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Remarkable thermoelectric performance in BaPdS₂ via pudding-mold band structure, band convergence, and ultralow lattice thermal conductivity

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Efficient thermoelectric materials require a rare and contraindicated combination of materials properties: large electrical conductivity, large Seebeck coefficient, and low thermal conductivity. One strategy to achieve the first two properties is via low-energy electronic bands containing both flat and dispersive parts in different regions of crystal momentum space, known as a pudding-mold band structure. Here, we illustrate that BaPdS₂ successfully achieves the pudding-mold band structure for the valence band, contributing to a large p-type thermoelectric power factor, due to its anisotropic crystal structure containing zig-zag chains of edge-sharing square planar PdS₄ units; large power factor is achieved for n-type doping as well due to band convergence. In addition, BaPdS₂ exhibits ultralow lattice thermal conductivity, and thus also achieves the third property, due to extremely soft and anharmonic interactions in its transverse acoustic phonon branch. We predict a remarkably large thermoelectric figure of merit, with peak values between 2 and 3 for two of the three crystallographic directions, suggesting BaPdS₂ warrants experimental investigation.

I. INTRODUCTION

The figure of merit for a thermoelectric material, which can convert a temperature gradient into electrical current, is

$$ZT = \frac{S^2 \sigma}{\kappa_{el} + \kappa_L} T,\tag{1}$$

where S, σ , T, κ_{el} , and κ_{L} are the Seebeck coefficient, electrical conductivity, temperature, electrical contribution to thermal conductivity, and lattice contribution to thermal conductivity, respectively. 1,2 One strategy to achieve a large ZT is via engineering the electronic band structure, i.e., the electron energy ε as a function of crystal momentum \mathbf{k} .³ In particular, a band with both flat (small $\nabla_{\mathbf{k}}\varepsilon$) and dispersive (large $\nabla_{\mathbf{k}}\varepsilon$) parts along a direction in k-space has been shown to enhance σS^2 if the electron chemical potential μ lies at an energy separating the two regions.⁴ In such a "pudding-mold" band structure, the band velocity difference leads to large S, and large σ is achieved due to the dispersive part of the band and a large Fermi surface. 4 More generally, one can consider a broader definition of a puddingmold band structure in which the flat and dispersive regions can along different directions in k-space, which naturally occurs for low-dimensional bulk and nanostructured crystals.⁵⁻⁷ The pudding-mold band structure has been used to explain the promising thermoelectric performance of Na_xCoO_2 , 4 SnSe, $^{8-10}$ and recently-proposed Fe-based Heusler compounds¹¹ among others.

Recently, the promising thermoelectric performance of several Pd oxide compounds has been attributed in part to the pudding-mold band structure. The layered compound PbPdO₂, for example, exhibits a large Seebeck coefficient of 175 μ V/K at 600 K when hole doped. ¹² Based on first-principles calculations, hole-doped Bi₂PdO₄ was predicted to exhibit high power factor (σS^2) in addition to low κ_L . ¹³ In both of these compounds, achieving the

pudding-mold band structure appears to be connected to the presence of square planar Pd^{2+} . Based on this observation, we previously developed an inverse band structure design approach based on a materials database screening to search for other square planar compounds achieving the pudding-mold band structure.¹⁴ Although several chemistries were considered, we focused on oxide materials such as Ba_2PdO_3 and La_4PdO_7 in order to validate the approach.

The thermoelectric efficiency of oxides is often limited by (1) large κ_L , stemming from the low atomic mass of oxygen and rigid chemical bonding, and (2) low carrier mobility. 15,16 Therefore, in this work, we turn our attention to chalcogenide materials. Chalcogenides comprise many of the most-investigated and highest-performance thermoelectrics, such as PbTe, Bi₂Te₃, SnSe, Cu₂(S/Se), and $La_{3-x}Te_4$. ¹⁷⁻¹⁹ We investigate a chalcogenide material based on square planar Pd, namely BaPdS₂, with the aim of achieving low κ_L in addition to puddingmold band structure. BaPdS₂ is the only such square planar chalcogenide identified by our previous inverse band structure design approach other than metallic ${\rm EuPd_3S_4}^{20}$ and the binaries ${\rm PdS_2}^{21}$ and ${\rm PdSe_2}^{22}$ Although ${\rm BaPdS_2}$ was first synthesized in 1986,²³ it has not been explored as a thermoelectric material to the best of our knowledge. The presence of heavy Ba, in addition to the lack of oxygen, leads to the potential for low κ_L . Using electronic structure and transport calculations based on the density functional theory, we find that BaPdS₂ exhibits (1) large p-type power factor stemming from a pudding-mold valence band structure, (2) large n-type power factor due to band convergence in the conduction band, (3) ultralow κ_L due to extremely soft and anharmonic bonding, and (4) highly-anisotropic electronic and thermal transport behavior. We find remarkable predicted thermoelectric performance, with ZTlarger than 2 in certain crystallographic directions at high temperature, assuming optimal doping and using a reasonable value of electronic relaxation time τ (5 fs).^{11,24–28} Based on these results, we suggest BaPdS₂ warrants experimental investigation.

II. COMPUTATIONAL DETAILS

We perform density functional theory^{29,30} calculations using the generalized gradient approximation³¹ and the projector augmented wave method (Ba_sv, Pd, and S potentials)^{32,33} in the Vienna *ab initio* simulation package (VASP).^{34–37} A plane wave basis with 500 eV kinetic energy cutoff and uniform k-meshes with $\geq 700~k$ -points per Å⁻³ are employed. The ionic forces and total energy are converged to 10^{-3} eV/Å and 10^{-6} eV, respectively. High-symmetry k- and q-point paths are based on the conventions of Setyawan *et al.*³⁸

Semiclassical electronic transport calculations within the constant relaxation time (τ) approximation are performed using BOLTZTRAP³⁹ with k-point density of $3,500/\text{Å}^{-3}$. Although the use of an isotropic relaxation time is an approximation, previous studies suggest only minor anisotropy in τ even in highly anisotropic crystal structures.³⁹ Phonon calculations are performed using the direct approach in PHONOPY⁴⁰ with a 192-atom supercell chosen to be approximately cubic based on the algorithm of Erhart et al. 41 as implemented in ASE. 42 The mode Grüneisen parameter is computed using a $\pm 0.3\%$ volume difference. κ_L is computed via the Debye-Callaway model^{43–45} and compared to that of the minimum thermal conductivity model of Cahill et al. 46 κ_{el} is computed via the Wiedemann-Franz law. Additional details on the electronic and thermal transport calculations are included in the Supplemental Material.⁴⁷

III. RESULTS AND DISCUSSION

BaPdS₂, shown in Fig. 1(a) and Fig. 1(b), has a Ccentered orthorhombic unit cell with space group Cmcm. Isostructural to BaNiO₂, it consists of 1D chains of edgesharing PdS₄ square planar units (corresponding to a PdS₂ composition) connected in a zig-zag pattern. The computed lattice parameters, atomic positions, and formation energy are in excellent agreement with experimental values, as discussed in the Supplemental Material. The electronic band structure of BaPdS₂ is shown in Fig. 1(c). The valence band, which consists primarily of Pd d states, exhibits a pudding-mold band structure. It is dispersive in the direction along the 1D chains (e.g., Γ –Z) and flat along other directions (e.g., Γ –X and Γ -Y). In order words, the pudding-mold band structure is closely related to the highly-anisotropic crystal structure of BaPdS₂. We do not find a pudding-mold band structure for the conduction band, which consists of a hybridization of Pd d and S p orbitals. We note that an additional conduction band minimum between S and R is close in energy (\sim 130 meV) to the band edge at Γ .

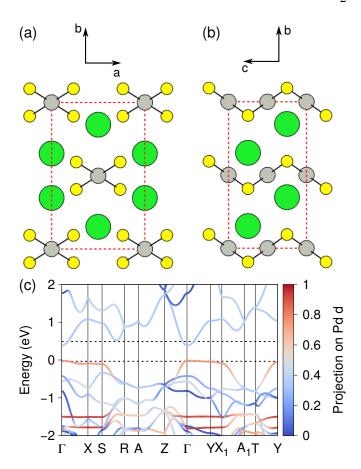


FIG. 1. Orthographic projection of the crystal structure of BaPdS₂ along the (a) **c** and (b) **a** axes of the conventional unit cell (red dashed lines). The green, gray, and yellow circles represent Ba, Pd, and S atoms, respectively. Black lines indicate the Pd–S bonds of the square planar units. (c) The electronic bands of BaPdS₂ colored by the Pd d character of the electronic states. The zero of energy is set to the valence band maximum, and the black horizontal dotted lines indicate the $\pm 10^{20}$ cm⁻³ doping levels.

The band structure of BaPdS₂ leads to notable electronic transport properties. Figure 2(a) illustrates the behavior of the average (over Cartesian directions) of $\sigma S^2/\tau$ as a function of carrier concentration for several temperatures. Results for BaPdS₂ are shown in comparison to those of SnSe, a high-performing thermoelectric material. $^{8,48-50}$ BaPdS $_2$ achieves comparable p-type power factor behavior to SnSe, albeit at larger (though still reasonable¹⁷) doping values. For example, at 700 K, the maximum predicted p-type $\sigma S^2/\tau$ is $\sim 3 \times 10^{11} \text{ W/(m \cdot \text{K}^2 \cdot \text{s})}$ for both BaPdS₂ (for 7×10^{20} holes/cm³) and SnSe (for 3×10^{20} holes/cm³). We also note that the p-type power factor behavior exhibits significantly less temperature dependence for BaPdS₂ than for SnSe. Although the conduction band does not exhibit pudding-mold qualities, we find even larger $\sigma S^2/\tau$ for n-type doping, though smaller than the corresponding SnSe behavior. For example, the peak n-type $\sigma S^2/\tau$

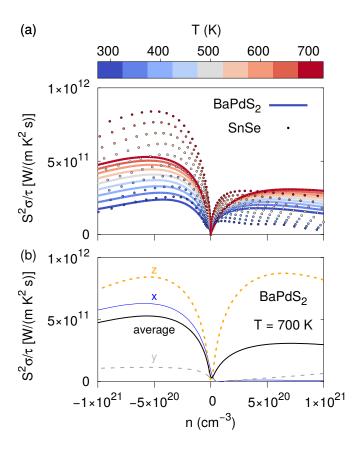


FIG. 2. (a) Power factor divided by electronic relaxation time as a function of carrier concentration for BaPdS₂ (lines) and SnSe (circles), averaged over the x, y, and z directions, for several values of T. (b) T=700 K power factor divided by electronic relaxation time versus carrier concentration in each direction, in addition to the average, for BaPdS₂.

for BaPdS₂ at 700 K is 5.3×10^{11} W/(m·K²·s), occurring at a doping value of $\sim 5.5 \times 10^{20}$ electrons/cm³; SnSe exhibits a significantly larger peak n-type $\sigma S^2/\tau$ of 8.4×10^{11} W/(m·K²·s) at around the same doping magnitude. The appreciable n-type power factor behavior for BaPdS₂ likely stems from the band convergence, $^{51-55}$ i.e., the small energy separation between the conduction band minima (1) at Γ and (2) between S and R. Although the conduction band minimum at Γ is singly degenerate, the band minimum between S and R has a higher degeneracy of 4, contributing to a large Seebeck coefficient.

Due to the structural anisotropy, the power factor behavior for BaPdS₂ is highly anisotropic. We illustrate the power factor behavior at 700 K, for example, in Fig. 2(b) for each direction. For hole doping, $\sigma S^2/\tau$ is only appreciable along the z direction (i.e., along the 1D chains), which is consistent with the pudding-mold band structure. In contrast, for electron doping, large $\sigma S^2/\tau$ is observed in both the z and x directions. A rationalization for the lower n-type power factor behavior in the y direction is discussed in the Supporting Information.

Having established the promising predicted electronic

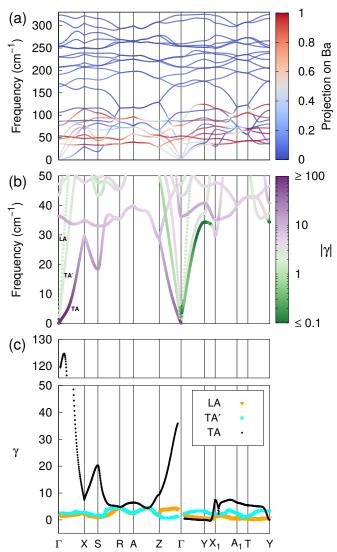


FIG. 3. (a) BaPdS₂ phonon bands colored by Ba contribution to the phonon eigenvector. (b) The low-frequency portion of the phonon dispersion with bands colored by the magnitude of the mode Grüneisen parameter. (c) Dispersion of the mode Grüneisen parameter $\gamma_{\mathbf{q}\nu}$ for the acoustic branches. Due to the large $\gamma_{\mathbf{q}\nu}$ values for the TA branch along Γ -X, we use a broken $\gamma_{\mathbf{q}\nu}$ axis for clarity.

Branch	v_x	v_y	v_z	$\Theta_{\Gamma X}$	$\Theta_{\Gamma Y}$	$\Theta_{\Gamma Z}$
TA	0.5	1.9	0.8	43	49	42
TA'	2.4	2.4	1.9	82	54	72
LA	4.1	3.7	3.6	99	115	72

TABLE I. Group velocity v at Γ (in km/s) and Debye temperature Θ (in K) of BaPdS₂ for each acoustic branch and direction.

transport properties of BaPdS₂, we now turn our attention to thermal transport. The phonon dispersion of BaPdS₂ is shown in Fig. 3(a). Low-frequency optical modes, primarily involving vibration of the heavy Ba atoms, are observed in the $\sim 30\text{--}110~\text{cm}^{-1}$ frequency range. For each acoustic branch, we compute the group velocity in the long-wavelength limit $(v_x, v_y, \text{ and } v_z)$ and Debye temperature $(\Theta_{\Gamma X}, \Theta_{\Gamma Y}, \text{ and } \Theta_{\Gamma Z})$ for each direction, as shown in Table I. Here, we define Θ as the acoustic phonon frequency at the zone boundary, and the acoustic branch definitions are given in the Supplemental Material. BaPdS₂ is a very soft material, exhibiting low group velocities (0.5–4.1 km/s) and Debye temperatures (42–115 K). For comparison, v of 1.0–2.9 and Θ of 19–72 are found in SnSe.⁸ The transverse acoustic (TA) branch of BaPdS₂ is particular soft, with $v=0.5~\mathrm{km/s}$ (0.8 km/s) and $\Theta = 43 \text{ K}$ (42 K) along the x (z) direction. Animations of the TA phonons along X (featuring rigid PdS_2 chains sliding in the z direction with respect to the others) and along Z (featuring oscillations of the individual PdS_2 chains in the x direction) are included in the Supplemental Material. We note that the behavior of the TA branch is significantly harder along the y direction in terms of the group velocity (1.9 km/s), though not the Debye temperature (49 K). Corresponding to PdS₂ chains sliding in the z direction, the TA branch along Y corresponds to a similar atomic motion to that along X; the significantly larger inter-chain distance in the y direction (10.8 Å) compared to that in x direction (6.9 Å) may be responsible for the very different phonon properties.

		BaP	dS_2	SnSe			
Branch	γ_{all}	$\gamma_{\Gamma X}$	$\gamma_{\Gamma Y}$	$\gamma_{\Gamma Z}$	$\gamma_{\Gamma X}$	$\gamma_{\Gamma Y}$	$\gamma_{\Gamma Z}$
TA	25.5	80.3	0.3	22.8	5.1	2.7	1.0
TA'	2.8	2.6	3.1	1.0	1.1	2.4	2.7
LA	2.3	2.1	1.2	4.0	5.9	1.3	3.3
Avg.	10.2	28.3	1.5	9.3	4.1	2.1	2.3

TABLE II. Averages (computed as $\sqrt{\langle \gamma^2 \rangle}$) of mode Grüneisen parameter $\gamma_{\mathbf{q}\nu}$ for BaPdS₂ and SnSe⁸ for different acoustic branches and directions. For BaPdS₂, the average over the entire high-symmetry path in the Brillouin zone is also given as γ_{all} .

To acquire a baseline assessment for the overall magnitude of the anharmonicity, i.e., the phonon-phonon scattering strength, we compute the mode Grüneisen parameter

$$\gamma_{\mathbf{q}\nu} = -\frac{\partial \omega_{\mathbf{q}\nu}/\omega_{\mathbf{q}\nu}}{\partial V/V},\tag{2}$$

where V is the volume and $\omega_{\mathbf{q}\nu}$ is the phonon frequency for crystal momentum \mathbf{q} and branch ν . The low-frequency phonons of BaPdS₂, colored by $|\gamma|$, are shown in Fig. 3(b), whereas the full dispersion of $\gamma_{\mathbf{q}\nu}$ for the acoustic branches is included in Fig. 3(c). The most prominent feature is that BaPdS₂ has gigantic $\gamma_{\mathbf{q}\nu}$ for

the TA branch along Γ -X (values up to ~ 125) and Γ -Z (values up to ~ 35), which indicates extremely anharmonic interactions for this acoustic branch in BaPdS₂. We perform tests to confirm the large computed $\gamma_{\mathbf{q}\nu}$ are not a numerical artifact, as discussed in the Supplemental Material. Averages of the mode Grüneisen parameter for each acoustic branch of $BaPdS_2$, averaged over Γ - $X, \Gamma - Y, \Gamma - Z,$ and the full high-symmetry path in the Brillouin zone, are shown in Table II, in which we also compare to literature results for SnSe.⁸ The average $\gamma_{q\nu}$ for the TA branch of $BaPdS_2$ is 80.3 in the x direction and 22.8 in the z direction, compared to much smaller (less anharmonic) values of 5.1 and 1.0 for SnSe in the corresponding directions. We note that a key exception to the large anharmonicity of the TA branch of BaPdS₂ is along the y direction, for which the average $\gamma_{\mathbf{q}\nu}$ is only 0.3, as compared to 2.7 for SnSe. In other words, the TA branch of $BaPdS_2$ is not only harder in the y direction than the other directions, but also much less anharmonic. Although the most remarkable finding is the extremely large $\gamma_{\mathbf{q}\nu}$ for the TA branch, we also find significant anharmonicity for other phonon branches. For example, the TA' (LA) branch exhibits appreciable $\gamma_{\mathbf{q}\nu}$ of 1.0–3.1 (1.2– 4.0), as compared to 1.1-2.7 (1.3-5.9) for SnSe. Large anharmonicity is also found in the lowest-frequency optical branch, with mode Grüneisen parameter values as large as ~ 12 , as shown in Fig. 3(b). This branch corresponds to oscillation of the Ba sublattice with respect to the PdS_2 chains in the z direction.

We employ the Debye-Callaway model, an approximate model taking into account normal and Umklapp acoustic phonon-phonon scattering via the mode Grüneisen parameter, as a means to estimate the lattice thermal conductivity of BaPdS₂. 43-45 We note that the low-frequency optical phonons with large $\gamma_{\mathbf{q}\nu}$, not taken into account in the Debye-Callaway model, may further lower κ_L compared to the values reported here. The contributions of each acoustic phonon branch to κ_L as a function of T in the x, y, and z directions are shown in Fig. 4(a)-4(c), respectively. BaPdS₂ exhibits ultralow lattice thermal conductivity (κ_L less than 1 W/m·K) in the x and z directions. In these directions, the predicted κ_L are of the same magnitude as that of SnSe along its smallest- κ_L direction (values of ~ 0.25 –0.7 W/m·K).⁸ For BaPdS₂, the main contributor to κ_L in the x direction is the LA branch, which exhibits the largest group velocity; the main contributor in the z direction is the TA' branch, which is the least anharmonic. Like the power factor, the thermal transport is highly anisotropic: in the y direction, κ_L is much larger (values of 4–8 W/m·K) than in the other directions, stemming primarily from the small $\gamma_{\Gamma Y}$ of 0.3 for the TA branch, as discussed above. In the x and z (but not y) directions, $BaPdS_2$ achieves the minimum possible κ_L , shown as red lines in Fig. 4(a)-4(c), estimated from the model of Cahill et al. Overall, BaPdS₂ is predicted to exhibit remarkably poor thermal transport in directions other than y.

As illustrated in Fig. 4(d)-4(f), the combination of

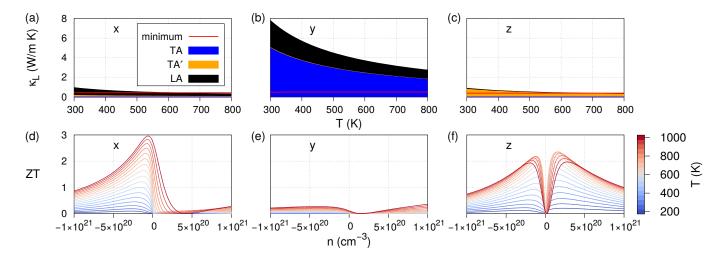


FIG. 4. (a)–(c) Stacked plots of BaPdS₂ acoustic branch contributions to κ_L versus T computed via the Debye-Callaway model for the (a) x, (b) y, and (c) z directions, respectively. Red lines indicate the minimum κ_L of the Cahill model. (d)–(f) ZT as a function of carrier concentration for different T in the (d) x, (e) y, and (f) z directions, respectively. Values are shown for $\tau = 5$ fs.

large power factor and low κ_L leads to impressive figure of merit for $BaPdS_2$, particularly in the x direction for electron doping and in the z direction for both electron and hole doping. At high temperature, we find peak ZT values of nearly 3 in the x direction (for 6.0×10^{19} electrons/cm 3) and values of nearly 2.3 in the z direction (for 1.1×10^{20} electrons/cm³ or 1.6×10^{20} holes/cm³). In contrast, much smaller peak ZT (< 1) is observed for $BaPdS_2$ in the y direction, due to the smaller power factor and larger κ_L . In order to obtain the predicted ZT values discussed above, we have chosen an electronic relaxation time (τ) of 5 fs. While an ab initio estimation of τ (e.g., from electron-phonon scattering calculations) will be important future work, we note that 5 fs is reasonable in terms of the order of magnitude 56,57 and can be considered a conservative estimate based on previous works. $^{11,24-28}$ Even with a less optimistic guess for the electronic scattering time, we still find quite large values for the thermoelectric figure of merit, albeit at larger doping. For example, assuming $\tau = 1$ fs, a peak ZT of 1.3 is achieved in the x direction for 2.0×10^{20} electrons/cm³. Therefore, even with the uncertainty in τ , our results strongly suggest BaPdS₂ merits experimental investigation.

IV. CONCLUSIONS

BaPdS₂, previously identified by our inverse band structure design approach, is a square planar sulfide material with remarkable, anisotropic thermoelectric properties. Due to a pudding-mold valence band structure and multiple conduction bands, $BaPdS_2$ exhibits impressive power factor behavior in two of the crystallographic directions. With heavy Ba atoms and extremely soft and anharmonic bonding, $BaPdS_2$ achieves ultralow lattice thermal conductivity in the x and z directions. We predict peak ZT values of 2–3 in the x direction (n-type) and the z direction (n- and p-type) for an electronic relaxation time of 5 fs. Our results strongly suggest $BaPdS_2$ warrants experimental investigation for its remarkable electronic, thermal transport, and thermoelectric properties.

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