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Abstract

Phonons are important carriers of energy and information in many cryogenic devices used for quantum information science and in fundamental physics experiments such as dark matter detectors. In these systems phonon behaviors can be dominated by interfaces and their atomic structures; hence, there is increasing demand for a more detailed understanding of interfacial phonon transport in relevant material systems. Previous studies have focused on understanding thermal transport over the entire phonon spectrum at and above room temperature. At ultralow temperatures, however, knowledge is missing regarding athermal phonon behavior due to the challenge in modeling the extreme conditions in micro-scale, heterogeneous cryogenic systems, as well as extracting single-phonon information from a large ensemble. In this work, we delineate the effects of interfacial atomic structures on phonon transport using a combination of classical molecular dynamics (MD) and phonon wave packet simulations, to illustrate the consistency and differences between the ensemble- and single-phonon dynamics. We consider three single-crystal Si surface reconstructions — (1×1) , $(\sqrt{3} \times \sqrt{3})$ and (7×7) — and model both experimentally observed Si(1 × 1)/Al interfaces and hypothesized Si($\sqrt{3} \times \sqrt{3}$)/Al and Si(7 × 7)/Al interfaces. The overall interfacial thermal conductance calculated from non-equilibrium MD shows that for the $Si(1 \times 1)/Al$ system, the presence of Al twin boundaries can hinder phonon transport and reduce thermal conductance by 2–12% relative to single-crystal Al; whereas the Si $(\sqrt{3} \times \sqrt{3})$ and (7×7) reconstructions can enhance it by 6-19%. Normal mode decomposition reveals that both the increase and decrease in conductance are related to inelastic phonon scattering. Single-phonon wave packet simulations predict phonon transport properties consistent with non-equilibrium MD, while further suggesting that phonon polarization conversion is significant even when elastic transmission dominates, and that the interfacial structures have anisotropic impacts on atomic vibrations along different lattice directions. Our findings suggest avenues for achieving selective phonon transport via controlling interfacial structures of materials using atomically precise fabrication techniques, and that the phonon wave packet formalism is a potentially powerful method for developing a detailed understanding of non-equilibrium phenomena in the low-temperature limit.

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12 I. INTRODUCTION

Progress in quantum information science (QIS) [1, 2] relies on improved understanding quantum excitations as information carriers for precision measurements and as energy carriers that transfer heat, thus enabling the achievement and maintenance of ultra-low working temperatures for devices [3, 4]. Phonons are important heat carriers in all solidtrastate materials, can be used as probes of material structures and properties, and also function as important signal transducers in high energy physics (HEP) experiments. To successfully utilize phonons in QIS and HEP devices for measurement and heat dissipation, we need to understand phonon scattering mechanisms, especially their interactions with interfaces and defects which can dominate phonon transport more than the properties of the bulk materials at the nanoscale [5–8].

The majority of studies of phonons have highlighted their role as heat carriers in thermal 23 ²⁴ transport, usually at and above room temperature [9]. According to the kinetic theory, 25 the thermal conductivity $k = 1/3 \cdot C_v v_g \lambda$ depends on the phonon heat capacity C_v , group $_{26}$ velocity v_g , and mean free path $\lambda = v_g \tau$ (and thus also the phonon scattering rate $1/\tau$). ²⁷ Matthiessen's rule suggests that phonon transport depends on the combination of all scat-²⁸ tering mechanisms, including phonon-phonon scattering in bulk phase single-crystal materi- $_{29}$ als [10-12], phonon scattering with inhomogeneities including defects and oxidation [13-15], 30 and phonon scattering with interfaces, including those between different materials such as $_{31}$ heterojunctions [16, 17] and those in the same material such as grain boundaries between ³² different crystallographic orientations [18, 19]. Direct experimental delineation of the rela-³³ tionship between phonon transport and the aforementioned factors in realistic systems is ³⁴ complicated by the presence of multiple types of inhomogeneities that can couple to each ³⁵ other in a complex nonlinear manner, as well as difficulties in characterization of defects ₃₆ and interfaces that are buried inside the systems. Therefore, theoretical calculations and 37 atomistic simulations can provide essential insights that complement experimental obser-³⁸ vations. Specifically, experiments have shown that while inhomogeneities generally reduce ³⁹ thermal conductivity in bulk materials [20], at the interface they can enhance the thermal 40 conductance in certain configurations [21, 22]. Findings from our prior modeling studies ⁴¹ suggested that both phenomena are due to the same phonon scattering mechanism, which ⁴² reduces the phonon mean free path in homogeneous materials but can bridge the phonon ⁴³ mismatch between different materials [23].

While the development in QIS devices is predicated by advanced manufacturing tech-44 45 niques [24] and our ability to approach ultra-low temperatures [25], correspondingly there ⁴⁶ is also increasing demand on knowledge of quantum-level particle physics under extreme ⁴⁷ conditions. For example, in Super Cryogenic Dark Matter Search (SuperCDMS) detectors, ⁴⁸ the initial energy deposition in semiconductors will convert to lattice vibrations, which prop-⁴⁹ agate as phonons and eventually cross an interface into metals to break the cooper-pairs and ⁵⁰ get detected [3]. Understanding the propagation and scattering of these athermal phonons ⁵¹ generated during this process is crucial for interpretation of the measurements [26–29]. Ad-⁵² vanced fabrication techniques have also made possible the realization of atomically precise ⁵³ micro/nanostructures [30, 31], which further emphasizes single-particle behaviors that are ⁵⁴ susceptible to local atomic structures or surface conditions that are precisely controlled. ⁵⁵ Therefore, understanding how structural and chemical inhomogeneities at the atomic level ⁵⁶ affect individual-phonon transport across interfaces is essential for the design of new mate-57 rials and devices. However, in previous studies usually it is the overall phonon transport ⁵⁸ properties, such as the interfacial thermal conductance, that are measured or predicted, ⁵⁹ while the detailed single-phonon scattering mechanism with atomic-level material struc-60 tures is missing. The conditions at or above room temperature where phonons follow the ⁶¹ Bose-Einstein distribution also deviate from the QIS domain where athermal phonons dom-⁶² inate, leaving the phonon transport mechanisms at ultra-low temperatures generally less 63 well-understood.

In this study, we focus on one heterojunction that is widely used for QIS and HEP systems: the Si/Al interface, and investigate the effect of its atomic structures on phonon transport and the underpinning phonon scattering mechanism. Si exhibits several well-known surface reconstructions that introduce many interesting properties such as unique electron-surface phonon coupling [32, 33]. However, the effects of these reconstructions on interfacial thermal transport in a heterojunction is not known. Recent developments in molecular beam re epitaxy (MBE) have made possible the fabrication of atomically perfect Si/Al interfaces ri grown on Si substrates with different surface reconstructions [30, 31], which sets the realistic reference for our simulations. We consider a total of four different interfaces, which ra are summarized in Table. I. To determine the commonalities and differences between transport mechanisms for phonons from the entire spectrum and individual athermal phonons,

Name	Si surface	Al type	Note
$\rm Si(1 \times 1)$ -Al _p	(1×1)	Perfect single crystal	MBE grown
$Si(1\times 1)\text{-}Al_t$	(1×1)	Twin boundaries	MBE grown
$Si(\sqrt{3}\times\sqrt{3})\text{-}Al_p$	$(\sqrt{3} \times \sqrt{3})$	Perfect single crystal	Hypothesized
$\rm Si(7 \times 7)\text{-}Al_p$	(7×7)	Perfect single crystal	Hypothesized

TABLE I: The four types of Si/Al interfaces considered in our study. "p" stands for "perfect" as in "perfect single crystal", and "t" stands for "twin" as in "twin boundary".

⁷⁶ we conduct both classical non-equilibrium molecular dynamics (NEMD) and phonon wave 77 packet simulations. NEMD is a powerful technique to understand thermal transport inte-⁷⁸ grated over all atomic-vibrational modes [34], which intrinsically includes all the phonon ⁷⁹ scattering mechanisms, both elastic and inelastic, and predicts reasonable thermal proper-⁸⁰ ties with well-developed interatomic potentials. The phonon occupation in classical MD ⁸¹ deviates from quantum-mechanical theory, but can be refined by corrections for comparison with experiments [23, 35]. We first calculate and compare the overall interfacial thermal 82 ⁸³ conductance at 100 and 200 K for all four interfaces. It is noteworthy that NEMD requires ⁸⁴ a notable temperature gradient established across the simulation system, which makes it ⁸⁵ impractical to conduct such simulations in the milli-Kelvin range. Therefore, we choose ⁸⁶ two temperatures to illustrate the trend of thermal transport. In addition, NEMD can only ⁸⁷ predict overall thermal properties that are a summation of all phonons; thus, we also apply ⁸⁸ normal mode decomposition (NMD) to delineate the spectral heat flow for an overview of ⁸⁹ the phonon transport mechanisms. To approach the ultra-low temperature limit and the ⁹⁰ corresponding details of single-phonon behaviors, we further apply the phonon wave packet ⁹¹ approach where atoms remain motionless except when excited by the wave packet. We ana-⁹² lyze the phonon scattering mechanism through the resulting atomic-vibrational spectra and ⁹³ direction-resolved atomic kinetic energies.

94 II. METHODOLOGY

⁹⁵ All the molecular dynamics (MD) simulations are performed using the LAMMPS pack-⁹⁶ age [36], which requires classical interatomic-pair potentials to describe interactions among

97 all atoms. Several pair potentials were evaluated for Si and Si/Al interfaces, such as the ⁹⁸ Tersoff potential [37] and a recently-developed angular dependence potential (ADP) [38]. ⁹⁹ However, the Tersoff potential cannot describe Si-Al interactions, while the ADP cannot correctly reproduce the Si surface reconstructions. The modified embedded atomic method 100 (MEAM) potential of Jelinek et al. [39] satisfies both our demands of correctly reproducing 101 structures of the Si surface reconstructions (as is demonstrated in the Results and Discussion 102 section) and being able to describe Si-Al interactions; thus, it is used for all pair-wise and 103 many-body atomic interactions. Our previous study showed that the bulk Si phonon spec-104 trum is stretched, and the cutoff frequency of 26 THz is over-predicted by 10 THz [23] [40]. 105 This leads to an overestimated population and thermal conductance of high-frequency Si 106 phonons. Even so, the MEAM potential can reproduce the surface structures and formation 107 energies of Si and Al with excellent accuracy, which are important for modeling such atom-108 ically precise interfaces. Therefore, we select the MEAM potential; its phonon-spectrum 109 overestimation can be quantified and corrected using the approach described in our previous 110 study [23], which considers deviations due to both the stretched phonon spectrum and the 111 equipartition of energy phonon distribution [41]. The current study focuses on investigat-¹¹³ ing the effect of interfacial atomic structures on phonon transport using both NEMD and ¹¹⁴ wave packet simulations; therefore, for consistency we analyze the results without applying ¹¹⁵ further adjustments.

A. Non-equilibirium molecular dynamics simulation and normal mode decompo-117 sition

A schematic of the system domain used for NEMD computation is shown in Fig. 1 a. The structure is periodic in all three dimensions. The single-crystal Si and Al blocks are of the and crystallographic orientations and joined through their (111) planes, corresponding to the heterojunction grown by McSkimming *et. al.* [30]. To facilitate building of the atomic structures, we define orthorhombic supercells of Si (12 atoms, Fig. 1 b) and Al (6 atoms, Fig. 1 c) with their three orthogonal lattice vectors along [111] ($a_{Si} = 9.41$ Å, $a_{A1} = 6.98$ Å), [$\overline{1}10$] ($b_{Si} = 3.84$ Å, $b_{A1} = 2.85$ Å), and [$\overline{1}\overline{1}2$] ($c_{Si} = 6.65$ Å, $c_{A1} = 4.93$ Å). The Si block is 24-unit cells long while the Al block is 32-unit cells long, forming a 45 nmtice long system with two materials of approximately the same length. The cross-sectional area, ¹²⁷ which is parallel to the interface, is specific for each of the four interfaces. For the two types ¹²⁸ of experimentally obtained Si/Al interfaces: Si(1 \times 1)-Al_p ("p" stands for "perfect" as in "perfect single crystal"), and Si(1 \times 1)-Alt ("t" stands for "twin" as in "twin boundary") 129 with (112) Al twin boundaries, the cross-section of our simulation box consists of 12×6 Si 130 $_{131}$ or 16×8 Al unit cells so that the interfacial lattice mismatch is small (1.2%). In the case of $_{132}$ Si(1 \times 1)-Al_t, we construct two twin boundaries in the simulation box as they must appear ¹³³ in pairs under periodic boundary conditions, resulting in a density of one twin boundary ¹³⁴ per 20.0 Å. We also consider two additional types of Si/Al interfaces. Although the Si $_{135}$ ($\sqrt{3} \times \sqrt{3}$) and (7 × 7) surface reconstructions are not preserved during the MBE growth ¹³⁶ and they eventually become (1×1) [30], they may be preserved by physically joining pre-¹³⁷ fabricated Si and Al (111) surfaces, and provide a useful hypothetical comparison. For ¹³⁸ Si($\sqrt{3} \times \sqrt{3}$)-Al_p, we use the same simulation-box cross-section as employed for the (1 × 1) ¹³⁹ reconstruction, because it can represent the surface reconstruction with two adatoms in every ¹⁴⁰ 3×2 boundary unit cells. For Si(7×7)-Al_p, a much larger cross-section consisting of 21×21 $_{141}$ Si or 28×28 Al unit cells is used to satisfy both the periodicity of the surface reconstruction $(7 \times 7 \text{ Si unit cells})$ and the Si/Al interfacial lattice mismatch (based on a three-to-four ¹⁴³ ratio). The cross-sections of our modeling systems are large enough to eliminate the finite ¹⁴⁴ size effect, as they are at least four times the 4×4 Si (100) unit cell area demonstrated ¹⁴⁵ to be sufficient by Landry and co-workers [42]. The length dependence of the interfacial ¹⁴⁶ thermal transport still exists, but extension to other lengths can readily be made using the ¹⁴⁷ linear extrapolation method introduced by Schelling and coworkers [43].

Thermal rectification has been demonstrated to exist at interfaces between materials of asymmetric thermal conductivities [44–46], as is true for our case. Here, we only consider the overall heat transfer from Si to Al as it more closely represents the phonon excitations and transport in cryogenic devices such as transition-edge sensors and quantum bits, which the background and focus for our study. Initially the system is relaxed in a constant number of atoms, pressure, and temperature (NPT) ensemble for 100 ps under zero pressure the initial temperature (100 or 200 K). Then atoms in a region of 15 Å at both ends of the simulation domain are fixed to break periodicity in the heat transfer direction, and heat is inserted into a region of 15 Å adjacent to the fixed end of Si while it is extracted from a region of the same size next to the fixed end of Al, forming a constant heat flux J of the constant 150 W/m² which is consistently used for every simulation. It is noteworthy that the ¹⁵⁹ heat flux is large in order to generate distinct temperature differences across the interface ¹⁶⁰ for more facile analysis, consistent with our previous studies [23, 47]. The simulation then ¹⁶¹ proceeds in a constant number of atoms, volume, and energy (NVE) ensemble for 0.8 ns ¹⁶² to reach steady state, and then another 6 ns for production. Temperature is averaged and ¹⁶³ recorded every 10 Å along the heat-transfer direction during the production run, from which ¹⁶⁴ the interfacial thermal transport efficiency is evaluated.

The NMD is based on the time domain normal mode analysis initially developed by Zhou and coworkers [48]. Here we apply the modified version as presented in our previous study [23]. The MD expression of heat flux (unit: W/m^2) is:

$$J = \left\langle \sum_{ia} (E_{ia} \mathbf{v}_{ia} - \mathbf{S}_{ia} \mathbf{v}_{ia}) \right\rangle / V, \tag{1}$$

where *ia* refers to the atomic index, E is the total energy (potential and kinetic), **S** is the Cauchy stress tensor and V is the total volume [49–51]. Projection of the heat flux onto each phonon mode is then accomplished by expressing the atomic velocities using the time derivative of phonon normal mode amplitude Φ :

$$\mathbf{v}_{jl,\mathbf{k},\nu,t} = \frac{1}{(Nm_j)^{0.5}} \mathbf{e}_{j,\mathbf{k},\nu} \exp(i\mathbf{k}\mathbf{r}_{jl}) \dot{\Phi}_{jl,\mathbf{k},\nu,t}.$$
(2)

where ν refers to the phonon branch, **k** and **r** are the trajectories in the reciprocal and real spaces respectively, j and l refer to the jth basis atom in the lth unit cell, m is the atomic mass, N is the total number of unit cells, t is time and **e** is the phonon eigenvector. Compared with the original NMD [48] which uses the convoluted $\Phi_{\mathbf{k},\nu,t}$, our approach is specifically adjusted for applications to non-periodic systems by further projecting Φ onto each atom:

$$\Phi_{jl,\mathbf{k},\nu,t} = \left(\frac{m_j}{N}\right)^{0.5} \mathbf{e}_{j,\mathbf{k},\nu}^* \exp(-i\mathbf{k}\mathbf{r}_{jl})\mathbf{u}_{jl,t},\tag{3}$$

 $_{165}$ where **u** is the atomic displacement and e^* is the complex conjugate of the corresponding $_{166}$ phonon eigenvector.

¹⁶⁷ To make clear distinctions among different cases, we apply NMD to interfacial atoms ¹⁶⁸ and choose only the surface layers of Si or Al atoms as the boundary groups for analyses. ¹⁶⁹ Each boundary group is treated as a single supercell evaluated at the Γ point only, where ¹⁷⁰ the phonon eigenvectors are acquired using the PHONON package [52] in LAMMPS for ¹⁷¹ consistency with NEMD. For Al, this is always the first layer of boundary Al atoms (two $_{172}$ atoms per boundary unit cell). For the Si (1×1) reconstruction, this is the first two layers 173 of boundary atoms — four atoms per boundary unit cell, marked by red-dashed boxes in ¹⁷⁴ Figs. 2 a & b. The Si $(\sqrt{3} \times \sqrt{3})$ reconstruction has adatoms in addition to the boundary 175 group of the (1×1) reconstruction, resulting in 26 atoms in every 3×2 boundary unit cells (red-dashed box in Fig. 2 c). The Si (7×7) reconstruction is more complicated. The dimer-176 adatom stacking fault (DAS) surface structure [53, 54] has a period of 7×7 Si unit cells, 177 and the cross-section of our simulation domain contains nine of these repeatable structures. 178 Still, the boundary group of the Si (1×1) reconstruction is present in the DAS structure, 179 with some atoms removed or rearranged. Therefore, we choose all the atoms in the first 180 two boundary layers plus those adatoms as the boundary group, resulting in 204 atoms per 181 7×7 Si boundary unit cells or 1836 atoms in the entire simulation domain (red-dashed box 182 ¹⁸³ in Fig. 2 d). The trajectories, energies (potential and kinetic), and Cauchy stress tensors of ¹⁸⁴ the boundary group atoms are recorded during the NEMD production run, which are then ¹⁸⁵ used in NMD to calculate the spectral phonon heat flow.

186 B. Phonon wave packet simulations

The phonon wave packet methodology is especially suited to investigate single-phonon propagation and scattering mechanisms, as all atoms remain motionless except for those in the phonon wave packet. Such a simulation method provides a good approximation of the ultra-low temperature regime in which the equilibrium thermal phonon population is exponentially suppressed. Here we apply the wave packet approach initially developed by Schelling *et. al.* [55, 56]; the atoms are displaced according to:

$$\mathbf{u}_{jl} = A \cdot \mathbf{e}_{j\nu}(\mathbf{k}_0) \cdot \exp[i\mathbf{k}_0(\mathbf{x}_l - \mathbf{x}_0) - i\omega_{\nu}t] \cdot \exp[-\eta^2(\mathbf{x}_l - \mathbf{x}_0 - \mathbf{v}_{g\nu}t)^2].$$
(4)

Here, \mathbf{u}_{jl} represents the displacement of basis atom j in the unit cell l located at \mathbf{x}_l , and $\mathbf{e}_{j\nu}(\mathbf{k}_0)$ is the eigenvector of the phonon with a wave vector of \mathbf{k}_0 from branch ν projected onto the same atom. The eigenvectors are also evaluated using the PHONON package for consistency with NEMD and NMD, as are the phonon frequency ω and group velocity \mathbf{v}_g . The wave packet is localized in space around \mathbf{x}_0 with a spatial extent of $\sim 1/\eta$. The initial atomic velocity can be found through the time derivative of Eq. (4):

$$\mathbf{v}_{jl} = \frac{\partial \mathbf{u}_{jl}}{\partial t}|_{t=0} = \mathbf{u}_{jl}[-i\omega_{\nu} + 2\eta^2(\mathbf{x}_l - \mathbf{x}_0)\mathbf{v}_{g\nu}].$$
(5)

187 Here we only consider phonons that are perpendicularly incident on the interface — ef-188 fectively one-dimensional (1D) wave packet simulations, as \mathbf{k} , \mathbf{x} and \mathbf{v}_g are always along the same direction (Si [111]). We select η so that the wave packet is 20-Si unit cells long 189 (~18.8 nm) along x. This corresponds to a spread of $0.1(2\pi/a_{Si})$ around \mathbf{k}_0 in the reciprocal 190 space. The amplitude A is selected so that the average energy E_{avg} inside the wave packet 191 is 0.192, 0.384 or 3.84 meV/atom. The three different energies are used to represent the 192 same phonons but of different populations, which can be correlated with their conditions 193 at different temperatures. According to the equipartition of energy distribution [41], at any 194 ¹⁹⁵ given temperature energy is evenly distributed among every phonon mode. Therefore, wave packet simulations of different phonon modes but with the same E_{avg} are directly compara-196 ble as they correctly represent the phonon distribution at the same temperature. The 1D 197 wave packet simulations are performed for all four interfaces. With perpendicular incidence 198 the cross-section of the system becomes irrelevant. Therefore, we reduce the cross-section to 199 the smallest-possible area, while expanding the length of the system to allow more space for 200 ²⁰¹ phonon propagation. Each interface consists of a 60-unit cell long Si block and an 80-unit cell long Al block (~110 nm in total). The Si(1 × 1)-Al_p and Si($\sqrt{3} \times \sqrt{3}$)-Al_p cases have cross-sections of 3×3 Si or 4×4 Al unit cells, while that of the Si(1×1)-Al_t case is 3×6 Si or 4×8 Al unit cells to ensure consistent density of twin boundaries with the NEMD simulations. The Si(7 \times 7)-Al_p interface has the same cross-section as the NEMD simula-²⁰⁶ tion, which is already the smallest possible. The atomic structures are first fully relaxed 207 at 10 mK, with periodic boundary conditions in all three dimensions and regions of 15 Å ²⁰⁸ fixed at both ends, to generate an optimized system at an ultra-low temperature. We select ²⁰⁹ a series of phonon modes along the Γ – L symmetry line based on the spectral heat flow ²¹⁰ and displace the atoms accordingly, which will be discussed in the Results and discussion section. It should be noted that each simulation illustrates one phonon mode perpendicu-211 larly scattering with one interface. Initially the phonon wave packet is placed at a specific 212 distance from the interface, which is chosen according to its modal group velocity, so that it 213 needs to travel for at least 2.5 ps before scattering at the interface. The simulation proceeds 214 ²¹⁵ for at least 15 ps to ensure that the transmitted and reflected phonons have sufficiently ²¹⁶ propagated away from the interface and can be separately identified. Some long-wavelength ²¹⁷ phonons will already be reflected a second time at the fixed ends during this period, but ²¹⁸ this part of the simulation is not used for analysis.



FIG. 1: a) Illustration of the system domain for simulation. Heat is inserted in the source region at the left end of Si and is extracted from the sink region at the right end of Al; the

temperature profile across the system provides an evaluation of the heat transfer performance. b) Orthorhombic unit cell of Si consisting of 12 atoms, with its three vectors along [111], [110] and [112], respectively. This unit cell serves as the monolithic building block to generate the Si blocks in our simulations by repeating itself along the three lattice vectors. c) Same as b) but for Al with 6 atoms.

219 III. RESULTS AND DISCUSSION

220 A. Structures of Si/Al interfaces

An important test of classical MD simulations to ensure the classical force field is working properly is the resultant atomic structures of the materials. The relaxed interfacial structure (Fig. 2 a) of Si(1 × 1)-Al_p matches the experimental transmission electron microscope image in Ref. 30 perfectly, while that of Si(1 × 1)-Al_t (Fig. 2 b) shows more deformation. This originates from the substantially larger density of Al twin boundaries in our simulation (1/20.0 Å), which greatly amplifies their influence on atomic structures as well as phonon transport. However, in the simulated and experimental systems, the interfacial structures are qualitatively the same; therefore, we expect exaggerated yet consistent impact from the the simulated phonon transport. For the Si($\sqrt{3} \times \sqrt{3}$)-Al_p (Fig. 2 c) and Si(7 × 7)-Al_p (Fig. 2 d) cases, we first test the surface structures with a Si slab. We find that the MEAM potential can accurately reproduce the Si surface reconstructions. When the two hypothesized interfaces are formed (supposedly by pressing the pre-fabricated surfaces together), we find that the Si surface reconstructions are preserved in our simulations. Furthermore, the Al boundary atoms stay in their intrinsic lattice points. There is no obvious deformation or intermixing induced by the Si adatoms and the interfacial voids caused by them, which is consistent with the relatively high point-substitution energy (7.67 eV) for an Al atom in a Si host lattice as suggested by the MEAM potential [39].

B. Interfacial thermal conductance from NEMD simulations

The interfacial thermal conductance G_{int} (Table. II) is calculated from the temperature 239 profiles using the same method (Fig. 3) as in our previous study [23], where the linear 240 temperature profile from the bulk phases of the materials are extrapolated to the interface 241 to calibrate the temperature difference ΔT . The change in interfacial thermal conductance 242 $_{243} \Delta G_{\text{int}}$ is calculated using the Si(1 × 1)-Al_p case at the same temperature as a reference. The $_{244}$ Al twin boundaries are shown to reduce $G_{\rm int}$ compared with perfect single-crystal Al. A ²⁴⁵ straightforward understanding is that the disorder introduced by the twin boundaries are hindering phonon transmission, while the minimum interfacial intermixing is not sufficient 246 to enhance phonon transport [57]. The reduction at 200 K (-2.7%) is smaller than at 100 K 247 (-12.6%) which indicates that inelastic phonon transport, which becomes greater at higher 248 249 temperatures, is likely making a significant contribution. In contrast, $Si(\sqrt{3} \times \sqrt{3})$ -Al_p and $_{250}$ Si(7 × 7)-Al_p surprisingly show increased G_{int} despite the interfacial voids that reduce the ²⁵¹ effective contact area. We show below that this is mainly due to enhanced transport from ²⁵² Si phonons beyond the Al phonon cutoff frequency.

²⁵³ C. Spectral phonon transport from NMD

To further understand the phonon transport mechanism, we perform NMD on the boundary group of Si and Al atoms as selected in the Methodology section above (Fig. 2). The results are shown in Fig. 4 along with the corresponding phonon density of states (PDOS).



FIG. 2: Atomic structures of the Si(1 × 1)-Al_p (a), Si(1 × 1)-Al_t (b), Si(√3 × √3)-Al_p (c) and Si(7 × 7)-Al_p (e) interfaces obtained from our simulations. The red-dashed boxes indicate the boundary group of atoms selected for NMD analyses. d) The view of boundary group atoms of the Si(√3 × √3) block along the [111] direction. Adatoms of the Si (√3 × √3) surface reconstruction are highlighted. f) The same as d), but for the Si(7 × 7) block. The structures of the Si(1 × 1)-Al_p and Si(1 × 1)-Al_t interfaces match perfectly with the experimental observations in Ref. 30. The Si surface reconstructions are preserved at the hypothesized Si(√3 × √3)-Al_p and Si(7 × 7)-Al_p interfaces.

We apply the auto-correlation technique [58, 59] to calculate the PDOS:

$$D_{\alpha\beta}(\omega) = \int_0^\tau \Gamma_{\alpha\beta}(t) \cos(\omega t) dt, \tag{6}$$

where

$$\Gamma_{\alpha\beta}(t) = \frac{\left\langle \sum_{i_{\alpha\beta}=1}^{N_{\alpha\beta}} \mathbf{v}_{i_{\alpha\beta}}(t) \mathbf{v}_{i_{\alpha\beta}}(0) \right\rangle}{\left\langle \sum_{i_{\alpha\beta}=1}^{N_{\alpha\beta}} \mathbf{v}_{i_{\alpha\beta}}(0) \mathbf{v}_{i_{\alpha\beta}}(0) \right\rangle}.$$
(7)



FIG. 3: Temperature profile of $Si(1 \times 1)$ -Al_p at 100 K from the NEMD simulation. The interfacial thermal conductance G_{int} is calculated from the difference in temperature ΔT across the Si/Al interface.

TABLE II: G_{int} of the considered interfaces at 100 and 200 K. ΔG_{int} is the percentage difference relative to the Si(1 × 1)-Al_p result at the same temperature.

Temperature	Interface	$G_{\rm int}~({\rm GW/m^2K})$	ΔG_{int}	
	$\rm Si(1 \times 1)$ - $\rm Al_p$	0.2021	_	
100 IZ	$\rm{Si}(1 \times 1)\text{-}\rm{Al}_t$	0.1767	-12.6%	
100 K	${\rm Si}(\sqrt{3} \times \sqrt{3}) - {\rm Al}_p$	0.2407	18.8%	
	$\rm Si(7 imes7) ext{-}Al_p$	0.2359	16.7%	
	$\rm Si(1 \times 1)$ -Al _p	0.2502	_	
000 IZ	$\rm{Si}(1 \times 1)$ - \rm{Al}_t	0.2435	-2.7%	
200 K	${\rm Si}(\sqrt{3} \times \sqrt{3}) - {\rm Al}_p$	0.2985	19.3%	
	$\rm Si(7 imes7) ext{-}Al_p$	0.2651	6.0%	

Here $D(\omega)$ refers to the PDOS, t is time, ω is phonon frequency, i is the atomic index, α ²⁵⁵ is the atom type, β is the region to which the atom belongs, $N_{\alpha\beta}$ is the total number of ²⁵⁶ α atoms in region β , and \mathbf{v} is the atomic velocity. Eq. (6) is evaluated over a period of ²⁵⁷ time τ , which affects the frequency resolution of the spectrum and is ideally infinity. In our ²⁵⁸ simulation we choose τ to be 25 ps, which equals a time step of 0.125 fs multiplied by 2×10^5 ²⁵⁹ steps.

In the Si(1 \times 1)-Al_p case (Fig. 4 a), a plateau in the J accumulation curve appears 260 ²⁶¹ for phonons from 7.8–10.9 THz, indicating that phonons in this range — high frequency longitudinal acoustic (LA) and low-frequency longitudinal optical (LO) Si phonons — barely 262 transfer heat across the interface. The Al cutoff frequency overlaps with this plateau region, 263 beyond which the high-frequency Si phonons contribute more to G_{int} at 200 K (50.9%) than 264 100 K (42.9%), consistent with inelastic phonon scatterings being more significant at higher 265 temperatures. The PDOS plot (Fig. 4 c) reveals that there is no available phonon state 266 in the Si boundaries within the plateau region. In bulk Si the LA phonons span from 0– 267 16 THz and the LO phonons span from 16–26 THz, where they have modes with overlapping 268 frequency at the boundaries of the first Brillouin zone. Consequently, there are available 269 phonons across the entire 0-26 THz spectrum. However, in the boundary atomic layers 270 with the Si (1×1) surface reconstruction, a gap of 7.8–10.9 THz opens between LA and 271 $_{272}$ LO phonons, resulting in the plateau in the J accumulation curve. The highest frequency ²⁷³ optical phonons in bulk Si, represented by the peak around 25 THz in the PDOS plot, are ²⁷⁴ also absent in the boundary and correspondingly there are more states within the 11–19 THz ²⁷⁵ range. This explains the negligible spectral heat flow carried by phonons above 23 THz.

The Si(1×1)-Al_t case (Fig. 4 b) shows an overall qualitatively similar curve with Si(1×1)-276 $_{277}$ Al_p. However, low-frequency (< 5 THz) Si phonons below the Al cutoff show an opposite trend with a larger contribution to G_{int} at 200 K (59.7%) than 100 K (49.8%), and their 278 spectral heat flow exceeds that of their Al counterparts at 200 K. This surprising phenomenon 279 indicates inelastic scattering mechanisms that are specific to such interfaces with Al twin 280 boundaries, where Si phonons are spontaneously combining into higher-frequency Al phonons 281 when they cross the interface. We present an interpretation by investigating the elastic 282 ²⁸³ transmission from the perspective of the acoustic mismatch model [60], as illustrated in ²⁸⁴ Fig. 5. When phonons of the same mode are incident on the interface with the same angle $_{285} \alpha$, they will have the same refraction angle β and travel in the same direction after the elastic ²⁸⁶ scattering. However, if the Al is polycrystalline as it is in our case with twin boundaries, ²⁸⁷ the same phonons incident on the grains with different crystallographic orientations will ²⁸⁸ be scattered into different directions (refraction angles $\beta \neq \gamma$). Phonons that are spatially ²⁸⁹ close before the interfacial scattering may quickly collide with each other and combine. ²⁹⁰ Consequently, some of the elastic phonon transmission that occurs across the Si(1 × 1)-²⁹¹ Al_p interface is converted to inelastic transport if twin boundaries are introduced in the Al ²⁹² lattice, because the boundaries can channel phonons into each other. Similar to all inelastic ²⁹³ phonon scatterings, this phonon transport becomes more significant at higher temperatures, ²⁹⁴ which explains the larger contribution from low-frequency Si phonons at 200 K.

The Si($\sqrt{3} \times \sqrt{3}$)-Al_p and Si(7 \times 7)-Al_p cases both show smoother J accumulation 295 ²⁹⁶ curves than the Si(1 \times 1) cases, where the plateau feature is significantly reduced. In the $_{297}$ Si $(\sqrt{3} \times \sqrt{3})$ -Al_p case (Fig. 4 e) the Si phonons carry heat up to 25 THz, in contrast to other ²⁹⁸ cases where the spectral heat flow becomes negligible beyond 22 THz. This is consistent with there being more phonon states available above 23 THz in the Si boundary with the $_{300}$ ($\sqrt{3} \times \sqrt{3}$) surface reconstruction (Fig. 4 g). These observations indicate that both the $_{301}$ ($\sqrt{3} \times \sqrt{3}$) and (7 × 7) surface reconstructions enable more channels for interfacial thermal 302 transport from high-frequency LA and low-frequency LO Si phonons beyond the Al cutoff ³⁰³ frequency, and the $(\sqrt{3} \times \sqrt{3})$ surface reconstruction further extends the additional channels $_{304}$ to high-frequency Si optical phonons. The J accumulation curves at the two temperatures are surprisingly consistent for each of these two cases, indicating that the spectral heat flow is 305 only slightly dependent on temperature and that high-temperature enhanced inelastic trans-306 port is probably negated by the simultaneously increased disorder in the surface adatoms. It is also noteworthy that the classical equipartition of energy distribution overestimates the 308 population of high-frequency phonons, the extent of which becomes greater as temperature 309 decreases. Therefore, the inelastic phonon transport at 100 K is overestimated to a greater 310 ³¹¹ extent than at 200 K. This contributes to the consistent spectral heat flow at both temper-³¹² atures which has been observed in classical MD simulations from previous studies [23, 61], ³¹³ while in reality high-frequency Si phonons are expected to contribute more at 200 K.



FIG. 4: Spectral heat flows J accumulation acquired from NMD for the Si(1 × 1)-Al_p (a),
Si(1 × 1)-Al_t (b), Si(√3 × √3)-Al_p (e), and Si(7 × 7)-Al_p (f) interfaces. The corresponding PDOS of the Si atoms used to calculate J is plotted underneath each case, respectively (c, d, g and h). The dependence of J on temperature is clearly manifested at the Si(1 × 1)-Al_p and Si(1 × 1)-Al_t interfaces, while it is less significant at the Si(√3 × √3)-Al_p and

 $Si(7 \times 7)$ -Al_p interfaces. Plateaus are observed for interfaces with Si (1×1) reconstructions,

and the PDOS reveals that this is caused by absence of phonon states within this frequency range. In contrast, this plateau feature is significantly reduced for interfaces with Si $(\sqrt{3} \times \sqrt{3})$ and (7×7) reconstructions, which show spectral heat flows that are almost temperature-independent. The Si $(\sqrt{3} \times \sqrt{3})$ -Al_p interface shows increment in cumulative Jfor phonons with frequencies higher than 23 THz that is consistent with available phonon states in this frequency range, which are absent at the other three interfaces

states in this frequency range, which are absent at the other three interfaces.

³¹⁴ D. Detailed interfacial scattering mechanism from single-phonon wave packet sim-³¹⁵ ulations

While NMD is illustrative of the general phonon transport over the entire frequency 317 spectrum, it cannot manifest the detailed phonon scattering mechanism. For example, it 318 can tell whether a phonon in a certain frequency range transfers heat or not and how much



FIG. 5: Illustration of phonons of the same mode with the same incidence angle α scattering with the Si(1 × 1)-Al_p (a) and Si(1 × 1)-Al_t (b) interfaces, respectively. The two different grains of polycrystalline Al have different refraction angles β and γ , respectively. The transmitted phonons will collide with each other as their propagation directions are non-parallel.

³¹⁹ it carries, but not what the phonon becomes after it goes across the interface. To explore ³²⁰ the detailed scattering mechanism, we conduct phonon wave packet simulations. Again, we ³²¹ only consider the incidence of Si phonons for consistency with the phonon excitations and transport in cryogenic devices. In addition, only in this way can we consider the scattering 322 of phonons beyond the cutoff frequency of the material they are incident on, as Si has a 323 broader spectrum than Al. Because the bulk phase of all Si blocks in our simulations are 324 constructed by repeating the same Si 12-atom unit cell (Fig. 1 b), we can initiate consistent 325 Si phonon wave packets across all four interfaces. We select specific phonons based on the 326 ³²⁷ spectral heat flow from NMD, with eight phonons chosen from the four branches — LA, LO, transverse acoustic (TA) and transverse optical (TO) — that cover all critical ranges: the 328 low-frequency range where f < 5 THz (LA-I, TA-I), the range below the Al cutoff frequency 329 to which the Si(1 \times 1)-Al_t case shows unique Si phonon inelastic scattering (LA-II, TA-II), 330 the plateau in the $Si(1 \times 1)$ -Al_p case (LA-III), the slope after the plateau (LO-I, TO-I), and 331 the highest-frequency optical phonon available (LO-II). All these phonons have the same 332 reduced momentum of $\mathbf{k} = (0.25, 0, 0)$ in the folded first-Brillouin zone determined by the 333 ³³⁴ 12-atom unit cell. It should be noted that due to the specific phonon dispersion along $\Gamma - L$, ³³⁵ not all branches are available in every range. For example, there is no transverse phonon ³³⁶ in the same range with LA-III; so we only select phonons that are representative of their ³³⁷ frequency range rather than trying to cover all possibilities. It is also noteworthy that many



FIG. 6: a) Illustrations of the TA-I phonon ($E_{avg}=0.192 \text{ meV}$) wave packet propagation and scattering with the Si(1 × 1)-Al_p interface, represented by snapshots of the atomic kinetic energy at t=0, 5.675 and 8.250 ps. The kinetic energy at each position is averaged over all local atoms. b) The corresponding atomic-vibrational spectra of the initial Si phonons, reflected Si phonons, and transmitted Al phonons.

338 of these phonon modes are not pure longitudinal or transverse modes, and our classification ³³⁹ is determined by examining the phonon eigenvectors according to the approach in Ref. 62. The first 2.5 ps in the simulation before the phonon scatters with the interface is used to 340 ³⁴¹ confirm the coherence of the phonons, namely that the phonon does not scatter into other phonons during propagation. We apply Eq. (6) again to calculate the atomic-vibrational 342 ³⁴³ spectrum. For a wave packet simulation, the integral can only be performed over certain periods of time to correctly represent the phonons of interest. In our simulation a time step 344 equals 0.125 fs, thus the first 2×10^4 steps representing $\tau = 2.5$ ps are used to characterize the 345 initial phonon propagation. We find that the frequencies of all phonons remain consistent 346 during propagation at all three energies during this period, except for LA-I which breaks 347 down when $E_{avg}=3.84$ meV/atom and the corresponding result is not considered. 348

An example of the wave packet simulation and the corresponding atomic-vibrational spectra are shown in Fig. 6, illustrating the TA-I phonon with $E_{avg}=0.192$ meV/atom across the Si(1 × 1)-Al_p interface. The frequency and energy transmission of all eight selected phonons across the four interfaces are listed in Table. III. Generally the results are consistent with NMD. For example, LA-III phonons have negligible transmission across the Si(1 × 1)-Al_p interface as the spectral heat flow forms a plateau at the frequency f=8.61 THz, while it has increased transmission for the Si($\sqrt{3} \times \sqrt{3}$)-Al_p and Si(7 × 7)-Al_p interfaces as the plateau is absent. The wave packet simulation reveals that the LA-III phonons are almost the Si(1 × 1)-Al_p interface by the Si(1 × 1)-Al_p interface, but not the others. LA-III phonons also $_{358}$ show a surprisingly increased transmission for the Si(1 \times 1)-Al_t interface, which will be $_{359}$ discussed later. When E_{avg} takes the two smaller values indicative of relatively small phonon ³⁶⁰ population at low temperatures, elastic transmission dominates, and the LO-I phonon can $_{361}$ hardly transfer energy across any interface. However, when E_{avg} takes the largest value, ³⁶² the LO-I phonon at elevated temperatures shows significantly enhanced inelastic transport across all four interfaces, which is consistent with the non-zero slope of the spectral heat 363 flow curves at f=14.68 THz. Details beyond NMD are also revealed. Compared with 364 $Si(1 \times 1)$ -Al_p, interfaces with Al twin boundaries or Si surface reconstructions reduce the 365 transmission of low-frequency Si LA phonons, while they have varied effects on the TA 366 phonons. Interfacial transport of LO phonons benefits from the Si surface reconstructions 367 but not from the Al twin boundaries, whereas the TO phonons experience trivial impact as 368 their ability to transfer energy into Al remains negligible. 369

By performing the vibrational spectrum analyses and distinguishing the atomic vibra-370 tions along all three directions, we gain more insight into the detailed phonon-interface 371 scattering mechanism. We list the direction-resolved phonon vibrational energy in Ta-372 ble. IV, where all energies are normalized by the total energy of their respective original 373 Si phonon. Here we also elaborate on four phonon modes that most clearly manifest the 374 differences among the four interfaces. Shown in Fig. 7 are the vibrational spectra of the 375 LA-II phonon before scattering with the interface, the reflected part, and the transmitted 376 $_{377}$ part. When $E_{avg}=0.192$ meV/atom, we only show the Si(1 \times 1)-Al_p case as there is no visible difference at the other interfaces. Elastic phonon transmission is observed for all four 378 379 interfaces, and the detailed direction-resolved phonon vibrational energy shows significant polarization conversions [63], which indicates that the longitudinal and transverse atomic 380 vibrations transform into each other during the scattering. When $E_{avg}=3.84$ meV/atom, 381 inelastic scattering becomes more significant as the LA-II phonons are reflected into several 382 higher-frequency phonons by the interfaces, which have integer multiples of the frequency 383 of the original Si phonon. This indicates that inelastic scatterings are dominated by linear 384 combinations among the original phonons. The Al phonon wave packets show vibrational spectra with different frequency distributions from the initial Si phonons, also indicating inelastic phonon scattering, while elastic transmission still dominates as the majority of the phonons retain their original frequency. 388

³⁸⁹ The LA-III phonon shows significantly enhanced transport across interfaces with Al twin

	Frequency	$Si(1 \times 1)$ - Al_p		$Si(1 \times 1)$ -Al _t			$\mathbf{Si}(\sqrt{3} \times \sqrt{3})$ - $\mathbf{Al}_{\mathbf{p}}$			$\mathbf{Si}(7 imes7) extsf{-}\mathbf{Al}_{\mathbf{p}}$			
	(THz)												
E_{avg} (me	V/atom)	0.192	0.384	3.84	0.192	0.384	3.84	0.192	0.384	3.84	0.192	0.384	3.84
LA-I	2.82	76.6%	67.2%	_	72.4%	63.0%	_	37.9%	33.1%	—	33.6%	28.7%	_
TA-I	2.84	34.9%	33.0%	18.6%	37.3%	35.1%	28.6%	14.3%	15.9%	16.8%	46.4%	45.5%	36.5%
TA-II	6.03	9.5%	9.4%	7.5%	2.6%	2.6%	2.4%	0.4%	0.4%	0.3%	2.0%	1.9%	2.3%
LA-II	6.98	49.5%	48.6%	29.2%	41.1%	39.1%	30.3%	21.7%	21.7%	22.0%	14.5%	14.4%	13.7%
LA-III	8.61	0.8%	0.8%	1.4%	10.4%	10.5%	10.8%	7.5%	7.5%	9.3%	5.3%	5.3%	4.9%
LO-I	14.68	0.1%	0.2%	6.0%	0.2%	0.2%	5.2%	0.2%	0.2%	12.3%	0.2%	0.3%	10.0%
TO-I	16.23	0.0%	0.0%	0.0%	0.1%	0.1%	0.0%	0.1%	0.0%	0.0%	0.1%	0.1%	0.1%
LO-II	23.83	0.1%	0.2%	0.9%	0.2%	0.2%	1.2%	0.1%	0.2%	1.0%	0.3%	0.4%	7.3%

TABLE III: The frequency and energy transfer rate of the eight considered phonons across the four interfaces.

TABLE IV: The direction-resolved kinetic energy distributions of the selected Si phonons before scattering with the interface and of the Al phonons that are generated from interfacial scattering. All energies are normalized by the total energy of their respective original Si phonon.

		$Si(1 \times 1)-Al_p$			$Si(1 \times 1)$ - Al_t			${f Si}(\sqrt{3} imes\sqrt{3}){f -Al_p}$			$\mathbf{Si}(7 \times 7)$ - $\mathbf{Al}_{\mathbf{p}}$		
Mode and E_{avg} (meV/atom)		E_x	E_y	E_z	E_x	E_y	E_z	E_x	E_y	E_z	E_x	E_y	E_z
	Si	100.0%	0.0%	0.0%									
LA-I	Al, 0.192	72.9%	1.9%	1.9%	62.9%	2.4%	7.2%	34.8%	1.7%	1.5%	18.5%	7.6%	7.5%
	Al, 0.384	62.1%	2.6%	2.6%	51.7%	3.2%	8.1%	29.0%	2.2%	1.9%	15.6%	6.6%	6.5%
	Si	89.0%	0.0%	11.0%									
LA-II	Al, 0.192	27.7%	7.4%	14.5%	15.9%	12.1%	13.0%	10.3%	6.3%	5.1%	6.8%	3.7%	4.0%
	Al, 3.84	14.9%	6.2%	8.1%	11.0%	9.3%	10.4%	9.9%	6.5%	5.6%	6.3%	3.5%	3.8%
	Si	87.2%	0.0%	12.8%									
LA-III	Al, 0.192	0.5%	0.2%	0.1%	4.2%	2.3%	4.0%	4.4%	2.7%	0.4%	2.1%	1.5%	1.7%
	Al, 3.84	0.9%	0.2%	0.2%	4.5%	2.5%	3.8%	4.5%	4.1%	0.8%	1.9%	1.4%	1.6%
IOI	Si	100.0%	0.0%	0.0%									
LO-1	Al, 3.84	5.4%	0.3%	0.3%	3.3%	0.7%	1.2%	6.1%	3.1%	3.1%	4.1%	3.0%	2.9%
	Si	0.3%	6.8%	92.9%									
TA-I	Al, 0.192	2.7%	5.1%	27.1%	7.0%	8.1%	22.3%	0.2%	1.6%	12.5%	7.0%	7.4%	31.9%
	Al, 3.84	5.3%	5.4%	7.9%	6.9%	6.0%	15.7%	5.2%	5.7%	5.9%	6.7%	7.5%	22.4%
	Si	0.2%	91.6%	8.2%									
TA-II	Al, 0.192	3.4%	4.3%	1.8%	0.9%	0.9%	0.8%	0.2%	0.1%	0.1%	0.8%	0.7%	0.6%
	Al, 3.84	3.0%	3.0%	1.6%	0.8%	0.9%	0.7%	0.1%	0.1%	0.1%	0.8%	0.8%	0.7%



FIG. 7: The atomic-vibrational spectra (logarithm scale) acquired for the initial Si phonon, reflected Si phonon, and transmitted Al phonon from the LA-II phonon wave packet simulations. The spread of frequency in the initial Si phonon results from the finite size truncation of the wave packet's spatial span. All cases except the Si(1 × 1)-Al_p case with E_{avg} =0.192 meV show significant inelastic phonon scattering, with peaks appearing in the vibrational spectra of the reflected phonon at frequencies that are integer multiples of the frequency of the incidence Si phonon (except for the peak at ~25 THz which represents trivial fractions of unintended phonons resulting from inaccuracy of the eigenvectors of the original phonon).

³⁹⁰ boundaries or Si surface reconstructions at both $E_{avg}=0.192$ and 3.84 meV/atom. The ³⁹¹ vibrational spectra (Fig. 8) show that, compared with the LA-II phonon, scatterings between ³⁹² the LA-III phonons and the interfaces are more consistently elastic at both energy levels as ³⁹³ no phonons of other frequencies are manifested in the atomic-vibrational spectra, despite ³⁹⁴ the LA-III phonon having much lower transmission. The direction-resolved kinetic energies ³⁹⁵ reveal that the Al twin boundaries and Si surface reconstructions significantly enhance both



FIG. 8: Same with Fig. 7 but for the LA-III phonon. Compared with the LA-II phonon, the LA-III phonon shows more consistent elastic transmission at both energy levels for all four interfaces. It should be noted that in the $Si(1 \times 1)$ -Al_p cases the phonons are almost entirely reflected, but the low-transmission into Al is elastic.

polarization transmission and conversion. This trend seems to contradict the fact that a 396 plateau of J accumulation is observed at the $Si(1 \times 1)$ -Al_t interface in this frequency range 397 at 100 and 200 K. It is thus noteworthy that although we consider three different E_{avg} 's 398 to represent phonons at different temperatures, each interface still has the same respective 399 atomic structure optimized at 0 K across all simulations. Compared with the J accumulation 400 curve at 200 K, the plateau is shifted towards high frequencies at 100 K; thus, we can expect 401 the plateau to be shifted even further towards high frequencies at 0 K which will likely result 402 $_{403}$ in a non-zero slope at f=8.61 THz, and that the two methods are still consistent. However, 404 one should note this limitation of the wave packet approach in representing high temperature 405 systems.

406 Lastly, we compare the two TA phonons. In addition to their different frequencies, their

407 atoms are also vibrating in different planes: in the TA-I phonon atoms mainly vibrate ⁴⁰⁸ in the $(\overline{1}10)$ plane, whereas the TA-II phonons mainly vibrate in the $(\overline{1}\overline{1}2)$ plane. Vi-409 brational spectra (Fig. 9) show that for the TA-I phonon, elastic scatterings dominate at 410 all four interfaces when $E_{avg}=0.192 \text{ meV/atom}$, while inelastic scatterings dominate when $E_{avg}=3.84 \text{ meV/atom}$. In contrast, the TA-II phonons (Fig. 10) elastically scatter with all 412 four interfaces at both energies. Still, at the higher energy $E_{avg}=3.84 \text{ meV}/\text{atom a noticeable}$ ⁴¹³ portion of the TA-II phonons experience inelastic scattering through linear combination, as is manifested by the second peak of the transmitted phonon at the frequency twice that 414 ⁴¹⁵ of the original Si phonon in the vibrational spectra. The direction-resolved kinetic energy ⁴¹⁶ reveals that both phonons experience significant polarization conversion at the interface, ex-⁴¹⁷ cept for TA-I phonons in the Si($\sqrt{3} \times \sqrt{3}$)-Al_p case where the Al phonons are still vibrating ⁴¹⁸ largely along the initial directions. Comparing the LA-I and LA-II phonons, it is obvious ⁴¹⁹ that frequency can affect the phonon transport. In addition, we speculate that there is directional anisotropy in polarization conversion that can be linked to the interfacial struc-420 tures. The comparison between TA-I and TA-II across the $Si(1 \times 1)$ -Al_p interface suggests 421 that this type of interface is more amenable to transport of vibrations in the $(\overline{1}10)$ plane than in the $(\overline{1}12)$ plane, which are perpendicular to each other. The other two surface reconstructions have the same symmetries with bulk Si and Al along [111]: C_3 rotation symmetry about the $[1\,1\,1]$ direction and mirror symmetry about the $(\overline{1}\,1\,0)$ plane (Fig. 2). Consequently, there is no additional anisotropy confinement from the surfaces. Therefore, $_{427}$ our results from Table. IV indicate that the Si (7×7) reconstruction enhances the transport $_{428}$ of $(\overline{1}10)$ plane vibrations, while further suppressing the $(\overline{1}\overline{1}2)$ plane vibrations. The Si $_{429}$ $(\sqrt{3} \times \sqrt{3})$ reconstruction suppresses both, $(\overline{1}\,\overline{1}\,2)$ more so than $(\overline{1}\,1\,0)$. The Al twin bound- $_{430}$ aries, on the other hand, destroy the C_3 rotation symmetry and introduces an additional mirror symmetry about the (112) plane. The impact on phonon transport is similar with 431 $_{432}$ that of the Si (7×7) reconstruction, and we notice that they both have mirror symmetry about the $(\overline{1}10)$ and $(\overline{1}\overline{1}2)$ planes. 433

⁴³⁴ Compared with NEMD and NMD, the wave packet simulations provide more details ⁴³⁵ about the phonon scattering mechanisms, especially single-phonon behavior at ultra-low ⁴³⁶ temperatures. However, the 1D wave packet simulation has its limitations because only ⁴³⁷ perpendicular incidence is considered. Besides limiting the case studies, it also contributes to ⁴³⁸ another issue with the analysis, as all phonon momentums are confined along the same line.



FIG. 9: Same with Fig. 7 but for the TA-I phonon. Inelastic scattering is significant for all four interfaces when $E_{avg}=3.84$ meV.

⁴³⁹ While this may not impose any additional constraints on elastic transmission according to ⁴⁴⁰ AMM, it suppresses inelastic scattering by eliminating all possibilities where the scattering ⁴⁴¹ phonons do not have collinear momenta. As a result, the elastic transmission may be ⁴⁴² overestimated, because the phonons are forced to transmit or get reflected if there is no ⁴⁴³ allowed collinear inelastic transition. Still, the 1D approach is able to predict consistent ⁴⁴⁴ trends with NEMD or NMD, which are methods that include all incident angles, and can ⁴⁴⁵ provide more insights as demonstrated by our study.

446 IV. SUMMARY

We have identified the influence on interfacial phonon transport from the structures at across four atomically precise Si/Al interfaces — Si(1 × 1)-Al_p, Si(1 × 1)-Al_t, Si($\sqrt{3} \times \sqrt{3}$)-Al_p and Si(7 × 7)-Al_p — and the detailed underlying phonon scattering mechanisms with



FIG. 10: Same with Fig. 7 but for the TA-II phonon. Compared with the TA-I phonon, the TA-II phonon shows more consistent elastic transmission at both energy levels for all four interfaces. A noticeable feature is that a second peak showing a frequency that is twice that of the original phonon appears for all four interfaces in the transmitted phonon when $E_{avg}=3.84$ meV, which is most significant for the Si(7 × 7)-Al_p interface. This indicates collinear inelastic combination of the phonons.

⁴⁵⁰ a combination of NEMD, NMD and single-phonon wave packet simulations. The NEMD ⁴⁵¹ results at 100 and 200 K show that the Al twin boundaries reduce the interfacial thermal ⁴⁵² conductance by 12.6% and 2.7%, respectively, relative to the Si(1 × 1)-Al_p interface. In ⁴⁵³ contrast, the Si ($\sqrt{3} \times \sqrt{3}$) and (7 × 7) reconstructions can enhance phonon transport by ⁴⁵⁴ 18.8–19.3% and 6.0–16.7%, respectively. By conducting NMD, we reveal the spectral phonon ⁴⁵⁵ heat flow, which shows that the Al twin boundaries are converting the elastic transmission ⁴⁵⁶ of low-frequency Si phonons across the otherwise ideal interface into inelastic scattering by ⁴⁵⁷ channeling the phonons into each other. The Si ($\sqrt{3} \times \sqrt{3}$) and (7 × 7) reconstructions, in ⁴⁵⁸ contrast, enhance high-frequency Si phonon transport by providing more channels for them 459 to inelastically convert into Al phonons. To further approach the low-temperature context 460 and reveal more details about the phonon scattering mechanism, we conduct phonon wave ⁴⁶¹ packet simulations, compatible with the same system domains. Generally NMD and wave 462 packet simulations show consistent results, while the latter provide more details showing that 463 elastic transmission or reflection is predominant for Si phonons at low temperatures, while ⁴⁶⁴ inelastic transport becomes more significant as the temperature increases. The significance of the inelastic scattering also depends on the phonon mode, because at the same temperature 465 inelastic scattering is strong for some phonon modes, while it is negligible for others. By 466 467 illustrating the direction-resolved atomic vibrations in the phonon wave packet, we find significant polarization conversion which depends on temperature and interfacial structures. 468 We find that the Al twin boundaries and Si surface reconstructions have anisotropic influence 469 on the transport of atomic vibrations in the $(\overline{1}10)$ and $(\overline{1}\overline{1}2)$ planes. While the reference Si 470 (1×1) reconstruction is more admissible to the former than the latter, the (7×7) amplifies 471 this effect while $(\sqrt{3} \times \sqrt{3})$ in turn suppresses both. The effect of the Al twin boundaries $_{473}$ is similar to the Si (7×7) reconstruction, and we speculate that this is related to the 474 symmetries they have in common.

While certain limitations exist in our simulations regarding the classical phonon distribu-475 476 tion and exaggerated density of twin boundaries, our model still predicts meaningful results 477 that provide guidance. Further, more realistic approximations can be made by applying corrections to the phonon population and linear interpolation to predict interfaces with a lower 478 and more realistic density of twin boundaries. Despite these limitations, the results indicate 479 460 that it should be possible to selectively control phonon transport via surface/interface engi-⁴⁸¹ neering in real systems. While our ability to utilize phonons as probes of material properties, 482 interfaces, and defects in QIS devices is predicated by our understanding of their scattering ⁴⁸³ mechanism, it also in turn demands advanced design of experiments in the cryogenic do-⁴⁸⁴ main that can provide more accurate measurements to deepen our knowledge. While it is currently impossible to directly simulate the long wavelengths representative of milli-Kelvin 486 temperatures, we expect the trends extracted from modeling increasingly long-wavelength phonons will inform which types of interface properties (e.g. twin boundaries) will impact 487 488 thermal conductance. We believe these results will lead to an increased understanding of 489 thermal conductivity at ultra-low temperatures, especially when phonons interact with fi-⁴⁹⁰ nite defects causing deviations from standard mismatch models. Comparison of experimental ⁴⁹¹ measurements with well-controlled interfaces to trends from NEMD and single-phonon wave ⁴⁹² packet simulations are crucial to understand the optimization of interfaces for devices. Our ⁴⁹³ wave packet approach is also well prepared to handle ultra-low temperature single-phonon ⁴⁹⁴ physics, with the potential to be expanded to 2D for a greater modeling space.

495 V. ACKNOWLEDGEMENT

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