^{0}\rangle). \quad (10)$$

In absence of CP violation in mixing, which is a very small effect, the heavy eigenstate is CP odd ($|B_{s,H}\rangle \approx |B_s^{odd}\rangle$) and the light one is CP even ($|B_{s,L}\rangle \approx |B_s^{even}\rangle$), in this case one has $p = 1/\sqrt{2}$ and $q = -1/\sqrt{2}$.

If we expand the eigenvalues of $M^s$ and $\Gamma^s$ in powers of $|\Gamma_{12}^s/M_{12}^s| \approx 5 \cdot 10^{-3}$ in the SM, we can express the mass and decay rate differences as

$$\Delta M_s := M_H - M_L = 2|M_{12}^s| \left(1 - \frac{|\Gamma_{12}^s|^2}{8|M_{12}^s|^2} \sin^2\phi_{12}^s + \cdots\right), \quad (11)$$
$$\Delta \Gamma_s := \Gamma_H - \Gamma_L = 2|\Gamma_{12}^s| \cos\phi_{12}^s \left(1 + \frac{|\Gamma_{12}^s|^2}{8|M_{12}^s|^2} \sin^2\phi_{12}^s + \cdots\right), \quad (12)$$

with the mixing phase

$$\phi_{12}^s := \arg \left(\frac{M_{12}^s}{\Gamma_{12}^s}\right). \quad (13)$$

In contrast to $\phi_M$ and $\phi_{\Gamma}$, this phase difference is physical. We follow here the definition given in (Aaij et al., 2013c). In some references, for example (Anikeev et al., 2001; Lenz and Nierste, 2007), $\phi_{12}^s$ is denoted as $\phi_s$. However, in the literature the notation $\phi_s$ is often used for different quantities, also related to CP violation in interference. We will define the phase that appears in interference in Section IV. The correction factor $1/8|\Gamma_{12}^s/M_{12}^s|^2 \sin^2\phi_{12}^s$ in Eq. (11) and Eq. (12) is of the order of $6 \cdot 10^{-11}$ in the Standard Model and the current experimental bound for this factor is smaller than $5 \cdot 10^{-5}$, thus it can be safely neglected. Diagonalisation of $M^s$ and $\Gamma^s$ gives also

$$q = -e^{-i\phi_M} \left[1 - \frac{1}{2} \frac{\Gamma_{12}^s}{M_{12}^s} \sin\phi_{12}^s + O\left(\frac{|\Gamma_{12}^s|^2}{|M_{12}^s|^2}\right)\right]$$
$$= -\frac{V_{ts}^*V_{tb}}{V_{ts}V_{tb}} \left[1 - \frac{a_{ts}^s}{2}\right] + O\left(\frac{|\Gamma_{12}^s|^2}{|M_{12}^s|^2}\right), \quad (14)$$

with the notation

$$a_{ts}^s = \frac{|\Gamma_{12}^s|}{|M_{12}^s|} \sin\phi_{12}^s. \quad (15)$$

Later on, in Section III, we will see that $a_{ts}^s$ equals the so-called flavour-specific CP asymmetry. From Eq. (14) it follows also that, in the absence of CP violation in mixing, $q/p = -1$. In Eq. (14) again all terms of order $|\Gamma_{12}^s/M_{12}^s|^2$ can be discarded, many times also the term of order $a_{ts}^s$ is not necessary.

2. Time evolution of neutral mesons

We now consider the time evolution of the flavour eigenstates of the $B_s^0$ mesons. $|B_s^0(t)\rangle$ denotes a meson at time $t$ that was produced as a $B_s^0$ meson at time $t_0$. Such an expansion does not hold in the charm system, because there $\Delta M$ and $\Delta M$ are of a similar size.

A more detailed discussion of the $B_s^0$ mixing system and its time evolution can be found in e.g. (Anikeev et al., 2001).
\[ t = 0. \] At a later time \( t \), \(|B_s^0(t)\) will have components both of \(|B_s^0\) and \(|\bar{B}_s^0\):

\[ |B_s^0(t) = g_+(t)|B_s^0 + \frac{q}{p} g_-(t)|\bar{B}_s^0 \], \hspace{1cm} (16)

\[ |\bar{B}_s^0(t) = \frac{p}{q} g_-(t)|B_s^0 + g_+(t)|\bar{B}_s^0 \], \hspace{1cm} (17)

with the coefficients

\[
g_+(t) = e^{-iM_st}e^{-\frac{i}{2}} \times \left[ \frac{\Delta \Gamma_s t}{4} \cos \frac{\Delta M_st}{2} - i \sinh \frac{\Delta \Gamma_s t}{4} \sin \frac{\Delta M_st}{2} \right], \hspace{1cm} (18)
\]

\[
g_-(t) = e^{-iM_st}e^{-\frac{i}{2}} \times \left[ - \sinh \frac{\Delta \Gamma_s t}{4} \cos \frac{\Delta M_st}{2} + i \cosh \frac{\Delta \Gamma_s t}{4} \sin \frac{\Delta M_st}{2} \right]. \hspace{1cm} (19)
\]

Here we used the averaged mass \( M_{B_s} \) and decay rate \( \Gamma_s \):

\[
M_s = \frac{M_H^s + M_L^s}{2}, \hspace{1cm} \Gamma_s = \frac{\Gamma_H^s + \Gamma_L^s}{2}. \hspace{1cm} (20)
\]

Next we consider the time evolution of the decay rate for a \( B_s^0 \) meson, that was initially (at time \( t = 0 \)) tagged as a \( B_s^0 \) flavour eigenstate into an arbitrary final state \( f \).

\[
\Gamma [B_s^0(t) \rightarrow f] = N_f |A_f|^2 (1 + |\lambda_f|^2) e^{-\Gamma t}\left\{ \frac{\cosh \frac{\Delta \Gamma_s t}{2}}{2} + \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos \left( \frac{\Delta M_st}{2} \right) \right\}
\]

\[
\hspace{1cm} - \frac{2 \Re(\lambda_f)}{1 + |\lambda_f|^2} \frac{\sinh \left( \frac{\Delta \Gamma_s t}{2} \right)}{2} - \frac{2 \Im(\lambda_f)}{1 + |\lambda_f|^2} \sin \left( \frac{\Delta M_st}{2} \right) \right\} \hspace{1cm} (21)
\]

is given by

\[
\lambda_f = \frac{q}{p} \frac{\tilde{A}^f}{A} \approx -\frac{V_{ts}V_{tb}^*}{V_{ts}V_{tb}} \frac{\tilde{A}^f}{A} \left[ 1 - \frac{\alpha_s^2}{2} \right]. \hspace{1cm} (22)
\]

For the terms appearing on the right-hand-side of Eq. (21) the following definitions are typically used

\[
A_{\text{dir}}^{\text{CP}} = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \hspace{1cm} (23)
\]

\[
A_{\text{mix}}^{\text{CP}} = -\frac{2 \Im(\lambda_f)}{1 + |\lambda_f|^2}, \hspace{1cm} (24)
\]

\[
A_{\Delta \Gamma} = -\frac{2 \Re(\lambda_f)}{1 + |\lambda_f|^2}. \hspace{1cm} (25)
\]

\[ \lambda_f \] describes effects related to direct CP violation, which is described in Section V. This can be seen

\[ \text{FIG. 1 Standard Model diagrams for the transition between } B_s^0 \text{ and } \bar{B}_s^0 \text{ mesons. The contribution of internal on-shell particles (only the charm and the up quark can contribute) is denoted by } \Gamma_{12}; \text{ the contribution of internal off-shell particles (all depicted particles can contribute) is denoted by } M_{12}. \]
by neglecting CP violation in mixing, i.e. assuming \(|q/p| = 1\) and considering the decay into a final state \(f\), that is a CP eigenstate, i.e. \(\bar{f} = \gamma_{CP} f\).

With these assumptions we get \(|\lambda_f| = |\bar{A}_f|/|A_f|\).

A non-vanishing value for \(A_{\text{dir}}^{\text{CP}}\) is obtained for \(|\lambda_f| \neq 1\) and this corresponds now to \(|\bar{A}_f| \neq |A_f|\), which is equivalent to direct CP violation. \(A_{\text{mix}}^{\text{CP}}\) encodes effects due to interference between mixing and decay, which is discussed in Section III and \(A_{\Delta \Gamma}\) is a correction factor, due to a finite value of the decay rate difference \(\Delta \Gamma\), \(A_{\Delta \Gamma}\) also appears in the definition of the effective lifetimes \(\tau^{\text{eff}}\)\footnote{The total lifetime of the \(B_s^0\) mesons is defined as \(\tau(B_s^0) = 1/\Gamma_s = 2/(\Gamma_L + \Gamma_L^{\text{eff}})\). But, the decay of a \(B_s^0\) meson is actually a superposition of a decay of a \(B_s^0\) meson and a \(B_s^0\) meson. Fitting such a decay with only one exponential PDF leads to the effective lifetime, which differs from the total lifetime.}:

\[
\tau^{\text{eff}} = \tau_{B_s^0} \frac{1}{1 - y_s^2} \left( \frac{1 + 2 M_s^0 / \Gamma_s}{1 + \Delta \Gamma y_s / \Gamma_s} \right)
\]

(27)

with

\[
\tau_{B_s^0} = \frac{1}{\Gamma_{B_s^0}} \quad \text{and} \quad y_s = \frac{\Delta \Gamma_s}{2 \Gamma_{B_s^0}}.
\]

(28)

Such lifetimes can also be used to determine \(\Delta \Gamma_s\), examples of theoretical derivation can be found in (Dunietz, 1995; Dunietz et al., 2001; Hartkorn and Moser, 1999) and will be discussed in Section II.B. In general \(A_{\text{dir}}^{\text{CP}}, A_{\text{mix}}^{\text{CP}}\) and \(A_{\Delta \Gamma}\) are governed by non-perturbative effects and there are no simple expressions for these quantities in terms of basic Standard Model parameters. These three quantities are, however, not independent and the following relation holds

\[
(\bar{A}_{\text{dir}}^{\text{CP}})^2 + (\bar{A}_{\text{mix}}^{\text{CP}})^2 + (\bar{A}_{\Delta \Gamma})^2 = 1.
\]

(29)

Under certain circumstances, we get, however, simplified expressions for \(A_{\text{dir}}^{\text{CP}}, A_{\text{mix}}^{\text{CP}}\) and \(A_{\Delta \Gamma}\):

1. In the case of flavour-specific decays that are discussed in Section III, we have \(\bar{A}_f = 0\) and thus \(\lambda_f = 0\), hence we get

\[
A_{\text{dir}}^{\text{CP}} = 1, \quad A_{\text{mix}}^{\text{CP}} = 0, \quad A_{\Delta \Gamma} = 0,
\]

(30)

\[
\tau^{\text{eff}} = \tau_{B_s^0} \frac{1}{1 - y_s^2}.
\]

(31)

2. In Section IV we will introduce so-called golden modes, which have only one contributing CKM structure and one considers the decay into a CP eigenstate \(f\). In that case we have \(|\lambda_f| = 1\) and thus the simple relations

\[
A_{\text{dir}}^{\text{CP}} = 0, \quad A_{\text{mix}}^{\text{CP}} = -\Im(\lambda_f), \quad A_{\Delta \Gamma} = -\Re(\lambda_f).
\]

(32)

Moreover the real and imaginary parts of \(\lambda_f\) are now given by simple combinations of CKM elements, which will be discussed in Section IV.

After discussing the decay of a \(B_s^0\) meson into the final state \(f\), we consider next the time evolution of the decay rate for a \(B_s^0\) meson into the same final state \(f\). It is given by

\[
\Gamma [B_s^0(t) \to f] = N_f |A_f|^2 (1 + |\lambda_f|^2) (1 + a_s^\text{eff}) e^{-\Gamma_f t} \left\{ \frac{\cosh (\Delta \Gamma_s/2 t)}{2} - \frac{1 - |\lambda_f|^2 \cos (\Delta M_s t)}{1 + |\lambda_f|^2/2} - \frac{2 \Re(\lambda_f) \sinh (\Delta \Gamma_s/2 t)}{1 + |\lambda_f|^2/2} + \frac{2 \Im(\lambda_f) \sin (\Delta M_s t)}{1 + |\lambda_f|^2/2} \right\}.
\]

(33)

The common pre-factors, i.e. \(N_f\) and \(|A_f|^2 (1 + |\lambda_f|^2)\), typically cancel in CP asymmetries and we do not need to know their value. This is very advantageous because the hadronic quantity \(A_f\) is notoriously difficult to calculate. Nevertheless, a dependence on the parameter \(\lambda_f\) will still be left in CP asymmetries. As already stated, in general this parameter cannot be calculated from first principles. Making, however, some additional assump-

\[|\tilde{f}| = CP |f|\]

(34)

With the definitions

\[
\bar{A}_f = \langle f | H_{\text{eff}} | B_s^0 \rangle, \quad \lambda_f = \frac{q}{p} \tilde{A}_f
\]

(35)
The above formulae can be used to extract the observables $\Delta M_s$, $\Delta \Gamma_s$ and $a_{K}^0$ from experiment, which can then be compared with the theory predictions. According to Eq. (11) and Eq. (12) these three observables are related to the matrix elements $\Gamma_{12}^*$ and $M_{12}^*$, thus a Standard Model calculation of the three mixing observables requires a calculation of the box diagrams in Fig. 1.

3. Theoretical determination of $M_{12}^*$

The calculation of the Standard Model value for $M_{12}^*$ is straight-forward. In principle there are nine different combinations of internal quarks in the box diagrams, thus we get

$$M_{12}^* \propto \lambda^2 F(u, u) + \lambda_u \lambda_c F(u, c) + \lambda_u \lambda_t F(u, t) + \lambda_c \lambda_t F(c, u) + \lambda_c \lambda^2 F(c, c) + \lambda_c \lambda_t F(c, t) + \lambda_t \lambda_u F(t, u) + \lambda_t \lambda_c F(t, c) + \lambda_t^2 F(t, t),$$

with the CKM unitarity, i.e. $\lambda_u + \lambda_c + \lambda_t = 0$, we get

$$M_{12}^* \propto \lambda^2 [F(c, c) - 2F(u, c) + F(u, u)] + 2\lambda_c \lambda_t [F(c, t) - F(u, t) - F(u, c) + F(u, u)] + \lambda_t^2 [F(t, t) - 2F(u, t) + F(u, u)].$$

From this equation one sees clearly the arising GIM cancellation (Glashow et al., 1970) in all three terms: if all masses would be equal, each of the three terms would vanish. Because of that also any constant term in the functions $F(x, y)$ cancels in $M_{12}^*$ and only the mass dependent terms will survive. An explicit calculation shows that $F(x, y)$ grows strongly with the masses (see Eq. (42)), thus there is a very severe GIM cancellation in the first two terms ($m_u/M_W$ and $m_c/M_W$ can be very well be approximated by zero), while the third term will give a sizable contribution ($m_t/M_W > 1$). Since the CKM structures have all a similar size ($\lambda_c \propto \lambda^2 \propto \lambda_t$, with the Wolfenstein parameter $\lambda$ (Wolfenstein, 1983)) we get to a very good approximation

$$M_{12}^* \propto \lambda^2 F(t, t) - 2F(u, t) + F(u, u)$$

$$\propto \lambda^2 S_0 \left( \frac{m_u^2}{M_W^2} \right),$$

where $S_0$ denotes the Inami-Lim function (Inami and Lim, 1981):

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x \ln x}{2(1-x)^2}.$$

In that respect it is sometimes stated that only the top quark contributes to $M_{12}^*$. Formally the process of calculating $M_{12}^*$ can be viewed as performing an operator product expansion (OPE) by integrating out the heavy W boson and the heavy top quark. Since both of these masses are far above the hadronic scale and the $b$ quark mass, there is no doubt in the applicability of the OPE. This will change in the discussion of $\Gamma_{12}^*$. The complete calculation of $M_{12}^*$ yields

$$M_{12}^* = \frac{G_F^2}{12\pi^2} \lambda^2 M_W^2 S_0(x_t) B_{f_B} \eta_B,$$

with simple pre-factors: the Fermi-constant $G_F$, the masses of the W boson, $M_W$, and of the $B_s$ meson, $M_{B_s}$ and the normalisation factor $1/12\pi^2$. As we have seen above there is only one CKM structure contributing $\lambda_t = V_{tb}^* V_{tb}$. The CKM elements are the only place in Eq. (43) where an imaginary part can arise. By writing

$$\lambda^2 = |\lambda_t^2| \frac{\lambda_t}{\lambda_t^2} = |\lambda_t^2| e^{i\phi_M}$$

we get the explicit dependence of the phase $\phi_M$ on CKM parameters, which was already stated in Eq. (5).
As discussed above, the result of the 1-loop diagrams given in Fig. 1 is denoted by the Inami-Lim function $S_0(x_1 = (\bar{m}_t(m_t))^2/M^2_{\text{MS}})$, where $\bar{m}_t(m_t)$ is the $\overline{\text{MS}}$-mass (Bardeen et al., 1978) of the top quark. Perturbative 2-loop QCD corrections are compressed in the factor $\bar{n}_B \approx 0.84$, they have been calculated by (Buras et al., 1990). Performing the calculation of $M^2_{\eta_2}$ one gets a spinor operator for each external quark in the box diagram. Together with the arising Dirac matrices they form the four quark $\Delta B = 2$ operator

$$ Q = \gamma_\mu (1 - \gamma_5) b^\alpha \times \gamma_\beta (1 - \gamma_5) b^\beta. \tag{45} $$

$\alpha$ and $\beta$ are the colour indices of the $b$ and $s$ quark spinors. All hadronic effects that describe the binding of the quarks into meson states as well as the non-perturbative QCD effects contributing to the transition of the $B^0_s$ meson into the $\bar{B}^0_s$ meson and vice versa are encoded in the hadronic matrix element of the operator $Q$. The hadronic matrix element\textsuperscript{7} of this operator is parametrised in terms of a decay constant $f_{B_s}$ and a bag parameter $B$:

$$ \langle Q \rangle = \langle \bar{B}^0_s | Q | B^0_s \rangle = \frac{8}{3} M^2_{\eta_2} f_{B_s} B(\mu), \tag{46} $$

The factor $8/3 = 2(1+1/N_c)$ stems from the colour structure. It ensures that the bag parameter $B$ obtains the value one in vacuum insertion approximation\textsuperscript{8}. We also indicated the renormalisation scale dependence of the bag parameter; in our analysis we take $\mu = m_t$.

Sometimes a different notation for the QCD corrections and the bag parameter is used in the literature (e.g. by the Flavour Lattice Averaging Group (FLAG): (Aoki et al., 2014)), ($\bar{n}_B \bar{B}$) instead of ($\bar{n}_B, B$) with

$$ \bar{n}_B \bar{B} := \bar{n}_B \bar{B} = \eta_B \bar{B} \tag{47} $$

$$ = \eta_B \alpha_s(\mu)^{-\frac{\alpha_s(\mu)}{4\pi}} \left[ 1 + \frac{\alpha_s(\mu)}{4\pi} \right] \frac{5165}{3174} B, \tag{48} $$

$$ \bar{B} = 1.51599 B. \tag{49} $$

The parameter $\bar{B}$ has the advantage of being renormalisation scale and scheme independent.

\textsuperscript{7} Throughout this review we will use the conventional relativistic normalisation for the $B^0_s$ meson states, i.e. $\langle B^0_s | B^0_s \rangle = 2E$V ($E$: energy, $V$: volume).

\textsuperscript{8} The matrix element in Eq. (46) can be rewritten, by inserting a complete set of states between the two currents of the operator $Q$, given in Eq. (45). Next this expression is equated to the contribution of the vacuum state only times a correction factor $B$ (bag factor), that corrects for the neglect of all higher states in the sum. Setting the bag parameter to one, corresponds to the vacuum insertion approximation. Many lattice evaluations show, that this assumption seems to be very well justified (Bazavov et al., 2016). The remaining matrix elements of the form $\langle B^0_s | \gamma_\mu (1 - \gamma_5) b^\gamma | 0 \rangle$ are proportional to $f_{B_s, p_\mu}$, where $p_\mu$ is the four-momentum of the $B^0_s$ meson.

A commonly used Standard Model prediction of $\Delta M_s$ was given by (Lenz and Nierste, 2011)

$$ \Delta M^{\text{SM,2011}}_s = (17.3 \pm 2.6) \text{ ps}^{-1}. \tag{50} $$

Using the most recent numerical inputs ($G_F, M_W, M_b$, and $m_b$ from the Particle Data Group (PDG) (Olive et al., 2014), the top quark mass from (ATLAS and Collaborations, 2014), the non-perturbative parameters from FLAG (web-update of (Aoki et al., 2014)) and CKM elements from the CKMfitter group [ web-update of (Charles et al., 2005)], [similar values can be taken from the UTfit group (Bona et al., 2006b)], we predict the mass difference of the neutral $B^0_s$ mesons to be

$$ \Delta M^{\text{SM,2015}}_s = (18.3 \pm 2.7) \text{ ps}^{-1}. \tag{51} $$

Here the dominant uncertainty comes from the lattice predictions for the non-perturbative parameters $B$ and $f_{B_s}$, giving a relative error of 14%. This input did not change compared to the 2011 prediction from (Lenz and Nierste, 2011). The uncertainty in the CKM elements contributes about 5% to the error budget. The CKM parameters were determined assuming unitarity of the $3 \times 3$ CKM matrix. For some new physics models this assumption might have to be given up, leading to larger CKM uncertainties. The uncertainties due to $m_t$, $m_b$ and $\alpha_s$ can be safely neglected at the current stage. A detailed discussion of the input parameters and the error budget is given in Appendix B.

There is, however, a word of caution: in the above theory prediction (51) we use the non-perturbative value from FLAG $f_{B_s} \sqrt{B} = 216 \pm 15 \text{ MeV}$\textsuperscript{9} (with $N_f = 2 + 1$ active flavours in the lattice simulations). However, only one number – from the HPQCD Collaboration (Gamiz et al., 2009) – is included in the FLAG average. It would of course be advantageous to have more numbers from different collaborations and there are currently some more (mostly preliminary) numbers on the market:

$$ f_{B_s} \sqrt{B} \approx 200 \text{ MeV} \Rightarrow \Delta M^{\text{HPQCD}}_s \approx 15.7 \text{ ps}^{-1}, \tag{52} $$

$$ f_{B_s} \sqrt{B} \approx 211 \text{ MeV} \Rightarrow \Delta M^{\text{ETMC}}_s \approx 17.4 \text{ ps}^{-1}, \tag{53} $$

$$ f_{B_s} \sqrt{B} \approx 227 \text{ MeV} \Rightarrow \Delta M^{\text{Fermilab}}_s \approx 20.2 \text{ ps}^{-1}. \tag{54} $$

HPQCD updated their results in (Dowdall et al., 2014) and for our numerical estimate in Eq. (52) we had to read off the numbers from Fig. 3 in their proceedings (Dowdall et al., 2014). Their investigations suggest a possible error of about 5% for $f_{B_s} B$ in the near future, which would be a major improvement. The ETMC number stems from (Carrasco et al., 2014), it is obtained with only two active flavours in the lattice simulation. The

\textsuperscript{9} This value is derived from the FLAG value of $f_{B_s} \sqrt{B}$. It is by accident equal to the value of $f_{B_s} \sqrt{B}$ quoted from FLAG.
During the refereeing process for this review, Fermilab-MILC presented final results in (Bazavov et al., 2016). The numerical effect of these new inputs on mixing observables was studied in (Jubb et al., 2016).

Fermilab-MILC number is an update for the LATTICE 2015 conference of (Bouchard et al., 2011)\(^\text{10}\). The range of the above numbers seems to be nicely covered by the current FLAG average, but it would of course be very interesting to have final numbers and an average for the values given in Eq. (52), Eq. (53) and Eq. (54). There is also a large value from RBC-UKQCD presented at LATTICE 2015, \(f_B \sqrt{B} = 262\) MeV (update of (Aoki et al., 2015)). However, this number is obtained in the static limit and currently missing \(1/m_b\) corrections are expected to be very sizable. Thus we do not give a value of \(\Delta M_s\) for this lattice value. For our numerical analysis, we only use the value from FLAG. In summary, an uncertainty of about \(\pm 5\%\) might be feasible for the theory prediction of \(\Delta M_s\) taking future lattice improvements into account.

4. Heavy Quark Expansion

The calculation of the decay rate difference \(\Delta \Gamma_s\) is more involved. In the box diagrams depicted in Fig. 1, we have to take into account not only the internal up and charm quarks. Integrating out all heavy particles (in this case only the \(W\) boson) we are not left with a local \(\nabla B = 2\) operator as in the case of \(M_{i2}\), but with a bi-local object depicted in Fig. 2. To get to the level of local operators, which is needed for being able to make a theory prediction, a second operator product expansion is required. The second OPE relies on the smallness of the parameter \(\Lambda/m_b\), where \(\Lambda\) is expected to be of the order of the hadronic scale \(\Lambda_{QCD}\) and \(m_b\) is the \(b\) quark mass. More precisely the HQE is an expansion in \(\Lambda\) normalised to the momentum release of the decay given by \(\sqrt{M_i^2 - M_f^2}\), with the initial mass \(M_i\) and the final state masses \(M_f\). For massless final states an expansion in \(\Lambda/m_b\) is generally expected to converge, while for a transition like \(b \to c \bar{c}s\) it is not a priori clear, whether \(\Lambda/\sqrt{m_b^2 - 4m_s^2}\) is small enough to get a converging series. Thus the validity of this so-called heavy quark expansion (HQE) has to be tested by comparisons of experiment and theory. The formulation of the HQE is based on work by Voloshin and Shifman in (Khoze and Shifman, 1983), (Shifman and Voloshin, 1985), (Bigi and Uraltsev, 1992a), (Blok and Shifman, 1993a), (Blok and Shifman, 1993b) and in detail described in (Lenz, 2014)\(^\text{11}\). The HQE applies also for lifetimes and totally inclusive decay rates of heavy hadrons. Historically there had been several discrepancies between experiment and theory that questioned the validity of the HQE:

- In the mid-nineties the missing charm puzzle (see e.g. (Lenz, 2000) for a brief review), a disagreement between experiment and theory about the average number of charm quarks produced per \(b\)-decay, was a hot topic. A possible interpretation could be new physics, but a violation of quark hadron duality, i.e. a violation of the validity of the HQE, was also considered to solve this discrepancy, in particular in the decay \(b \to c \bar{c}s\). This issue has now been resolved, by more precise data and improved theory predictions (see (Krinner et al., 2013)), leading to a nice agreement between experiment and theory within uncertainties.

- For a long time the measured \(\Lambda_b\) lifetime was considerably shorter than its predicted value (according to estimates of the HQE - see e.g. (Bigi et al., 1997; Voloshin, 2000)). This issue has been resolved by recent measurements, mostly by the LHCb Collaboration ((Aaij et al., 2013i, 2014j,k)) but also by the Tevatron experiments (e.g. (Aaltonen et al., 2014)). The history of the \(\Lambda_b\) lifetime - HFAG quoted 2003 a value of \(\tau_{HFAG}^{\Lambda_b} = (1.229 \pm 0.080)\) ps, which is about 3 standard deviations away from the 2015 average of \(\tau_{HFAG}^{\Lambda_b} = (1.466 \pm 0.010)\) ps - and also (sometimes embarrassing) theoretical attempts to obtain low theory values are discussed in detail in the review of (Lenz, 2014). The low experimental values reported in the early measurements are mostly determined using semileptonic decays with an undetectable neutrino (Stone, 2014),

\(^{10}\) During the refereeing process for this review, Fermilab-MILC presented final results in (Bazavov et al., 2016). The numerical effect of these new inputs on mixing observables was studied in (Jubb et al., 2016).

\(^{11}\) See e.g. (Bigi et al., 1989) and (Bigi et al., 1997), for early reviews.
while new measurements use non-leptonic decays with x-fuilly reconstructed final states. The huge range in the theory predictions for the Λ_b lifetime stems from our missing knowledge about the size of the hadronic matrix elements. Some theory groups tried to create some extraordinary large enhancements of these matrix elements in order to describe the experimental data, while other groups, including, for example, Bigi and Uraltsev, stuck to theory estimates that were in conflict with the old measurements, but agree perfectly with the new ones. The current status of lifetimes is depicted in Fig. 3. No lifetime puzzle exists anymore. The theoretical precision is strongly limited by a lack of up-to-date values for the arising non-perturbative parameters. For the Λ_b-baryon the most recent lattice numbers stem from 1999 (Di Pierro et al., 1999) and for the B mesons the most recent numbers are from 2001 (Becirevic, 2001). This lack of theoretical investigations limits also our current knowledge about the intrinsic precision of the HQE.

• Since $\Delta \Gamma_s$ is dominated by a $b \to c\bar{c}s$ transition, the applicability of the HQE was in particular questioned for $\Delta \Gamma_s$, see e.g. (Ligeti et al., 2010) and the discussion in (Lenz, 2011) and the references therein. In the last years this was also related to the unexpected measurement of a large value of the di-muon asymmetry by the D0 collaboration (Abazov et al., 2010a,b, 2011, 2014). In 2012 the issue of $\Delta \Gamma_s$ was solved experimentally by a direct measurement of this quantity by the LHCb Collaboration (Aaboud et al., 2014) average, combining values from LHCb, ATLAS, CMS, D0 and CDF, in perfect agreement with the HQE prediction from (Lenz and Nierste, 2011) which is based on on the calculations of (Beneke et al., 1999a, 2003; Ciuchini et al., 2003; Lenz and Nierste, 2007). This will be discussed in detail below.

All in all the HQE has been experimentally proven to be very successful and one could try next to test its applicability also for charm-physics, see e.g. (Bobrowski et al., 2010; Lenz and Rauh, 2013) for some recent investigations. Charm studies would be very helpful for assessing the intrinsic uncertainties of the HQE. Having more confidence in the validity of HQE, it can now also be applied to quantities that are sensitive to new physics, in particular to the semileptonic CP asymmetries, which will be discussed in Section III. A very recent study of the possible size of duality violating effects (i.e. deviations from the HQE expectations) can be found in (Jubb et al., 2016).

5. Theoretical determination of $\Gamma_{12}^{s}$

According to the HQE, the off-diagonal element $\Gamma_{12}^{s}$ of the $B_s^0$ mixing matrix can be expanded as a power series in the inverse of the heavy $b$-quark mass $m_b$ and the strong coupling $\alpha_s$:

$$\Gamma_{12}^{s} = \frac{\Lambda^3}{m_b^4} \left( \Gamma_{3}^{s,0} + \frac{\alpha_s}{4\pi} \Gamma_{3}^{s,1} + \cdots \right) + \frac{\Lambda^4}{m_b^4} \left( \Gamma_{4}^{s,0} + \cdots \right) + \cdots .$$  \hspace{1cm} (55)

$\Lambda$ denotes a hadronic scale, which is assumed to be of the order of $\Lambda_{QCD}$, but its actual value has to be determined by a non-perturbative calculation. Each of the $\Gamma_i^{s,j}$ is a product of perturbative Wilson coefficients and non-perturbative matrix elements. In $\Gamma_3^{s}$ these matrix elements arise from dimension 6 four quark operators, in $\Gamma_4^{s}$ from dimension 7 operators and so on. The leading term in Eq. (55), $\Gamma_{3}^{s,0}$, was calculated already quite long ago by (Ellis et al., 1977), (Hagelin, 1981), (Franco et al., 1982), (Chau, 1983), (Buras et al., 1984) and (Khoze et al., 1987). Here three different 4 quark operators arise; besides $Q$ from Eq. (45) these are

$$Q_S = \bar{s}\alpha_s + \bar{s}\gamma_5 b\alpha_s + \bar{s}\gamma_5 b\alpha_s,$$  \hspace{1cm} (56)

and each factor $\Gamma_{12}^{s,xy}$ has contributions of the three operators $Q$, $Q_S$ and $Q_{S'}$

$$\Gamma_{12}^{s,xy} = \Gamma_{12}^{s,Q} \langle Q \rangle + \Gamma_{12}^{s,Q_S} \langle Q_S \rangle + \Gamma_{12}^{s,Q_{S'}} \langle Q_{S'} \rangle.$$  \hspace{1cm} (59)

The matrix elements of the newly arising operators are typically parameterised as

$$\langle Q_S \rangle = \langle \bar{B}_s | Q_S | B_s \rangle = - \frac{5}{3} M_B^2 f_B^2 \xi_B^{s} ,$$  \hspace{1cm} (60)

with the modified bag parameters

$$B_X^{s} = \frac{M_B^2}{[m_b(m_b) + m_s(m_s)]} B_X \approx 1.57706 B_X .$$  \hspace{1cm} (62)

In the vacuum insertion approximation, the unmodified bag parameters are equal to one. More reliable values can be obtained by using non-perturbative methods like QCD sum rules\footnote{A QCD sum rule determination of $\langle Q \rangle$ is given e.g. in (Korner et al., 2003). However, we will not use the number obtained there in our analysis.} or lattice QCD. $Q$, $Q_S$ and $Q_{S'}$ were
In the Standard Model only the following relation holds: 
\[ e^{-1} \text{ independent (see e.g. (Beneke et al.)} \]

It was found, that these three operators are not independent (see e.g. (Beneke et al., 1996)) and that the following relation holds:

\[ Q \equiv O_1, Q_S \equiv O_2, \tilde{Q}_S \equiv O_3 \, . \quad (63) \]

In the case of (Bouchard et al., 2011) there is also an additional factor 4 present.

\[ Q \equiv 4O_1, Q_S \equiv 4O_2, \tilde{Q}_S \equiv 4O_3 \, . \quad (64) \]

(Becirevic et al., 2002) and (Carrasco et al., 2014) use the same definitions of the bag parameters as we do

\[ B \equiv B_1, B_s \equiv B_2, \tilde{B}_s \equiv B_3 \, , \quad (65) \]

while (Dowdall et al., 2014) and (Bouchard et al., 2011) use the modified bag parameters

\[ B \equiv B_1, B_s' \equiv B_2, \tilde{B}_s' \equiv B_3 \, . \quad (66) \]

It was found, that these three operators are not independent (see e.g. (Beneke et al., 1996)) and that the following relation holds

\[ R_0 = Q_S + \alpha_1 \tilde{Q}_S + \frac{\alpha_2}{2} Q = 0 + \mathcal{O} \left( \frac{\Lambda}{m_b} \right) \, , \quad (67) \]

with the coefficients (obtained in (Beneke et al., 1999a) using the renormalisation scheme described there)

\[ \alpha_1 = 1 + \frac{\alpha_s(\mu)}{3\pi} \left( 12 \ln \frac{\mu}{m_b} + 6 \right) \, , \quad (68) \]

\[ \alpha_2 = 1 + \frac{\alpha_s(\mu)}{3\pi} \left( 6 \ln \frac{\mu}{m_b} + \frac{13}{2} \right) \, . \quad (69) \]

With the help of Eq. (67) one can substitute one of the three operators; historically \( \tilde{Q}_S \) was eliminated, obtaining

\[ \Gamma^{s,xy}_{12} = \left[ \Gamma_s Q_{xy} - \frac{1}{2} \alpha_1 \Gamma_s \tilde{Q}_S \right] (Q) + \left[ \Gamma_s Q_S - \frac{1}{\alpha_1} \Gamma_s \tilde{Q}_S \right] (Q_S) + \mathcal{O} \left( \frac{\Lambda}{m_b} \right) \, , \quad (70) \]

which was denoted in the literature as

\[ \Gamma^{s,xy}_{12} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[ G^{s,xy}(Q) - C_S^{s,xy} (Q_S) \right] + \Gamma_{12}^{s,xy} \frac{1}{m_b} \, . \quad (71) \]

where the Wilson coefficients \( G^{s,xy} \) and \( C_S^{s,xy} \) contain the result of the calculation of the box diagrams with internal on-shell up and/or charm quarks; \( xy \in \{ uu, uc, cc \} \).

Neglecting the mass of the charm quark and penguin contributions, \( G^{s,xy} \) and \( C_S^{s,xy} \) read in LO-QCD

\[ G^{s,xy} = 3 C_1^2 + 2 C_1 C_2 + \frac{1}{2} C_2^2 \, , \quad (72) \]

\[ G_S^{s,xy} = - (3 C_1^2 + 2 C_1 C_2 - C_2^2) \, , \quad (73) \]

where \( C_{1,2} \) denote the \( \Delta B = 1 \) Wilson coefficients of the effective Hamiltonian describing \( b \) quark decays (in

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13 In the Standard Model only \( Q \) contributes to \( \Delta M_s \), while in extensions of the Standard Model additional contributions of new operators can appear. The whole set of these operators is called SUSY-basis and typically denoted by \( O_1 \ldots O_5 \). It turns out, however, that all these five operators are also needed for a precise standard model prediction of \( \Delta \Gamma_s \).
our notation $C_2$ corresponds to the colour allowed operator). Early LO-QCD estimates of $G_{\pi}^{s,xy}$ and $G_{S}^{s,xy}$ can be found in (Ellis et al., 1977), (Hagelin, 1981), (Franco et al., 1982), (Chau, 1983), (Buras et al., 1984) and (Khoze et al., 1987). NLO QCD corrections, i.e. $\Gamma_{3}^{(1)}$ in Eq. (55), were done for the first time in (Beneke et al., 1999a), they turned out to be quite large. This work was also a proof of the IR-safety of the HQE by direct calculation. The corresponding NLO-QCD diagrams are shown in Fig. 4. General arguments for such a proof were given already in the seminal paper of (Bigi and Uraltsev, 1992), which resolved the theoretical issues that were prohibiting a systematic expansion in the inverse of the heavy $b$ quark mass. Five years later the calculation of the QCD corrections was confirmed and also sub-leading CKM structures were included by (Beneke et al., 2003) and (Ciuchini et al., 2003). In these papers the full expressions for $G_{\pi}^{s,xy}$ and $G_{S}^{s,xy}$ are given; they also include contributions from the QCD penguin operators $Q_1, Q_3$ and the chromo-magnetic penguin operator $Q_8$. (Beneke et al., 2002) found that the use of $\bar{m}_c(m_b)$ (charm mass at the bottom mass scale) instead of $\bar{m}_c(m_c)$, sums up large logs of the form $m_c^2/m_b^2$ to all orders; we will thus use the parameter $\tilde{z}$ in our numerical analysis, given by

$$\tilde{z} = \left( \frac{\bar{m}_c(m_b)}{\bar{m}_c(m_c)} \right)^2.$$  

In Eq. (71) the term $\Gamma_{12,1/m_b}^{s,xy}$ denotes sub-leading $1/m_b$ corrections to $\Gamma_{12}$ - in Eq. (55) these terms were called $\Gamma_{4}^{s,(0)}$. Such sub-leading $1/m_b$ corrections were first calculated by (Beneke et al., 1996) and they also turned out to be quite sizable. The operators arising in $\Gamma_{4}^{s,(0)}$ are of dimension 7 (e.g. four quark operators with one derivative), they are denoted by $R_0$, $R_1$, $R_2$ and $R_3$, as well as the colour-rearranged counterparts $\tilde{R}_1$, $\tilde{R}_2$ and $\tilde{R}_3$, see e.g. (Lenz and Nierste, 2007) for more details. The operators $R_0, R_1$ and $R_1$ can be reduced to four quark operators (see e.g. the definition of $R_0$ in Eq. (67)) and thus they can be studied with current lattice technologies; their results can be deduced from (Becirevic et al., 2002), (Bouchard et al., 2011), (Carrasco et al., 2014) and (Dowdall et al., 2014), who were calculating the full five-dimensional SUSY basis of $\Delta B = 2$ operators. All of those five independent operators ($Q, Q_S, \tilde{Q}_S, R_1$ and $\tilde{R}_1$) contribute to $\Gamma_{12}$. The genuine dimension 7 operators $R_2, R_3, \tilde{R}_2$ and $\tilde{R}_3$ are considerably more complicated. For the corresponding matrix elements currently no lattice determination is available, so we have to rely on vacuum insertion approximation, i.e. the bag parameters $B_{R_2}, B_{R_3}, B_{\tilde{R}_2}$ and $B_{\tilde{R}_3}$ are set to one. First steps towards a non-perturbative determination of these matrix elements within the framework of QCD sum rules have been done by (Mannel et al., 2007, 2011). Here a more complete study would be very desirable, because - as will be seen below - these parameters give currently the dominant uncertainty in $\Gamma_{12}^{s}$. The precision of the theory prediction can be further improved by using ratios of theoretical expressions and by choosing an optimal operator basis:

- $\Gamma_{12}$ depends on $f_{B_s} B_s$, which is currently not very well-known. Thus, it might be advantageous to consider the ratio $\Gamma_{12}/M_{12}^s$, where the decay constant cancels. One gets from this ratio

$$\text{Re} \left( \frac{\Gamma_{12}}{M_{12}^s} \right) = - \frac{\Delta \Gamma_{s}}{\Delta M_{s}}, \quad \text{Im} \left( \frac{\Gamma_{12}}{M_{12}^s} \right) = a_{s}.$$  

The ratio $\Gamma_{12}/M_{12}^s$ can be further modified by using the CKM unitarity ($\lambda_u + \lambda_c + \lambda_t = 0$):

$$\frac{\Gamma_{12}}{M_{12}^s} = \frac{\lambda_u^2 \Gamma_{12,uc}^{s,cc}}{\lambda_t} + \left( \frac{\lambda_u}{\lambda_t} \right)^2 \frac{\Gamma_{12,uc}^{s,cc}}{M_{12}^s} + \frac{\lambda_u}{\lambda_t} \frac{\Gamma_{12,uc}^{s,uc}}{M_{12}^s} + \frac{\lambda_u^2}{\lambda_t} \frac{\Gamma_{12,uc}^{s,uc}}{M_{12}^s}$$

$$= -10^{-4} \left[ c + a \lambda_u \lambda_t + \left( \frac{\lambda_u}{\lambda_t} \right)^2 \right].$$

where $\tilde{M}_{12}^s$ is defined in such a way that only the CKM-dependence of $M_{12}^s$ in Eq. (43) is split off. Eq. (78) introduces the $a, b$ and $c$ notation of (Beneke et al., 2003). In the ratios $\Gamma_{12}/M_{12}^s$ - which are the building blocks of the parameters $a, b$ and $c$ - many quantities cancel, in particular the decay constant $f_{B_s}$, the mass of the $B_s$ meson and the Fermi constant. We get

$$\Gamma_{12,xy}^{s,xy} = \frac{\pi m_b^2}{6 M_{W} S_{0}(x_t) \eta_{B}} \left[ 8 G_{\pi}^{s,xy} + 5 G_{S}^{s,xy} B_{u}^{f} + O \left( \frac{1}{m_b} \right) \right].$$

Now the first term in Eq. (79), proportional to $G_{\pi}^{s,xy}$ is completely free of any non-perturbative contribution. It can be completely determined in perturbative QCD. Because of all these cancellations $a, b$ and $c$ are theoretically quite clean and they are also almost identical for $B_d$ and $B_s$ mesons, except for differences in the primed bag factors and in the $1/m_b$ corrections. The way of writing $\Gamma_{12}/M_{12}^s$ in Eq. (77) and Eq. (78) can be viewed as a Taylor expansion in the small ratio of CKM parameters, $\lambda_u/\lambda_t$, for which we get the following numerical values

$$\frac{\lambda_u}{\lambda_t} = -8.0486 \cdot 10^{-3} + 1.81082 \cdot 10^{-2} I,$$

$$\left( \frac{\lambda_u}{\lambda_t} \right)^2 = -2.63126 \cdot 10^{-4} - 2.91491 \cdot 10^{-4} I.$$
Moreover a pronounced GIM ((Glashow et al., 1970)) cancellation is arising in the coefficients $a$ and $b$ in Eq. (78). With the newest input parameters described in Appendix A, we get for the numerical values of $a$, $b$ and $c$:

\[
\begin{align*}
c &= -48.0 \pm 8.3 \quad (-49.5 \pm 8.5), \\
a &= +12.3 \pm 1.4 \quad (+11.7 \pm 2.0), \\
b &= 0.079 \pm 0.012 \quad (+0.24 \pm 0.06).
\end{align*}
\]  

The numbers in brackets denote the corresponding values for the $B^0$ system. Putting all this together, we see that the real part of $\Gamma_{12}/M_{12}$ is absolutely dominated by the coefficient $c$, while for the imaginary party only $a$ and to a lesser extent $b$ are contributing. We get

\[
\Re \left( \frac{\Gamma_{12}}{M_{12}} \right) = 10^{-4} \left( c + a \Re \left( \frac{\lambda_u}{\lambda_t} \right) + b \Re \left( \frac{\lambda_u^2}{\lambda_t^2} \right) \right)
\]

\[
\Rightarrow \frac{\Delta \Gamma_s}{\Delta M_s} \approx -10^{-4} c,
\]

\[
\Im \left( \frac{\Gamma_{12}}{M_{12}} \right) = 10^{-4} \left( a \Im \left( \frac{\lambda_u}{\lambda_t} \right) + b \Im \left( \frac{\lambda_u^2}{\lambda_t^2} \right) \right)
\]

\[
\Rightarrow a_{ts} \approx 10^{-4} a \Im \left( \frac{\lambda_u}{\lambda_t} \right).
\]

So for a determination of only $\Delta \Gamma_s$ (or also $\Delta \Gamma_d$) to a good approximation the first term of Eq. (77) - or equivalently the coefficient $c$ - is sufficient.

- Unfortunately it turned out after the calculation of the NLO-QCD and the sub-leading $1/m_b$ corrections that $\Delta \Gamma_s$ is not very well-behaved (see Lenz, 2004)): all corrections are quite large and they have the same sign. Surprisingly this problem could be solved to a large extent by using $Q$ and $\tilde{Q}_S$ as the two independent operators instead of $Q$ and $Q_S$, which is just a change of the operator basis, see (Lenz and Nierste, 2007). As an illustration of the improvement we discuss the real part of the ratio $\Gamma_{12}/M_{12}$ and split up the terms according to Eq. (79). We leave only the ratio of bag parameters as free parameters, while we else insert all Standard Model parameters according to the values given in the appendix. We get now for $\Delta \Gamma_s/\Delta M_s$ in the old (operators $Q$ and $Q_S$) and the new basis (operators $Q$ and $\tilde{Q}_S$):

\[
\frac{\Delta \Gamma_s}{\Delta M_s}^{\text{Old}} = 10^{-4} \cdot \left( 2.6 + 69.7 \frac{B_{S}}{B} - 24.3 \frac{B_{R}}{B} \right),
\]

\[
\frac{\Delta \Gamma_s}{\Delta M_s}^{\text{New}} = 10^{-4} \cdot \left( 44.8 + 16.4 \frac{B_{S}}{B} - 13.0 \frac{B_{R}}{B} \right).
\]
where $B_R$ is an abbreviation for all seven bag parameters of the dimension 7 operators. In the old basis the first term, which has no dependence on non-perturbative lattice parameters, is almost negligible. The second term, that depends on the ratio of the matrix elements of the operators $Q_S$ and $Q$ is by far dominant and the third term, that describes $1/m_b$ corrections gives an important negative contribution. In the new basis the first term, being completely free of any non-perturbative uncertainties, is numerical dominant. The second term is sub-leading and the $1/m_b$ corrections became smaller and undesired cancellations therein are less pronounced. Therefore, the second formulation has a much weaker dependence on the badly known bag parameters, also on the dimension seven ones. If all bag parameters were known precisely, then such a change of basis has no effect, but since $B_R$ is unknown and the ratios $B_S/B$ and $B'_S/B$ are much less known compared to the exact value one (stemming from $B/B$), now a basis, where the coefficients of $B_R/B$ and $B'_S/B$ are small, gives results with a much better theoretical control. For more details we refer the reader to (Lenz and Nierste, 2007).

$1/m_b$ corrections for the sub-leading CKM structures in $\Gamma_{12}$ (Dighe et al., 2002) and $1/m_b^2$ corrections for $\Delta \Gamma_s$ (Badin et al., 2007) were also determined; their numerical effect is small. A commonly used Standard Model Prediction for $\Delta \Gamma_s$ was given by (Lenz and Nierste, 2011)

$$\Delta \Gamma_s^{\text{SM,2011}} = (0.087 \pm 0.021) \text{ ps}^{-1}. \quad (89)$$

We take the most recent numerical inputs from the following sources: $G_F$, $M_W$, $M_B$, and $m_b$ from the PDG (Olive et al., 2014), the top quark mass from (ATLAS and Collaborations, 2014), the non-perturbative parameters from FLAG (web-update of (Aoki et al., 2014)) and $B_S/B$, $B_{s0}$, $B_{s1}$ and $B_{s2}$ from (Becirevic et al., 2002), (Bouchard et al., 2011), (Carrasco et al., 2014) and (Dowdall et al., 2014) and CKM elements from CKMfitter (web-update of (Charles et al., 2005)) - similar values can be taken from UTfit (Bona et al., 2006b). With these new values we predict the decay rate difference of the neutral $B_s$ mesons to be

$$\Delta \Gamma_s^{\text{SM,2015}} = (0.088 \pm 0.020) \text{ ps}^{-1}. \quad (90)$$

The dominant uncertainty stems from the dimension 7 bag parameter $B_{R_2}$ (about 15%), closely followed by $f_B \sqrt{B}$ (about 14%) and the renormalisation scale dependence, which contributes about 8% to the error budget. A detailed listing of all the contributing uncertainties can be found in Appendix B. In order to reduce the theory uncertainty to a value between 5% and 10%, a non-perturbative determination of $B_{R_2}$, a calculation of NNLO-QCD corrections (denoted by $\Gamma_5^{(2)}$ in Eq. (55)), a first step in this direction, has been done by (Asatryan et al., 2012) and by $\Gamma_4^{(1)}$ and more precise values of the matrix elements of the operators $Q_s, Q_S$ and $Q_{s0}$ are mandatory. All of this seems to be feasible in the next few years.

In the discussion of the dimuon asymmetry in Section III we will also need several mixing quantities from the $B^0$ sector. Their calculation within the Standard Model is analogous to the one in the $B^0_s$ sector. We present here numerical updates of the predictions given in (Lenz and Nierste, 2011). The input parameters are identical to the ones in the $B^0_s$ system, except $f_{B_s} \sqrt{B}$, $B_s/B$, $M_{B^0}$ and $m_d$, which can found in the same literature as the values for the $B^0_s$ system. Our new predictions are

$$\Delta M_{d_s}^{\text{SM,2015}} = (0.528 \pm 0.078) \text{ ps}^{-1}, \quad (91)$$
$$\Delta \Gamma_d^{\text{SM,2015}} = (2.61 \pm 0.59) \cdot 10^{-3} \text{ ps}^{-1}, \quad (92)$$

$$\left(\frac{\Delta \Gamma_d}{\Gamma_d}\right)^{\text{SM,2015}} = (3.97 \pm 0.90) \cdot 10^{-3}, \quad (93)$$
$$\Re \left(\frac{\Gamma_{12}}{M_{12}^2}\right)^{\text{SM,2015}} = (-49.4 \pm 8.5) \cdot 10^{-4}. \quad (94)$$

A detailed error analysis is given in Appendix B.

B. Experiment: Mass and decay rate difference $\Delta M_s$ and $\Delta \Gamma_s$.

Experimental studies of $\Delta M_s$ and $\Delta \Gamma_s$ and their comparison with the theoretical predictions of Eq. (51) and Eq. (90) constitute an important SM test. In addition, $\Delta M_s$ together with the mass difference $\Delta M_d$ of the $B^0$ meson can be used to evaluate the ratio of the CKM parameters $|V_{td}/V_{ts}|$. These elements are not likely to be measurable with high precision in tree-level decays involving a top-quark, because the top quark is too short-lived to form a hadron (see e.g. (Olive et al., 2014)), but the ratio between $\Delta M_d$ and $\Delta M_s$ provides a theoretically clean and precise constraint. Using the results discussed below, and un-quenched lattice calculations, Ref. (Olive et al., 2014) quotes

$$\left|\frac{V_{td}}{V_{ts}}\right| = 0.216 \pm 0.001 \pm 0.011, \quad (95)$$

where the first error stems from experiment and the second from theory. Therefore, the measurement of $\Delta M_s$, although not directly related to CP violation, contributes significantly to the test of the unitarity of the CKM matrix (Amhis et al., 2014).

The measurement of $\Delta M_s$ and $\Delta \Gamma_s$ eluded experimentalists for a very long time. A relatively large value of $|V_{ts}|$ results in a high oscillation frequency of $B^0_s$ mesons and numerous transitions from particle to anti-particle during its lifetime. Therefore, a high precision of the
proper decay length measurement is required to be sensitive to $\Delta M_s$. On the other side, the measurement of $\Delta \Gamma_s$ is also challenging because $\Delta \Gamma_s/\Gamma_s = O(10\%)$.

The measurement of $\Delta M_s$ was attempted by many experiments during more than 20 years; the CDF collaboration at Fermilab first succeeded to perform it with a statistical significance exceeding five standard deviations (Abulencia et al., 2006).

From a technical point of view, the measurement of $\Delta M_s$ requires these essential components:

- identification of the flavor of the $B^0_s$ meson at the time of production;
- identification of the flavor of the $B^0_s$ meson when it decays;
- measurement of its proper lifetime.

To measure the final state of the $B^0_s$ meson decay, a flavour-specific transition is used. The simplest flavour-specific state is the semileptonic decay $B^0_s \to D^+_s \mu^+ \nu_\mu$ since the muon usually provides an excellent possibility for an efficient selection of such decays during both the data taking and the subsequent analysis. However, the precision of the proper lifetime measurement in this decay mode is rather poor because of the missing neutrino taking some part of the $B^0_s$ momentum. Figure 5 shows the proper decay time resolution for different decay modes as a function of the $B^0_s$ proper decay time in the CDF measurement. The resolution in the semileptonic decay channel deteriorates very quickly with the increase of the proper time. Therefore, the ability of an experiment to reconstruct hadronic $B^0_s$ decays such as $B^0_s \to D^+_s \pi^+$ plays a crucial role in the $\Delta M_s$ measurement.

The identification of the $B^0_s$ initial state, also known as the initial flavour tagging (IFT), was first developed and used at hadron colliders by the CDF (Abulencia et al., 2006) and D0 (Abazov et al., 2006b) experiments at the Tevatron. In the LHCb implementation of the IFT (Aaij et al., 2012g, 2013f, 2015a), the special capabilities of the detector, such as the particle identification and efficient reconstruction of secondary decays, are extensively used.

Technically, the IFT is divided into opposite-side (OS) and same-side (SS) tagging. At LHC, where the gluon splitting dominates the $b\bar{b}$ production and the $b$ quarks are considerably boosted, the “opposite-side” is actually not “opposite” at all. Therefore, the naming of the two IFT methods is nowadays largely historical and does not reflect the actual topology of the $b\bar{b}$ events. The OS tagging is based on the correlation of the flavours of two produced $B$ hadrons, while the SS tagging exploits the correlation of the flavour of the $B^0_s$ meson and the charge of additional particles produced in the hadronisation of the initial $b$ quark. The performance of the IFT is quantified by the tagging power $P$, which is expressed as $P = \epsilon (1 - 2w)^2$, where $\epsilon$ is the tagging efficiency and $w$ is the wrong-tag probability. The tagging power multiplied by the total number of events in the analysis corresponds to the effective statistics used to measure $\Delta M_s$.

The performance of the IFT in different experiments is presented in Table I. It includes the results of the ATLAS (Aad et al., 2016) and CMS (Khachatryan, 2015) collaborations, who use the IFT for the measurement of CP violation. It can be seen that the tagging power never exceeds few percents meaning that a large statistics should be collected to obtain the significant measurement of $\Delta M_s$. In general, the tagging power improves with a better understanding of the underlying event and with the refinement of multi-variate tagging methods.

![Figure 5](image_url) The proper decay time resolution measured by the CDF collaboration. The plot is taken from Ref. (Abulencia et al., 2006).

<table>
<thead>
<tr>
<th>Experiment method</th>
<th>$P$ (%)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDF OS</td>
<td>1.8 ± 0.1</td>
<td>(Abulencia et al., 2006)</td>
</tr>
<tr>
<td>CDF SS</td>
<td>3.7±0.9</td>
<td>(Abulencia et al., 2006)</td>
</tr>
<tr>
<td>D0 OS</td>
<td>2.48 ± 0.22</td>
<td>(Abazov et al., 2006b)</td>
</tr>
<tr>
<td>LHCb</td>
<td>2.55 ± 0.14</td>
<td>(Aaij et al., 2013f)</td>
</tr>
<tr>
<td>LHCb SS</td>
<td>1.26 ± 0.17</td>
<td>(Aaij et al., 2013f)</td>
</tr>
<tr>
<td>ATLAS OS</td>
<td>1.49 ± 0.02</td>
<td>(Aad et al., 2016)</td>
</tr>
<tr>
<td>CMS OS</td>
<td>1.307 ± 0.032</td>
<td>(Khachatryan, 2015)</td>
</tr>
</tbody>
</table>

TABLE I Performance of the initial flavour tagging in different experiments. The numbers shown correspond to the same-side (SS) or the opposite-side (OS) tagging power ($P$). The uncertainty shown is the combination of the statistical and systematic uncertainties. In general, the same-side flavour tagging depends upon the mode being investigated. CDF finds a SS tagging power of $P = (4.8 ± 1.2\%)$ (Abulencia et al., 2006) in the semileptonic decay sample.
The period of oscillation of the $B^0_s$ meson corresponding to $\Delta M_s = 17.76 \text{ ps}^{-1}$ is $T = 2\pi/\Delta M_s \approx 350 \text{ fs}$. To measure it reliably and thus extract $\Delta M_s$, the precision of the proper lifetime measurement should be at least four times better. The precision of the proper decay length measurement in the CDF experiment was about 100 fs, while for the LHCb experiment it is about 44 fs. This excellent performance together with large statistics collected by the LHCb experiment in the LHC Run I results in a much better precision of the $\Delta M_s$ measurement. They also succeeded to obtain a clear oscillation pattern in the proper decay length distribution, which is shown in Fig. 6.

![FIG. 6 Proper decay time distribution for the selected $B^0_s$ decays candidates tagged as mixed (different flavour at decay and production; red, continuous line) or unmixed (same flavour at decay and production; blue, dotted line). The data and the fit projections are plotted in a signal window around the reconstructed $B^0_s$ mass of $5.32 - 5.55 \text{ GeV}/c^2$. The plot is taken from Ref. (Aaij et al., 2013h).](image)

The first double sided bound at 90 % C.L on the $\Delta M_s$ value was obtained by the D0 collaboration (Abazov et al., 2006a). Soon after that the CDF collaboration reported the actual measurement of this quantity (Abulencia et al., 2006)

$$\Delta M_s^{\text{CDF}} = 17.77 \pm 0.10(\text{stat}) \pm 0.07(\text{syst}) \text{ ps}^{-1}. \quad (96)$$

Later, the LHCb collaboration performed the most precise single-experiment measurement of $\Delta M_s$ (Aaij et al., 2013h)

$$\Delta M_s^{\text{LHCb}} = 17.768 \pm 0.023(\text{stat}) \pm 0.006(\text{syst}) \text{ ps}^{-1}. \quad (97)$$

The combination of all $\Delta M_s$ measurements by the HFAG (Amhis et al., 2014) gives

$$\Delta M_s^{\text{HFAG 2015}} = 17.757 \pm 0.021 \text{ ps}^{-1}. \quad (98)$$

The currently most precise measurement of $\Delta \Gamma_s$ consists in the simultaneous study of the proper decay length and angular distributions of the decay $B^0_s \rightarrow J/\psi K^+ K^-$ which mainly includes the $B^0_s \rightarrow J/\psi f_0(980)$ final state. For simplicity, this study is denoted as $B^0_s \rightarrow J/\psi f_0(980)$ channel in the following discussion, although it should be remembered that the addition of the non-resonant contribution is required for an appropriate analysis of data. Both the CP-even and CP-odd $B^0_s$ states contribute in this decay mode and therefore its properties are sensitive to both the $B^0_s$ width difference and the phase $\phi_s$ (defined in Section IV.A) describing CP violation in the interference of decay and mixing.

All collider experiments at the Tevatron and LHC perform the measurement of $\Delta \Gamma_s$ in the $B^0_s \rightarrow J/\psi f_0(980)$ decay. The first results were obtained by the CDF (Aaltonen et al., 2012) and D0 (Abazov et al., 2012a) collaborations, who largely developed the measurement technique. The ATLAS (Aad et al., 2016), CMS (Khachatryan, 2015) and LHCb (Aaij et al., 2015h) collaborations continue this study at LHC, where a significantly larger statistics is collected and much more data are expected in the future.

As for $\Delta M_s$, the measurement of $\Delta \Gamma_s$ in $B^0_s \rightarrow J/\psi f_0(980)$ decay requires IFT and the proper decay length of the $B_s$ meson. In addition, the study of the angular distributions of the $B^0_s$ decay products is needed. This is the reason why this analysis is sensitive to the quality of the data description by the simulation. All experiments succeed in achieving an excellent understanding of their detectors.

The measurements of $\Delta \Gamma_s$ using $J/\psi f_0(K^+ K^-)$ are summarised in Table II. It also includes the world average value obtained by the HFAG (Amhis et al., 2014), which is found to be

$$\Delta \Gamma_s = 0.079 \pm 0.006 \text{ ps}^{-1} (B^0_s \rightarrow J/\psi f_0). \quad (99)$$

An alternative approach to determine $\Delta \Gamma_s$ relies upon the direct measurement of the effective lifetime of $B^0_s$ decays to pure CP eigenstates. The extraction of $\Delta \Gamma_s$ with this method is discussed in detail in Ref.(Fleischer and Knecht, 2011a).

To first order in $y_s \equiv \Delta \Gamma_s/(2\Gamma_s)$, we have (Amhis et al., 2014)

$$\tau_{\text{single}}(B^0_s \rightarrow CP - \text{ even}) \approx \frac{1}{\Gamma_L} \left( 1 + \frac{\phi_s}{2} y_s \right) \quad (100)$$

$$\tau_{\text{single}}(B^0_s \rightarrow CP - \text{ odd}) \approx \frac{1}{\Gamma_H} \left( 1 - \frac{\phi_s}{2} y_s \right) \quad (101)$$

where $\tau_{\text{single}}$ is the effective lifetime of the $B^0_s$ decaying to a specific CP-eigenstate state $f$. This formula assumes that $A_{\Gamma_s}^{\text{CP-\text{EVEN}} = \cos \phi_s}$ and $A_{\Gamma_s}^{\text{CP-\text{ODD}} = -\cos \phi_s}$, where the mixing angle $\phi_s$ will be defined in Section IV.A. Thus, the decay width measured in the CP-even final state, such as $B^0_s \rightarrow K^+ K^-$ and $B^0_s \rightarrow D^+ D^-$, is approximately equal to $1/\Gamma_L(s)$. Similarly, the CP-odd decay modes $B^0_s \rightarrow J/\psi K^0_s$ and $B^0_s \rightarrow J/\psi f_0(980)$ provide measurements of $1/\Gamma_H(s)$, thus $\Delta \Gamma_s$ can be obtained as the difference of these two quantities. There
are several subtleties that need to be taken into account when using this method to measure $\Delta \Gamma_s$. For example, the decays $B_s^0 \to K^+K^-$ and $B_s^0 \to J/\psi K_S^0$ may suffer from CP violation due to interfering tree and loop amplitudes. Thus Ref. (Amhis et al., 2014) uses only the effective lifetimes obtained for $D_s^+D_s^-$ (CP-even), and $J/\psi f_0$, $J/\psi \pi$ (CP-odd) decays to obtain

$$\tau_{\text{single}}(B_s^0 \to CP - \text{even}) = 1.379 \pm 0.031 \text{ ps} \quad (102)$$

$$\tau_{\text{single}}(B_s^0 \to CP - \text{odd}) = 1.656 \pm 0.033 \text{ ps} \quad (103)$$

Table III summarises the current values as well as the average values of $1/\Gamma_{Z}^L$ and $1/\Gamma_{Z}^H$ reported in Ref. (Amhis et al., 2014). Note that the effective lifetimes measured in $B_s^0 \to K^+K^-$ and $B_s^0 \to J/\psi K_S^0$ have not been used in these averages because of the difficulty in quantifying the penguin contribution in these modes. These effective lifetimes correspond to

$$\Delta \Gamma_s = 0.121 \pm 0.020. \quad (104)$$

This value is higher by two standard deviations than the one shown in Eq. (99). However, this difference should be considered with caution. The value in Eq. (104) is obtained with theoretical assumptions and external input on weak phases and hadronic parameters.

Using these data in conjunction with the $J/\psi/\psi(K^+K^-)$ determinations of $\Delta \Gamma_s$, the current experimental average is (Amhis et al., 2014)

$$\Delta \Gamma_s^{\text{HFAG 2015}} = 0.083 \pm 0.006 \text{ ps}^{-1} \quad (105)$$

The comparison of different lifetime measurements of CP eigenstates, which can be used to extract $\Delta \Gamma_s$ is presented in Fig. 7.

At the end of this section we would like to compare the experimental and theoretical numbers for the mass difference and the decay rate difference. For the experimental value of the mass difference we take the value from Eq. (98) and for the value of the decay width difference we take Eq. (105). For the theory value, we take the more precise prediction of the ratio $\Delta \Gamma_s/\Delta M_s$. We find a very good agreement for experiment and theory

$$\frac{(\Delta \Gamma_s/\Delta M_s)_{\text{Exp}}}{(\Delta \Gamma_s/\Delta M_s)_{\text{SM}}} = 0.00467(1 \pm 0.072) \quad (106)$$

$$= 0.97 \pm 0.07 \pm 0.17. \quad (107)$$

In the last line the first error is the experimental and the second the theoretical. This results proves that the heavy quark expansion is working in the $B$-sector with a precision of at least 20%, also for the decay channel $b \to c\bar{c}s$, which seems to be most sensitive to violations of quark hadron duality. Assuming that there are no new physics effects in $\Delta M_s$ and taking into account that the ratio $\Delta \Gamma_s/\Delta M_s$ is theoretically cleaner than $\Delta \Gamma_s$ alone, we get an improved prediction for $\Delta \Gamma_s$

$$\Delta \Gamma_s^{\text{SM,2015b}} = \left( \frac{\Delta \Gamma_s}{\Delta M_s} \right)^{\text{SM}} \cdot \Delta M_s^{\text{Exp}} = 0.085 \pm 0.015 \text{ ps}^{-1}. \quad (108)$$

This is the most precise theory value for $\Delta \Gamma_s$ that can

<table>
<thead>
<tr>
<th>Exp.</th>
<th>$\Delta \Gamma_s$ (ps$^{-1}$)</th>
<th>$\Gamma_s$ (ps$^{-1}$)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDF</td>
<td>0.068 ± 0.026 ± 0.009</td>
<td>0.654 ± 0.008 ± 0.004</td>
<td>(Aaltonen et al., 2012)</td>
</tr>
<tr>
<td>D0</td>
<td>0.163±0.065</td>
<td>0.693±0.018</td>
<td>(Abazov et al., 2012a)</td>
</tr>
<tr>
<td>ATLAS</td>
<td>0.083±0.011 ± 0.007</td>
<td>0.677±0.003 ± 0.003</td>
<td>(Aad et al., 2016)</td>
</tr>
<tr>
<td>CMS</td>
<td>0.095±0.013 ± 0.007</td>
<td>0.6704±0.0043 ± 0.0051</td>
<td>(Khachatryan, 2015)</td>
</tr>
<tr>
<td>LHCb</td>
<td>0.0805±0.0091 ± 0.0033</td>
<td>0.6603±0.0027 ± 0.0015</td>
<td>(Aaij et al., 2015h)</td>
</tr>
<tr>
<td>HFAG</td>
<td>0.079±0.006</td>
<td>0.6649±0.0022</td>
<td>(Amhis et al., 2014)</td>
</tr>
</tbody>
</table>

**TABLE II Measurements of $\Delta \Gamma_s$ in $B_s^0 \to J/\psi$ decay.** The last line gives the world average value obtained by HFAG.
in semileptonic decays are also called semileptonic CP asymmetries. Examples for such decays are e.g. 

\[ B^0_s \rightarrow K^+ K^- \] 

and 

\[ B^0_s \rightarrow D^+_s D_s^- \] 

can be further simplified as 

\[ a_{s}^s = -2 \left( \frac{|q|}{|p|} - 1 \right) \]

\[ = 3 \left( \frac{\Gamma_{12}^s}{M_{12}^s} \right) \frac{1}{\tan \phi_{12}} \sin \phi_{12} \cdot \] (113)

For the SM prediction of the flavour specific asymmetries we can now simply use our determination of the ratio of the matrix elements \( M_{12}^s \) and \( \Gamma_{12}^s \) from the previous section, in particular we need only the coefficient \( a \) (b gives only a small correction) defined in Eq.(78) to get:

\[ a_{s}^s \approx \frac{1}{a} \cdot 10^{-4} \] .

(114)

The coefficient \( a \) was given given by the difference of the internal charm-charm loop and the internal up-charm loop. Using the exact expression for \( \Gamma_{12}^s/M_{12}^s \) the Standard Model prediction of \( a_{s}^s \) was given by (Lenz and Nierste, 2011)

\[ a_{s}^{s, SM, 2011} = (1.9 \pm 0.3) \cdot 10^{-5} \] .

(115)

With the most recent numerical inputs (\( G_F, M_W, M_{B_s} \) and \( m_b \) from the PDG (Olive et al., 2014), the top quark mass from (ATLAS and Collaborations, 2014), the non-perturbative parameters from FLAG (web-update of (Aoki et al., 2014) in Summer 2015) and \( B_S/B, B_{s_0}, B_{s_1}, B_{s_2} \) from (Becirevic et al., 2002), (Boudard et al., 2011), (Carrasco et al., 2014) and (Dowdall et al., 2014) and CKM elements from CKMfitter (web-update of (Charles et al., 2005) in Summer 2015) ( similar values can be taken from UTfit (Bona et al., 2006b) ) we predict the flavour specific CP asymmetries of the neutral \( B^0_s \) mesons to be

\[ a_{s}^{s, SM, 2015} = (2.22 \pm 0.27) \cdot 10^{-5} \] .

(116)

The dominant uncertainty stems from the renormalisation scale dependence, with 9\%, followed by the CKM quantum number.
dependence with 5% and the charm quark mass dependence with 4%. A detailed discussion of the uncertainties is given in Appendix B. Because of this small value and the proven validity of the HQE, the flavour specific asymmetries represent a nice null test, as any sizable experimental deviation from the prediction in Eq.(116) is a clear indication for new physics, see (Jubb et al., 2016) for a more detailed discussion of this point.

In addition we obtain the SM prediction for the mixing phase $\phi_{12}^\text{SM,2015}$:

$$
\phi_{12}^\text{SM,2015} = (4.6 \pm 1.2) \cdot 10^{-3} \text{ rad}\quad (117)
$$

$$
= 0.26^\circ \pm 0.07^\circ .
$$

In the discussion of the dimuon asymmetry below we also need the semileptonic CP asymmetry from the $B^0$ sector. Its calculation within the SM is analogous to the one of $a^\text{sl}_s$. We update the predictions given in (Lenz and Nierste, 2011), by using the same input parameters as for the $B^0_\text{s}$-system, except using $M_{B^0}$, $m_d$ and $B_S/B$. We get as new Standard Model values

$$
a^\text{sl,SM,2015}_s = (-4.7 \pm 0.6) \cdot 10^{-4},
$$

$$
\phi_{12}^\text{sl,SM,2015} = (-0.996 \pm 0.025) \text{ rad}
$$

$$
= -5.5^\circ \pm 1.4^\circ .
$$

A more detailed analysis of the uncertainties can be found in Appendix B. Measurements of the dimuon asymmetry triggered a lot of interest in $B^0$ and $B^0_\text{s}$ mixing, because early measurements seemed to indicate large new physics effects (Abazov et al., 2010a,b, 2011, 2014). Originally, the dimuon asymmetry $A_{CP}$ was considered to be given by a linear combination of the semileptonic CP asymmetries in the $B^0$ and the $B^0_\text{s}$ system (see e.g. (Abazov et al., 2010a,b, 2011))

$$
A_{CP} = C_d a^d_{s} + C_s a^s_d ,
$$

(121)

with $C_d$ and $C_s$ being roughly equal. The large deviation of the measured value of $A_{CP}$ from the calculated values of the linear combination of $a^d_s$ and $a^s_d$ seemed to be a hint for large new physics effects in the semileptonic CP asymmetries. In 2013 Borissov and Hoeiven (Borissov and Hoeiven, 2013) found that there is actually also an additional contribution from indirect CP violation. This led to the following new interpretation (also used in (Abazov et al., 2014))

$$
A_{CP} = C_d a^d_{s} + C_s a^s_d + C_{\Delta \Gamma_d} \frac{\Delta \Gamma_d}{\Gamma_d} ,
$$

(122)

Because of the small value of $\Delta \Gamma_d$ in the SM, see Eq.(92) and Eq.(93) the additional term did not solve the discrepancy. It was pointed out (Nierste, 2014), that the relation should be further modified to

$$
A_{CP} = C_d a^d_{s} + C_s a^s_d + \alpha C_{\Delta \Gamma_d} \frac{\Delta \Gamma_d}{\Gamma_d} ,
$$

(123)

where $\alpha \leq 1/2$. An interesting feature of this new interpretation is that a large enhancement of $\Delta \Gamma_d$ by new physics effects could explain the experimental value of the dimuon asymmetry, while huge enhancements of the semileptonic CP asymmetries are disfavoured by direct measurements, see next section. The investigation of (Bobeth et al., 2014) has further shown that enhancements of $\Delta \Gamma_d$ by several hundred per cent are not excluded by any other experimental constraint - such an enhancement could bring the dimuon asymmetry in agreement with experiment. One possible enhancement mechanism would be e.g. new $b \bar{d} \tau \tau$ transitions. Since two tau leptons are lighter than a $B^0$ meson such a new operator could contribute to $\Gamma_{12}^d$. This possibility can be tested by investigating $b \bar{d} \tau \tau$ transitions directly. In Fig. 8 we show the possible enhancement of $\Delta \Gamma_d$ due to new scalar (l.h.s.) and due to new vector (r.h.s.) $b \bar{d} \tau \tau$ operators. Currently enhancements within the yellow regions are allowed. In the case of vector operators $\Delta \Gamma_d$ can be enhanced to about 3.5 the SM value of $\Delta \Gamma_d$. The connection between a direct measurement of or a bound on $B^0 \to \tau^+ \tau^-$ is given by the red line. From Fig. 8 one can read off that a bound on $B^0 \to \tau^+ \tau^-$ of the order of $10^{-3}$ would limit the enhancement of $\Delta \Gamma_d$ to about 15% of the SM value in the case of scalar new physics operators and to about 50% of the SM value in the case of scalar new physics operators. Similar relations between a possible enhancement of $\Delta \Gamma_d$ and a direct search for $B^0 \to X_d \tau^+ \tau^-$ and $B^+ \to \pi^+ \tau^+ \tau^-$ are indicated by the blue line and the green line.

Another enhancement mechanism would be new physics effects in tree-level decays, which are typically neglected. Such studies were performed systematically in (Bobeth et al., 2015, 2014; Brod et al., 2015) and could also lead to sizable enhancements of $\Delta \Gamma_d$. Here a more precise measurement of $\Delta \Gamma_d$ would of course be very helpful.

B. Experiment: Semi-leptonic asymmetries $a^d_s$ and $a^s_d$, the di-muon asymmetry

The measurement of the flavour-specific charge asymmetry is conceptually simple. Essentially, it is given by the asymmetry between flavour-specific decays $B^0 \to f$ and $B^0_\text{s} \to f$. As the expected value of the asymmetry is tiny, great care needs to be taken to assess any potential source of asymmetry, for example, production dynamics, background sources, or detection asymmetry. The final state typically used for this measurement is the semi-leptonic decay

$$
B^0 \to D^{(*)}_s \mu^+ \nu,
$$

where the notation $(*)$ denotes the production of either $D^+_s$, $D^-_s$, or $D_{sJ}$ states. The published results consider only the decay $D_s \to \phi \pi$ with $\phi \to K^+ K^-$. The initial flavor of the $B^0_\text{s}$ meson is not determined and the measured quantity is

$$
A_{\text{meas}} = \frac{N(D^+_s \mu^+) - N(D^-_s \mu^-)}{N(D^+_s \mu^+) + N(D^-_s \mu^-)} ,
$$

(124)
where \( N(f) \) (\( f = D_s^+ \mu^+ + D_s^- \mu^- \)) is the number of reconstructed events in the final state \( f \). It can be expressed as

\[
N(f) \propto \int_0^{+\infty} [\sigma(B_s^0)\Gamma(B_s^0(t) \rightarrow f) + \sigma(B_s^0)\Gamma(B_s^0(t) \rightarrow f)] \epsilon(t) dt. \tag{125}
\]

This expression takes into account the absence of the initial flavour tagging, the possible difference in the production cross-sections \( \sigma(B_s^0) \) and \( \sigma(B_s^0) \), and time dependent reconstruction efficiency \( \epsilon(t) \) of the final state \( f \). The most important instrumental charge-asymmetries are related to differences between \( \mu^+ - \mu^- \), and \( \pi^+ - \pi^- \) detection efficiencies. The two opposite-charge kaons from \( \phi \) decay have almost the same momentum spectrum, and thus charge-dependent detection effects do not influence the measured asymmetry.

Using the expressions of the time evolution of \( B_s^0 \) mesons, assuming that the ratio of the reconstruction efficiencies \( r_s \equiv \epsilon(D_s^0 \mu^+, t)/\epsilon(D_s^0 \mu^-, t) \) does not depend on time, and neglecting the second order terms, the semi-leptonic charge asymmetry \( a_{s\mu} \) is related to \( A_{\text{meas}} \) as

\[
A_{\text{meas}} = \frac{a_{s\mu}}{2} - \frac{1 - r_s}{2} + \left( a_p - \frac{a_{s\mu}}{2} \right) I \tag{126}
\]

\[
I \equiv \int_0^{+\infty} e^{-\Gamma_s t} \cosh(\Delta M_s t) \epsilon(t) dt \int_0^{+\infty} e^{-\Gamma_s t} \cosh(\Delta M_s t/2) \epsilon(t) dt.
\]

Here \( a_p \) is the production asymmetry of the \( B_s^0 \) meson defined as

\[
a_p = \frac{\sigma(B_s^0) - \sigma(B_s^0)}{\sigma(B_s^0) + \sigma(B_s^0)}. \tag{127}
\]

The asymmetry \( a_p \) is zero at a \( p\bar{p} \) collider, while it does not exceed a few percent for the \( B_s^0 \) production at LHC (see (Aaij et al., 2012f, 2013d; Norrbin and Vogt, 2000)). Because of the large value of \( \Delta M_s \), the value of \( I \) is about 0.2%. As a result, the value of the third term in Eq. (126) is of the order of \( 10^{-4} \) and can be safely neglected. Thus, the main experimental task in the measurement of the \( a_{s\mu} \) is the determination of \( r_s \).

Measurements of the asymmetry \( a_{s\mu} \) have been reported by the D0 (Abazov et al., 2013) and LHCb (Aaij et al., 2014c) collaborations. Both D0 and LHCb collected large statistics using semi-leptonic \( B_s^0 \) decays. The number of reconstructed signal events in the D0 measurement is 215763 ± 1467. The corresponding \( \mu^{\pm} \phi \pi^{\mp} \) mass distribution is shown in Fig. 9. Recently, LHCb updated the measurement of \( a_{s\mu} \), using three \( fb^{-1} \) and including all the possible \( D_s \) decays to the \( K^{+}K^{-}\pi^{+} \) final state. The corresponding mass distribution is shown in Fig. 10.

The important feature of both experiments is a regular reversal of the magnet polarities. In the D0 experiment, the polarities of the toroidal and solenoidal magnetic fields (Abazov et al., 2006c) were reversed on average every two weeks so that the four solenoid-toroid polarity combinations are exposed to approximately the same integrated luminosity. D0 reported only results averaged over all the magnet polarities. The 1 \( fb^{-1} \) LHCb sample comprises approximately 40% of data taken with the magnetic field up, oriented along the positive \( y \)-axis in the LHCb coordinate system, and the rest with the opposite down polarity. The 2 \( fb^{-1} \) sample comprises equal amounts of data with the two magnet polarities. LHCb analyses data with magnetic field up and down separately, to allow a quantitative assessment of charge-dependent asymmetries. Figure 11 shows their measurement of the ratio \( r_s \) for two magnet polarities and the two data sets. It can be seen that the majority of the detection asymmetry changes sign with the reversal of the magnet polarity, and thus the final average of the two
samples is much less sensitive to detection asymmetry.

The resulting values of $a_{sl}^D$ obtained by the two experiments as well as their average are

$$a_{sl}^{D_0} = (-1.12 \pm 0.74 \pm 0.17) \times 10^{-2}$$

$$a_{sl}^{LHCb} = (+0.39 \pm 0.26 \pm 0.20) \times 10^{-2}$$

$$a_{sl}^{average} = (+0.17 \pm 0.30) \times 10^{-2}.$$  

Both results are consistent with the Standard Model expectation (116), albeit the uncertainty is still a factor of about 130 larger than the central value in the Standard Model.

The Babar, Belle, D0, and LHCb collaborations perform the independent measurement of the asymmetry $a_{sl}^d$.

FIG. 9 The weighted $K^+K^-\pi^\pm$ invariant mass distribution for the $\mu\phi\pi^\pm$ sample. The solid line represents the result of the fit and the dashed line shows the background parametrisation. The lower mass peak is due to the decay $D^{\pm} \rightarrow \phi\pi^\pm$ and the second peak is due to the $D_s^{\mp}$ meson decay. Note the suppressed zero on the vertical axis. The plot is taken from (Abazov et al., 2013).

FIG. 10 Invariant mass distributions of $K^+K^-\pi^\pm$ and (b) $K^+K^-\pi^-$ in the three Dalitz regions studied in the LHCb analysis, summed over both magnet polarities and data taking periods. Overlaid are the results of the fits, with signal and combinatorial background components as indicated in the legend. The plot is taken from (Aaij et al., 2016).

FIG. 11 Relative muon efficiency as a function of the muon momentum. The plot is taken from (Aaij et al., 2014c).

<table>
<thead>
<tr>
<th>Experiment</th>
<th>measured $a_{sl}^d$ (%)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHCb $D^{(*)}\mu X$</td>
<td>$-0.02 \pm 0.19 \pm 0.30$</td>
<td>(Aaij et al., 2015f)</td>
</tr>
<tr>
<td>D0 $D^{(*)}\mu X$</td>
<td>$+0.68 \pm 0.45 \pm 0.14$</td>
<td>(Abazov et al., 2012b)</td>
</tr>
<tr>
<td>BaBar $D^{(*)}\ell X$</td>
<td>$+0.29 \pm 0.84_{-1.61}^{+1.88}$</td>
<td>(Lees et al., 2013)</td>
</tr>
<tr>
<td>BaBar $\ell\ell$</td>
<td>$-0.39 \pm 0.35 \pm 0.19$</td>
<td>(Lees et al., 2015b)</td>
</tr>
</tbody>
</table>

TABLE IV Most recent measurements of the $CP$ violation parameter $a_{sl}^d$.

Their results are summarised in Table IV. The world average value of $a_{sl}^d$ is

$$a_{sl}^d (HFAG) = 0.0001 \pm 0.0020.$$  

The D0 experiment also reports a complementary measurement related to the semi-leptonic asymmetries of $B^0_s$ and $B^0$ mesons (Abazov et al., 2014). It performs the simultaneous study of the inclusive semi-leptonic charge asymmetry and of the like-sign di-muon charge asymmetry. These quantities are defined as

$$a = \frac{n^+ - n^-}{n^+ + n^-},$$

$$A = \frac{N^{++} - N^{--}}{N^{++} + N^{--}}.$$  

Here $n^+$ and $n^-$ are the number of events with the reconstructed positive or negative muon, respectively. $N^{++}$ and $N^{--}$ are the number of events with two positive or two negative muons, respectively. The asymmetries $a$ and $A$ are cast as

$$a = a_{CP} + a_{bkg},$$

$$A = A_{CP} + A_{bkg}.$$  

Here $a_{CP}$ and $A_{CP}$ are the asymmetries due to the genuine $CP$-violating processes, such as $CP$ violation in mixing of $B^0_s$ and $B^0$ mesons. The asymmetries $a_{bkg}$ and $A_{bkg}$ are...
A_{bkg} are produced by the background processes not related to CP violation. The main source of these asymmetries is the difference in the interaction cross-section of the positive and negative charged particles with the detector material. The main challenge in the D0 analysis is the accurate estimate of the background asymmetries $a_{bkg}$ and $A_{bkg}$ and the extraction of the values of $a_{CP}$ and $A_{CP}$.

The asymmetries $a_{CP}$ and $A_{CP}$ depend on both $a_{d}^{s}$ and $a_{a}^{s}$. Since the oscillation period of $B^{0}$ and $B^{0}_{s}$ mesons is significantly different, the contribution of $a_{d}^{s}$ and $a_{a}^{s}$ strongly depends on the decay time of collected $B$ mesons. This decay time is not measured in the inclusive analysis. Instead, the D0 experiment measures the asymmetries $a_{CP}$ and $A_{CP}$ in sub-samples containing the muons with different muon impact parameter. The division into the sub-samples according to the muon impact parameter is used to estimate the contribution of $a_{d}^{s}$ and $a_{a}^{s}$. In addition, the asymmetry $A_{CP}$ is sensitive to the width difference $\Delta \Gamma_{d}$ of $B^{0}$ meson (see (Borissov and Hoeneisen, 2013)) and this quantity is also obtained in the D0 analysis. Their result is

\begin{align}
    a_{d}^{s} &= (-0.62 \pm 0.43) \times 10^{-2}, \\
    a_{a}^{s} &= (-0.82 \pm 0.99) \times 10^{-2}, \\
    \frac{\Delta \Gamma_{d}}{\Gamma_{d}} &= (+0.50 \pm 1.38) \times 10^{-2}.
\end{align}

The correlation between the fitted parameters are

\begin{equation}
    \rho_{d,s} = -0.61, \quad \rho_{d,\Delta \Gamma} = -0.03, \quad \rho_{s,\Delta \Gamma} = +0.66.
\end{equation}

Although the central values of all three quantities are consistent with the SM prediction within the uncertainties, a deviation from the SM prediction by 3 standard deviations is reported because of the correlation between these observables.

The world knowledge of CP violating parameters in $B^{0}$ and $B^{0}_{s}$ mixing is summarised in Fig. 12, that shows that the individual measurements of $a_{d}^{s}$ and $a_{a}^{s}$ are consistent with the Standard Model. Only the D0 di-muon result suggests a deviation from Standard Model expectations in CP violation in neutral $B^{0}$ oscillations. Since this measurement is inclusive, other unknown effects not directly related to CP violation in $B^{0}_{s}$ mixing could contribute to it.

The LHCb experiment is currently taking data and is expected to collect an additional sample of $\sim 6$ fb$^{-1}$ in the current LHC run, and at least $50$ fb$^{-1}$ with an upgraded detector to be installed in 2020. Moreover Belle II will start taking data in a time scale comparable to the expected start of the LHCb upgraded detector. Therefore the prospects for increased precision in CP violating asymmetries in neutral $B$ meson decays are excellent.

![Figure 12](image_url)

**IV. CP VIOLATION IN INTERFERENCE**

**A. Theory**

In this section we discuss CP violating effects that arise from interference between mixing and decay, which is also called mixing-induced CP violation. Therefore we consider a final state $f$ in which in principle both the $B^{0}_{s}$-meson and the $B^{0}_{s}$-meson can decay. The corresponding decay amplitudes will be denoted by $A_{f}$ and $\bar{A}_{f}$, defined in Eq.(22). These amplitudes can have contributions from different CKM structures; their general structure looks like

\begin{equation}
    A_{f} = \sum_{j} A_{j} e^{i(\phi_{j}^{\text{strong}} + \phi_{j}^{\text{CKM}})} ,
\end{equation}

where $j$ sums over the different CKM contributions, $\phi_{j}^{\text{CKM}}$ denotes the corresponding CKM phase and $A_{j} e^{i\phi_{j}^{\text{strong}}}$ encodes the whole non-perturbative physics as well as the moduli of the CKM-elements. The calculation of the strong amplitudes and phases from first principles is a non-trivial problem, for which a general solution has not yet been developed. Currently several working tools are available in order to investigate this non-perturbative problem: QCD factorisation (QCDF; e.g. (Beneke et al., 1999b, 2000, 2001; Beneke and Neubert, 2003)), Soft Collinear Effective Theory (SCET; e.g. (Bauer et al., 2001, 2004, 2002)), light cone sum rules (LCSR; e.g. (Balitsky et al., 1989; Khodjamirian et al., 2003; Khodjamirian, 2001)) and perturbative QCD (pQCD; e.g. (Li and Yu, 1996; Yeh and Li, 1997)).
Considering the CP conjugate decay $B^0_s \rightarrow \bar{f}$, one finds
\[ A_f = -\sum_j A_j e^{(\phi_j^{\text{strong}} - \phi_j^{\text{CKM}})} , \] (141)
so only the CKM phase has changed its sign, while the strong amplitude and the strong phase remain unmodified. The overall sign is due to the CP properties of the $B^0_s$-mesons, defined in Eq.(8) and $f$ defined in Eq.(34).

In some CP asymmetries the hadronic amplitudes cancel to a good approximation in the ratios of decay rates. The corresponding decay modes are the so-called gold-plated modes, which were introduced e.g. by (Carter and Sanda, 1981) and (Bigi and Sanda, 1981). Later on we will see that gold-plated modes will appear, when the decay amplitude is governed by a single CKM structure. This could be the case in a decay like $B^0_s \rightarrow J/\psi \phi$, which is governed on quark-level by a $b \rightarrow c\bar{c}s$-transition. Such a transition has a large tree-level contribution and a suppressed penguin contribution, see Fig. 13. To a good first approximation the penguins can be neglected and we have a gold-plated mode, with a precise relation of the corresponding CP asymmetry to fundamental Standard Model parameters, including the CKM-couplings. In view of the dramatically increased experimental precision in recent years it turns out, however, that it is necessary to investigate the possible size of penguin effects, the so-called penguin pollution. This will be discussed below.

Let us go back to the general case, and consider the following time-dependent CP asymmetry for a $B^0_s \rightarrow f$ transition without any approximations concerning the structure of the decay amplitude:
\[ A_{CP,f}(t) = \frac{\Gamma (B^0_s(t) \rightarrow f) - \Gamma (B^0_s(t) \rightarrow \bar{f})}{\Gamma (B^0_s(t) \rightarrow f) + \Gamma (B^0_s(t) \rightarrow \bar{f})} . \] (142)

Inserting the time evolution given in Eq.(21) and Eq.(33) one finds\(^{15}\)
\[ A_{CP,f}(t) = -\frac{A_{CP}^{\text{dir}} \cos(\Delta M_s t) + A_{CP}^{\text{mix}} \sin(\Delta M_s t) + O (a_f \phi_s)}{\cos(\frac{\Delta M_s t}{2}) + A_{\Delta \tau} \sin(\frac{\Delta M_s t}{2})} . \] (143)

with $A_{CP}^{\text{dir}}$, $A_{CP}^{\text{mix}}$ and $A_{\Delta \tau}$ being defined in Eq.(24), Eq.(25) and Eq.(26). We can rewrite two of those definitions as
\[ A_{CP}^{\text{mix}} = -\frac{2|\lambda_f|}{1 + |\lambda_f|^2} \sin[\arg(\lambda_f)] + \frac{2|\lambda_f|}{1 + |\lambda_f|^2} \sin[\phi_s] , \] (144)
\[ A_{\Delta \tau} = -\frac{2|\lambda_f|}{1 + |\lambda_f|^2} \cos[\arg(\lambda_f)] - \frac{2|\lambda_f|}{1 + |\lambda_f|^2} \cos[\phi_s] , \] (145)

with the phase $\phi_s$ to be defined as
\[ \phi_s = -\arg(\lambda_f) = -\arg \left( \frac{q A_f}{p A_f} \right) \] (146)
\[ = -\pi + \phi_M - \arg \left( \frac{A_f}{A_f} \right) . \] (147)

This is the most general definition of the phase that appears in interference. However, in this form a measurement of $\phi_s$ does not enable us to connect the phase with fundamental parameters of the underlying theory. To do so, we either find some simplifications for the decay amplitudes or we have to evaluate the ratio of amplitudes non-perturbatively. Before discussing a particular simplification, we note that sometimes a different notation $(S_f$ for the coefficient that is arising in Eq.(143) with the term $\sin(\Delta M_s t)$ and $C_f$ or $A_f$ for the coefficient that is arising with the term $\cos(\Delta M_s t)$ - up to signs) is used
\[ -A_f = C_f \equiv A_{CP}^{\text{dir}} , \] (148)
\[ -S_f \equiv A_{CP}^{\text{mix}} . \] (149)

BaBar uses $C_f$ and Belle $A_f$. Expanding the hyperbolic functions in Eq.(143) for small arguments, i.e. small decay rate differences and/or short times, we can express the time-dependent CP asymmetry $A_{CP,f}(t)$ as
\[ A_{CP,f}(t) \approx \frac{S_f \sin(\Delta M_s t) - C_f \cos(\Delta M_s t)}{1 + A_{\Delta \tau} \frac{\Delta M_s t}{2} + \frac{1}{2} \left( \frac{\Delta M_s t}{2} \right)^2} . \] (150)

This formula holds in general, and no approximation on the corresponding decay amplitudes has been made yet. In this general case, the quantities $A_{CP}^{\text{dir}}$, $A_{CP}^{\text{mix}}$ and $A_{\Delta \tau}$ are unknown hadronic contributions that are very difficult to be determined in theory.

In the following we discuss the simplified case of the gold-plated modes. Here we consider the final state $f$ to be a CP eigenstate, i.e. $f = f_{CP} = \eta_{CP} f$ and we assume that only one CKM structure is contributing to the decay amplitude - by e.g. neglecting penguins. In this special case we get
\[ A_{f_{CP}} = A_f e^{i(\phi_j^{\text{strong}} + \phi_j^{\text{CKM}})} , \] (151)
\[ A_{f_{CP}} = \eta_{CP} A_{f_{CP}} = -\eta_{CP} A_f e^{i(\phi_j^{\text{strong}} - \phi_j^{\text{CKM}})} , \] (152)
\[ = A_{f_{CP}} = -\eta_{CP} e^{-2i\phi_j^{\text{CKM}}} . \] (153)

So in the case of gold-plated modes all hadronic uncertainties cancel exactly in the ratio of the two decay amplitudes in Eq.(153) and one is left with a pure weak CKM phase. Thus the parameter $\lambda_f$, which triggers the CP asymmetries is given by
\[ \lambda_{f_{CP}} = \frac{q A_{f_{CP}}}{p A_{f_{CP}}} = \eta_{CP} \frac{V_{ts} V_{tb}^*}{V_{ts} V_{tb}} e^{-2i\phi_j^{\text{CKM}}} . \] (154)

\(^{15}\) A more detailed derivation can be found in (Anikeev et al., 2001).
Therefore we have in the case of only one contributing CKM structure $|\lambda_{fCP}| = 1$ and thus
\[
A_{\text{CP}}^0 = 0 , \quad (155)
\]
\[
A_{\text{mix}}^f = + \sin (\phi_s) , \quad (156)
\]
\[
A_{\Delta \Gamma}^f = - \cos (\phi_s) , \quad (157)
\]
leading to the simplified formula for the asymmetry
\[
A_{\text{CP},f}(t) \approx \frac{\sin \phi_s \sin (\Delta M_st)}{\cos \phi_s \sinh (\frac{\Delta_M^2}{2}) - \cosh (\frac{\Delta_M^2}{2})} . \quad (158)
\]
This formula holds in the case of only one contributing CKM structure to the whole decay amplitude and the final state being a CP eigenstate. If the corresponding decay is triggered e.g. by a $b \to c\bar{c}s$ transition on quark-level, as in the case of $B_s^0 \to J/\psi f$, we get
\[
\phi_s = - \arg \left( \frac{V_{ts}^* V_{tb}}{V_{cs}^* V_{cb}} \right) . \quad (159)
\]
Thus a measurement of the mixing phase $\phi_s$ gives us direct information about the phases, i.e. the amount of CP violation, of the CKM elements. If in addition the final state has a CP eigenvalue $\eta_{CP} = +1$, then we get
\[
\phi_s = - 2 \beta_s , \quad (160)
\]
with the commonly used notation
\[
\beta_s = - \arg \left( - \frac{V_{ts}^* V_{tb}}{V_{cs}^* V_{cb}} \right) . \quad (161)
\]
\[
= 0.0183 \pm 0.0010 = (1.05 \pm 0.05)^\circ . \quad (162)
\]
Here we used a definition for $\beta_s$ that ensures that its numerical value is positive; sometimes a different sign is used. If there is only a modest experimental precision available, penguins can be neglected, to a first approximation, for the tree-level dominated $b \to c\bar{c}s$ decays like $B_s \to J/\psi f$ penguins, and we can use simplified formulae like Eq. (160).

However, we will see below that the current experimental precision in the determination of $\phi_s$ is of the order of $2^\circ$, which equals the SM expectation of $\phi_s^{\text{SM}} = (2.1 \pm 0.1)^\circ$. In view of this high experimental precision, it seems mandatory to determine the possible size of penguin contributions, in order to make profound statements about new physics effects in these CP asymmetries.

Let us examine the general expression for the decay amplitude without neglecting penguins. Examples for decays with both tree-level and penguin contributions are e.g. $B_s \to J/\psi f$ or $B_s \to K^- \pi^+$. The former is governed on quark-level by a $b \to c\bar{c}s$-transition, and the latter by a $b \to u\bar{d}\bar{d}$-transition. The tree-level components and penguin contributions to these decays are shown in Fig. 13.

A naive dimensional estimate (size of CKM couplings, number of strong couplings and colour counting) gives a small penguin contribution in the case of $B_s \to J/\psi f$ and a large penguin contribution in the case $B_s \to K^- \pi^+$. Thus $B_s \to J/\psi f$ is a prime candidate for a gold-plated mode, while in the case of $B_s \to K^- \pi^+$ direct CP violation, i.e. CP violation directly in the decay might be visible; this will be further discussed in Section V.

To become more quantitative, we take a closer look at the general structure of the decay amplitude of a $b \to c\bar{c}s$-transition. Using the effective Hamiltonian for $\Delta B = 1$ transitions (see e.g. (Buras, 1998) for a nice introduction) we get for the amplitude
\[
A_f \left( B_s^0 \to f \right) = \langle f | H_{\text{eff}} | B_s^0 \rangle , \quad (163)
\]
with the effective SM Hamiltonian for $b \to c\bar{c}s$ transitions
\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} [\lambda_u (C_1 Q_1^u + C_2 Q_2^u) + \lambda_c (C_1 Q_1^c + C_2 Q_2^c) + \lambda_t \sum_{i=3}^6 C_i Q_i ] + \text{h.c.} . \quad (164)
\]

The CKM structure is given as before by $\lambda_q := V_{qb} V_{qs}$; the decay $b \to c\bar{c}s$ proceeds via the current-current operators $Q_1$, $Q_2$ and the QCD penguin operators $Q_3, \ldots, Q_6$. $C_1, \ldots, C_6$ are the corresponding Wilson coefficients. When the current-current operators $Q_1^u, Q_2^u$ are inserted in a penguin diagram in the effective theory, they also contribute to $b \to c\bar{c}s$. Electro-weak penguins are neglected.

Therefore we have the following structure of the amplitude $A_f \left( B_s^0 \to f \right)$
\[
A_f = \frac{G_F}{\sqrt{2}} \left[ \lambda_u \sum_{i=1,2} C_i (Q_i^u)^{P} + \lambda_c \sum_{i=1,2} C_i (Q_i^c)^{T+P} + \lambda_t \sum_{i=3}^6 C_i (Q_i)^T \right] . \quad (165)
\]

$(Q)^T$ denotes the tree-level insertion of the local operator $Q_i$. $(Q)^P$ denotes the insertion of the operator $Q$ in a penguin diagram. Using further the unitarity of the CKM matrix, $\lambda_t = - \lambda_u - \lambda_c$, we can rewrite the amplitude in a form where only two different CKM structures are appearing.

\[
A_f = \frac{G_F}{\sqrt{2}} \frac{\lambda_c}{\lambda_c} \left[ \sum_{i=1,2} C_i (Q_i^u)^{T+P} - \sum_{i=3}^6 C_i (Q_i)^T \right] + \lambda_u \left[ \sum_{i=1,2} C_i (Q_i^u)^{P} - \sum_{i=3}^6 C_i (Q_i)^T \right] \quad (166)
\]
\[
\Rightarrow \quad A_f = A_f^{\text{Tree}} + A_f^{\text{Peng}} . \quad (167)
\]
In the last line we have defined separately a tree-level amplitude and a penguin amplitude. They are given by

$$A_f^{\text{Tree}} = \frac{G_F}{\sqrt{2}} \lambda_c \left[ \sum_{i=1,2} C_i(Q_i^u)^T_p - \sum_{i=3}^6 C_i(Q_i^u)^T \right]$$

$$= \left| A_f^{\text{Tree}} \right| e^{i\phi^{\text{QCD}} + \arg(\lambda_u)} , \quad (168)$$

$$A_f^{\text{Peng}} = \frac{G_F}{\sqrt{2}} \lambda_u \left[ \sum_{i=1,2} C_i(Q_i^d)^P - \sum_{i=3}^6 C_i(Q_i^d)^P \right]$$

$$= \left| A_f^{\text{Peng}} \right| e^{i\phi^{\text{QCD}} + \arg(\lambda_u)} . \quad (169)$$

Here we split up the amplitudes into their modulus and their phase. Sometimes it advantageous to split off the explicit dependence on the modulus of the CKM structure:

$$\left| A_f^{\text{Tree}} \right| = \frac{G_F}{\sqrt{2}} \left| \lambda_c \right| A_f^{\text{Tree}} , \quad (170)$$

$$\left| A_f^{\text{Peng}} \right| = \frac{G_F}{\sqrt{2}} \left| \lambda_u \right| A_f^{\text{Peng}} . \quad (171)$$

The strong amplitudes and the strong phases are in principle unknown. A first naive estimate of the size of the modulus can be done by investigating, what $\Delta B = 1$ Wilson coefficients are contributing. In the case of $B_s^0 \rightarrow J/\psi$ the large Wilson coefficients $C_1$ and $C_2$: the penguin amplitude is suppressed by $\lambda_u$ and further either by small penguin Wilson-coefficients $C_3...6$ or by a loop.

In general, i.e. without any approximations concerning the size of the hadronic effects, we get the ratio of decay amplitudes

$$\frac{\tilde{A}_f}{A_f} = -e^{-2i \arg(\lambda_u)} \frac{1 + re^{-i \arg(\lambda_u)}}{1 + re^{+i \arg(\lambda_u)}} \quad (172)$$

with $r$ being defined as

$$r = \frac{\left| \lambda_u \right|}{\left| \lambda_c \right|} \left| \frac{A_f^{\text{Peng}}}{A_f^{\text{Tree}}} e^{i(\phi^{\text{QCD}} - \phi^{\text{QCD}})} \right| . \quad (173)$$

In the case of $B_s^0 \rightarrow J/\psi$ the CKM part of $r$ is very small, it is given by $|\lambda_u/\lambda_c| \approx 0.02$. The hadronic part of $r$ is a non-perturbative quantity that can currently not be calculated from first principles. Before we turn to some quantitative investigations in the literature, we have a look at a naive estimate: $A_f^{\text{Peng}}$ and $A_f^{\text{Tree}}$ contain Wilson coefficients from the effective Hamiltonian. The penguin Wilson coefficients $|C_3...6|$ are typically smaller than 0.04, therefore one can neglect them in comparison to the Wilson coefficient $C_2 \approx 1$, see e.g. (Buchalla et al., 1996) for numerical values. Thus we are left with the tree-level insertion of the operator $Q_2$ in the case of $A_f^{\text{Tree}}$ and with the penguin insertion of the operator $Q_2$ in the case of $A_f^{\text{Peng}}$. Since we do not know the relative size, of these
two, we take the analogy of inclusive $b$-quark decays as a first indication of its size. For the inclusive decay $b \to c \bar{c}s$ it was found (see e.g. (Bagan et al., 1995; Krimmer et al., 2013; Lenz et al., 1997)) that $|Q|^2 \leq 0.05 (Q)^2$. Taking this value as an indication for the size of $\mathcal{A}_{\text{Peng}}/\mathcal{A}_{\text{Tree}}$ we get an estimate of $r$ of about $|r| \approx 0.001$. One should be aware, however, that this very naive estimate can easily be off by a factor of 10 and we also cannot quantify the size of the strong phase in this approach. Using the same methods for the decay $B_0^0 \to K^+\pi^-$ we would get a value of $\sqrt{\mathcal{A}_{\text{Peng}}/\mathcal{A}_{\text{Tree}}} \approx 0.1$, so roughly 100 times larger than in the case of $B_0^0 \to J/\psi\phi$. $B_0^0 \to K^-\pi^+$ is thus a first indication of its size in the case of $B_0^0 \to J/\psi\phi$. $B_0^0 \to K^-\pi^+$ is thus a prime candidate for decays where we are looking for large penguin effects, e.g. if we want to measure a direct CP asymmetry in the $B_0^0$ system. Our naive estimate does not take into account that these two channels proceed via different topologies; hence the factor 100 might have to be modified considerably. Nevertheless it seems that $r$ is a small number in the case of $B^0 \to J/\psi\phi$ and we can make a Taylor expansion in Eq.(172) to obtain

$$\frac{\mathcal{A}_f}{\mathcal{A}_0} \approx 1 - 2ir \sin \left( \frac{\lambda_e}{\lambda_c} \right) \left( 1 - 2ir \sin \left( \frac{\lambda_e}{\lambda_c} \right) \right)^{-1}$$

(174)

Investigating further Eq.(174) or Eq.(172), we see that the first term on the r.h.s. gives rise to $-2\beta_3$ in the CP asymmetry in Eq.(158). The second term (proportional to $r$), corresponds to the SM penguin pollution, which we denote by $\delta_{\text{Peng},\text{SM}}$. Therefore the experimentally measured phase $\phi_s$ has two contributions in the Standard Model:

$$\phi_s = -2\beta_3 + \delta_{\text{Peng},\text{SM}},$$

(175)

where the Standard Model penguin is given by

$$e^{i\delta_{\text{Peng},\text{SM}}} \approx 1 - 2ir \sin \left( \frac{\lambda_e}{\lambda_c} \right) e^{i(\phi_{\text{Peng},\text{SM}} - \phi_{\text{Tree}})}.$$  

(176)

Inserting the above approximations for $B^0 \to J/\psi\phi$ we get as a very rough estimate of the penguin pollution:

$$e^{i\delta_{\text{Peng},\text{SM}}} \approx 1 - 0.002ie^{i(\phi_{\text{Peng},\text{SM}} - \phi_{\text{Tree}})}$$

$$\Rightarrow \delta_{\text{Peng},\text{SM}} \approx 0 \pm 0.002 = 0 \pm 0.1^\circ.$$  

(177)

Thus very naively we expect a penguin pollution of at most $\pm 0.1^\circ$ in the case of $B^0 \to J/\psi\phi$. This very rough estimate could, however, be easily modified by a factor of e.g. 10, due to non-perturbative effects and then we would be close to the current experimental uncertainties. Thus more theoretical work has to be done to quantify the size of penguin contributions. There are now several strategies to achieve this point:

1. Measure $\phi_s$ in different decay channels: assuming that penguins are negligible, different determinations should give the same value for the mixing phase. Until now we have focused on the extraction of the phase $\phi_s$ from the decay $B^0_0 \to J/\psi\phi$. This final state is an admixture of CP-even and CP-odd components. To extract information on $\Delta \Gamma_s$ and $\phi_s$ an angular analysis is required, see the discussion in Section II.B and Section IV.B or the early references: (Dighe et al., 1999, 1996; Dunietz et al., 2001). Moreover the $J/\psi\phi(\to K^+K^-)$ final state can be investigated for non-resonant $K^+K^-$ contributions in order to increase the statistics. The phase $\phi_s$ has also been determined in different $b \to c\bar{c}s$-channels, like $B^0_0 \to J/\psi\pi^+\pi^-$ (including $B^0_0 \to J/\psi f_0$), see e.g. (Colangelo et al., 2011; Fleischer et al., 2011a; Stone and Zhang, 2009, 2013; Zhang and Stone, 2013), $B^0_0 \to J/\psi \eta$ (see e.g. (Di Donato et al., 2012; Dunietz et al., 2001; Fleischer et al., 2011b)) and $B^0_0 \to D^{(*)+}D^{(*)-}$ (see e.g. (Dunietz et al., 2001; Fleischer, 2009b)) as a cross-check. Here $B^0_0 \to J/\psi f_0$ is a CP-odd final state, $J/\psi \eta$ and $D^{(*)+}D^{(*)-}$ are CP-even final states, and $D^{(*)+}D^{(*)-}$ is again an admixture of different CP components. Getting different values for $\phi_s$ from different decay modes would point towards different and large penguin contributions in the individual channels. The different experimental results will be discussed in the next section: they show no significant deviations within the current experimental uncertainties, but there is also plenty of space left for some sizable differences.

2. Measure the phase $\phi_s$ for different polarisations of the final states in $B^0_0 \to J/\psi\phi$: potential differences might originate from penguins, which in general contribute differently to different polarisations, see (Faller et al., 2009a; Fleischer, 1999a). Such an analysis was done in (Aaij et al., 2015) and within the current experimental uncertainties no hint for a polarisation dependence of $\phi_s$ was found:

$$\phi_{s,\|} - \phi_{s,\perp} = -0.13 \pm 0.26 \pm 0.52,$$

(178)

$$\phi_{s,\perp} - \phi_{s,\|} = -0.80 \pm 0.21 \pm 0.34.$$  

(179)

On the other hand one sees that effects of the order of $2^\circ$, which would be as large as the whole SM prediction for $\phi_s$, are not ruled out yet. A further discussion of this result was done by (De Bruyn and Fleischer, 2015).

3. Compare the decay $B^0_0 \to J/\psi\phi$ to a decay with a similar hadronic structure, but a CKM enhanced penguin contribution: differences in the phase $\phi_s$ extracted from $B^0_0 \to J/\psi\phi$ and from the new decay might then give experimental hints for the size of the penguin contribution. Exchanging the $s$-quark line in Fig.13 with a $d$-quark line one arrives at decays like $B^0_s \to J/\psi K_S$.
(see e.g. (Fleischer, 1999b)) or \(B_0^0 \to J/\psi K^*(892)\) (see e.g. (De Bruyn and Fleischer, 2015; Dunietz et al., 2001; Faller et al., 2009a)). In the first decay there is only one vector particle in the final state, while the latter case we have two (\(K^*(892)\) is a vector meson) as in the case of \(B^0_s \to J/\psi\phi\). Thus we consider here only the decay \(B^0_s \to J/\psi K^*(892)\).

The analogous modes in \(B^0\) decays are \(B^0 \to J/\psi K_S \leftrightarrow B^0 \to J/\psi \pi^0\). The extraction of penguin pollution via this relation was discussed in e.g. (Ciuchini et al., 2005, 2011; Faller et al., 2009b). To get an idea of the size of the penguin uncertainties, we note that (Ciuchini et al., 2011) found a possible Standard Model penguin pollution of about \(\pm 1.1^\circ\) in the gold-plated mode \(B^0 \to J/\psi K_S\).

Coming back to the \(B^0_s\) system, the relative size of the penguin contributions in the decays \(B^0_s \to J/\psi K_S\) and \(B^0_s \to J/\psi K^*(892)\), compared to the tree-level components, are larger by a factor of about \(1/\lambda^2 \approx 25\) than in \(B^0_s \to J/\psi\phi\). This enhancement of the penguin contribution might manifest itself in different values for the extracted values of the phase \(\phi_s\).

A disadvantage of these decays is that they are more difficult to measure, because they proceed on quark-level via \(b \to c\bar{c}d\), whose branching ratio is suppressed by a factor of about \(\lambda^2 \approx 1/25\) compared to \(B^0_s \to J/\psi\phi\). This is the reason, why the CP asymmetries in \(B^0_s \to J/\psi K_S\) and the one in \(B^0_s \to J/\psi K^*(892)\) have only been determined recently with large uncertainties by (Aaij et al., 2015g) and by (Aaij et al., 2015d). The corresponding branching ratios have been measured earlier by the LHCb Collaboration (Aaij et al., 2012c,e). (De Bruyn and Fleischer, 2015) discuss some strategies to extract the size of penguin pollution without having the full knowledge about these CP asymmetries. A further drawback of this method is that the size of the hadronic effects due to the exchange of a \(\phi\)-meson with a \(K^*(892)\)-meson cannot be quantified from first principles. Finally there are also penguin annihilation and weak exchange topologies contributing to \(B^0_s \to J/\psi\phi\), that are not present in the \(B^0_s \to J/\psi K^*(892)\) case, see e.g. (Faller et al., 2009a). Whether it is justified to neglect such contributions can e.g. be tested by decays like \(B^0 \to J/\psi\phi\) that proceed only via weak-exchange and annihilation topologies. Experimental constraints on \(B^0 \to J/\psi\phi\) from Belle (Liu et al., 2008), BaBar (Lees et al., 2015a) and LHCb (Aaij et al., 2013c) indicate, however, no unusual enhancement of annihilation or weak exchange contributions.

4. Compare the decay \(B^0_s \to J/\psi\phi\) with a decay which is related to it via a symmetry of QCD: having now a symmetry might add confidence in obtaining some control over the effect of exchanging the initial and final states mesons with other mesons. Such a symmetry is the flavour symmetry \(SU(3)_F\), i.e. a symmetry of QCD under the exchange of \(u\)-, \(d\)- and \(s\)-quarks. The application of these symmetries is quite widespread, see e.g. (Bhattacharya et al., 2013; Ciuchini et al., 2005, 2011; De Bruyn and Fleischer, 2015; Faller et al., 2009a; Fleischer, 1999b; Jung, 2012; Ligi et al. and Robinson, 2015) for some examples related to \(B\)-meson decays. Again a word of caution: it is currently not clear how well the \(SU(3)_F\)-symmetry is working and how large the corrections are, see e.g. (Faller et al., 2009a; Frings et al., 2015) for some critical comments. On the other hand a comparison of experimental data finds that \(SU(3)_F\) might work quite well, for some of these decay channels, see e.g. (De Bruyn and Fleischer, 2015).

A sub-group of \(SU(3)_F\), which is supposed to work particularly well, is the so-called \(U\)-spin symmetry, i.e. the invariance of QCD under the exchange of the \(s\)-quark with a \(d\)-quark, see e.g. (De Bruyn and Fleischer, 2015; Fleischer, 1999a,b). Substituting the \(s\)- and \(\bar{s}\)-quark on the l.h.s. of Fig.13 with down-type quarks one gets, e.g. (Fleischer, 1999a).

\[
B^0_s \to J/\psi\phi \leftrightarrow B^0 \to J/\psi\rho^0 , J/\psi\pi^0 .
\] (180)
guin contributions and a similar structure as $B^0 \to J/\psi \pi^0$ and $B^0_{s} \to J/\psi K_S$; tree and penguin contributions to $B^0 \to J/\psi \pi^+ \pi^-$, which contains $B^0 \to J/\psi p^0$ are depicted in Fig. 14. $B^0 \to J/\psi p^0$ is discussed further e.g. by (DeBruyn and Fleischer, 2015) and there are also first measurements of the mixing induced CP asymmetries by the LHCb Collaboration (Aaij et al., 2015e). In this decay we have again two vector mesons in the final state, as in the case $B^0_s \to J/\psi \phi$. Thus here we do not consider the decay $B^0 \to J/\psi p^0$ any further. However this decay gave important constraints on the penguin pollution in $B^0 \to J/\psi K_S$ - as it was explained above.

Applying U-spin symmetry to the $B^0$ system one gets e.g.

$$B^0 \to J/\psi K_S \Leftrightarrow B^0_s \to J/\psi K_S.$$  \hspace{1cm} (181)

The decay $B^0_s \to J/\psi K_S$ was already mentioned above for estimating penguin uncertainties in $B^0 \to J/\psi \phi$. It is, however, much better suited for the decay $B^0 \to J/\psi K_S$, see (De Bruyn and Fleischer, 2015; Faller et al., 2009b; Fleischer, 1999b). Further experimental studies of this decay were performed by (Aaij et al., 2013g).

Currently symmetry considerations put a quite strong bound on the penguin pollution; (De Bruyn and Fleischer, 2015) (see also (Fleischer, 2015)) get for the decay $B^0_s \to J/\psi \phi$ the following possible size of penguin pollution:

$$\delta_{\mathrm{Peng,SM}}^{J/\psi \phi} = [0.08^{+0.56}_{-0.72} \text{(stat)} + 0.15 \text{(SU(3))}]^{0}. \hspace{1cm} (182)$$

This bound is currently dominated by statistical uncertainties stemming from experiment and it will thus be getting stronger in the future by improved measurements, even without theoretical improvements.

5. Investigate purely penguin induced decays: an example for a decay that has no tree-level contribution is $B^0_s \to \phi \phi$, which is governed by a $b \to s\bar{s}s$ quark level transition. Traditionally such decays are considered to be most sensitive to new physics effects. The decay $B^0_s \to \phi \phi$ has contributions from an $u$, $c$ and $t$ penguin. Its amplitude reads

$$A_f(B^0_s \to \phi \phi) = \frac{G_F}{\sqrt{2}} \lambda_u \left[ \sum_{i=1,2} C_i(Q^u_i)^P - \lambda_s \sum_{i=3}^6 C_i(Q_i)^T \right] \hspace{1cm} \lambda_c = \frac{\lambda_u}{\lambda_c}$$

Using again the unitarity of the CKM matrix, we can rewrite the amplitude in a form where only two different CKM structures are appearing.

$$A_f = \frac{G_F}{\sqrt{2}} \lambda_c \left[ \sum_{i=1,2} C_i(Q^u_i)^P - 6 \sum_{i=3}^6 C_i(Q_i)^T \right]$$

Neglecting the second term, proportional to $\lambda_u/\lambda_c$ we get the same result as in the case of the gold-plated mode $B^0_s \to J/\psi \phi$: the measured mixing phase is $\phi_s = -2\beta_s$. In the case of $B^0_s \to \phi \phi$ this might, however, not be a very good approximation. Our leading term is now given by the difference of the charm penguins and the top penguins, which will give a small contribution compared to the large tree-level term in the case of $B^0 \to J/\psi \phi$. The sub-leading term is suppressed by $\lambda_u/\lambda_c$, which is a small number, but the hadronic part is now the difference of up penguins and top penguins, which is of a similar size as leading term. Thus deviations of the measured phase in $B^0_s \to \phi \phi$ from $-2\beta_s$ might tell us something about unexpected non-perturbative enhancements of the up quark penguins compared to the charm quark penguins. More advanced theory investigations can be found in (Bartsch et al., 2008; Beneke et al., 2007; Cheng and Chua, 2009; Datta et al., 2012). First measurements (see (Aaij et al., 2014c)) have still a sizable uncertainty, but they show no significant deviation of the mixing phase $B^0_s \to \phi \phi$ from $-2\beta_s$.

6. Try to do a calculation from first principles. Very recently penguin effects were estimated in that manner by (Frings et al., 2015) by proofing the infrared safety of the penguin contributions in a factorisation approach. This study suggests that penguin contributions to $\phi_s$ in the case of $B_s \to J/\psi \phi$ should be smaller than about $1^\circ$. First steps in such a direction have been performed by (Boos et al., 2004) and they were pioneered by (Bander et al., 1979). In the framework of pQCD this was attempted recently by (Liu et al., 2014). Most of the current investigations point toward a maximal size of SM penguin effects of about $\pm 1^\circ$, which is unfortunately very close to the current experimental precision of about $\pm 2^\circ$. Thus more theoretical work has to be done in that direction. Note that the LHCb constraint from the study of the decay $B^0 \to J/\psi \rho$ (Aaij et al., 2015e) gives a limit on penguin effects at about $1^\circ$.
B. Experiment

The experimental study of the CP-violating phase $\phi_s$ has been pursued vigorously and considerable experimental progress has been achieved. The main channels used are $B^0_s \to J/\psi h^+ h^-$, where the $h^+ h^-$ system in general may comprise many states with different angular momenta. Many studies focus on the “golden mode,” $B^{+}_s \to J/\psi\phi$, discussed in Section II.B, which also contains the references to the latest experimental results. The analysis of this final state provides the constraint on both $\Delta \Gamma_s$ and $\phi_s$ and is therefore presented as a two-dimensional confidence level contour.

The determination of $\phi_s$ requires the CP-even and CP-odd components to be disentangled by analysing the differential distribution $d\Gamma/dtd\Omega$, where $\Omega \equiv (\cos \theta_h, \cos \theta_\mu, \chi)$, defined as (a) $\theta_h$ is the angle between the $h^+$ direction in the $h^+ h^-$ rest frame with respect to the direction of the $h^+ h^-$ pair in the $B^0_s$ rest frame, (b) $\theta_\mu$ is the angle between the $\mu^+$ direction in the $J/\psi$ frame with respect to the $J/\psi$ direction in the $B^0_s$ rest frame, and (c) $\chi$ is the angle between the $J/\psi$ and the $h^+ h^-$, as shown in Fig. 15.

The decay $B^0_s \to J/\psi K^+ K^-$ proceeds predominantly via $B^{0}_s \to J/\psi\phi$ with the $\phi$ meson decaying subsequently to $K^+ K^-$. In this case, the $B^0_s$ decays into two vector particles, and the $K^+ K^-$ pair is in a P-wave configuration. The final state is then the superposition of CP-even and CP-odd states, depending upon the relative orbital angular momentum of the $J/\psi$ and the $\phi$. The final state can be produced also with $K^+ K^-$ pairs in an S-wave configuration, as pointed out by Stone and Zhang (Stone and Zhang, 2009). This S-wave component is CP-odd.

The decay width can be expressed in terms of four time-dependent complex amplitudes $A_i(t)$. Three of them arise from the P-wave configuration, and, correspond to the relative orientation of the linear polarization vectors of the $J/\psi$ and $\phi$ mesons, $0, \perp, ||$ (see (Aaij et al., 2015f)), and one of them corresponds to the S-wave configuration. The distributions of decay angles and time for a $B^0_s$ meson produced at time $t \equiv 0$ can be expressed in terms of ten terms, corresponding to the four polarization amplitudes and their interference terms. The expressions for the decay rate $d\Gamma(B^0_s)/dtd\Omega$ is invariant under the transformation

$$ (\phi_s, \Delta \Gamma_s, \delta_0, \delta_1, \delta_\perp, \delta_S) \to (\pi - \phi_s, -\Delta \Gamma_s, -\delta_1, \pi - \delta_\perp, -\delta_S). $$

(185)

Here, the convention $\delta_0 = 0$ is chosen. Thus in principle there is a two-fold ambiguity in the results. This is removed by performing fits in bins of $m_{h h}$, see (Xie et al., 2009). Thus the LHCb collaboration performs the fit to the distribution $dn/d\Omega$ in bins of $m_{h h}$ to resolve this ambiguity. The projections of the decay time and angular distributions obtained from the analysis of the 3 fb$^{-1}$ LHCb data set is shown in Fig. 16, and the corresponding fit parameters are summarised in Table V. Note that the mixing parameter $\Delta M_s$ is not constrained from other measurements in this fit and is consistent with world averages.

This decay mode has been studied also in the general purpose detectors at the Tevatron (Aaltonen et al., 2012; Abazov et al., 2012a) and LHC (Aad et al., 2014; Khachatryan, 2015). The analysis method is similar to the one described before. Fig. 17 shows the fit projections obtained with the recent CMS measurements reported in Ref. (Khachatryan, 2015).

Another channel (see (Stone and Zhang, 2009)) was recognized to provide complementary information on $\phi_s$, namely $B^0_s \to J/\psi f_0$, with $f_0 \to \pi^+ \pi^-$. The original appeal of this mode is that it was assumed to be predominantly an S-wave decay, and thus not in need of the complex multidimensional fit just described. The study of the Dalitz plot of $B^{0}_s \to J/\psi \pi^+ \pi^-$ (Aaij et al., 2012a, 2014i) revealed a more complex resonant structure. A combination of five resonant states are required to described the data (see (Aaij et al., 2014i)): $f_0(980)$, $f_0(1500)$, $f_0(1790)$, $f_0(1270)$, and $f'_2(1525)$. The data are compatible with no additional non-resonant (NR) component, as well as a combination of these 5 reso-

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FIG. 16 Decay-time and helicity-angle distributions for $B^0_s \to J/\psi K^+ K^-$ decays (data points) with the one-dimensional fit projections overlaid. The solid blue line shows the total signal contribution, which is composed of CP-even (long-dashed red), CP-odd (short-dashed green) and S-wave (dotted-dashed purple) contributions. The figure is taken from Ref. (Aaij et al., 2015h).

nances plus significant NR component, with a fit fraction of $(5.9\pm1.4)\%$. The latter solution is shown in Fig. 18. Thus the most recent study of CP violation in $B^0_s \to J/\psi \pi^+ \pi^-$ uses the formalism developed in Ref. (Zhang and Stone, 2013). Their approach is to cou-
The current experimental uncertainty in \( \phi_s \) is commensurate with the central value of the SM prediction given in Eqs. (160,161).
shown in Fig. 21. They perform an unbinned maximum likelihood fit to $B^0_s \to J/\psi\phi$, thus providing an excellent CP violation in this decay has been studied by LHCb (Aaij et al., 2013a). They perform an unbinned maximum likelihood fit to $d\Gamma/(dt d\cos \theta_1 d\cos \theta_2 d\Phi)$, where $t$ is the decay time and $\theta_{1,2}$ is the angle between the $K^+$ track momentum in the $B^0_s$ rest frame and the $\phi_{1,2}$ meson parent momentum in the $B^0_s$ rest frame, and $\Phi$ is the angle between the two $\phi$ decay planes. The background is taken into account by assigning a weight to each candidate derived with an sPlot technique (see (Pivk and Le Diberder, 2005)), using the invariant mass of the four $K$ meson system as a discriminating variable. The resulting fit projections are shown in Fig. 22. The CP-violating phase is found to be in the interval $[-2.46, -0.76]$ rad at 68% confidence level. The $p$-value of the SM prediction is 16%. The precision of the $\phi_s$ determination is dominated by the statistical uncertainty and is expected to improve with more data. The current results are based on a sample of 1 fb$^{-1}$.

V. CP VIOLATION IN DECAYS AND DIRECT MEASUREMENTS OF $\gamma$

A. Theory

CP violation in decays, also called direct CP violation, can arise if we have $|A_f| \neq |\bar{A}_f|$. In that case we expect the following CP asymmetry

$$A_{\text{dir CP}, f}(t) = \frac{\Gamma(B^0_s(t) \to \bar{f}) - \Gamma(B^0_s(t) \to f)}{\Gamma(B^0_s(t) \to f) + \Gamma(B^0_s(t) \to \bar{f})},$$

(188)

to give a non-vanishing value. Inserting the time evolution for the decay rates from Eq.(21) and Eq.(37), we get a complicated expression, that vanishes, however for $|A_f| = |\bar{A}_f|$, $|A_f| = |\bar{A}_f|$ and neglecting terms of order $a^2_s$. Neglecting mixing in a first step, i.e. setting $\Delta M_s$ and $\Delta \Gamma_s$ equal to zero, we get the simplified expression

$$A_{\text{dir CP}, f}(t) = \frac{|\bar{A}_f|^2 - |A_f|^2}{|A_f|^2 + |\bar{A}_f|^2}.$$  

(189)
Using the definitions in Eq.(167) and in Eqs.(168,169) we can write the two amplitudes as

\[
A_f = |A_f^{\text{Tree}}| e^{i\phi_f^{\text{Tree}} + \phi_f^{\text{QCD}}} + |A_f^{\text{Peng}}| e^{i\phi_f^{\text{Peng}} + \phi_f^{\text{QCD}}},
\]

(190)

\[
\tilde{A}_f = |A_f^{\text{Tree}}| e^{i\phi_f^{\text{Tree}} - \phi_f^{\text{QCD}}} + |A_f^{\text{Peng}}| e^{i\phi_f^{\text{Peng}} - \phi_f^{\text{QCD}}},
\]

(191)

and we find for \(A_{\text{dirCP},f}(t)\) the following expression

\[
A_{\text{dirCP},f}(t) = \frac{2 |A_f^{\text{Tree}}| |A_f^{\text{Peng}}| \sin (\phi_f^{\text{Peng}} - \phi_f^{\text{QCD}}) \sin [\arg(\lambda_u) - \arg(\lambda_c)]}{|A_f^{\text{Tree}}|^2 + |A_f^{\text{Peng}}|^2 + 2 |A_f^{\text{Tree}}| |A_f^{\text{Peng}}| \cos (\phi_f^{\text{Peng}} - \phi_f^{\text{QCD}}) \cos [\arg(\lambda_u) - \arg(\lambda_c)]}
\]

\[
= \frac{2|r| \sin (\phi_f^{\text{Peng}} - \phi_f^{\text{QCD}}) \sin [\arg(\lambda_u) - \arg(\lambda_c)]}{1 + |r|^2 + 2|r| \cos (\phi_f^{\text{Peng}} - \phi_f^{\text{QCD}}) \cos [\arg(\lambda_u) - \arg(\lambda_c)]},
\]

(192)

where \(|r|\) gives the modulus of the ratios of the penguin amplitude and the tree amplitude, analogous to Eq.(173).
mixing, shows that we can have a direct CP violation in decay only, if we have at least two different CKM contributions with different weak and different strong phases. The size of the CP asymmetry is also proportional to the modulus of the penguin contributions normalised to the tree contributions. Thus such an asymmetry could in principle, e.g. arise in the decays $B_s^0 \rightarrow K^-\pi^+$ and $B_d^0 \rightarrow K^+\pi^-$ (see Fig. 13), where we expected large penguins. Using the definition of the CKM angle $\gamma$

$$\gamma = \arg \left( -\frac{V_{ud}V^{*}_{ub}}{V_{cd}V^{*}_{cb}} \right), \quad (193)$$

we can write to a very good approximation

$$A_{\text{dirCP},f}(t) = \frac{2|r| \sin \left( \phi_{\text{Peng}} - \phi_{\text{Tree}} \right) \sin \gamma}{1 + |r|^2 - 2|r| \cos \left( \phi_{\text{Peng}} - \phi_{\text{Tree}} \right) \cos \gamma}. \quad (194)$$

If $|r|$ and the strong phases were known, this direct CP asymmetry could be used to determine the CKM angle $\gamma$. We already pointed out several times the difficulty of determining these hadronic parameters from a first principle calculation. Further strategies to determine $\gamma$ will be discussed below. On the other hand, using a measured value of $\gamma$, the direct CP asymmetry can give indications about the size of hadronic parameters, which is a very useful input in the investigation of penguin pollution.

Another possibility in the search for direct CP violation is the investigation of final states, that are common to $B_s^0$ and $B_s^0$, as e.g. in $B_s^0 \rightarrow J/\psi \phi$ or $B_s^0 \rightarrow K^+K^-$. According to the definition of the asymmetry in Eq. (142) the coefficient of $\cos(\Delta M_s t)$ will be proportional to $A_{\text{dir}CP}$, which describes direct CP violation and which is non-zero if $|\lambda_f| \neq 1$. Here again the ratio $r$ will be the crucial parameter.

It is worth also mentioning that $B_s^0$ decays provide also information about the CKM phase $\gamma$, which was defined in Eq. 193. This phase is directly proportional to the amount of CP violation in the SM. Thus any measurement of $\gamma$ is a measurement of CP violation.

In the case of the tree-level decay $B_s^0 \rightarrow D_s^+ K^+$ the extraction of $\gamma$ was e.g. discussed in (Aleksan $et$ $al.$, 1992; De Bruyn $et$ $al.$, 2013; Dunietz and Sachs, 1988; Fleischer, 2003; Fleischer and Ricciardi, 2011; Gligorov, 2011). $B_s^0 \rightarrow D_s^+ K^-$ proceeds via a colour-allowed $\bar{b} \rightarrow \bar{u}c\bar{s}$ transition and $B_s^0 \rightarrow D_s^- K^+$ proceeds via a colour-allowed $\bar{b} \rightarrow \bar{c}u\bar{s}$ transition, see Fig. 23. Doing a naive counting of powers of the Wolfenstein parameter $\lambda$ one expects that both amplitudes have a similar size, while the phase difference is given by the CKM angle $\gamma$, which is more or less the phase of the CKM element $V_{ub}$. From the diagrams in Fig. 23 one sees that both the $B_s^0$ and $B_s^0$-meson can decay into the same final state. Thus an interference between mixing and decay can arise, and in the end the value of $\phi_s + \gamma$ can be extracted from measuring CP asymmetries. Such an extraction of $\gamma$ became very popular, using $B^- \rightarrow DK^-$, because tree-level decays are supposed not to be affected by new physics effects. In view of the increasing experimental precision this assumption should, however, be challenged. A recent study (Brod $et$ $al.$, 2015) found that current experimental bounds on different flavour observables that are dominated by tree-level effects, allow beyond SM effects to be as large as about ±0.1 in the tree-level Wilson coefficients $C_1$ and $C_2$. A new physics contribution to the imaginary part of $C_1$ of about 0.1 would modify the measurement of $\gamma$ from tree-level decays by about 4° (Brod $et$ $al.$, 2015), which is smaller.
than the current experimental uncertainty of $\gamma$ (about $7^\circ$ according to Eq.(198)), but larger than the expected future uncertainty of about $1^\circ$ (Abe et al., 2010; Collaboration, 2011). Here clearly more studies are necessary in order to constrain the possible space for new physics effects in tree level decays. Currently $B_s^0 \to D_s^\pm K^\mp$ decays lead to value of $\gamma = 115^{+26}_{-19}^\circ$ (see (Aaij et al., 2014d)), which is not competitive. An extraction of this angle from $B^0 \to \pi^+\pi^-$, $B_s^0 \to K^+K^-$ and $B_{d,s} \to \pi^\pm K^{\mp}$ decays, which have also loop contributions was e.g. discussed in (Ciuchini et al., 2012; Fleischer, 1999c, 2007a; Fleischer and Kneigjens, 2011b). Assuming the SM value for $\beta_s$ and neglecting Standard Model penguins one gets a very precise value of $\gamma = 63.5^{+2.3}_{-0.7}^\circ$ (see (Aaij et al., 2013b)). For this decay the usual argument about the theoretical cleanliness of the extraction does, however, not hold. Finally (Bhattacharya and London, 2015) discussed also the extraction of the CKM angle $\gamma$ from three-body decays $B^0, B_s^0 \to K_S h^+h^-$ with $(h = K, \pi)$.

B. Experiment

The discovery of CP violation in charmless two-body decays of $B^0$ and $B^+$ mesons by the BaBar and Belle experiments provide very interesting data, whose impact is difficult to ascertain in view of the challenges in determining precisely the hadronic matrix element relating the observed asymmetries with fundamental phases. The first observables of interests are the direct CP asymmetries. So far flavor SU(3) symmetry has been used to provide at least a theoretical framework to related such asymmetries measured in different decays. First principle calculations of the hadronic matrix elements involved will enable to fully exploit these measurements to test SM predictions. The study of direct CP asymmetries in $B_s^0$ decays provides valuable additional constraints.

The LHCb collaboration has measured CP violation asymmetries in $B_s^0 \to K^-\pi^+$ ((Aaij et al., 2012b)) and $B_0^0 \to K^+K^-$ ((Aaij et al., 2013b)). These measurements share the same level of complexity as the measurements of asymmetries mediated by the interference between $B_{s}^0 - \bar{B}_{s}^0$ mixing and CP violation in direct decays: they require a determination of the flavor of the decaying $B_{s}^0$, a time-dependent analysis to disentangle $A_{CP}$ from the $B_0^0$ production asymmetry, in addition to a careful determination of all the instrumental asymmetries discussed before. An important advantage that enables the LHCb experiment to perform these measurements with high precision is the excellent hadron identification efficiency and purity provided by the ring imaging Cherenkov (RICH) detectors (see (Adinolfi et al., 2013)).

As an illustration, Fig. 24 shows the invariant mass spectra for different species of $B \to hh$ final states. There is excellent separation between different particle species.

Using the formalism of Ref. (Aaij et al., 2013d), the CP asymmetry is related to the raw asymmetry through

$$A_{CP} = A_{raw} - A_\Delta$$

with

$$A_\Delta(B_s^0 \to K^+\pi^-) = -A_D(K^+\pi^-) + \kappa_s A_P(B_s^0)$$

where $A_D$ represents the detection efficiency asymmetry, that is derived from raw asymmetries measured for decays with known $A_{CP}$, $\kappa_s = -0.033 \pm 0.003$ (Aaij et al., 2012b), and $A_P$ is the $B_{s}^0 - \bar{B}_{s}^0$ production asymmetry, derived from a fit to the time-dependent measured asymmetry. The parameter $\kappa_s$ accounts for the dilution of the effect of the production asymmetry due to the fast $B_0^0$ oscillations and is given by

$$\kappa_s = \frac{\int_0^\infty e^{-\Gamma_s t} \cos(\Delta m_s t) \epsilon(B_s^0 \to K\pi, t) dt}{\int_0^\infty e^{-\Gamma_s t} \cosh(0.5\Gamma_s t) \epsilon(B_s^0 \to K\pi, t) dt}$$

$A_P$ introduced an oscillatory component that makes it possible to measure the production asymmetry unambiguously. Note that $A_P$ has a very marginal effect.
FIG. 24 Panels (a) and (b) show the invariant mass spectra obtained using the event selection adopted for the best sensitivity to $A_{CP}(B^0 \to K^+\pi^-)$; panels (c) and (d) show the invariant mass spectra obtained using the event selection adopted for the best sensitivity to $A_{CP}(B_s^0 \to K^+\pi^-)$. Panels (a) and (c) show the $K^+\pi^-$ invariant mass, while panels (b) and (d) the $K^-\pi^+$ invariant mass. The plot is taken from (Aaij et al., 2013d).

on $A_{CP}$ because the fast flavor oscillations greatly diminish the correlation between the flavor at decay time with the flavor at production time. The final result is $A_{CP}(B^0 \to K^+\pi^-) = 0.27 \pm 0.04 \pm 0.01$ where the first error is statistical and the second systematic.

The study of the CP asymmetry and branching fraction of the decay $B_s^0 \to K^-\pi^+$, combined with the knowledge of the corresponding observables in $B^0 \to \pi^+\pi^-$ can in principle be used to determine the CKM angle $\gamma$, defined in Eq. (193), or $-2\beta_s$, defined in Eq. (161), if U-spin be a valid symmetry of the strong interaction. The LHCb collaboration, using their measurements of CPV observables in $B^0_s \to K^-\pi^+$, performs two analyses to determine either $\gamma$ or $\beta_s$ (Aaij et al., 2013b). Here we quote the first analysis, that uses the measured value of $\beta_s$ (and neglecting Standard Model penguins) to derive

$$\gamma = (63.5_{-6.7}^{+7.2})^°. \quad (198)$$

This value is consistent with the $\gamma$ value derived from tree-level decays. Further understanding of U-spin symmetry breaking as well as penguin pollution is needed to assess the impact of this measurement.

The decay $B_s^0 \to D_s^0K^-$ is sensitive to the angle $\gamma$ of the Cabibbo-Kobayashi-Maskawa matrix. This is an example of a determination of $\gamma$ from a tree-level process, and thus, in principle, not sensitive to effects induced by most new-physics models currently considered. Other such determinations of $\gamma$ from tree-level mediated processes have been performed at the $B$-factories and LHCb, through the study of $B^0 \to D^-\pi^+$, and $B^0 \to D^-K^+$ decays. In these decays, the ratio $r_{D^0(s)} \equiv \mathcal{A}(B^0 \to D(s)^-\pi^+)/\mathcal{A}(B^0 \to D(s)^+\pi^-)/\mathcal{A}(B^0 \to D(s)^+\pi^-)$ is small, $r_{D^0(s)} \approx 0.02$, while in the case of $B_s^0 \to D_s^+K^-$ the interfering amplitudes are of the same order of magnitude. Moreover, the decay width difference in the $B_s^0$ system, $\Delta\Gamma_s$, is non-zero, which allows a determination of $\gamma - 2\beta_s$ from the sinusoidal and hyperbolic terms in the decay time evolution, up to a two-fold ambiguity.

The measurement is sensitive to the combination $\gamma - 2\beta_s$, and, as we have seen that $\beta_s$ is now measured with great precision from the study of $B_s^0 \to J/\psi h^+h^-$, if Standard Model penguins are neglected, it can be directly translated into a measurement of $\gamma$. This decay has been studied by the LHCb collaboration using 1 fb$^{-1}$ of data and the measurement requires a fit to the decay-time distribution of the selected $B_s^0 \to D_s^-K^+$ candidates. It is a very challenging measurement because it requires the determination of the time-dependent efficiency, as well as the determination of the flavor of the decaying $B_s$. The kinematically similar mode $B_{s0}^- \to D_{s0}^-\pi^+$ helps in constraining the time-dependent efficiency and the flavor tagging performance. In order to derive the CP-violating parameters, two different approaches have been pursued: the first, labelled sfit (see (Xie, 2009)), consists of a statistical method to subtract background in maximum likelihood fit, without relying on any separate sideband or simulation for background modelling, whereas the second, labelled cfit separates signal from background by fitting for these two components with separate PDFs. Fig. 25 shows the results of the time-dependent fits, and Tab. VII shows the fitted values of the CP observables in this decay.

FIG. 25 Result of the decay-time (top) sfit and (bottom) cfit to the $B_s^0 \to D_s^-K^+$ candidates. This figure is taken from Ref. (Aaij et al., 2014d).

The study of $B_s^0$ decays into two vector particles
(B_0 \to V_1 V_2), with the vector particles decaying into two pseudo-scalar mesons, has three helicity states that are allowed by angular momentum conservation, with amplitudes identified as $H_{+1}$, $H_{-1}$ and $H_0$. It is convenient to map these amplitudes in terms of three transversity states to be considered, identified as “longitudinal” ($0$), “perpendicular” ($\perp$), and “parallel” ($\parallel$). They are related as

$$
A_0 = H_0 \\
A_\perp = \frac{H_{+1} - H_{-1}}{\sqrt{2}} \\
A_\parallel = \frac{H_{+1} + H_{-1}}{\sqrt{2}}
$$

(199)

Two such decays have been studied at LHCb: $B_0 \to \phi \phi$ (discussed in the previous section), and $B^0 \to K^*\bar{K}^*$. The study of the CP asymmetries and polarisation fractions in $B^0 \to K^*\bar{K}^*$ (see (Aaij et al., 2015c)) takes a somewhat different approach. In view of the limited statistics, rather than trying to implement a flavor tagged time-dependent analysis, a study of the triple product and direct CP violation asymmetry is performed with a time-integrated analysis of $B^0 \to K^*\bar{K}^*$, without determining the flavor of the decaying $B^0$. In $B$ mesons decays there are two possible triple products

$$
T_1 = (\hat{n}_{V_1} \times \hat{n}_{V_2}) \cdot \hat{p}_{V_1} = \sin \phi \\
T_2 = 2(\hat{n}_{V_1} \cdot n_{V_2})(\hat{n}_{V_1} \times n_{V_2}) \cdot \hat{p}_{V_1} = \sin 2\phi
$$

(200)

They are found to be compatible with the Standard Model.

VI. MODEL INDEPENDENT CONSTRAINTS ON NEW PHYSICS

Indirect searches for new physics effects can be performed by assuming certain extensions of the SM and calculating then the contribution of this model to different flavour observables, e.g. $M_{12}^{s,\text{NP}}$. Combining these calculations with the SM contributions (e.g. $M_{12}^{s,\text{SM}}$) one gets a theory prediction for flavour observables that depends on unknown parameters $x, y, ...$ of the considered new physics model. Currently a comparison of experimental numbers and these new theory predictions enables to bound the parameter space of new physics models, e.g.

$$
\Delta M^s_{12} = 2 \left| M_{12}^{s,\text{NP}}(x, y, ...) + M_{12}^{s,\text{SM}} \right| .
$$

(201)

In future, this program could lead to a discovery of new physics effects, provided there is a sufficient control over the theoretical uncertainties. But also if physics beyond the SM will be first found by direct detection of new particles, the above discussed investigations will be crucial in order to determine the flavour couplings of the new model. There is a huge literature determining contributions of specific new physics models to the observables discussed in this review, in particular $B_s^0$ mixing. We present here some examples, not an exhaustive list: super-symmetric contributions were discussed e.g. by (Altmannshofer and Carena, 2012; Buras et al., 2011; Crivellin et al., 2011; Endo et al., 2011; Endo and Yokozaki, 2011; Girrbach et al., 2011; Hayakawa et al., 2012; Ishimori et al., 2011; Kaburaki et al., 2011; Kawashima et al., 2009; Kifune et al., 2008; Kubo and Lenz, 2010; Wang et al., 2011, 2010); contributions of $2$ Higgs-double models were discussed e.g. by (Chang et al., 2015; Dutta et al., 2012; Urban et al., 1998); extra dimensions were discussed in (Datta et al., 2011; Goertz and Pfoh, 2011); L-R symmetric models were discussed e.g. in (Bertolini et al., 2014; Lee and Nam, 2012); extended gauge sectors were discussed e.g. in (Alok et al., 2011; Chang et al., 2014, 2011; Fox et al., 2011; Kim et al., 2013; Li et al., 2012; Sahoo et al., 2011, 2013); additional fermions\textsuperscript{16} which were discussed e.g. in (Alok and Gangal, 2012; Botella et al., 2012).

In order to minimise the risk of betting on the wrong model, we will discuss here a little more in detail the model-independent approach, where one tries to identify new physics effects without assuming a specific model. To start, it seems to be reasonable to assume that new physics only acts in mixing, in particular in $M_{12}$, but not in tree-level decays. For simplicity we also assume no penguin contributions. Later on we will soften these restrictions. Thus we postulate a general modification of $M_{12}^s$ by an a priori arbitrary complex parameter $\Delta_s$, while $\Gamma_{12}^s$ is just given by the SM prediction

$$
M_{12}^s = M_{12}^{s,\text{SM}}|e^{i\phi_1^s}|, \\
\Gamma_{12}^s = \Gamma_{12}^{s,\text{SM}}.
$$

(202)

(203)

\textsuperscript{16} A sequential, chiral, perturbative fourth generation of fermions is already excluded by experiment, see e.g. (Buchkremer et al., 2012; Djouadi and Lenz, 2012; Eberhardt et al., 2012a,b,c; Kuflik et al., 2013). This exclusion holds, however, not for e.g. vector-like quarks or a combination of a fourth chiral family with an additional modification of the SM, see e.g. (Lenz, 2013).
Such a modification changes the mixing observables in the following way\textsuperscript{17}
\begin{equation}
\Delta M_{12}^{\text{Exp}} = 2 \frac{m_{12}^{s,\text{SM}}}{M_{12}^{s,\text{SM}}} |\Delta s|, \quad (204)
\end{equation}
\begin{equation}
\Delta \Gamma_{12}^{\text{Exp}} = 2 \frac{\Gamma_{12}^{s,\text{SM}}}{M_{12}^{s,\text{SM}}} \cos \left( \phi_{12}^{s,\text{SM}} + \phi_{s}^{\Delta} \right), \quad (205)
\end{equation}
\begin{equation}
\phi_{sl}^{s,\text{Exp}} = \frac{\frac{\Gamma_{12}^{s,\text{SM}}}{M_{12}^{s,\text{SM}}} \sin \left( \phi_{12}^{s,\text{SM}} + \phi_{s}^{\Delta} \right)}{|\Delta s|}. \quad (206)
\end{equation}

Also the phases $\phi_{12}^{s}$ and $\phi_{s}$ will get new contributions
\begin{equation}
\phi_{12}^{s,\text{Exp}} = \phi_{12}^{s,\text{SM}} + \phi_{s}^{\Delta}, \quad (207)
\end{equation}
\begin{equation}
\phi_{s}^{\text{Exp}} = -2 \beta_{s} + \phi_{s}^{\Delta}. \quad (208)
\end{equation}

Now a comparison of experimental numbers and SM predictions can be used to obtain the bounds on the complex parameter $\Delta_{s}$. If there is no new physics present, the comparison should result in $\Delta_{s} = 1 + 0 \times i$. For a specific new physics model the parameter $\Delta_{s}$ can also be explicitly calculated in dependence on the new physics parameters $x, y, ...$. One gets
\begin{equation}
\Delta_{s} = \frac{M_{12}^{s,\text{NP}}(x, y, ...)}{M_{12}^{s,\text{SM}}} + \frac{M_{12}^{s,\text{SM}}}{M_{12}^{s,\text{SM}}} |\Delta s|. \quad (209)
\end{equation}

General model-independent investigations, using the above introduced notation, were done e.g. in (Lenz et al., 2011, 2012; Lenz and Nierste, 2007) and (Charles et al., 2015, 2014). Below we discuss different approaches. Early investigations actually pointed towards large deviations from the SM. Unfortunately more data brought the extracted value for $\Delta_{s}$ in perfect agreement with the SM. The most recent result of such a investigation is depicted in Fig. 26\textsuperscript{18}. For completeness we show also the result for the $B_{d}^{0}$-system. The constraint from the mass difference, Eq.(204), is denoted by the orange ring. The finite size of the ring is mostly due to the theory uncertainty of $\Delta M_{s}$. In the case of $B_{d}^{0}$ mesons we have two rings, due to two different values for the CKM parameters $\rho$ and $\eta$ in the CKM-fit. The purple region stems from the measurement of the phase $\phi_{s}$. According to Eq.(208) this constrains also $\phi_{s}^{\Delta}$. Here one has to keep in mind that this assumes no sizable SM penguins and also no new physics penguins. The dark-grey area stems from the semileptonic asymmetries. Here we are currently limited by the experimental precision. The overlap region of all experimental bounds is plotted in red. All in all we find in both mixing systems a perfect agreement with the SM, but there is still some sizable space (of the order of 10% in $|\Delta s|$ and several degrees in the phase $\phi_{s}^{\Delta}$) for new physics effects in $B_{d}^{0}$ and $B_{d}^{0}$ mixing. It is entertaining and may be instructive - in the view of the currently discussed deviations of experiment and SM - to show the corresponding plots from 2010 (Lenz et al., 2011) in Fig. 27. Here a quite clear hint for new physics effects can be seen, actually in both mixing systems, which unfortunately vanished completely in the last years.

Similar investigations had been performed by (Fox et al., 2008) and the UTfit group (see e.g. the web-update of (Bevan et al., 2013; Bona et al., 2006a, 2008). In their notation one has
\begin{equation}
C_{B_{d}^{0}} e^{2i\phi_{B_{d}^{0}}} = \Delta_{s}, \quad (210)
\end{equation}
\begin{equation}
C_{B_{d}^{0}} = |\Delta_{s}|, \quad (211)
\end{equation}
\begin{equation}
\phi_{B_{d}^{0}} = \frac{1}{2} \phi_{s}^{\Delta}. \quad (212)
\end{equation}

Having only two parameters $C_{B_{d}^{0}}$ and $\phi_{B_{d}^{0}}$ for parameterising new physics effects in $B_{d}^{0}$ mixing corresponds to making the same assumptions as above: no new physics effects in $\Gamma_{12}^{s}$ and neglecting penguins. Investigating all available mixing observables UTfit finds the following preferred parameter ranges
\begin{equation}
C_{B_{d}^{0}} = 1.052 \pm 0.084, \quad (213)
\end{equation}
\begin{equation}
\phi_{B_{d}^{0}} = -2.0^{\circ} \pm 3.2^{\circ}, \quad (214)
\end{equation}

Again, everything seems to be perfectly consistent with the SM, while leaving room for sizable new physics effects, i.e. of the order of 10% in $C_{B_{d}^{0}}$ and of the order of a factor of 10 in the phase $\phi_{B_{d}^{0}}$. The corresponding allowed parameter regions for the $B_{d}^{0}$-system read
\begin{equation}
C_{B_{d}^{0}} = 1.07 \pm 0.17, \quad (215)
\end{equation}
\begin{equation}
\phi_{B_{d}^{0}} = -2.0^{\circ} \pm 3.2^{\circ}, \quad (216)
\end{equation}

yielding similar conclusions as in the $B_{d}^{0}$-system. Sometimes these bounds are transferred into bounds on a hypothetical new physics scale. (Charles et al., 2014) use the following notation for a deviation of $M_{12}^{s}$ from its SM value
\begin{equation}
M_{12}^{s} = M_{12}^{s,\text{SM}}(1 + h_{s} e^{2i\sigma_{s}}), \quad (217)
\end{equation}

Assuming further the operator
\begin{equation}
\frac{C_{ij}^{2}}{\Lambda^{2}} (\bar{q}_{i} L \gamma^{\mu} q_{j} L)^{2}, \quad (218)
\end{equation}
to describe the new physics contribution to $B_{d}^{0}$ mixing (i.e. $i = s$ and $j = b$), they find
\begin{equation}
h_{s} \approx \frac{C_{sb}^{2}}{\Lambda^{2} s b} \left( \frac{4.5 \text{ TeV}}{\Lambda} \right)^{2}, \quad (219)
\end{equation}
\begin{equation}
\sigma_{s} = \text{arg} (C_{sb} s b^{s}). \quad (220)
\end{equation}

\textsuperscript{17}The correction factor $1/8|F_{12}^{s,\text{SM}}/M_{12}^{s,\text{SM}}|^{2} |\Delta s|^{2} \sin \phi_{12}^{s}$ in Eq.(11) and Eq.(12) stays still small.

\textsuperscript{18}These plots are taken from the CKMfit page (Summer 2014 - see (Charles et al., 2005)).
Here $C_{ij}$ denotes the size of the new physics couplings and $\Lambda$ is the mass scale of new physics. Both of these parameters are a priori unknown, because new physics has not been detected yet and an investigation of current experimental bounds on $B_0^s$ mixing gives only information about the ratio $C^2_{ij}/\Lambda^2$, but not about the individual values of the couplings and of the scale. $\lambda_{ab} = V_{ts}^* V_{tb}$ denotes the CKM structure of the SM contribution to $B_0^s$ mixing.

To make some statements about the new physics scale additional assumptions have to be made. Assuming that the coefficients $C_{ab}$ have the same size as the CKM couplings, i.e. $C_{ab} = \lambda_{ab}$ (Charles et al., 2014) got a new physics scale $\lambda$ of about 19 TeV. Assuming instead $C_{ab} = 1$ the new physics scale increases to roughly 500 TeV. In particular the second scale is far above the direct reach of LHC and thus $B_0^s$ mixing could in principle probe new physics scales that are far from being accessible via direct measurements. On the other hand one should not forget that the assumption about the size of the coupling is in principle arbitrary. If the new physics couplings are very small then also the new physics scale that can be probed is very low.

In order to fulfill our final goal of unambiguously disentangling hypothetical new effects from mixing observables a strict control over uncertainties is mandatory. Also the assumptions made above, have to be challenged. First of all we have to include penguins, both from the SM as well as from new sources, this will modify the phase $\phi_s$ to

$$\phi_s = -2\beta_s + \phi_s^\Delta + \delta_{\text{Eng,SM}} + \delta_{\text{Eng,NP}}.$$  

(221)

SM penguins are expected to contribute at most up to $1^\circ$, while new physics penguins are less constrained. General new physics effects in $M_{12}^s$ will be treated as above

$$M_{12}^s = M_{12}^{s,\text{SM}} |\Delta_s| e^{i\phi_s^\Delta}.$$  

(222)

In addition we will also allow new effects in $\Gamma_{12}^s$, encoded by the parameter $\Delta_s$

$$\Gamma_{12}^s = \Gamma_{12}^{s,\text{SM}} |\Delta_s| e^{-i\phi_s^\Delta},$$  

(223)

resulting in a modified mixing phase $\phi_{12}^s$

$$\phi_{12}^s = \phi_{12}^{s,\text{SM}} + \phi_s^\Delta + \Delta_s.$$  

(224)

(225)
FIG. 27 Bounds on the complex parameters $\Delta_d$ (left) and $\Delta_s$ (right) from different mixing observables with data available till 2010. The point $\Delta_q = 1 + 0i$ corresponds to the SM. Plots are taken from (Lenz et al., 2011). Unfortunately this quite clear hint for new physics effects has completely vanished by now.

New contributions to $\Gamma_{12}^s$ can be due to new penguins and/or new contributions to tree-level decays. For a long time new physics effects in tree-level decays were considered to be negligible. Due to the dramatically improved experimental precision, this possibility has, however, to be considered. Taking only experimental constraints into account and no bias from model building, first studies performed by (Bobeth et al., 2014), (Bobeth et al., 2015) and (Brod et al., 2015) find that the tree-level Wilson coefficients $C_1$ and $C_2$ can easily be affected by new effects of the order of 0.1. Such a deviation can sometimes have dramatic effects, e.g. a modification of the imaginary part of $C_1$ by about 0.1 would modify the extracted value of the CKM angle $\gamma$ by about $4^\circ$, see (Brod et al., 2015), which is larger than the expected future experimental uncertainty. Thus these possibilities should be taken into account for quantitative studies about new physics effects in mixing. For a future disentangling of new effects in mixing, penguins and tree-level decays clearly more theoretical work has to be done.

The above modification of $M_{12}^s$ (see Eq.(222)) and $\Gamma_{12}^s$ (see Eq.(223)) changes the mixing observables in the following way\(^{19}\).

$$\Delta M_s = 2 \left| \frac{M_{12}^{s,\text{SM}}}{M_{12}^{\text{SM}}} \right| \cdot |\Delta_s|,$$  
(226)

$$\Delta \Gamma_s = 2 \left| \frac{\Gamma_{s,\text{SM}}^{s,\text{SM}}}{\Gamma_{12}^{s,\text{SM}}} \right| \cdot |\Delta_s| \cdot \cos \left( \phi_{12}^{s,\text{SM}} + \phi_{s}^\Delta + \tilde{\phi}_{s}^\Delta \right),$$  
(227)

$$\alpha_{st}^s = \left| \frac{\Gamma_{12}^s}{\Gamma_{12}^{s,\text{SM}}} \right| \cdot \frac{\Delta s}{\Delta s} \cdot \sin \left( \phi_{12}^{s,\text{SM}} + \phi_{s}^\Delta + \tilde{\phi}_{s}^\Delta \right),$$  
(228)

First steps in that direction have been done in the analysis of (Lenz et al., 2012), where in Scenario IV new physics in $\Gamma_{12}^s$ was introduced by the parameter $\delta_s$.

$$\delta_q = \frac{\Gamma_{12}^s}{\Gamma_{12}^{s,\text{SM}} \left( \frac{\Gamma_{12}^{s,\text{SM}}}{\Gamma_{12}^s} \right)^2} \cdot \frac{\left| \Delta_s / \Delta_s \right|^2 \sin \phi_{12}^s}{\sin \phi_{12}^s},$$  
(229)

\(^{19}\) Again, the correction factor $1/8|\Gamma_{12}^{s,\text{SM}} / M_{12}^{s,\text{SM}}|^2 \cdot \left| \Delta_s / \Delta_s \right|^2 \sin \phi_{12}^s$ stays small.
This parameter is related to mixing observables in the following way:

\[
\Re (\delta_s) = \frac{\Delta \Gamma - \Delta M}{\Delta M^{\text{SM}}} , \quad \Im (\delta_s) = -\frac{a_{s1}^s}{\Delta M^{\text{SM}}} .
\] (230)

In 2012 the fit of (Lenz et al., 2012) seemed to prefer some deviations of \( \Im (\delta_s) \) and \( \Re (\delta_s) \), which were mostly triggered by an interpretation of the dimuon asymmetry, which was commonly accepted at that time commonly, but turned out to be incomplete.

In future these model independent investigations should include new physics effects in \( M_{12}^s \), in \( \Gamma_{12}^s \), and in penguins. Doing the latter might also include a combination of \( \Delta B = 2 \) and \( \Delta B = 1 \) observables.

\textbf{VII. CONCLUSION/OUTLOOK}

The study of CP violation phenomena in the \( B_s^0 \) system has been the focus of experimental and theoretical efforts. It was started by the Tevatron experiments CDF and D0, who made the first measurements in this system. Among their main achievements are the measurement of the \( B_s^0 \) meson mass difference \( \Delta M_s \) (Abulencia et al., 2006) and the study of the semi leptonic charge asymmetry of the \( B_s^0 \) meson \( a_s^s \) (Abazov et al., 2012b, 2013, 2014). The measured value of \( a_s^s \) based on the study of \( B_s^0 \to D_s^+ \mu^- \bar{\nu}_\mu \) is still contributing to the average with the LHCb result, based on one-third of the run 1 data. The Tevatron experiments also initiated the studies of other CP-violating phenomena, such as the mixing phase \( \phi_s \) in the \( B_s^0 \to J/\psi \phi \) decay, albeit with large uncertainties.

The pioneering work of the Tevatron experiment is continued and refined at the LHC, with a new level of precision allowed by high statistics, improved detector performance, and new analysis techniques. In particular, the LHCb experiment has performed the most precise measurement of all types of CP violation (see (Aaij et al., 2012b, 2014c, 2015h)), as well as that of \( \Delta M_s \) and \( \Delta \Gamma_s \) (Aaij et al., 2013h). They measure the CKM angle \( \gamma \) not only in \( B^0 \) decays previously studied by the \( e^+e^- \) \( B \)-factories, but also in \( B_s^0 \) decays both in tree-mediated processes, and in loop-mediated processes. Finally, they observe direct CP violation in several \( B_s^0 \) channels.

The current data do not confirm CP violation in the \( B_s^0 \) system in excess of the SM prediction, as it was originally hoped for. Still, some room for new physics manifestations remains. In CP violation in the interference of decays and mixing quantified by the angle \( \phi_s \), the experimental uncertainty is getting very close to the SM central value. In this respect, the emphasis on understanding small corrections such as penguin pollution is a field of active investigation in the theoretical and experimental community. The theory prediction for CP violation in mixing is still orders of magnitude smaller than the experimental uncertainty. The level of understanding of the SM expectations for mixing observables and CP violating phenomena in the \( B_s^0 \) system is now very advanced. Experimental studies have not only proven the CKM-mechanism to be the primary source of quark-mixing and CP violation, but they have also confirmed the validity of theoretical approaches such as the HQE to an unprecedented accuracy.

The uncertainty on the theory prediction for the mass difference \( \Delta M_s \) is about \( \pm 15\% \), thus allowing for new effects of the same order in this observable. To improve the accuracy in \( \Delta M_s \) further, more precise lattice evaluations of bag parameters and decay constants are mandatory. In this respect, an uncertainty of about \( \pm 5\% \) seems to be achievable in the next years. The calculation of the width difference according the HQE seems on less solid theoretical grounds. The assumption of quark hadron duality was questioned many times, see e.g. (Ligeti et al., 2010) or the discussion by (Lenz, 2011), and deviations of more than 100\% were discussed. Such a failure of the HQE is now clearly ruled out. The measurement of the width difference \( \Delta \Gamma_s \) has shown that the HQE works also in the most challenging channel - \( b \to c \bar{c}s \) - with an accuracy of at least 20\%\(^{21}\). For further independent tests of the precision of the HQE, lattice determinations of the matrix elements that arise in lifetime difference of different \( b \) hadrons, like \( \tau(B^+)/\tau(B^0) \), \( \tau(B^0_s)/\tau(B^0) \) and \( \tau(\Lambda_b)/\tau(B^0) \) are urgently needed, see the detailed discussion in (Lenz, 2014). Here it might also be insightful to study the charm sector, in particular the ratio \( \tau(D^+)/\tau(D_0) \) and \( \tau(D^+_s)/\tau(D_0) \). To reduce the uncertainty on the theory prediction of \( \Delta \Gamma_s \) a first non-perturbative determination of dimension 7 matrix elements is needed, i.e. the bag parameters \( B_{R_0}, B_{R_2}, B_{R_4}, B_{R_2} \), and \( B_{R_4} \). Currently, these parameters contribute the biggest individual uncertainty. Next, more precise lattice values of the complete SUSY-basis of \( \Delta B = 2 \) four quark operators are needed\(^{22}\). In parallel to these non-perturbative improvements, NNLO-QCD corrections\(^{23}\) have to be calculated (i.e. \( \Gamma_3^{(2)} \) and \( \Gamma_4^{(1)} \) in our notation). Having all these improvements at hand, a final accuracy of about 5\% for the \( \Delta \Gamma_s \) prediction might also be feasible in the next years\(^{24}\).

---

20 Here we assume an accuracy of lattice values for dimension six operators considerably below 5\%.

21 For very recent estimates of the possible size of duality violating effects, see (Jubb et al., 2016).

22 While preparing this paper a new study of the Fermilab Lattice and MILC Collaborations was made public (Bazavov et al., 2016).

23 See (Asatryan et al., 2012) for a first step in that direction.

24 Here we assume an accuracy of lattice values for dimension six operators considerably below 5\%, an accuracy of about 10\% for the bag parameters \( B_R \) of the dimension seven operators and a reduction of the renormalisation scale dependence by at least a factor of two due to NNLO-QCD corrections.
The current experimental uncertainty on $a^s_{1}$ is still about a factor of 130 larger than the tiny central value of the Standard Model expectations, thus still allowing plenty of room for new physics effects. Turning to indirect CP violation, we find that the current experimental precision is coming close to SM central value and also to the intrinsic theoretical uncertainties due to penguin contributions. In principle the weak phase $\phi_s$ measured e.g. in $B^0 \to J/\psi \phi$ is a null test similar to the semileptonic asymmetries. In practice the theory prediction of the latter one is much more robust than the one for $\phi_s$. To fully exploit the improving experimental precision extended studies of penguin effects and a quantification of them are mandatory.

All LHC experiments expect to continue data taking at least up to 2030. The LHCb collaboration is currently engaging in a detector upgrade that should increase its sensitivity by a factor of 10, with a combination of operating at higher instantaneous luminosity, and the implementation of a purely software based trigger system, which will have to process the full 30 MHz of inelastic collisions delivered by the LHC. The physics opportunities offered by such an upgrade have been quantified by (LHCb, 2014) assuming a total integrated luminosity of 50 fb$^{-1}$ (phase I upgrade) are given. For a comparison we show also the current theory uncertainty of the Standard Model predictions, given in Eqs. (160,161) and Eq. (116). The theory error in $\phi_s$ holds only for neglecting penguins.

<table>
<thead>
<tr>
<th>Observable</th>
<th>LHCb 2018 Upgrade</th>
<th>Theory Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_s (B^0 \to J/\psi \phi)$</td>
<td>0.025</td>
<td>0.009</td>
</tr>
<tr>
<td>$a^s_{1}$ ($10^{-3}$)</td>
<td>1.4</td>
<td>0.5</td>
</tr>
<tr>
<td>$\phi_s^{eff}(B_s^0 \to \phi \phi)$</td>
<td>0.10</td>
<td>0.018</td>
</tr>
<tr>
<td>$\gamma(B^0 \to D_s K)$</td>
<td>11°</td>
<td>2.0°</td>
</tr>
</tbody>
</table>

TABLE VIII Statistical sensitivity of the LHCb upgrade to key observables discussed in this paper. For each observables the projected sensitivity at the end of Run II and with a luminosity of 50 fb$^{-1}$ (phase I upgrade) are given. For a comparison we show also the current theory uncertainty of the Standard Model predictions, given in Eqs. (160,161) and Eq. (116). The theory error in $\phi_s$ holds only for neglecting penguins.

The current experimental uncertainty on $a^s_{1}$ is still about a factor of 130 larger than the tiny central value of the Standard Model expectations, thus still allowing plenty of room for new physics effects. Turning to indirect CP violation, we find that the current experimental precision is coming close to SM central value and also to the intrinsic theoretical uncertainties due to penguin contributions. In principle the weak phase $\phi_s$ measured e.g. in $B^0 \to J/\psi \phi$ is a null test similar to the semileptonic asymmetries. In practice the theory prediction of the latter one is much more robust than the one for $\phi_s$. To fully exploit the improving experimental precision extended studies of penguin effects and a quantification of them are mandatory.

The operators

\begin{align*}
O_1 & = \bar{q} \gamma^\mu p \gamma^\nu q \frac{1}{m_q} \epsilon_{\mu\nu}\eta_{\nu} \\
O_2 & = \bar{q} \gamma^\mu p \gamma^\nu q \frac{1}{m_q} \epsilon_{\mu\nu}\eta_{\nu}
\end{align*}

are mandatory. The huge statistics, which will become available during the next ten years, will also allow to measure the CP violating phenomena in other channels like $B^0_s \to J/\psi \eta$. Advancement in theory, in particular in lattice QCD and other approaches to constrain the hadronic matrix elements needed to access fundamental quantities, are expected to follow a similar path. Thus, a new exciting era of $B^0_s$ meson studies is ahead of us.

## VIII. ACKNOWLEDGEMENTS

We would like to thank Ikaros Bigi for many helpful comments on the manuscript. M.A. would like to thank the US National Science Foundation for their support, and S. Stone and P. Koppenburg for useful discussions. A.L. would like to thank Christine Davies, Tomomi Ishikawa, Andreas Jüttner and James Simone for helpful information about lattice inputs, Gilberto Tetlalmatz-Xolocotzi for creating the diagrams in Fig. 1, Fig.13 and Fig.23, checking some of the numerics and proof-reading, Matthew Kirk for providing Fig. 3, spotting a tricky sign error and proof-reading, Lucy Budge, Jonathan Cullen and Xiu Liu for checking some of the numerical updates.

### Appendix A: Numerical input for theory predictions

In this appendix we list all input parameters that were used for our numerical updates of several Standard Model predictions. We start with listing some very well-known parameters in Table IX that are mostly taken from the PDG (Olive et al., 2014). Next we list in Table X some not so well determined input parameters. For lattice values our standard reference is FLAG (Aoki et al., 2014). In the case of $B_S/B$ FLAG did not provide an average, so we took the values from (Becirevic et al., 2002), (Bouchard et al., 2011), (Carrasco et al., 2014) and (Dowdall et al., 2014) and did our own naive average. For $B_{R_0}$ we took the preliminary value that can be read off the plots given by (Dowdall et al., 2014). $B_{R_1}$ and $\bar{B}_{1}$ can be deduced from (Becirevic et al., 2002), (Bouchard et al., 2011), (Carrasco et al., 2014) and (Dowdall et al., 2014). The operators $R_1$ and $\bar{R}_1$ are denoted by $O_1$ and $O_3$ in

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_W$</td>
<td>80.385(15) GeV</td>
<td>PDG 2015</td>
</tr>
<tr>
<td>$M_Z$</td>
<td>91.1876(21) GeV</td>
<td>PDG 2015</td>
</tr>
<tr>
<td>$G_F$</td>
<td>1.1663787(6) $10^{-5}$ GeV$^{-2}$</td>
<td>PDG 2015</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>6.58211928(15) $10^{-25}$ GeV$^2$</td>
<td>PDG 2015</td>
</tr>
<tr>
<td>$M_{B^s}$</td>
<td>5.3667(4) GeV</td>
<td>PDG 2015</td>
</tr>
<tr>
<td>$\bar{m}_{s}(\bar{m}_u)$</td>
<td>4.18(3) GeV</td>
<td>PDG 2015</td>
</tr>
<tr>
<td>$\bar{m}_{c}(\bar{m}_c)$</td>
<td>1.275(25) GeV</td>
<td>PDG 2015</td>
</tr>
<tr>
<td>$\bar{m}_{s}(2\text{GeV})$</td>
<td>0.0935(25) GeV</td>
<td>PDG 2015</td>
</tr>
<tr>
<td>$\alpha_s(M_Z)$</td>
<td>0.1185(6)</td>
<td>PDG 2015</td>
</tr>
</tbody>
</table>

TABLE IX List of precisely known input parameters needed for an update of the theory prediction of different mixing observables.

We would like to thank Ikaros Bigi for many helpful comments on the manuscript. M.A. would like to thank the US National Science Foundation for their support, and S. Stone and P. Koppenburg for useful discussions. A.L. would like to thank Christine Davies, Tomomi Ishikawa, Andreas Jüttner and James Simone for helpful information about lattice inputs, Gilberto Tetlalmatz-Xolocotzi for creating the diagrams in Fig. 1, Fig.13 and Fig.23, checking some of the numerics and proof-reading, Matthew Kirk for providing Fig. 3, spotting a tricky sign error and proof-reading, Lucy Budge, Jonathan Cullen and Xiu Liu for checking some of the numerical updates.

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the lattice literature
\[ R_1 \equiv \frac{m_s}{m_b} O_4, \tilde{R}_1 \equiv \frac{m_s}{m_b} O_5. \]  
(A1)

The Fermilab-MILC Collaboration (Bouchard et al., 2011) uses again an additional factor 4
\[ R_1 \equiv \frac{m_s}{m_b} 4 O_4, \tilde{R}_1 \equiv \frac{m_s}{m_b} 4 O_5. \]  
(A2)

Moreover one has to be aware of different normalisation factors used in the definition of the matrix elements. In e.g. (Beneke et al., 1996) and (Lenz and Nierste, 2007)
\[ \langle R_1 \rangle = \frac{7}{3} \frac{m_s}{m_b} M_{B_0}^2 f_{B_0}^2 B_{R_1}, \]  
(A3)
\[ \langle \tilde{R}_1 \rangle = \frac{5}{3} \frac{m_s}{m_b} M_{B_0}^2 f_{B_0}^2 \tilde{B}_{R_1}, \]  
(A4)

was used. This definition ensures that in vacuum insertion approximation the bag parameters \( B_{R_1} \) and \( \tilde{B}_{R_1} \) have the value one. In the lattice literature different normalisation factors, compared to \( \frac{7}{3} \) and \( \frac{5}{3} \), are used. (Becirevic et al., 2002) and (Carrasco et al., 2014) have
\[ \langle R_1 \rangle = \frac{2}{3} \frac{m_s}{m_b} M_{B_0}^2 f_{B_0}^2 B_{R_1}, \]  
(A5)
\[ \langle \tilde{R}_1 \rangle = \frac{2}{3} \frac{m_s}{m_b} M_{B_0}^2 f_{B_0}^2 \tilde{B}_{R_1}, \]  
(A6)

while (Bouchard et al., 2011) and and (Dowdall et al., 2014) use
\[ \langle R_1 \rangle = \frac{2}{3} \frac{m_s}{m_b} M_{B_0}^2 f_{B_0}^2 B_4, \]  
(A7)
\[ \langle \tilde{R}_1 \rangle = \frac{2}{3} \frac{m_s}{m_b} M_{B_0}^2 f_{B_0}^2 \tilde{B}_5. \]  
(A8)

For the top-quark mass we did not take the PDG value, but a first combination of TeVatron and LHC results, presented by (ATLAS and Collaborations, 2014). \( \Lambda_{QCD}^{(5)} \) we derived from the NLO running of \( \alpha_s \) using \( \alpha_s(M_Z) \) and \( M_Z \) given above as an input. The values of the CKM-elements were taken from the web-update of the CKMfitter group (Charles et al., 2005), similar results are given by UTfit (Bona et al., 2006b). Here the value of \( V_{ub} \) is taken from the fit and not from either an inclusive or an exclusive determination. Finally we also present in Table XI a list of additional lattice determinations for \( f_{B_s \sqrt{B}} \) and \( \bar{B}_s / B \), given by HQQCD (LATTICE 2014 update by (Dowdall et al., 2014)), ETMC (Carrasco et al., 2014), the LATTICE 2015 update from the Fermilab-MILC Collaboration (Bouchard et al., 2011) and the LATTICE 2015 update from RBC-UKQCD of (Aoki et al., 2015).

### Appendix B: Error budget of the theory predictions

In this appendix we compare the error budget or our new Standard Model predictions with the ones given in 2011 by (Lenz and Nierste, 2011) and the ones given in 2006 by (Lenz and Nierste, 2007).

The error budget for the updated Standard Model prediction of \( \Delta M_{K}^{SM} \) is given in Table XII. For the mass difference we observe no improvement in accuracy compared to the 2011 prediction, because the by far dominant uncertainty (close to 14%) stems from \( f_{B_s \sqrt{B}} \) and here the inputs are more or less unchanged. This will change as soon as new lattice values will be available. The next important uncertainty is the accuracy of the CKM element \( V_{cb} \), which contributes about 5% to the error budget. If one gives up the assumption of the unitarity of the 3 times 3 CKM matrix, the uncertainty can go up considerably. The uncertainties due to the remaining parameters, play no important role. All in all we are left with an overall uncertainty of close to 15%, which has to be compared to the experimental uncertainty of about 1 per mille. This situation leaves currently some space for new physics contributions to the mass difference \( \Delta M_{K} \).

With future improvements on the non-perturbative parameters a theoretical uncertainty in the range of 5% till 10% is feasible.

Next we study the error budget of the decay rate difference \( \Delta \Gamma_s \) in Table XIII. The uncertainty in the decay rate difference also did not change considerably compared to 2011. The dominant uncertainty is still the unknown bag parameter of the power suppressed operator \( R_2 \). This input did not improve since 2011. Here and in (Lenz and Nierste, 2011) and (Lenz and Nier-
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Collaboration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_B \sqrt{B}$</td>
<td>200(5 – 10) MeV</td>
<td>HPQCD</td>
</tr>
<tr>
<td>$B_S/B$</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td>$B_{R_1}/B$</td>
<td>1.98</td>
<td></td>
</tr>
<tr>
<td>$B_{R_2}/B$</td>
<td>1.48</td>
<td></td>
</tr>
<tr>
<td>$f_B \sqrt{B}$</td>
<td>211(8) MeV</td>
<td>ETMC</td>
</tr>
<tr>
<td>$B_S/B$</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td>$B_{R_1}/B$</td>
<td>1.46</td>
<td></td>
</tr>
<tr>
<td>$B_{R_2}/B$</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
<td>$f_B \sqrt{B}$</td>
<td>227(7) MeV</td>
<td>Fermi-MILC</td>
</tr>
<tr>
<td>$B_S/B$</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
<td>$B_{R_1}/B$</td>
<td>1.60</td>
<td></td>
</tr>
<tr>
<td>$B_{R_2}/B$</td>
<td>1.17</td>
<td></td>
</tr>
<tr>
<td>$f_B \sqrt{B}$</td>
<td>262(7) MeV</td>
<td>RBC-UKQCD</td>
</tr>
</tbody>
</table>

**Table XI** List of additional and mostly preliminary determinations of lattice parameters needed for an update of the theory prediction of different mixing observables. Some of the values given here were simply read off plots provided by the different collaborations. The error of the RBC-UK evaluation cannot be estimated currently, because of missing corrections.

<table>
<thead>
<tr>
<th>$\Delta M^s_{\text{SM}}$</th>
<th>This work</th>
<th>LN 2011</th>
<th>LN 2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central Value</td>
<td>18.3 ps$^{-1}$</td>
<td>17.3 ps$^{-1}$</td>
<td>19.3 ps$^{-1}$</td>
</tr>
<tr>
<td>$\delta(f_B \sqrt{B})$</td>
<td>13.9%</td>
<td>13.5%</td>
<td>34.1%</td>
</tr>
<tr>
<td>$\delta(V_{cb})$</td>
<td>4.9%</td>
<td>3.4%</td>
<td>4.9%</td>
</tr>
<tr>
<td>$\delta(m_t)$</td>
<td>0.7%</td>
<td>1.1%</td>
<td>1.8%</td>
</tr>
<tr>
<td>$\delta(\alpha_s)$</td>
<td>0.1%</td>
<td>0.4%</td>
<td>2.0%</td>
</tr>
<tr>
<td>$\delta(\gamma)$</td>
<td>0.1%</td>
<td>0.3%</td>
<td>1.0%</td>
</tr>
<tr>
<td>$\delta(V_{ub}/V_{cb})$</td>
<td>0.1%</td>
<td>0.2%</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\delta(m_b)$</td>
<td>&lt; 0.1%</td>
<td>0.1%</td>
<td>~ ~ ~</td>
</tr>
<tr>
<td>$\Sigma \delta$</td>
<td>14.8%</td>
<td>14.0%</td>
<td>34.6%</td>
</tr>
</tbody>
</table>

**Table XII** List of the individual contributions to the theoretical error of the mass difference $\Delta M_s$ within the Standard Model and comparison with the values obtained in (Lenz and Nierste, 2011) and (Lenz and Nierste, 2007).

dall et al. (2014), ETMC (Carrasco et al. (2014), the LATTICE 2015 update from the Fermilab-MILC Collaboration (Bouchard et al. (2011)) and the LATTICE 2015 update from RBC-UKQCD of (Aoki et al. (2015)) - that seem to indicate that $f_B \sqrt{B}$ can be determined with an uncertainty as low as 5% in the near future. In most of these works not only the matrix element of $Q$, but also the full $\Delta B = 2$ operator basis is studied. This will provide improved values for the bag parameters $B_S$, $B_S$, $B_{R_1}$, $B_{R_1}$, and $B_{R_2}$, via Eq.(67). Number three in the error budget is the dependence on the renormalisation scale, here a calculation of NNLO-QCD corrections would be necessary to further reduce the error. First steps of such an endeavour were done by (Asatryan et al. (2012)). The next important dependence is the CKM element $V_{cb}$, which leads currently to an uncertainty of about 5%. In the ratio $\Delta \Gamma^s_{\text{SM}} / \Delta M^s_{\text{SM}}$ one of the dominant uncertainties, the dependence on $f_B \sqrt{B}$ is cancelling and we get for the error budget the values given in Table XIV. For $\Delta \Gamma^s / \Delta M_s$ we see a tiny improvement in the theoretical precision compared to 2011. The dominant uncertainty is given by the unknown matrix element of the dimension 7 operator $R_2$, followed by the uncertainty due to the renormalisation scale dependence. The overall uncertainty is currently 17.3%, which is also the final theoretical uncertainty that can currently be achieved for $\Delta \Gamma^s$. Future investigations, i.e. non-perturbative deter-
The error budget for the semileptonic CP asymmetries is minimizations of the matrix element of $R_2$ and NNLO-QCD corrections might bring down this uncertainty to maybe 5%.

The error budget for the semileptonic CP asymmetries is finally listed in Table XV. Here we witness some sizable reduction of the theory error. This quantity does not depend on $f_B\sqrt{B}$ and has only a weak dependence on $R_2$, thus the two least known parameters in the mixing sector do not affect the semileptonic asymmetries. The increase in precision stems mostly from better known CKM elements in particular of $V_{ub}$, in comparison to 2011. Currently the dominant uncertainty stems from the renormalisation scale dependence followed by the dependence on $V_{ub}$. For a reduction of the overall theoretical uncertainty considerably below 10% a NNLO-QCD calculation is mandatory.

Finally we present in Table XVI also the theory errors for the observables in the $B_s^0$-sector.

**REFERENCES**


<table>
<thead>
<tr>
<th>$\Delta \Gamma_s^{SM}/\Delta M_s^{SM}$</th>
<th>this work</th>
<th>LN 2011</th>
<th>LN 2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta(B_{R2})$</td>
<td>14.8%</td>
<td>17.2%</td>
<td>15.7%</td>
</tr>
<tr>
<td>$\delta(\mu)$</td>
<td>8.4%</td>
<td>7.8%</td>
<td>9.1%</td>
</tr>
<tr>
<td>$\delta(B_s)$</td>
<td>2.1%</td>
<td>4.8%</td>
<td>3.1%</td>
</tr>
<tr>
<td>$\delta(B_{R0})$</td>
<td>2.1%</td>
<td>3.4%</td>
<td>3.6%</td>
</tr>
<tr>
<td>$\delta(\zeta)$</td>
<td>1.1%</td>
<td>1.5%</td>
<td>1.9%</td>
</tr>
<tr>
<td>$\delta(m_b)$</td>
<td>0.8%</td>
<td>1.4%</td>
<td>1.6%</td>
</tr>
<tr>
<td>$\delta(B_{R1})$</td>
<td>0.7%</td>
<td>1.9%</td>
<td>−−−</td>
</tr>
<tr>
<td>$\delta(B_{R2})$</td>
<td>0.6%</td>
<td>0.5%</td>
<td>−−−</td>
</tr>
<tr>
<td>$\delta(B_{R1})$</td>
<td>0.5%</td>
<td>0.8%</td>
<td>−−−</td>
</tr>
<tr>
<td>$\delta(B_{R2})$</td>
<td>0.2%</td>
<td>0.2%</td>
<td>−−−</td>
</tr>
<tr>
<td>$\delta(\alpha_s)$</td>
<td>0.2%</td>
<td>0.8%</td>
<td>0.1%</td>
</tr>
<tr>
<td>$\delta(m_s)$</td>
<td>0.1%</td>
<td>1.0%</td>
<td>0.1%</td>
</tr>
<tr>
<td>$\delta(\gamma)$</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.1%</td>
</tr>
<tr>
<td>$\delta(V_{ub})$</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>$\sum \delta$</td>
<td>17.3%</td>
<td>20.1%</td>
<td>18.9%</td>
</tr>
</tbody>
</table>

**TABLE XIV** List of the individual contributions to the theoretical error of the ratio $\Delta \Gamma_s/\Delta M_s$ within the Standard Model and comparison with the values obtained in (Lenz and Nierste, 2011) and (Lenz and Nierste, 2007).

<table>
<thead>
<tr>
<th>$\Delta M_d^{SM}$</th>
<th>$\Delta \Gamma_d^{SM}$</th>
<th>$a_d^{l,SM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central Value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta(B_{R2})$</td>
<td>−−−</td>
<td>14.4%</td>
</tr>
<tr>
<td>$\delta(\mu)$</td>
<td>−−−</td>
<td>13.7%</td>
</tr>
<tr>
<td>$\delta(V_{ub})$</td>
<td>4.9%</td>
<td>4.9%</td>
</tr>
<tr>
<td>$\delta(B_s)$</td>
<td>−−−</td>
<td>4.0%</td>
</tr>
<tr>
<td>$\delta(B_{R0})$</td>
<td>−−−</td>
<td>2.5%</td>
</tr>
<tr>
<td>$\delta(\zeta)$</td>
<td>−−−</td>
<td>1.1%</td>
</tr>
<tr>
<td>$\delta(m_b)$</td>
<td>0.0%</td>
<td>0.8%</td>
</tr>
<tr>
<td>$\delta(B_{R1})$</td>
<td>−−−</td>
<td>0.0%</td>
</tr>
<tr>
<td>$\delta(B_{R2})$</td>
<td>−−−</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\delta(m_s)$</td>
<td>−−−</td>
<td>0.0%</td>
</tr>
<tr>
<td>$\delta(\gamma)$</td>
<td>0.2%</td>
<td>0.2%</td>
</tr>
<tr>
<td>$\delta(\alpha_s)$</td>
<td>0.0%</td>
<td>0.1%</td>
</tr>
<tr>
<td>$\delta(V_{ub})$</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>$\sum \delta$</td>
<td>14.8%</td>
<td>22.7%</td>
</tr>
</tbody>
</table>

**TABLE XV** List of the individual contributions to the theoretical error of the semileptonic CP asymmetries $a_d^{l}$ within the Standard Model and comparison with the values obtained in (Lenz and Nierste, 2011) and (Lenz and Nierste, 2007).

<table>
<thead>
<tr>
<th>$\Delta M_d^{SM}$</th>
<th>$\Delta \Gamma_d^{SM}$</th>
<th>$a_d^{l,SM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central Value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta(B_{R2})$</td>
<td>−−−</td>
<td>9.5%</td>
</tr>
<tr>
<td>$\delta(\mu)$</td>
<td>−−−</td>
<td>9.5%</td>
</tr>
<tr>
<td>$\delta(V_{ub})$</td>
<td>5.0%</td>
<td>11.6%</td>
</tr>
<tr>
<td>$\delta(\zeta)$</td>
<td>−−−</td>
<td>7.9%</td>
</tr>
<tr>
<td>$\delta(B_{R1})$</td>
<td>1.1%</td>
<td>1.2%</td>
</tr>
<tr>
<td>$\delta(m_b)$</td>
<td>1.0%</td>
<td>2.0%</td>
</tr>
<tr>
<td>$\delta(B_{R2})$</td>
<td>0.5%</td>
<td>0.2%</td>
</tr>
<tr>
<td>$\delta(B_s)$</td>
<td>0.3%</td>
<td>0.6%</td>
</tr>
<tr>
<td>$\delta(B_{R0})$</td>
<td>0.2%</td>
<td>0.3%</td>
</tr>
<tr>
<td>$\delta(m_s)$</td>
<td>0.1%</td>
<td>0.1%</td>
</tr>
<tr>
<td>$\delta(\gamma)$</td>
<td>0.1%</td>
<td>0.0%</td>
</tr>
<tr>
<td>$\delta(\alpha_s)$</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>$\delta(V_{ub})$</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>$\sum \delta$</td>
<td>12.2%</td>
<td>17.3%</td>
</tr>
</tbody>
</table>

**TABLE XVI** List of the individual contributions to the theoretical error of the mixing quantities $\Delta M_d$, $\Delta \Gamma_d$ and $a_d^{l,SM}$ in the $B_d$-sector.


Aaij, R., et al. (LHCb) (2012f), “Measurements of the branching fractions and CP asymmetries of \(B^\pm \rightarrow J/\psi \pi^\pm\) and \(B^{\pm} \rightarrow \psi(2S)\pi^{\pm}\) decays,\” Phys. Rev. **D85**, 091105, arXiv:1203.3592 [hep-ex].


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De Bruyn, Kristof, and Robert Fleischer (2015), “A Roadmap to Control Penguin Effects in $B_{d,s}^0 \to J/\psi K^\pm$ and $B_{s}^0 \to J/\psi \phi$,” JHEP 03, 145, arXiv:1412.6834 [hep-ph].


Fleischer, Robert, Robert Kneegjens, and Giulia Ricciardi (2011b), “Exploring CP Violation and $\eta, \eta'$ Mixing with the $B_{s,d} \rightarrow J/\psi \eta^/$ and $B_{s,d} \rightarrow J/\psi \eta'$ Systems,” Eur. Phys. J. C71, 1798, arXiv:1110.5490 [hep-ph].


