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Energy gap of the even-denominator fractional quantum Hall state in bilayer graphene

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Bernal bilayer graphene hosts even denominator fractional quantum Hall states thought to be described by a Pfaffian wave function with nonabelian quasiparticle excitations. Here we report the quantitative determination of fractional quantum Hall energy gaps in bilayer graphene using both thermally activated transport and by direct measurement of the chemical potential. We find a transport activation gap of 5.1 K at $B = 12$ T for a half-filled $N = 1$ Landau level, consistent with density matrix renormalization group calculations for the Pfaffian state. However, the measured thermodynamic gap of 11.6 K is smaller than theoretical expectations for the clean limit by approximately a factor of two. We analyze the chemical potential data near fractional filling within a simplified model of a Wigner crystal of fractional quasiparticles with long-wavelength disorder, explaining this discrepancy. Our results quantitatively establish bilayer graphene as a robust platform for probing the nonabelian anyons expected to arise as the elementary excitations of the even-denominator state.

Nonabelian anyons[1] are thought to enable fault tolerant topological quantum bits through their non-trivial braiding statistics [2, 3]. In an ideal scenario, the error rate of such qubits is limited only by the density of thermally excited quasiparticles present in the system. Such processes—analogue to quasiparticle poisoning in superconducting qubits—are exponentially suppressed at low temperature by an Arrhenius law, $n_{\text{qp}} \propto \exp(-\Delta_{\text{qp}}/2k_B T)$, where Δ_{qp} is the energy gap for nonabelian quasiparticles and T is temperature. The energy gap is thus a key figure of merit for candidate nonabelian states. According to numerical calculations [4, 5], nonabelian ground states are the leading candidates to describe the even denominator fractional quantum Hall (FQH) states observed in the second orbital Landau level of single-component systems such as GaAs quantum wells [6]. While these numerical results are thought to be reliable, the small energy gaps measured for these states in GaAs [7–10] have hampered experimental efforts to directly probe nonabelian statistics via fusion and braiding of individual quasiparticles.

Within the simplest model of bilayer graphene, the $N = 0$ and $N = 1$ orbital levels are both pinned to zero energy[11]. Combined with the spin- and valley degeneracies native to graphene quantum Hall systems[12], this produces an eight-fold degeneracy—a seemingly inauspicious arena for the single-component physics of nonabelian FQH states. However, as a wealth of experimental work has shown, all of these degeneracies are lifted by the combination of electronic interactions and the applied displacement field[13–23]. In particular, broad domains of density and displacement field are characterized by partial filling of a singly degenerate $N = 0$ or $N = 1$ Landau level. In the $N = 1$ regime, an incompressible state is observed at half-integer filling[18, 21–23],

which calculations show should be described by a nonabelian Pfaffian ground state[22, 24–26]. Prior measurements of energy gaps have found activation gaps as large as 1.8 K at $B = 14$ T; however, precise comparisons of activation and thermodynamic gaps to theoretical expectations have not been previously reported.

Here we report energy gaps for both odd- and even-denominator FQH states in bilayer graphene using both transport and chemical potential measurements. Thermally activated transport measures the energy cost of creating a physically separated quasiparticle-quasihole pair. We measure activated transport using a Corbino-like geometry[27, 28], which directly probes the conductivity of the gapped, insulating bulk. Chemical potential measurements record a jump at incompressible filling factors known as the thermodynamic gap, which—in the clean limit—measures the difference between adding charge $\pm e$ to the gapped system. We measure the thermodynamic gap using a direct-current charge sensing technique based on a double-layer device[29, 30]. Combining these techniques, we find several new features, including weak FQH states at $\nu = 5/11$, $\nu = 6/11$ and $\nu = 5/9$ of a partially filled $N=1$ Landau level. Moreover, both schemes show an energy gap for a half-filled single component Landau level that is several times larger than reported to date for a candidate nonabelian state in any system [7–9, 22, 23, 31–33]. Notably, these measurement schemes effectively average over $\sim 10 \mu\text{m}^2$ sized areas, a testament to the exceptional uniformity of the electron gas in bilayer graphene.

Fig. 1A shows a schematic of the experimental geometry used to measure the chemical potential μ . A graphene bilayer hosting the FQH system of interest is separated by a 62 nm-thick hexagonal boron nitride (hBN) dielectric from a graphene monolayer that functions as a sensor. Both layers

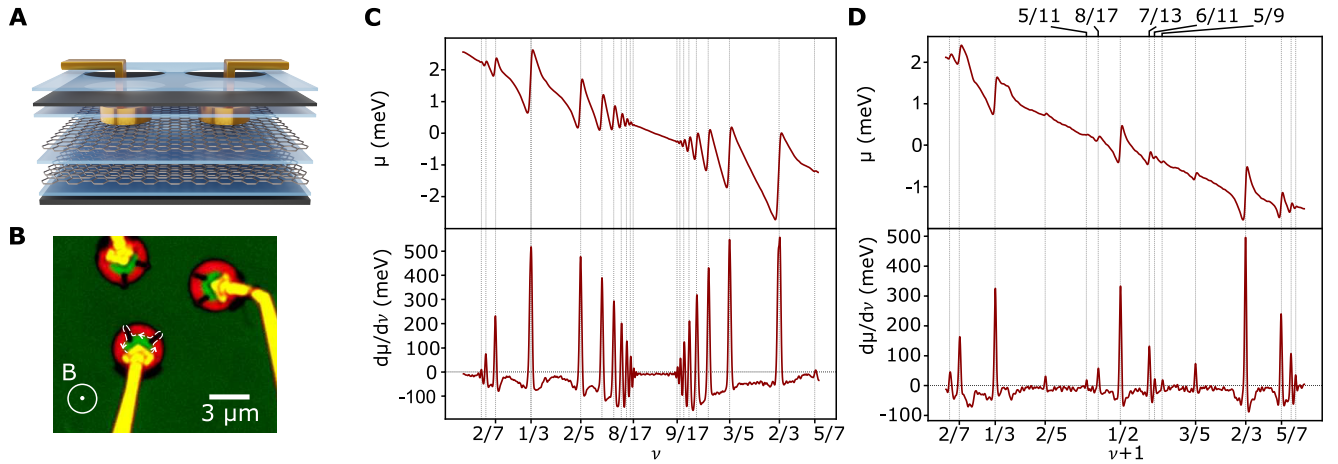


FIG. 1. **Chemical potential and inverse compressibility of bilayer graphene fractional quantum Hall states.** (A) Device schematic showing the hBN layers (blue), top and bottom graphite gates (dark grey), monolayer graphene detector layer connected to Corbino contacts, and bilayer graphene sample layer. (B) Optical image of the Corbino contacts to the monolayer graphene detector. White dashed lines show the trajectory of a chiral edge state along trenches etched through the device, which ensures contact between the metal and dual gated sample bulk. (C) The top panel shows the measured μ at $B = 13.8$ T and $T = 50$ mK in the partially filled $N=0$ level spanning $0 < \nu < 1$. The bottom panel shows the inverse compressibility, $d\mu/d\nu$, calculated by numerically differentiating the data in the top panel. (D) The same as C, but for the partially filled $N=1$ orbital Landau level spanning $-1 < \nu < 0$.

are encapsulated by additional hBN dielectrics and graphite gates, creating a four plate geometry that allows independent control of the carrier density on both the monolayer detector and bilayer sample layer. We measure Corbino transport in the detector layer, where a FQH state functions as a sensitive detector of the local potential. An optical image of the Corbino contacts is shown in Fig. 1B. As described in detail in the supplementary information, monitoring transport in the sensor layer allows us to precisely determine μ of the bilayer sample. An advantage of our technique is that it avoids finite-frequency modulation of the carrier density, allowing us to accommodate charge equilibration times as large as a second. The current technique is less invasive than previous capacitance measurements[22], requiring no modulation of the sample density and reducing heating due to cryogenic semiconductor amplifiers.

Figs. 1C-D show μ and $d\mu/d\nu$ measured in our bilayer graphene device at $B = 13.8$ T. In the $N = 0$ Landau level, incompressible spikes are observed at fillings corresponding to the two- and four-flux ‘Jain’ sequence[34], with denominators as high as 17. In the $N = 1$ orbital, a different hierarchy is observed, including a prominent state at $\nu + 1 = 1/2$ along with states at $8/17$ and $7/13$ filling. This sequence is consistent with a Pfaffian state at half filling and abelian ‘daughter’ states built from its elementary excitations[22, 35]. Additional peaks are observed at fillings consistent with the four-flux Jain sequence, at $3/5$ and $2/5$, and finally several weaker states at $5/11$, $6/11$ and $5/9$ which were not previously reported. Away from these incompressible fillings, the compressibility is negative throughout the partially filled Landau level[36, 37]. Additional negative compressibility is observed near the incompressible states, associated with the formation of Wigner

crystals of fractionally charged quasiparticles at low quasiparticle density.

Fig. 2A shows the two terminal conductance (G) in a second sample consisting of a dual gated bilayer with Corbino-like geometry (see supplementary). Measurements are taken at $B = 12$ T in a partially filled $N=1$ Landau level corresponding to filling factors $0.25 \lesssim \nu + 3 < 0.75$ (see supplementary information). The activation gap and chemical potential measurements were not performed in the same filling factor range due to constraints arising from the electrical contacts to the bilayer or sensor layer. However, the states originate from the same orbital level and differ only in their valley isospin. Consequently, they are treated theoretically in the same way, taking into account orbital levels and a single spin/valley component. The three most prominent FQH states, at $\nu + 3 = 1/3, 1/2$, and $2/3$, all show vanishing conductance at the lowest temperatures. Fig. 2B shows the minimal conductance for $\nu + 3 = 1/2$ and $2/3$ as a function of temperature, along with fits to an Arrhenius law $G \propto n_{qp} \propto e^{-\Delta_{qp}/2k_B T}$. For the $1/2$ state, the activation gap is found to be 5.1K at $B=12$ T, considerably larger than previous measurements in other two-dimensional electron systems[7–10, 32, 33]. The increase relative to the GaAs $\nu = 5/2$ state is expected due to the higher electron density and lower dielectric constant of the VdW platform. Interestingly, the gap is also much larger than previously reported in bilayer graphene[22, 23], under the same conditions. We attribute this to the Corbino geometry used here, which directly probes the bulk conductivity, avoiding systematic underestimates resulting from poor equilibration of electrical contacts with the chiral edge states[27, 28].

We may compare the result for the activation gap with a numerical calculation that accounts for the microscopic de-

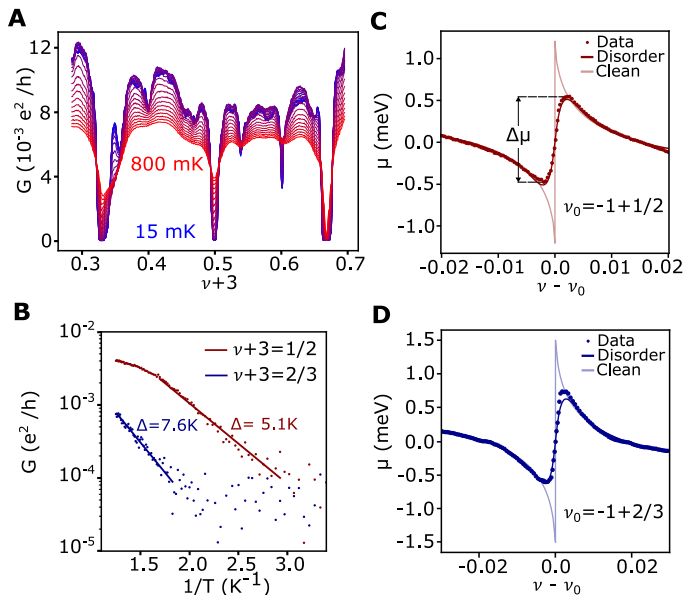


FIG. 2. **Comparison of activation and thermodynamic gaps in a partially filled $N = 1$ Landau level.** (A) Two terminal conductance measured in a Corbino geometry as a function of filling factor at $B = 12$ T for different temperatures. The temperature spacing is 5mK. (B) Activation gap from the Arrhenius fit for $\nu = -3 + 1/2$ (red) and $\nu = -3 + 2/3$ (blue). (C) Chemical potential measurement near $\nu_0 = -1 + 1/2$ (red dots) at $B = 13.8$ T. Theory fit using the Wigner crystal model in the clean limit (light red line) and in the disordered limit (red line). (D) Chemical potential measurement near $\nu_0 = -1 + 2/3$ (blue dots) at $B = 13.8$ T. Theory fit using the Wigner crystal model in the clean limit (light blue line) and in the disordered limit (blue line).

tails of bilayer graphene, accomplished using the density matrix renormalization group (DMRG) [38, 39]. Following Ref. [22], these calculations are conducted on an infinite cylinder within a 4-band model of BLG and account for mixing between the $N = 0$ and 1 Landau levels, screening from the gates, and—crucially—screening due to inter-Landau level transitions, which is treated within the random phase approximation (see supplementary information). We obtain a quasiparticle gap $\Delta_{\text{qp}}^{\text{DMRG}} = 0.011E_C$, where the Coulomb energy scale E_C depends on both the magnetic field and the dielectric constant for hBN, which we take as $\epsilon_{\text{hBN}} = \sqrt{\epsilon_{xy}\epsilon_z} = 4.5$ [30]. The calculated gap is 5.6 K at 12 T, within 10% of the experimental value.

The jump in chemical potential at fractional filling, $\Delta\mu$, provides an alternative measurement of the FQH energy gaps, as shown in Figs. 2C-D measured at $B = 13.8$ T. For the $1/2$ state in Fig.2C, the $\Delta\mu$ jump of 1.0 meV corresponds to a temperature of 11.6 K. In the clean limit, $\Delta\mu$ corresponds to the energy cost of adding a whole electron to the gapped system, and is expected to be e/e^* times larger than the quasiparticle gap, where e^* is the quasiparticle charge. At $\nu = -1/2$, where $e/e^* = 4$, the quasiparticle gap $\Delta_{\text{qp}}^{\mu} = \Delta\mu/4 = 2.9$ K implied by the thermodynamic measurement is significantly smaller than $\Delta_{\text{qp}}^{\text{act}} \approx 5.1$ K, even before accounting for the

small difference in B between Figs. 2A and C. A similar discrepancy is seen at $\nu + 1 = 2/3$, where $\Delta_{\text{qp}}^{\text{act}} = (7.6 \pm 0.5)$ K but the quasiparticle gap from thermodynamic measurements is $\Delta_{\text{qp}}^{\mu} = 5.2$ K.

We attribute the discrepancy to the contrasting role of disorder on the thermodynamic and activation gaps. In the simplest model for activated transport[40] disorder does not reduce the activation gap, while in more detailed models the activation gap is reduced by an amount that depends on the spatial correlations of the disorder potential[41, 42]. On the other hand, disorder always reduces the thermodynamic gap as it produces in-gap localized states which result in a finite compressibility. So while disorder affects both gaps, it does so through different mechanisms, and we expect the reduction of the thermodynamic gap to be larger. To assess this hypothesis, we compare our data against a phenomenological model for $\mu(\nu)$ that accounts for both the disorder and quasiparticle interactions. Our model assumes that the compressible states adjacent to the incompressible FQH states are Wigner crystals of fractionally charged quasiparticles[37, 43]. As a starting point, we compute the energy density $\mathcal{E}(\nu)$ of this pristine Wigner crystal under the assumption that the fractional point charges e^* form a triangular lattice and interact through an effective Coulomb potential which accounts for screening from the gates as well as the dielectric response of the parent gapped state. In the disorder-free limit, we obtain theoretical $\mu(\nu)$ curves in which an infinitely-sharp jump of $\Delta\mu = \frac{e}{e^*}\Delta_{\text{qp}}$ is flanked by the negative compressibility of the screened Wigner crystal (see supplementary information). As shown in Figs. 2C-D, we find this disorder-free model provides a good fit to the data at moderate quasiparticle densities, where the compressibility is strongly negative.

To account for disorder, we make the assumption that the disorder potential varies slowly in comparison with both the inter-quasiparticle distance and the distance to the gates. As described in the supplementary material, this allows us to make a local density approximation; $\mu(\nu)$ can then be solved for explicitly given the interaction energy density $\mathcal{E}(\nu)$ and the disorder distribution $P[V_D]$, which we assume to be a Gaussian of width Γ . We note that these assumptions may not be correct. For example, it will not be the case if the disorder arises from dilute Poisson-distributed charge impurities in the hBN. Nevertheless, it results in a tractable model that accounts for the competition between disorder and interactions.

Fits to this model are shown in Fig.2C-D near $\nu = -1 + 1/2$ and $\nu = -1 + 2/3$. The fit is parameterized by the quasiparticle gap $\Delta_{\text{qp}}^{\text{fit}}$, a phenomenological parameter χ which accounts for the dielectric response of the parent state, and the disorder broadening Γ (see supplementary information). We find quantitative agreement between the Wigner crystal model and experiment, providing strong evidence for a Wigner crystal of fractionalized quasiparticles. From the fit we infer $\Delta_{\text{qp}}^{\text{fit}} = 7$ K for the $1/2$ state, within 20% of $\Delta_{\text{qp}}^{\text{DMRG}} = 6.0$ K. The same analysis for the $\nu_0 = -1 + 2/3$ gives $\Delta_{\text{qp}}^{\text{fit}} = 11.6$ K, again within 20% of the $\Delta_{\text{qp}}^{\text{DMRG}} = 11.7$ K. For both fillings, we

find $\Gamma = (1.0 \pm 0.5)$ meV, consistent with previous estimates for the Landau level broadening [27, 28]. The comparison between experimental and theoretical gaps is summarized in Table I.

Filling $\nu + 1$	B	$\Delta_{\text{qp}}^{\text{act}}$	Δ_{qp}^{μ}	$\Delta_{\text{qp}}^{\text{fit}}$	$\Delta_{\text{qp}}^{\text{DMRG}}$
$\frac{1}{2}$	12 T	5.1 K	—	—	5.6 K
	13.8 T	—	2.9 K	7.0 K	6.0 K
$\frac{2}{3}$	12 T	7.6 K	—	—	10.8 K
	13.8 T	—	5.2 K	11.6 K	11.7 K

TABLE I. Comparison of the quasiparticle gaps at $1/2$ and $2/3$ filling in the $N = 1$ Landau level as determined by DMRG calculations $\Delta_{\text{qp}}^{\text{DMRG}}$, thermally activated transport $\Delta_{\text{qp}}^{\text{act}}$, the chemical potential jump Δ_{qp}^{μ} , and from the fit to the Wigner crystal model $\Delta_{\text{qp}}^{\text{fit}}$.

Given the rather large discrepancies between experiment and numerics in GaAs[44]—particularly at half filling—the level of agreement we find for both activated and thermodynamic gaps with numerical modeling is encouraging. We note that several sources may account for the remaining quantitative discrepancies in our work. These including differences in inter-Landau level screening strength at $\nu \sim -3$ relative to $\nu \sim -1$ [45], as well as possible spin textures in the excitation spectrum, which can lower the activation gap but are not accounted for in our modeling. For $\Delta_{\text{qp}}^{\text{fit}}$, moreover, the phenomenological nature of our model for disorder may not capture the microscopic physics at a quantitative level. Finally, we note that the discrepancy between theory and experiment is greater at $2/3$ than at $1/2$, perhaps due to the greater quasiparticle charge resulting in a greater effect of the disorder potential.

Fig. 3A shows the μ measured at different temperatures near the $\nu + 1 = 1/2$ gap. We focus on the strong temperature dependence of $\Delta\mu$, plotted for several incompressible filling factors in Fig. 3B (see also the supplementary information). We fit the low temperature limit of $\Delta\mu(T)$ using the Sommerfeld expansion $\Delta\mu(T) = \Delta_0 - bT^2 + \dots$, which is justified so long as the quasiparticles experience short-range repulsion. The fitted values Δ_0 and b are reported in Figs. 3C and D, respectively.

Notably, the $\nu = -1 + 1/2$ state shows anomalously strong temperature dependence, manifesting as a large value of the b parameter. Note that this state becomes compressible at about 800mK in Fig. 3A, which corroborates with the deviation from activated transport at high temperature in Fig. 2A. According to the Maxwell relation $\left. \frac{d\mu}{dT} \right|_n = -\left. \frac{ds}{dn} \right|_T$, this suggests an anomalous contribution to the entropy in the dilute quasiparticle limit. Anomalous entropy is expected in the vicinity of nonabelian states[46] owing to the topological degeneracy of a dilute gas of nonabelian anyons. However, this contribution is considerably smaller than the anomalous entropy we observe. This implies that the anomalous entropy near $\nu = 1/2$ —at least at the filling factors corresponding to the extrema in μ —does not arise solely from the topologi-

cal degeneracy. Notably, these extrema occur at a density of quasiparticles where the average inter-quasiparticle distance is larger than the distance to the gate. Disorder is expected to dominate this regime, as inter-quasiparticle interactions are screened. Crudely, if disorder is more important than the long-range Coulomb interaction, we expect $b \propto (e/e^*)^2/\Gamma$, where Γ is the strength of the disorder. However, determining the prefactor requires understanding the thermodynamics of a Coulomb glass of fractionalized particles in an unknown disorder distribution, a challenge we leave to future work.

In closing, we note that a related manuscript reports scanning tunneling microscopy to study the same bilayer graphene FQH states studied here[47]. In that work, the gate voltage δV_g over which the FQH gaps appear provides a local measurement of the thermodynamic gap. Those authors find $4\Delta_{\text{qp}}^{\text{STM}} = 30$ K for the $1/2$ state at $B = 14$ T. This result is consistent with the intrinsic gap inferred from our WC model, $4\Delta_{\text{qp}}^{\text{WC}} \sim 28$ K, as expected for a local measurement that probes the chemical potential at length scales smaller than the disorder correlation length. The large intrinsic gaps manifesting across several experimental techniques show that bilayer graphene is an ideal platform to explore the intrinsic physics of nonabelian anyons in the solid state.

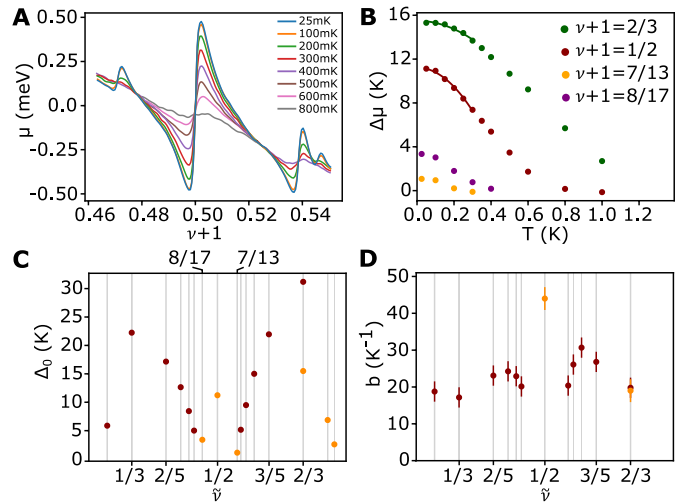


FIG. 3. **Temperature dependent μ near fractional filling.** Measurements are performed at $B = 13.8$ T. (A) Chemical potential near half filling of an $N = 1$ Landau level at several different temperatures. (B) Chemical potential jump across the incompressible states as a function of temperature for different filling factors in an $N = 1$ LL (dots). The solid lines are a low temperature fit, $\Delta\mu(T) = \Delta_0 - bT^2$. (C) Chemical potential jump Δ_0 extracted from the fit for different fractional states in the $N = 0$ ($\tilde{\nu} = \nu$, red dots) and $N = 1$ ($\tilde{\nu} = \nu + 1$, orange dots) orbital Landau levels. (D) Temperature decay parameter b extracted from the fit.

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