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Origin of the Low-Energy Enhancement of the γ-ray Strength Function

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The low-energy enhancement (LEE) in γ-ray strength functions has been experimentally identified in a large number of nuclei during the past two decades; however, the origin of the enhancement is not fully understood. Building on previous theoretical work, we investigate the LEE and its relation to the scissors mode (SM) with an independent theoretical approach. We apply a novel angular-momentum-projected shell-model method that explicitly endows degrees of freedom to describe the scissors motion. Taking the recently measured γ-ray strength functions in Neodymium isotopes as examples, we find that the LEE arises from a quasi-free scissors motion appearing only in weakly-deformed nuclei, which can be viewed as an approximate free-rotation of neutrons with respect to protons. This leads us to propose a new type of collective motion, *scissors rotation*, to contrast the scissors vibration widely known in well-deformed nuclei. The observed LEE is naturally interpreted as the first evidence for this collective excitation mode.

The γ-ray strength function (γSF) [1] describes the dependence of the average γ-ray strength on its energy in electromagnetic transitions of nuclei. It gives the probability for a compound nucleus to decay through γ-ray emissions and is therefore an invaluable ingredient for the statistical description of γ-ray involved reactions. A well-known example is the calculation of (n,γ) cross sections with the Hauser-Feshbach model [2], for which the γSF is a required input. The (n,γ) reaction in the rapid (r-process) and intermediate neutron capture processes (i-process) [3] is particularly important in the synthesis of heavy elements in the universe [4, 5]. Over the years, the γSF has attracted considerable experimental and theoretical attention [6].

Twenty years ago, Voinov *et al.* [7], using the Oslo method, observed that the γSFs in ^{56,57}Fe exhibit an exponential increase as the γ-ray energy approaches zero. This unexpected feature, termed the low-energy enhancement (LEE) in the γSF, was later confirmed independently by an alternative experimental approach [8–10]. The LEE has also been shown to be of dipole [11, 12] and possibly of magnetic character [10]. The occurrence of LEE has been found in a large number of nuclei with weak deformation, ranging from the pf shell to the rare-earth region, which was summarized in Ref. [13] and more recently reported in Refs. [14–20]. The existence of the LEE can have a significant impact on nucleosynthesis outcomes. It has been estimated that (n,γ) cross sections in neutron-rich Fe, Mo, Cd isotopes [21], as well as Yb isotopes [5], can increase by an order of magnitude or more due to the presence of the LEE.

Since its discovery, the origin of the LEE has been an active field of research. The finite-temperature random-phase-approximation (RPA) suggested an E1 nature of the LEE, arising from transitions between the thermally unblocked orbitals above the Fermi level and the continuum [22]. The calculated γSF by the RPA showed a dramatic decrease as the γ-ray energy approaches zero, which has not been confirmed experimentally. In contrast to the RPA, existing investigations based on the nuclear shell model suggest an M1 nature of the LEE [23].

The first shell model study [24] on the γSF for the Mo isotopes near closed shell [25] suggested that the occurrence of the LEE is connected to the shears mechanism [26], with the M1 transitions generated by reorientation of the proton particles and neutron holes in the high-j single-particle orbitals. It was found later that orbitals with small j's also produce some M1's, making a weaker LEE [27]. Another shell-model study on the open-shell Fe isotopes [28] found a weaker LEE and a scissors-like resonance in its γSF, with an approximate "conservation" of the total M1 strength of the two, indicating a possible connection of the LEE with the well-known scissors motion [29]. Such a bimodal struc-

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ture, explained by Frauendorf as originating from differences in deformation-induced splitting of Nilsson single-particle levels [30], shows up in a systematic way in spherical shell-model calculations. Other theoretical studies [13, 31, 32] appear to show similar connection discussed in [28]. The coexistence of the LEE and the scissors resonance has been observed in the Sm isotopes [15, 33, 34], with the "conservation" of the M1 strength supported within large error bars [15]; however, the LEE in the Sm isotopes was not observed in the earlier and later works [35, 36]. Further, the conservation in total M1 strength was not supported by the more recent work on Nd isotopes [17]. Hence, the relation between the LEE and the scissors motion is not clear.

It is also not clear how an interpretation using the shears mechanism [26] can be reconciled with the scissors motion [29], although both involve a reorientation between neutrons and protons. The shears mechanism favors specific mass regions [26], while the scissors motion is extensively established in deformed nuclei [37]. Note that both interpretations are conjectures, not conclusions supported directly by shell model results.

Different from the conventional shell model approach [24, 28], the shell model based on angular momentum projection separately for neutrons and protons [38, 39], which is an extension from the original projected shell model (PSM) [40], explicitly contains the important degree of freedom relevant to the scissors motion. As a generator coordinate method, the PSM can provide a clear illustration for the collective motion in terms of microscopic wave functions, which may serve as an unique means to understand the LEE. The framework in [38, 39] was later improved to account for quasiparticle excitations [41], by which the shears mechanism, as another tentative explanation for the LEE, can also be described.

In this Letter, we apply the angular-momentum-projected model framework in [41] to explore the physical origin of the LEE. The rare-earth nucleus ^{144}Nd is taken as an example where a recent experiment found a significant LEE [17]. Two more deformed isotopes, ^{148,150}Nd, are also calculated for comparison.

The present model starts from angular momentum projections for the intrinsic neutron and proton configurations, respectively, while one can refer to Ref. [42] and Supplemental Material [43] for more details. The calculated M1 γSF for ^{144,148,150}Nd are shown in Fig. 1. To be consistent with the Oslo-method analysis, only theoretical M1 transitions below the neutron separation energy are included. Combined with the E1 component from the generalized/standard Lorentzian (GLO/SLO) fit of the giant/pygmy dipole resonances [17], the total γSF shows a qualitative agreement with the experimental data [17, 45]. For ^{144}Nd in panel (a), a pronounced enhancement appears for $E_\gamma \leq 2$ MeV, showing a clear signal for the LEE. The LEE evolves to the scissors resonance (SR) at 2-3 MeV while moving towards the well deformed region, as can be seen from panels (b) and (c) in Fig. 1 for heavier Nd isotopes. Noticeably, none of our calculated Nd isotopes indicates a significant bimodal

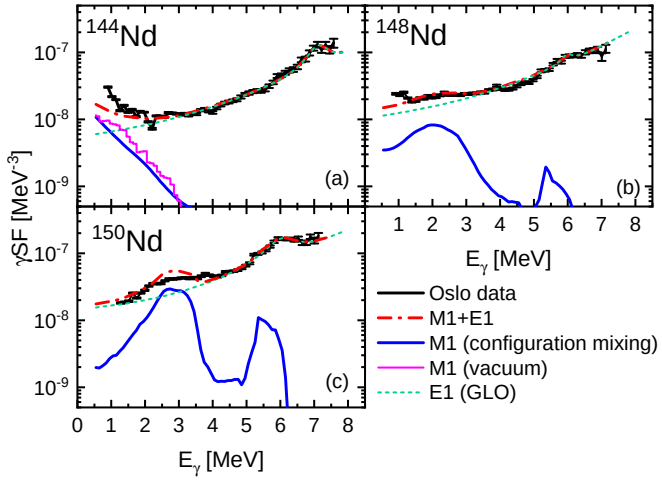


FIG. 1. (Color online) Comparison of experimental γSF data from the Oslo-method analysis (black solid line with error bars) [17], taken from the IAEA database [45], to the theoretical result (red dash dotted line) combined by the M1 γSF (blue solid line) from the present calculation with full configuration mixing, and the E1 strength function (green dashed line) from the GLO/SLO fit to the giant/pygmy dipole resonances. The magenta steps for ^{144}Nd gives the M1 γSF from the same calculation but with the qp vacuum configuration only.

LEE-SR structure as predicted in [28]. Rather, in agreement with data [17], the well-deformed ^{150}Nd shows a double-peak structure as a SR with another one at 5-6 MeV, presumably a spin-flip resonance.

It is interesting to explore whether the LEE emerged depends on specific configurations, as expected by the shears mechanism [24], or if it is related to the scissors motion that is collective and does not depend on any specific configurations. The calculated M1 γSF shown in Fig. 1 (pink dotted line) is obtained with inclusion of 2qp configurations. We now repeat the calculation but excluding all 2qp ones from the configuration space, leaving the qp vacuum only. The blue histogram in Fig. 1 gives the M1 γSF from such a calculation, in which the shears mechanism should be absent. However, as can be seen, besides statistical fluctuations, the LEE behavior remains similar. This result undoubtedly excludes the shears mechanism as the leading source for the LEE. On the other hand, the occurrence of the LEE may relate to the scissors motion, which is the only intrinsic degree of freedom with a single vacuum configuration.

The scissors motion in well-deformed nuclei is known as a harmonic oscillation of the scissors angle between the symmetry axes of protons and neutrons [46]. Such an oscillation generates the scissors resonance near $E_\gamma \sim 3$ MeV in the γSF [17]. However, the scissors motion in weakly deformed nuclei, where the LEE emerges, is less investigated. To find the origin of the LEE in weakly-deformed nuclei, we may first search for the evolution of the scissors motion from strong to weak deformation, by comparing the spectra and $B(M1)$s calculated for *different* deformations. As the calculations in Fig. 1 suggest

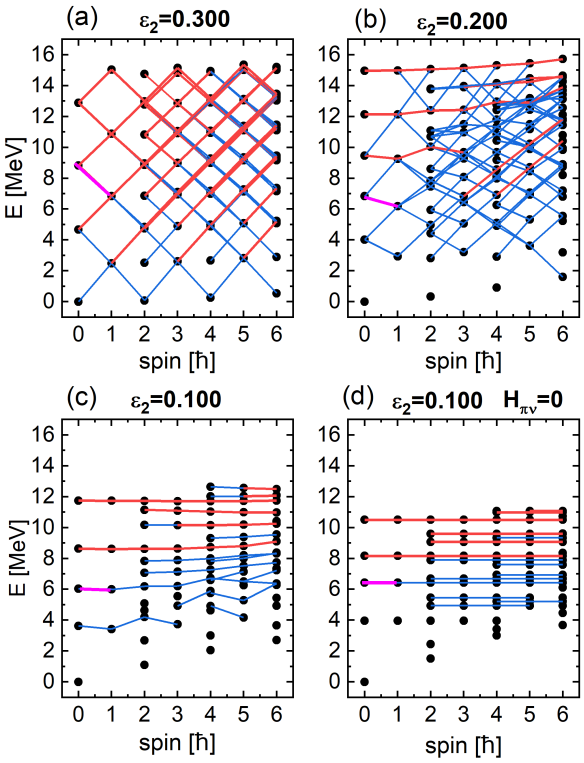


FIG. 2. (Color online) The spectra and the M1 transitions calculated with the qp vacua. Panels (a)-(c) show results obtained with deformations $\varepsilon_2 = 0.3$, 0.2 and 0.1, respectively, while panel (d) is the same as (c) except for a vanishing neutron-proton interaction. Dots in the plots represent the eigenstates and lines connecting them represent M1 transitions. For a given I state, we count both $B(M1, I \to I-1)$ and $B(M1, I \to I+1)$. The red (thick) lines are for M1 transitions with $B(M1) > 5\mu_N^2$ and the blue (thin) lines are for those with $1\mu_N^2 < B(M1) < 5\mu_N^2$. Transitions with $B(M1) < 1\mu_N^2$ are not shown here. The magenta line highlights the M1 transition connecting the second 1^+ and third 0^+ states that will be analyzed later in Fig. 3.

the LEE can be described with the qp vacua only, our following discussion is based on this approximation. In Fig. 2, the calculated energy levels and the connecting M1 transitions, with varying deformations (ε_2), are shown for excitations up to 15 MeV.

Interestingly, the obtained M1 patterns in Fig. 2 show dramatic changes with deformation. For a large deformation $\varepsilon_2 = 0.3$ in (a), the M1 connections form a rhombus pattern. The regularity in the pattern originates from the harmonicity of the scissors vibration, resulting in equidistant excited-bands (cf. Fig. 1 of Ref. [38]). The average transition energy between neighboring bands is ∼2.5 MeV, corresponding to the scissors resonance in the γSF.

As ε_2 decreases, the rhombus-like structure gets distorted (Fig. 2(b)), or even disappears (Fig. 2(c)). While remains of the rhombus can still be recognized in (b) in the lower energy region, it can hardly be seen in the higher energy region. The disappearance of the rhombus pattern in (b) and (c) is a manifestation of anharmonicity in the scissors spectrum. Notably, as the rhombus-like structure fades away, new regularity emerges as band-like structures in the upper part of panel (b), which expands to the lower part as the deformation further decreases (panel (c)). These new bands consist of near-degenerate states connected by strong M1, and therefore, are the dominate source of the LEE.

The radical differences in the M1 patterns of Figs. 2(a) and (c) motivate us to consider the limiting case depicted in Fig. 2(d), where the states within any M1 sequence are exactly degenerate. The results in (d) are obtained by eliminating the neutron-proton quadrupole interaction ($H_{\pi\nu}$), which is the only neutron-proton interaction in our model. Without the n-p interaction, the orientation of neutrons and protons is completely independent of each other, and their angular momenta, I_ν and I_π, become good quantum numbers. The total wave function can then be simply written as

$$|\Psi^I_M\rangle = \sum_{M_\nu} C^{I_\nu, I_\pi, I}_{M_\nu, M-M_\nu, M} |\psi^{I_\nu}_{M_\nu}\rangle \otimes |\psi^{I_\pi}_{M-M_\nu}\rangle. \quad (1)$$

The states $|\psi^{I_\nu}_{M_\nu}\rangle$ and $|\psi^{I_\pi}_{M-M_\nu}\rangle$ are eigenstates of the neutron and proton subsystem, respectively:

$$\hat{H}_\nu |\psi^{I_\nu}_{M_\nu}\rangle = E_\nu |\psi^{I_\nu}_{M_\nu}\rangle, \quad \hat{H}_\pi |\psi^{I_\pi}_{M-M_\nu}\rangle = E_\pi |\psi^{I_\pi}_{M-M_\nu}\rangle, \quad (2)$$

and the total energy, $E = E_\pi + E_\nu$, is independent of I. Therefore, all of the coupled states in a band with $|I_\nu - I_\pi| \le I \le I_\nu + I_\pi$ are *exactly* degenerate. We have confirmed numerically that the $B(M1)$s connecting these degenerate states are of similar magnitude to those in the M1 sequences shown in Fig. 2(c).

The striking similarity between Fig. 2(c) for a weakly-deformed case and (d) for an idealized limit of the "free" scissors motion suggests a possible mechanism for the regularity emerged in Fig. 2(c): The near-degenerate states with strong M1 connections, thus the LEE, may arise from a "quasi-free scissors motion", which is less confined and close to be free.

To reinforce the above discussion, we further show how the probability density distributions of the scissors angle [43] varies with deformation. The results shown in Fig. 3 are chosen for two representative states, 0^+_3 and 1^+_2 in the scissors spectra, connected by a strong M1 transition, as highlighted in Fig. 2.

The scissors angle distribution for a large deformation $\varepsilon_2 = 0.3$ in Fig. 3(b) looks almost identical to the analytical result for the harmonic scissors vibration [46] shown in Fig. 3(a). This unambiguously confirms the harmonic nature of the scissors vibration for the large deformation case. As deformation decreases, the scissors angle distribution starts to spread over a wider range, as shown in Figs. 3(b)-(d). The one with $\varepsilon_2 = 0.1$ in Fig. 3(d) even becomes similar to the analytical result of free scissors motion shown in Fig. 3(e). With decreasing deformation, the scissors motion evolves from the limit of harmonic vibration toward the limit of free scissors motion, in accordance with the observation in Fig. 2.

The enhanced freedom in the scissors motion with decreasing deformation can also be reflected by the infor-

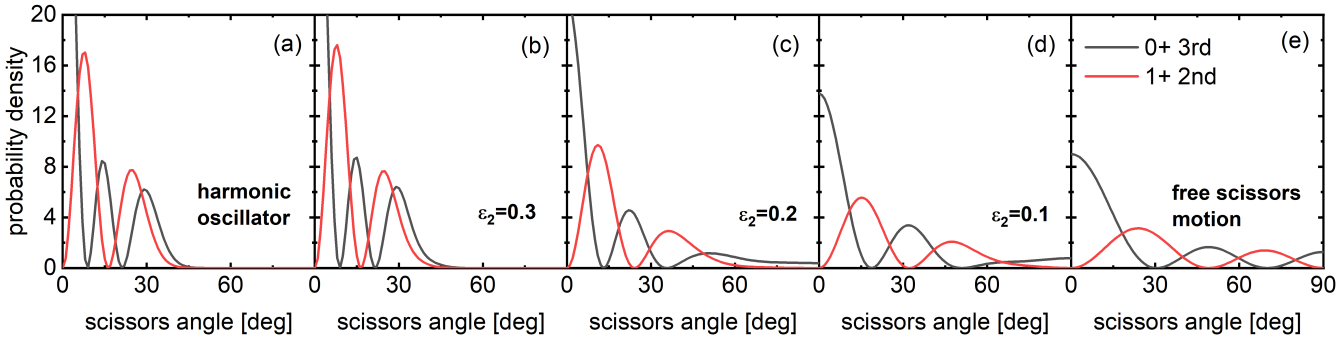


FIG. 3. (Color online) Probability density distributions of the scissors angle for the third 0^+ state, 0^+_3, and the second 1^+ state, 1^+_2, in Fig. 2, connected by a strong M1 transition. Panel (a) gives the analytical result of the harmonic oscillator limit and panel (e) gives the analytical result of the free scissors motion. The numerical results from the angular-momentum-projected model with the qp vacuum configuration for deformations $\varepsilon_2 = 0.3, 0.2$ and 0.1 are shown in panels (b), (c) and (d), respectively.

mation entropy [47] of the wave functions, as seen in the Supplemental Material [43].

The emergence of the "quasi-free scissors motion" in weakly-deformed systems is easy to understand with the illustration in Fig. 4. Fig. 4(a) represents an energy curve as a function of the scissors angle for well-deformed nuclei, which can be viewed as the collective potential for scissors motion, arising from the energy cost as the spatial overlap between neutron and proton bodies reduces (see Fig. 4(d)). The potential thus acts like a "restoring force" [48] for the scissors motion.

In cases of large deformation, the n-p spatial overlap decreases quickly with the scissors angle, and the scissors potential shown in Fig. 4(a) increases rapidly. The collective motion is thus well confined by the potential, with the probability density distribution (colored curves) being harmonic due to its small amplitude, a recognized feature for the scissors vibration [46].

In weakly-deformed cases, the decrease of the n-p overlap with scissors angles is overall slower, as illustrated in Fig. 4(e.2), and the potential curve shown in Fig. 4(b) becomes more flat. Compared to the well-deformed case, the collective motion in Fig. 4(b) carries out in a larger range of the scissors angle, being close to that of exactly free scissors motion shown in Fig. 4(c), obtained with $\hat{H}_{\pi\nu} = 0$ as illustrated in Fig. 4(f). Such an idea may also apply to a large deformation (see Fig. 4(e.1)), if the neutron-proton interaction would be much weaker than what is typically assumed.

The completely-flat "potential curve" in Fig. 4(c) means that the energy of the scissors system is independent of the scissors angle, which is essentially the *Euler angle* between neutrons and protons. This flatness lets us think of a rotational motion in free space, in which the energy of the system is independent of its Euler angle with respect to the laboratory system. According to the analysis for our collective wave function [43], a free scissors motion is just a free rotation of neutrons (protons) seen from the intrinsic frame of the protons (neutrons). Therefore, the evolution from Fig. 4 (a) to (c), corresponding to changes from large to weak deformation, may be viewed as an evolution from the scissors vibration to

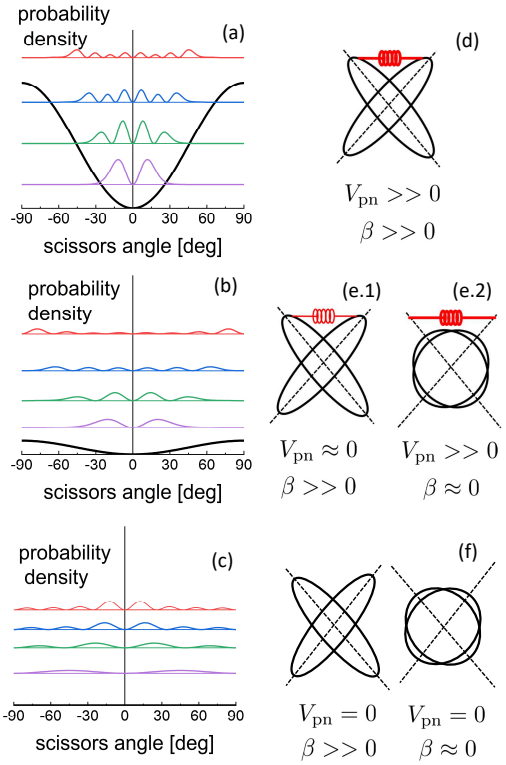


FIG. 4. (Color online) Illustration of the scissors motion in strongly and weakly deformed nuclei, as well as in the limit case: a free scissors motion. (a) Schematic plot of the scissors potential (black line) as function of the scissors angle, together with corresponding scissors angle distribution (colored lines), for well deformed nuclei with a strong restoring force as illustrated in the cartoon representation in (d). (b) Similar to (a), but for well deformed nuclei with a weak restoring force, as illustrated in the cartoon representation in (e.1), or for weakly deformed nuclei with a strong restoring force, as illustrated in (e.2). (c) Similar to (a), but for the limit case of free scissors motion, obtained with a vanishing neutron-proton interaction for well deformed and near spherical nuclei, as illustrated in (f).

"scissors rotation". The LEE, whose origin has remained elusive, can be interpreted as a manifestation of such a scissors rotation.

We note that the usual terminology for nuclear "rotation" [49] refers to the motion of the whole system in Euler angles. However, the concept of "scissors rotation" here refers to the relative rotation between the proton and neutron systems, which is a motion in the scissors angle, a degree of freedom totally different from the usual rotation.

In conclusion, the present work intended to solve the challenging problem to understand the underlying physics of the LEE observed in the γSF, a quantity frequently used to obtain nuclear capture reactions for fundamental research and applications. By using a novel shell model constructed in the angular-momentum-projected framework and taking the weakly-deformed nucleus ^{144}Nd as an example, we found that the LEE arises from a quasi-free scissors motion defined in the intrinsic frame where the neutrons (protons) rotate almost freely with respect to the body system of protons (neutrons). The transition in the γSF, from the scissors resonance to the LEE, can be described as an evolution from the scissors vibration to a completely new kind of collective mode proposed in the present work: the scissors rotation. The experimentally observed LEE can be regarded as the first experimental evidence for this collective excitation mode.

This explanation further establishes an ultimate relationship between the LEE and the scissors motion, which was initially indicated in Ref. [28]. Our work naturally explains the fact that most of the nuclei with observed LEE are weakly deformed. It predicts that observation of the LEE should not be confined in selected mass regions: The LEE is a universal effect occurring in all weakly-deformed nuclei across the nuclear chart.

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[1] G. A. Bartholomew, E. D. Earle, A. J. Fergusson, J. W. Knowles, M. A. Lone, Adv. Nucl. Phys. **7**, 229 (1973).
[2] W. Hauser and H. Feshbach, Phys. Rev. **87**, 366 (1952).
[3] M. Wiedeking and S. Goriely, Phil. Trans. R. Soc. A 382, 20230125 (2024).
[4] J. J. Cowan, C. Sneden, J. E. Lawler, A. Aprahamian, M. Wiescher, K. Langanke, G. Martínez-Pinedo, F.-K. Thielemann, Rev. Mod. Phys. **93**, 015002 (2021).
[5] M. Arnould, S. Goriely, Prog. Part. Nucl. Phys. **112**, 103766 (2020).
[6] S. Goriely *et al*, Eur. Phys. J. A **55**, 172 (2019).
[7] A. Voinov *et al*, Phys. Rev. Lett. **93**, 142504 (2004).
[8] M. Wiedeking, L. A. Bernstein, M. Krtička, D. L. Bleuel, J. M. Allmond, M. S. Basunia *et al*, Phys. Rev. Lett. **108**, 162503 (2012).
[9] M. Krtička, M. Wiedeking, F. Bečvář, S. Valenta, Phys. Rev. C **93**, 054311 (2016).
[10] M. D. Jones *et al*, Phys. Rev. C **97**, 024327 (2018).
[11] A. C. Larsen, N. Blasi, A. Bracco, F. Camera, T. K. Eriksen, A. Gorgen *et al*, Phys. Rev. Lett. **111**, 242504 (2013)
[12] A C Larsen *et al*, J. Phys. G: Nucl. Part. Phys. 44, 064005 (2017).
[13] J. E. Midtbø, A. C. Larsen, T. Renstrøm, F. L. Bello Garrote, E. Lima, Phys. Rev. C **98**, 064321 (2018).
[14] S. N. Liddick *et al*, Phys. Rev. C **100**, 024624 (2019).
[15] F. Naqvi *et al*, Phys. Rev. C **99**, 054331 (2019).
[16] V. W. Ingeberg *et al*, Eur. Phys. J. A **56**, 68 (2020).
[17] M. Guttormsen *et al*, Phys. Rev. C **106**, 034314 (2022).
[18] V. W. Ingeberg *et al*, Phys. Rev. C **106**, 054315 (2022).
[19] M. Markova, A. C. Larsen, G. M. Tveten, P. vonNeumann-Cosel, T. K. Eriksen, F. L. Bello Garrote *et al*, Phys. Rev. C **108**, 014315 (2023).
[20] V. W. Ingeberg *et al*, submitted to Phys. Rev. C.
[21] A. C. Larsen, S. Goriely, Phys. Rev. C **82**, 014318 (2010).
[22] E. Litvinova, N. Belov, Phys. Rev. C **88**, 031302(R) (2013).
[23] K. Sieja, Phys. Rev. Lett. **119**, 052502 (2017).
[24] R. Schwengner, S. Frauendorf, A. C. Larsen, Phys. Rev. Lett. **111**, 232504 (2013).
[25] M. Guttormsen *et al*, Phys. Rev. C **71**, 044307 (2005).
[26] S. Frauendorf, Rev. Mod. Phys. **73**, 463 (2001).
[27] S. Frauendorf, R. Schwengner, Phys. Rev. C **105**, 034335 (2022).
[28] R. Schwengner, S. Frauendorf, B. A. Brown, Phys. Rev. Lett. **118**, 092502 (2017).
[29] N. Lo Iudice, F. Palumbo, Phys. Rev. Lett. **41**, 1532 (1978).
[30] S. Frauendorf, WSPC Proceeding of the 6th International Conference on Fission and Properties of Neutron-Rich Nuclei, eds. J. H. Hamilton, R. V. Ramayya, P. Talou, World Scientific: Singapore (2017), p. 37-46.
[31] A. C. Larsen *et al*, Phys. Rev. C **97**, 054329 (2018).
[32] P. Fanto, Y. Alhassid, Phys. Rev. C **109**, L031302 (2024).

[33] A. Simon *et al.*,Phys. Rev. C **93**, 034303 (2016).
[34] S. P. E. Magagula *et al.*, J. Phys.: Conf. Ser. **2586**, 012070 (2023)
[35] S. Siem *et al.*, Phys. Rev. C **65**, 044318 (2002).
[36] K. L. Malatji *et al*, Phys. Rev. C **103**, 014309 (2021).
[37] K. Heyde, P. von Neumann-Cosel, A. Richter, Rev. Mod. Phys. **82**, 2365 (2010).
[38] Y. Sun, C.-L. Wu, K. Bhatt, M. Guidry, D. H. Feng, Phys. Rev. Lett. **80**, 672 (1998).
[39] Y. Sun, C.-L. Wu, K. Bhatt and M. Guidry, Nucl. Phys. A **703**, 130 (2002).
[40] K. Hara, Y. Sum, Int. J. Mod. Phys. **4**, 637 (1995).
[41] C.-J. Lv, F.-Q. Chen, Y. Sun, M. Guidry, Phys. Rev. Lett. **129**, 042502 (2022).
[42] See Supplemental Material at https://journals.aps.org/prl/ supplemental/10.1103/PhysRevLett.129.042502 for more details on the angular-momentum-projected model framework adopted in this work.
[43] See Supplemental Material to the present Letter.
[44] P. Ring, P. Shuck, *The Nuclear Many-Body Problem*, Springer-Verlag, Berlin Heidelberg 1980.
[45] IAEA Nuclear Data Services, /https://www-nds.iaea.org/PSFdatabase.
[46] N. Lo Iudice, RIVISTA DEL NUOVO CIMENTO **23**, 9 (2000).
[47] V. Zelevinsky, B. A. Brown, N. Frazier, M. Horoi, Phys. Rep. **276**, 85 (1996).
[48] R. Nojarov, Z. Bochnacki, and A. Faessler, Z. Phys. A **324**, 289 (1986).
[49] A. Bohr, B. R. Mottelson, *Nuclear Structure, Vol. II: Nuclear Deformations* (W. A. Benjamin, 1975).