



This is the accepted manuscript made available via CHORUS. The article has been published as:

Accelerated Decay due to Operator Spreading in Bulk-Dissipated Quantum Systems

Tatsuhiko Shirai and Takashi Mori

Phys. Rev. Lett. **133**, 040201 — Published 24 July 2024

DOI: [10.1103/PhysRevLett.133.040201](https://doi.org/10.1103/PhysRevLett.133.040201)

Accelerated Decay due to Operator Spreading in Bulk-Dissipated Quantum Systems

Tatsuhiko Shirai¹ and Takashi Mori²

¹Waseda Institute for Advanced Study, Waseda University, Nishi Waseda, Shinjuku-ku, Tokyo 169-0051, Japan

²Department of Physics, Keio University, Kohoku-ku, Yokohama, Kanagawa 223-8522, Japan

Markovian open many-body quantum systems display complicated relaxation dynamics. The spectral gap of the Liouvillian characterizes the asymptotic decay rate towards the stationary state, but it has recently been pointed out that the spectral gap does not necessarily determine the overall relaxation time. Our understanding on the relaxation process before the asymptotically long-time regime is still limited. We here present a collective relaxation dynamics of autocorrelation functions in the stationary state. As a key quantity in the analysis, we introduce the instantaneous decay rate, which characterizes the transient relaxation and converges to the conventional asymptotic decay rate in the long-time limit. Our theory predicts that a bulk-dissipated system generically shows an accelerated decay before the asymptotic regime due to the scrambling of quantum information associated with the operator spreading.

Introduction.— It is a fundamental problem in nonequilibrium statistical physics to elucidate how a quantum system approaches to stationarity under dissipative couplings to large environments [1–3]. This problem has been investigated mainly for small quantum systems, whereas our understanding on the nonequilibrium dynamics of open *many-body* quantum systems has still been limited. The recent experimental progress using ultracold atoms and trapped ions has made us to introduce controlled dissipation to realize many-body quantum systems with desired properties [4–8]. The experimental background also motivates us to study the generic dynamical properties of open many-body quantum systems.

Recently, it was pointed out that open many-body quantum systems exhibit counterintuitive dynamical features. The dynamics of Markovian open quantum systems are generated by the Liouvillian superoperator of the celebrated Lindblad form [9, 10]. The spectral gap of the Liouvillian, which is simply called the Liouvillian gap, gives the asymptotic decay rate [3], and thus it is expected that the relaxation time is given by the inverse of the Liouvillian gap. However, it was found that the relaxation time can be much longer than the inverse of the Liouvillian gap when there is a conserved current in the bulk (it is always the case when the dissipation acts only at the boundaries of the system) [11–14]. The point is that the crossover time into the asymptotic regime is very long and even diverging in the thermodynamic limit. If the transient regime is dominant for the overall relaxation process, the decay rate in the transient regime determines the relaxation time. Hence, we should investigate the relaxation dynamics in the transient regime, which is the main focus of our work.

In this Letter, we discuss the transient dynamics of quantum many-body systems under *bulk dissipation*. In contrast to boundary dissipation, we find that the relaxation time is generically much *shorter* than the inverse of the Liouvillian gap in bulk-dissipated systems. Based on a rigorous inequality on the autocorrelation function, we introduce a key quantity called the instantaneous decay rate. We argue that the instantaneous decay rate exhibits three distinct dynamical regimes in the weak dissipation regime: (1) the accelera-

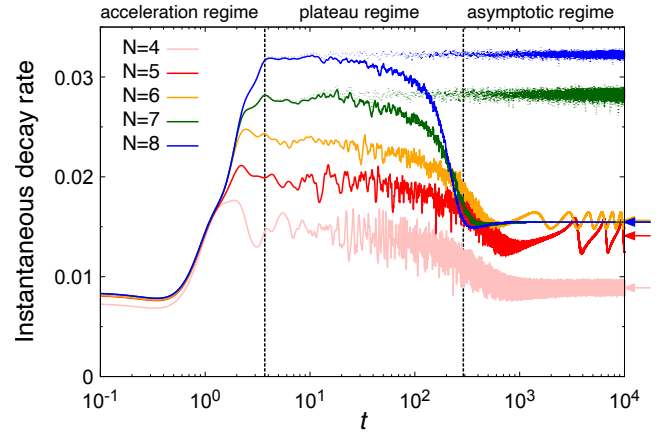


FIG. 1. Dynamics of the instantaneous decay rate $\kappa_A(t)$ with $\hat{A} = \sigma_1^z - \langle \sigma_1^z \rangle_{ss}$ for various system sizes in the bulk-dissipated Ising spin chain (see the model in Eq. (10) and Eq. (11)). Solid lines show $\kappa_A(t)$ for $N = 4, 5, 6, 7, 8$ from bottom to top, while dotted lines show $\kappa_A^{(0)}(t)$ for $N = 7, 8$. The arrows indicate the values of the Liouvillian gap.

tion regime, (2) the plateau regime, and (3) the asymptotic regime. Figure 1 illustrates dynamics of the instantaneous decay rate for various system sizes in a bulk-dissipated system. In the acceleration regime, the decay rate increases with time. This is a universal phenomenon under bulk dissipation, and this accelerated decay is explained by the operator-size growth (or the operator spreading) under unitary time evolution without dissipation. In the plateau regime, the decay rate is saturated at a constant value proportional to γN , where γ and N are the strength of dissipation and the system size, respectively. In the asymptotic regime, the decay rate converges to the asymptotic value, which is identical to the Liouvillian gap.

In the following, we first provide the general set up, and then present the main result. We demonstrate the relevance of the theoretical results in a bulk-dissipated spin chain.

Set up.— This work adopts the same set up as [15]. We consider a finite quantum system, whose density matrix $\rho(t)$

obeys the Lindblad equation [1]:

$$\frac{d\rho(t)}{dt} = \mathcal{L}\rho(t),$$

$$\mathcal{L}\rho = -i[\hat{H}, \rho] + \gamma \sum_k \left(\hat{L}_k \rho \hat{L}_k^\dagger - \frac{1}{2} \left\{ \hat{L}_k^\dagger \hat{L}_k, \rho \right\} \right), \quad (1)$$

where $\mathcal{L}$ is the Liouvillian superoperator, $\hat{H}$ is the bulk Hamiltonian, and $\{\hat{L}_k\}$ is a set of jump operators. The Planck constant $\hbar$ is set to unity. We assume that the stationary state is unique. The density matrix of the stationary state ρ_{ss} is described by $\mathcal{L}\rho_{ss} = 0$. The Lindblad equation ensures the complete positivity [9, 10].

Let $\hat{A}$ be an Hermitian operator. The autocorrelation function for $\hat{A}$ in the stationary state is expressed as [16]

$$C_{AA}(t) := \langle \hat{A}(t), \hat{A} \rangle_{ss}, \quad (2)$$

where $\langle \hat{A}, \hat{B} \rangle_{ss} = \text{Tr}(\hat{A}^\dagger \hat{B} \rho_{ss})$. We can assume $\langle \hat{A} \rangle_{ss} := \text{Tr}(\hat{A} \rho_{ss}) = 0$ without loss of generality. Here, $\hat{A}(t)$ represents the time-evolved operator in the Heisenberg picture: $\hat{A}(t) = e^{\tilde{\mathcal{L}}t} \hat{A}$, where the superoperator $\tilde{\mathcal{L}}$ is defined as

$$\tilde{\mathcal{L}}\hat{A} = i[\hat{H}, \hat{A}] + \gamma \sum_k \left(\hat{L}_k^\dagger \hat{A} \hat{L}_k - \frac{1}{2} \left\{ \hat{L}_k^\dagger \hat{L}_k, \hat{A} \right\} \right). \quad (3)$$

The eigenvalue spectrum of $\tilde{\mathcal{L}}$ is the same as that of $\mathcal{L}$. Since any eigenvalue has a non-positive real part, we sort the eigenvalues denoted by $\{\lambda_\alpha\}$ in the descending order: $0 = \lambda_0 > \text{Re } \lambda_1 \geq \text{Re } \lambda_2 \geq \dots$. The Liouvillian gap g is defined as

$$g = -\text{Re } \lambda_1. \quad (4)$$

The Liouvillian gap determines the asymptotic decay rate of $C_{AA}(t)$ as

$$|C_{AA}(t)| \sim e^{-gt} \quad (t \rightarrow \infty). \quad (5)$$

In this work, we discuss the transient dynamics before reaching the asymptotic regime. Recent work introduced the symmetrized Liouvillian to study the relaxation dynamics in a transient regime [15]:

$$\tilde{\mathcal{L}}_s = \frac{\tilde{\mathcal{L}} + \tilde{\mathcal{L}}^*}{2}, \quad (6)$$

where the superoperator $\tilde{\mathcal{L}}^*$ is the adjoint superoperator of $\tilde{\mathcal{L}}$ associated with the inner product $\langle \hat{A}, \hat{B} \rangle_{ss}$:

$$\langle \hat{A}, \tilde{\mathcal{L}}\hat{B} \rangle_{ss} = \langle \tilde{\mathcal{L}}^*\hat{A}, \hat{B} \rangle_{ss}, \quad (7)$$

which is expressed as $\tilde{\mathcal{L}}^*\hat{A} = \mathcal{L}(\hat{A}\rho_{ss})\rho_{ss}^{-1}$ [17]. We assume that ρ_{ss} is invertible for simplicity. The symmetrized Liouvillian is a non-positive and Hermitian superoperator (i.e., $\tilde{\mathcal{L}}_s \leq 0$ and $\tilde{\mathcal{L}}_s^* = \tilde{\mathcal{L}}_s$). The non-positivity is shown by using the complete positivity of $\mathcal{L}$. We sort the eigenvalues of $\tilde{\mathcal{L}}_s$ denoted by $\{s_\alpha\}$ in the descending order: $0 = s_0 \geq s_1 \geq$

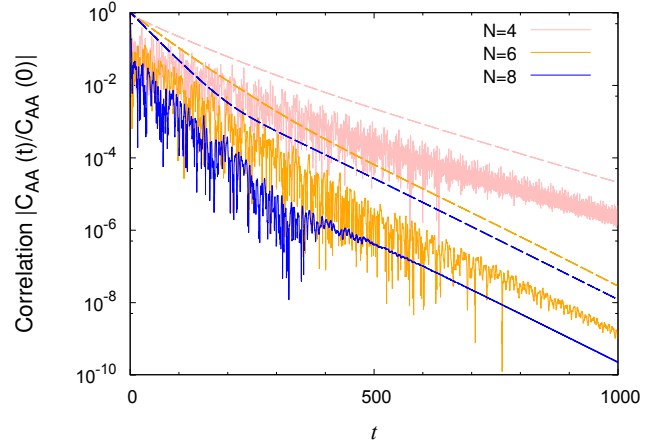


FIG. 2. Dynamics of the autocorrelation function $C_{AA}(t)$ with $\hat{A} = \hat{\sigma}_1^z - \langle \hat{\sigma}_1^z \rangle_{ss}$ for various system sizes in the bulk-dissipated Ising spin chain. Solid lines show $|C_{AA}(t)/C_{AA}(0)|$ for $N = 4, 6, 8$ from top to bottom. Dashed lines show the upper bound of the correlation functions in Eq. (12). Although the upper bounds are not tight, they correctly capture large decay rates appearing in the transient regime.

$s_2 \geq \dots$. $\tilde{\mathcal{L}}_s$ has a zero eigenvalue: $\tilde{\mathcal{L}}_s \hat{I} = 0$, where $\hat{I}$ is the identity operator. The spectral gap g_s of $\tilde{\mathcal{L}}_s$ is defined by

$$g_s = -s_1. \quad (8)$$

The spectral gap gives a simple bound on the autocorrelation function in the stationary state [15]:

$$|C_{AA}(t)| \leq e^{-g_s t} C_{AA}(0). \quad (9)$$

However, as we see in the following, we need a more sophisticated analysis beyond the spectral gap bound like Eq. (9) to describe generic relaxation dynamics of bulk-dissipated quantum many-body systems.

Instantaneous decay rate.—Let us investigate how the autocorrelation function in the stationary state decays. Although the discussion below is general, for clarity we consider a bulk-dissipated Ising spin chain under the periodic boundary condition [18]. The bulk Hamiltonian is given by

$$\hat{H} = \sum_{i=1}^N (h^z \hat{\sigma}_i^z + h^x \hat{\sigma}_i^x + J \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z), \quad (10)$$

where $\hat{\sigma}_i^{x,y,z}$ denotes the Pauli matrices at site i . We fix the parameters as $h^z = 0.9045$, $h^x = 0.809$, and $J = 1$, for which eigenstate thermalization hypothesis has been numerically shown to hold [19]. Each site i is associated with the jump operator $\hat{L}_i$ given by

$$\hat{L}_i = \hat{\sigma}_i^- = \frac{1}{2}(\hat{\sigma}_i^x - i\hat{\sigma}_i^y). \quad (11)$$

We assume weak dissipation and set $\gamma = 0.01$. This open quantum system has been implemented on Rydberg atoms under laser driving and dissipation [20, 21]. Each Rydberg

atom is regarded as a two-level system, where the spin up (down) corresponds to the Rydberg (ground) state. In this model, the Liouvillian gap is independent of the system size (i.e., $g \sim O(N^0)$). See Fig. 1.

Figure 2 shows the autocorrelation functions of a local operator $\hat{A} = \hat{\sigma}_1^z - \langle \hat{\sigma}_1^z \rangle_{ss}$ for various system sizes by solid lines. In the long-time limit, the autocorrelation function decays at the rate of the Liouvillian gap. There is a different decay rate appearing in the transient regime. The decay rate is much larger than the Liouvillian gap.

To understand how the transient dynamics emerges, we give an upper bound of the autocorrelation function:

$$|C_{AA}(t)| \leq e^{-\int_0^t ds \kappa_A(s)} C_{AA}(0). \quad (12)$$

Here, we introduce the *instantaneous decay rate* $\kappa_A(t)$. The instantaneous decay rate is defined by using the symmetrized Liouvillian $\tilde{\mathcal{L}}_s$ as

$$\kappa_A(t) = -\frac{\langle \hat{A}(t), \tilde{\mathcal{L}}_s \hat{A}(t) \rangle_{ss}}{\langle \hat{A}(t), \hat{A}(t) \rangle_{ss}}. \quad (13)$$

We prove the inequality. The autocorrelation function is generally bounded as follows:

$$|C_{AA}(t)| \leq \sqrt{\langle \hat{A}(t), \hat{A}(t) \rangle_{ss}} \sqrt{\langle \hat{A}, \hat{A} \rangle_{ss}}, \quad (14)$$

where we have used the Cauchy-Schwarz inequality $|\langle \hat{A}, \hat{B} \rangle_{ss}| \leq \sqrt{\langle \hat{A}, \hat{A} \rangle_{ss}} \sqrt{\langle \hat{B}, \hat{B} \rangle_{ss}}$. The time evolution of the quantity $\langle \hat{A}(t), \hat{A}(t) \rangle_{ss}$ is given by

$$\frac{d}{dt} \langle \hat{A}(t), \hat{A}(t) \rangle_{ss} = 2 \langle \hat{A}(t), \tilde{\mathcal{L}}_s \hat{A}(t) \rangle_{ss}. \quad (15)$$

Equation (15) is then formally solved as

$$\langle \hat{A}(t), \hat{A}(t) \rangle_{ss} = e^{-2 \int_0^t ds \kappa_A(s)} \langle \hat{A}, \hat{A} \rangle_{ss}. \quad (16)$$

By substituting it into Eq. (14), we obtain Eq. (12). The instantaneous decay rate is actually the decay rate of the quantity $\sqrt{\langle \hat{A}(t), \hat{A}(t) \rangle_{ss}}$, and it gives an upper bound on the autocorrelation function as in Eq. (12).

Now we list important properties of the instantaneous decay rate. Firstly, it is non-negative $\kappa_A(t) \geq 0$ since $\tilde{\mathcal{L}}_s$ is a non-positive superoperator. Secondly, $\kappa_A(t)$ is not smaller than the spectral gap g_s of $\tilde{\mathcal{L}}_s$:

$$\kappa_A(t) \geq g_s. \quad (17)$$

This is proved by using the following property:

$$g_s = \inf_{\hat{X} \neq 0, \langle \hat{X} \rangle_{ss} = 0} \frac{\langle \hat{X}, \tilde{\mathcal{L}}_s \hat{X} \rangle_{ss}}{\langle \hat{X}, \hat{X} \rangle_{ss}}. \quad (18)$$

The condition $\langle \hat{X} \rangle_{ss} = \langle \hat{I}, \hat{X} \rangle_{ss} = 0$ ensures that $\hat{X}$ is orthogonal to $\hat{I}$, which is the eigenvector of $\tilde{\mathcal{L}}_s$ corresponding to the zero eigenvalue, and thus g_s is expressed in the variational

form as in Eq. (18). Equation (17) indicates that the instantaneous decay rate gives a simple bound on the autocorrelation function, i.e. Eq. (9), but it cannot capture the three dynamic regimes manifest in Fig. 1.

Thirdly, the instantaneous decay rate converges to the asymptotic decay rate (i.e. the Liouvillian gap) in the long-time limit:

$$\lim_{t \rightarrow \infty} \kappa_A(t) = g, \quad (19)$$

when $\text{Tr}(\hat{A}\rho_1) \neq 0$ and $\text{Im } \lambda_n = 0$ for every n with $\text{Re } \lambda_n = -g$.

Accelerated decays due to operator spreading.— To gain insights into the relaxation dynamics in Fig. 2, we investigate how $\kappa_A(t)$ behaves at transient times for sufficiently small γ with a fixed system size N .

Figure 1 illustrates the dynamics of $\kappa_A(t)$ by solid lines. We find three distinct dynamic regimes: acceleration regime, plateau regime, and asymptotic regime. $\kappa_A(t)$ initially increases in the acceleration regime, has an almost constant value in the plateau regime, and decreases and converges to the Liouvillian gap in the asymptotic regime. The growth of $\kappa_A(t)$ implies the acceleration of the decay.

For weak dissipation, it is expected that the time evolution of $\hat{A}(t)$ appearing in the instantaneous decay rate, i.e. Eq. (13), would be well approximated by the unitary dynamics without dissipation. The instantaneous decay rate is thus approximated as

$$\kappa_A^{(0)}(t) = -\frac{\langle \hat{A}_0(t), \tilde{\mathcal{L}}_s \hat{A}_0(t) \rangle_{ss}}{\langle \hat{A}_0(t), \hat{A}_0(t) \rangle_{ss}}, \quad \hat{A}_0(t) = e^{i\hat{H}t} \hat{A} e^{-i\hat{H}t}. \quad (20)$$

Figure 1 shows the dynamics of $\kappa_A^{(0)}(t)$ for $N = 7$ and 8. The approximated one correctly reproduces the instantaneous decay rate in the acceleration regime and the plateau regime [22]. It indicates that the growth of the instantaneous decay rate arises due to the unitary dynamics and dissipation is relevant only for the crossover to the asymptotic regime.

We now explain the growth of the instantaneous decay rate from the viewpoint of operator spreading. Figure 3 gives a schematic of operator spreading in a one-dimensional quantum spin system. Let us consider a local operator $\hat{A}$ acting to a single site at initial time. In the figure, colored circles denote the sites where the operator $\hat{A}_0(t)$ nontrivially acts. Under unitary dynamics, the operator spreads over the system and the number of colored sites linearly increases with time [23, 24]. Here, the number of colored sites represents the operator size. Then, we consider the effect of bulk dissipation. The instantaneous decay rate is roughly estimated as the product of γ and the operator size. Suppose an operator $\hat{O}_i$ acting on a single site i . The operator decays with a decay rate of γ since the dissipation independently acts on every site with strength γ . Then, the operator acting to three sites such as $\hat{O}_{i-1}\hat{O}_i\hat{O}_{i+1}$ decays with a decay rate of 3γ . In this way, the instantaneous decay rate accelerates due to the operator spreading in the acceleration regime.

In the plateau regime, the instantaneous decay rate is saturated to a value proportional to N . The plateau regime is

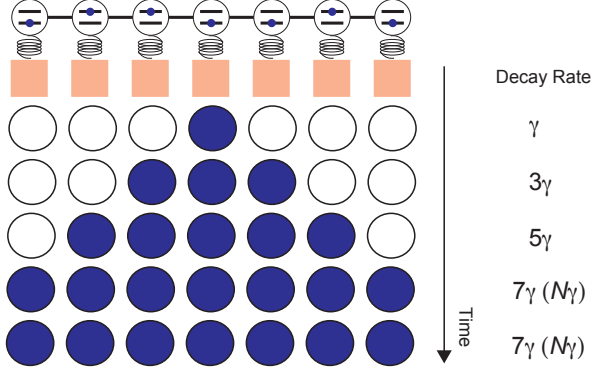


FIG. 3. Mechanism of accelerated decay in a bulk-dissipated quantum chain with $N = 7$ spins. The vertical axis denotes the time. Colored circles represent sites where an operator $\hat{A}^{(0)}(t)$ nontrivially acts.

also explained from operator spreading. In this regime, the operator $\hat{A}^{(0)}(t)$ spreads across the entire system, and hence the operator size is N . Using the relation between the operator size and the instantaneous decay rate, $\kappa_A(t) \sim \gamma N$. The dissipation exhibits a collective decay in this regime.

In the asymptotic regime, the instantaneous decay rate converges to the Liouvillian gap g . The oscillation around the Liouvillian gap in Fig. 1 implies that $\text{Im} \lambda_1 \neq 0$. We find in supplemental material that the crossover time into the asymptotic regime is inversely proportional to γ for weak dissipation [18]. Thus, the plateau regime is longer for weaker dissipation.

We develop a more quantitative theory explaining those three dynamical regimes. In the following argument, we assume $\gamma \ll N^{-1}$, which is relevant for this study. Later, we will briefly discuss what happens when $N \rightarrow \infty$ is taken at a fixed γ . The instantaneous decay rate $\kappa_A(t)$ obeys the following equation [18]:

$$\frac{d\kappa_A(t)}{dt} = v_A - 2\delta\kappa_A^2, \quad (21)$$

where

$$v_A = \frac{\langle \hat{A}(t), [\tilde{\mathcal{L}}, \tilde{\mathcal{L}}_s] \hat{A}(t) \rangle_{ss}}{\langle \hat{A}(t), \hat{A}(t) \rangle_{ss}}, \quad (22)$$

$$\delta\kappa_A^2 = \frac{\langle \hat{A}(t), (-\tilde{\mathcal{L}}_s - \kappa_A(t))^2 \hat{A}(t) \rangle_{ss}}{\langle \hat{A}(t), \hat{A}(t) \rangle_{ss}}.$$

This formula generalizes the result in [25] to any open quantum systems obeying Lindblad equations. v_A/γ represents how fast the operator size of $A(t)$ spreads with time and $(\delta\kappa_A/\gamma)^2$ roughly gives the variance of the operator size. Note that the time evolution of $A(t)$ in v_A is well approximated by the unitary dynamics without dissipation [18].

The formula accurately explains the observed three dynamic regimes in Fig. 1. In the acceleration regime, recent studies on random unitary circuits [23, 24] indicate $v_A \sim \gamma$ and $\delta\kappa_A^2 \sim \gamma^2 t$. Since $v_A \gg \delta\kappa_A^2$ for small γ ,

$$\kappa_A(t) \sim \gamma t \quad (t \lesssim N). \quad (23)$$

The system enters the plateau regime when the operator spreads across the entire system at $t \sim N$. Accordingly, $\kappa_A(t)$ is proportional to γN . In this regime, $v_A \approx 0$, whereas $\delta\kappa_A^2 \sim \gamma^2 N$ [18]. Thus, during the duration time Δt in the plateau regime, $\kappa_A(t)$ reduces by the amount proportional to $\gamma^2 N \Delta t$. This decrease is negligible compared with the plateau value $\sim \gamma N$ up to $\Delta t \sim \gamma^{-1}$. We therefore conclude that $\kappa_A(t)$ approximately remains constant up to $t \sim \gamma^{-1}$:

$$\kappa_A(t) \sim \gamma N \quad (N \lesssim t \lesssim \gamma^{-1}). \quad (24)$$

When $t \gtrsim \gamma^{-1}$, the variation of $\kappa_A(t)$ becomes the same order of magnitude as the plateau value. Then the system shows a crossover to the asymptotic regime and $\kappa_A(t)$ converges to the Liouvillian gap.

Here, we briefly explain the dynamics of $\kappa_A(t)$ in the thermodynamic limit (i.e., $N \rightarrow \infty$ before taking $\gamma \ll 1$). Equation (21) with $v_A \sim \gamma$ and $\delta\kappa_A^2 \sim \gamma^2 t$ implies that the operator-size growth stops at $t \sim \gamma^{-1}$ due to dissipation. The peak of $\kappa_A(t)$ is independent of γ and N . Subsequently, $\kappa_A(t)$ gradually approaches the Liouvillian gap $g \sim \gamma$. In this way, there is no plateau regime in the thermodynamic limit, but the amplification of dissipation via the operator spreading occurs, which yields a finite decay rate even for an infinitesimally small γ .

Finally we compare the exact autocorrelation function with its upper bound. In Fig. 2, we plot the autocorrelation $|C_{AA}(t)/C_{AA}(0)|$ by solid lines, whereas the upper bound $e^{-\int_0^t ds \kappa_A(s)}$ by dashed lines. The autocorrelation function shows a rapid decay due to the unitary dynamics at short times. This effect is not taken into account in the upper bound. Thus, we find that the inequality in Eq. (12) is not tight. However, the upper bound qualitatively reproduces the autocorrelation function at large times. In particular, the relaxation dynamics in the transient regime is well described by the plateau value of the instantaneous decay rate. The plateau value is proportional to the system size, which is consistent with the numerical observation that a larger system shows a faster decay. Thus, the autocorrelation function for a local operator exhibits a collective decay due to the operator spreading.

Summary and Discussion.— We studied the autocorrelation functions for a local operator in the stationary state in bulk-dissipated quantum many-body systems. In the transient regime, the correlation exhibits a fast relaxation with a decay rate much larger than the asymptotic one. We derived a rigorous upper bound on the autocorrelation function. We demonstrated that the instantaneous decay rate shows a plateau over a long time interval (Fig. 1), which explains the collective decay found in the autocorrelation function (Fig. 2).

We explained the mechanism of the accelerated decay from the operator spreading. This mechanism is generic in quantum many-body systems under weak bulk dissipation. In recent cold-atom experiments, bulk dissipation has been introduced in a controlled manner [8]. Ultracold atoms thus provide an experimental platform to confirm the collective relaxations predicted in this work. Also this set up is relevant to the field of quantum computation using noisy intermediate-scale quantum devices [26].

As a comparison of decay rates indicated by $O(N)$, we explain the difference with super-radiance. Super-radiance is a phenomenon that occurs when N atoms couple with a common dissipative environment [27]. Therefore, it is qualitatively different from the accelerated decay in this work, in which the dissipation independently acts on each site.

Although this work focuses on one-dimensional systems, the accelerated decay should also appear in high-dimensional systems. Namely, the instantaneous decay rate increases as $\kappa_A(t) \sim \gamma t^d$ where d is a spatial dimension in the acceleration regime, $\kappa_A(t) \sim \gamma N$ in the plateau regime, and $\kappa_A(t) = g$ in the asymptotic regime. The crossover time between the plateau regime and the asymptotic regime should be longer for weaker dissipation. It is a future problem to study the explicit γ dependence of the crossover time in two or three dimensional systems.

This work was supported by JSPS KAKENHI (Grant Numbers JP21H05185 and 23K13034) and by JST, PRESTO Grant No. JPMJPR2259.

-
- [1] H.-P. Breuer and F. Petruccione, *The theory of open quantum systems* (Oxford Univ. Press, 2002).
 - [2] H. Spohn, *Rev. Mod. Phys.* **52**, 569 (1980).
 - [3] M. Žnidarič, *Phys. Rev. E* **92**, 042143 (2015).
 - [4] S. Diehl, A. Micheli, A. Kantian, B. Kraus, H. P. Büchler, and P. Zoller, *Nature Physics* **4**, 878 (2008).
 - [5] F. Verstraete, M. M. Wolf, and J. Ignacio Cirac, *Nature Physics*

- 5**, 633 (2009).
- [6] J. T. Barreiro, M. Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller, and R. Blatt, *Nature* **470**, 486 (2011).
- [7] G. Barontini, R. Labouvie, F. Stubenrauch, A. Vogler, V. Guarrera, and H. Ott, *Phys. Rev. Lett.* **110**, 035302 (2013).
- [8] T. Tomita, S. Nakajima, I. Danshita, Y. Takasu, and Y. Takahashi, *Science Advances* **3**, e1701513 (2017), <https://www.science.org/doi/pdf/10.1126/sciadv.1701513>.
- [9] G. Lindblad, *Comm. Math. Phys.* **48**, 119 (1976).
- [10] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, *Journal of Mathematical Physics* **17**, 821 (1976).
- [11] T. Mori and T. Shirai, *Phys. Rev. Lett.* **125**, 230604 (2020).
- [12] T. Haga, M. Nakagawa, R. Hamazaki, and M. Ueda, *Phys. Rev. Lett.* **127**, 070402 (2021).
- [13] G. Lee, A. McDonald, and A. Clerk, *Phys. Rev. B* **108**, 064311 (2023).
- [14] T. Sawada, K. Sone, R. Hamazaki, Y. Ashida, and T. Sagawa, *Phys. Rev. Lett.* **132**, 046602 (2024).
- [15] T. Mori and T. Shirai, *Phys. Rev. Lett.* **130**, 230404 (2023).
- [16] H. Carmichael, *Statistical Methods in Quantum Optics 1: Master Equations and Fokker-Planck equations*, Vol. 1 (Springer Berlin, Heidelberg, 1998).
- [17] R. Alicki, *Reports on Mathematical Physics* **10**, 249 (1976).
- [18] See Supplemental Material at URL-will-be-inserted-by-publisher for the result of a bulk-dissipated Bose-Hubbard model, derivation of Eq. (21), and dissipation-strength dependence of instantaneous decay rate, which includes Ref. [28].
- [19] H. Kim, T. N. Ikeda, and D. A. Huse, *Phys. Rev. E* **90**, 052105 (2014).
- [20] C. Carr, R. Ritter, C. G. Wade, C. S. Adams, and K. J. Weatherill, *Phys. Rev. Lett.* **111**, 113901 (2013).
- [21] F. Letscher, O. Thomas, T. Niederprüm, M. Fleischhauer, and H. Ott, *Phys. Rev. X* **7**, 021020 (2017).
- [22] Note that there is a small deviation between $\kappa_A(t)$ and $\kappa_A^{(0)}(t)$ within the plateau regime, which linearly increases over time.
- [23] C. W. von Keyserlingk, T. Rakovszky, F. Pollmann, and S. L. Sondhi, *Phys. Rev. X* **8**, 021013 (2018).
- [24] A. Nahum, S. Vijay, and J. Haah, *Phys. Rev. X* **8**, 021014 (2018).
- [25] T. Schuster and N. Y. Yao, *Phys. Rev. Lett.* **131**, 160402 (2023).
- [26] J. Preskill, *Quantum* **2**, 79 (2018).
- [27] R. H. Dicke, *Phys. Rev.* **93**, 99 (1954).
- [28] M. Rigol, *Phys. Rev. Lett.* **103**, 100403 (2009).