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Current cosmological data exhibit discordance between indirect and some direct inferences of the present-day expansion rate, H_0. Early dark energy (EDE), which briefly increases the cosmic expansion rate prior to recombination, is a leading scenario for resolving this "Hubble tension" while preserving a good fit to cosmic microwave background (CMB) data. However, this comes at the cost of changes in parameters that affect structure formation in the late-time universe, including the spectral index of scalar perturbations, n_s. Here, we present the first constraints on axion-like EDE using data from the Lyman-α forest, i.e., absorption lines imprinted in background quasar spectra by neutral hydrogen gas along the line of sight. We consider two independent measurements of the one-dimensional Lyα forest flux power spectrum, from the Sloan Digital Sky Survey (SDSS eBOSS) and from the MIKE/HIRES and X-Shooter spectrographs. We combine these with a baseline dataset comprised of *Planck* CMB data and baryon acoustic oscillation (BAO) measurements. Combining the eBOSS Lyα data with the CMB and BAO dataset reduces the 95% confidence level (CL) upper bound on the maximum fractional contribution of EDE to the cosmic energy budget, $f_{\rm EDE}$, from 0.07 to 0.03 and constrains $H_0 = 67.9^{+0.4}_{-0.4}$ km/s/Mpc (68% CL), with maximum *a posteriori* value $H_0 = 67.9$ km/s/Mpc. Similar results are obtained for the MIKE/HIRES and X-Shooter Lyα data. Our Lyα-based EDE constraints yield H_0 values that are in $> 4\sigma$ tension with the SH0ES distance-ladder measurement and are driven by the preference of the Lyα forest data for n_s values lower than those required by EDE cosmologies that fit *Planck* CMB data. Taken at face value, the Lyα forest severely constrains canonical EDE models that could resolve the Hubble tension.

Introduction— The recent direct measurement of the current cosmic expansion rate from the SH0ES collaboration, $H_0^{\rm SH0ES} = 73.04 \pm 1.04$ km/s/Mpc [1], is in significant tension with the indirect inference from *Planck* measurements of the cosmic microwave background (CMB), $H_0^{Planck} = 67.36 \pm 0.54$ km/s/Mpc [2], as well as inferences from other CMB experiments [3, 4] and probes of large-scale structure (e.g., [5–10]). Although some local distance ladder probes do not exhibit this discordance [11–13], and thus the origin of this "Hubble tension" could be systematic, many new physics models have been proposed as solutions (see [14] for a review). One of the most popular candidates is early dark energy (EDE), in which a scalar field increases the cosmic expansion rate just prior to recombination, before rapidly decaying away so as to not further impact the late universe. EDE decreases the sound horizon at last scattering and thereby increases the value of H_0 inferred from CMB analyses [15–20].

Although EDE may resolve the Hubble tension, it does so at the expense of introducing or worsening other tensions when confronted with additional cosmological datasets [21–24] (see [25–28] for a different viewpoint). As discussed in [21, 29], EDE models produce an enhanced early Integrated Sachs-Wolfe (eISW) effect in the CMB, which must be compensated by larger values of the physi-

cal cold dark matter density $\Omega_c h^2$ and the scalar spectral index n_s (as compared to their values in ΛCDM) in order to fit the CMB data. Conversely, recent analyses of the Lyman-α forest — absorption features in the spectra of distant quasars due to neutral hydrogen along the line of sight — prefer values of n_s and $\Omega_m h$ that are lower than those of CMB datasets [30–32]. In this *Letter* we demonstrate that, taken at face value, recent Lyα datasets significantly constrain EDE models.

Model— We consider EDE composed of a scalar field with an axion-like potential [16, 33], $V(\phi) = m^2 f^2 (1 - \cos(\phi/f))^n$ where f is the axion decay constant, m is a mass scale, and n is a power-law index. Instead of parametrizing the model in terms of the physical parameters (m, f) and the initial field value ϕ_i, we use an effective parametrization defined by the maximum fractional contribution of the EDE field to the cosmic energy budget, $f_{\rm EDE}$, the critical redshift z_c at which the EDE field reaches this contribution, and the initial field displacement $\theta_i \equiv \phi_i/f$ [16, 17]. To be consistent with late-time observables, the EDE field must decay sufficiently rapidly after z_c, which requires $n \geq 2$ (thus excluding the standard axion with $n = 1$). In this work, we fix $n = 3$, which has been shown to fit current data [16, 33]. We compute theoretical predictions using `CLASS_EDE` [21],[1] a modifica-

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[1] https://github.com/mwt5345/class_ede

Lyα Dataset	$\bar{\Delta}_{L^2}$	$\bar{n}_L$	$\sigma_{\Delta_L^2}$	σ_{n_L}	ρ
eBOSS	0.310	-2.340	0.020	0.006	0.512
XQ-100/MIKE-HIRES	0.343	-2.388	0.033	0.021	0.694

TABLE I. Parameter values for the 2D Gaussian compressed likelihoods from the Lyα datasets used in this work.

tion of the Einstein-Boltzmann code `CLASS` [34, 35] that incorporates EDE dynamics at the background and linear perturbation level.

Datasets— Our baseline dataset consists of *Planck* 2018 measurements of the CMB temperature and polarization power spectra at small (TTTEEE) and large angular scales (lowl+lowE) [2, 36] and the CMB lensing potential power spectrum [37], and Baryon Acoustic Oscillation (BAO) measurements from BOSS DR12 [38], the SDSS Main Galaxy Sample [39], and 6dFGS [40].

Traditional analyses of Lyα forest flux power spectra interpolate between hydrodynamical simulations to make theory predictions. Given the computational complexity of such simulations, we instead model Lyα forest measurements using a compressed likelihood characterized by the amplitude $\Delta_L^2 \equiv k_p^3 P_{\mathrm{lin}}(k_p, z_p)/(2\pi^2)$ and slope $n_L \equiv (d\ln P_{\mathrm{lin}}(k,z)/d\ln k)|_{(k_p,z_p)}$ of the linear power spectrum P_{lin}, both evaluated at a pivot redshift $z_p = 3$ and wavenumber $k_p = 0.009$ s/km [41]. This likelihood is marginalized over astrophysical uncertainties due to baryons. As shown in [42], Δ_L^2 and n_L contain essentially all of the cosmological information in the Lyα forest flux power spectrum over the range of scales probed by the datasets considered here.

Our fiducial dataset is derived from the analysis of the 1D Lyα flux power spectrum of SDSS DR14 BOSS and eBOSS quasars [30], which we refer to as eBOSS Lyα. We fit a 2D Gaussian to samples from the $\Delta_L^2 - n_L$ contour shown in Fig. 20 of [30]. In the Supplemental Material [43] we show that the 2D Gaussian accurately models the contour. The log-likelihood, up to a constant, is

$$\log \mathcal{L} = -\frac{1}{2(1-\rho^2)}\left\{\Delta x^2 - 2\rho\Delta x\Delta y + \Delta y^2\right\}, \quad (1)$$

where $\Delta x \equiv (\Delta_L^2 - \bar{\Delta}_L^2)/\sigma_{\Delta_L^2}$ and $\Delta y \equiv (n_L - \bar{n}_L)/\sigma_{n_L}$. Here $(\bar{\Delta}_L^2, \bar{n}_L)$ and $(\sigma_{\Delta_L^2}, \sigma_{n_L})$ are the mean and errors of the 2D Gaussian, respectively, and ρ is the correlation coefficient between $\sigma_{\Delta_L^2}$ and σ_{n_L}. Our best-fit parameters describing the eBOSS Lyα dataset are shown in Table I.

The eBOSS Lyα constraints assume a ΛCDM cosmology with a prior of $H_0 = 67.3 \pm 1.0$ km/s/Mpc and three species of massless neutrinos. The H_0 prior and assumptions regarding the neutrino mass have negligible impact on our results because constraints in the $\Delta_L^2 - n_L$ plane are insensitive to the precise value of H_0 and $\sum m_\nu$ [42, 44] (see Appendix A of [42] and the Supplemental Material [43] of this work for more details). In the
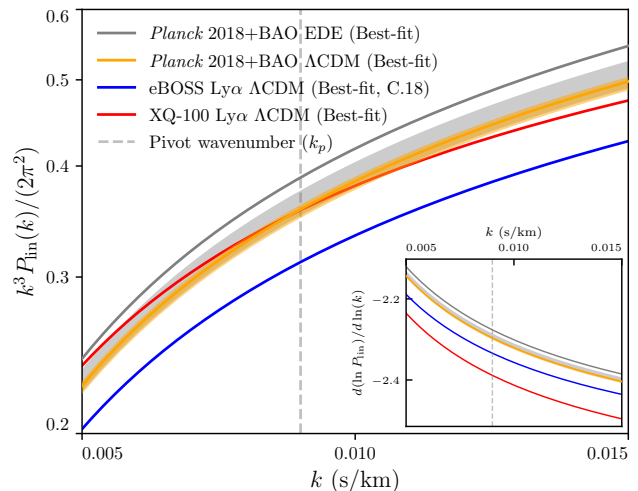

FIG. 1. Comparison of the best-fit linear matter power spectrum at $z_p = 3$ from the EDE (grey) and ΛCDM (orange) fits to the baseline CMB + BAO dataset with the best-fit ΛCDM cosmologies for the eBOSS (blue) [30] and XQ-100 (red) Lyα forest datasets. Shaded bands indicate the 68% CL from our baseline analyses; note that the best-fit EDE model lies outside the 68% CL due to prior-volume effects. The sub-panel shows the slope $(d\ln P_{\mathrm{lin}}(k,z)/d\ln k)|_{(k_p,z_p)}$ and the vertical line shows the Lyα pivot wavenumber ($k_p = 0.009$ s/km). EDE cosmologies that can resolve the Hubble tension and fit the baseline dataset require an enhanced amplitude and slope near the pivot scale relative to ΛCDM cosmologies. These requirements, particularly the steeper derivative, are in tension with the Lyα measurements. This figure is for illustrative purposes and thus does not include errors for the Lyα data.

Supplemental Material [43] we show that, for the range of scales probed by the Lyα datasets used here, the linear power spectrum for EDE cosmologies that are consistent with the baseline dataset can be mimicked at high precision by a ΛCDM cosmology; thus the ΛCDM assumption in the Lyα likelihood has little impact on our results.

We also consider measurements from the 1D Lyα forest flux power spectrum of the XQ-100 [45] and MIKE/HIRES quasar samples [46]. We fit a 2D Gaussian to the $\Delta_L^2 - n_L$ contour at $z_p = 3$ and $k_p = 0.009$ s/km derived from the analysis in Appendix A of [32].[2] The details of this likelihood are described in the Supplemental Material [43]. We refer to this dataset as XQ-100 Lyα. Table I includes the parameters for this likelihood.

Methodology— We sample from the EDE parameter posterior distributions using the Markov chain Monte Carlo (MCMC) code `Cobaya` [47].[3] We compute the effective Lyα parameters n_L and Δ_L^2 from the linear matter

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[2] The XQ100 and MIKE/HIRES samples have a pivot at higher redshifts, but the results can be converted to $z_p = 3$ since the rescaling is done in the matter-dominated era.
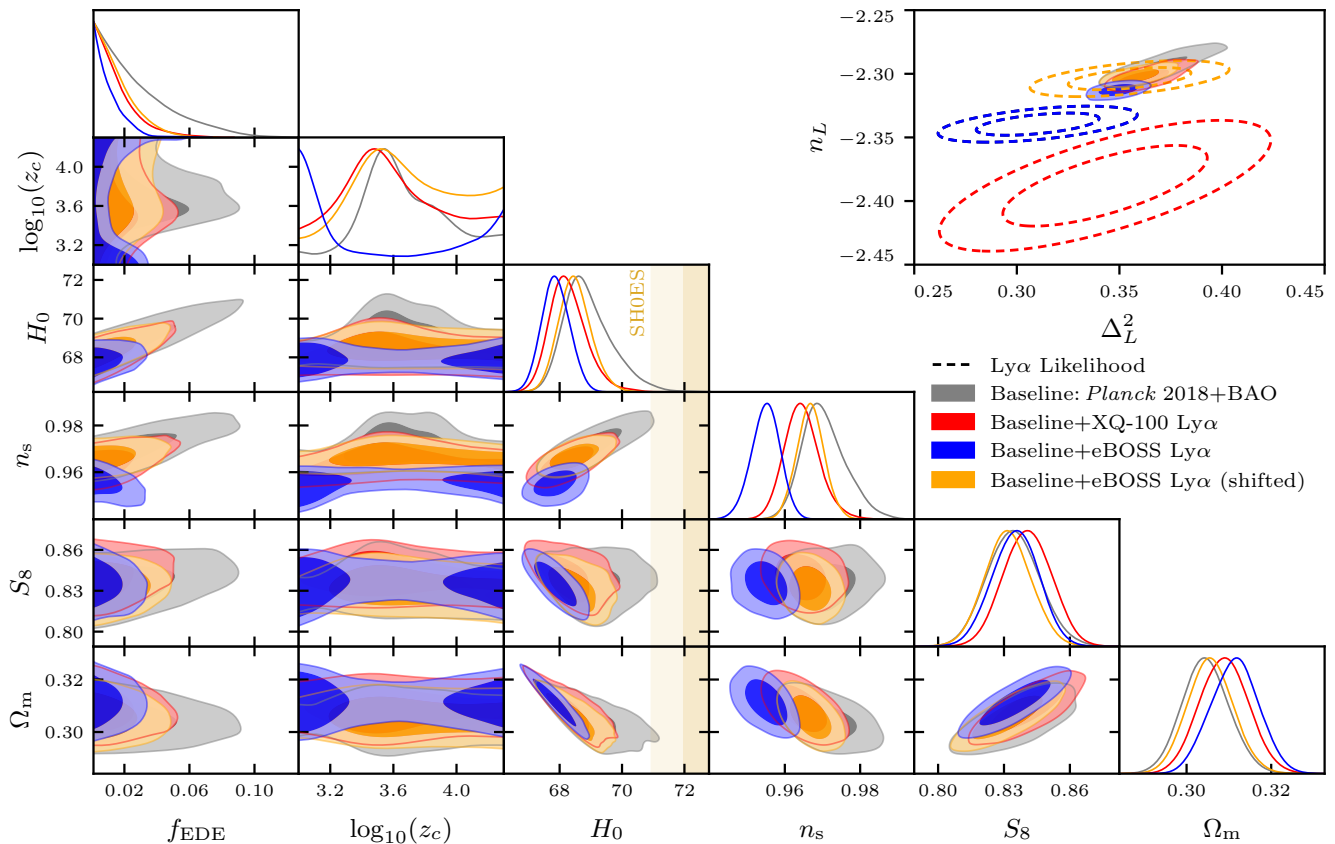
[3] https://github.com/CobayaSampler/cobaya

FIG. 2. Marginalized posteriors for a subset of EDE and ΛCDM parameters with and without Lyα data. The baseline dataset (grey) consists of *Planck* 2018 high-ℓ (TT+TE+EE) and low-ℓ (TT+EE) measurements, as well as BOSS DR12, SDSS MGS, and 6dFGS BAO data. Including eBOSS Lyα data (blue) or XQ-100 Lyα data (red) significantly reduces the upper bound on $f_{\rm EDE}$. H_0 values that are able to resolve the Hubble tension are strongly excluded by both analyses that include Lyα data. The top right panel shows constraints in the $\Delta_L^2 - n_L$ plane for each of these datasets, and the Lyα likelihoods alone (dashed lines). Although both Lyα likelihoods are in significant tension with the baseline analysis, our conclusions are unchanged even if we artificially shift the center of the eBOSS Lyα likelihood to the posterior mean of the baseline ΛCDM analysis (orange).

power spectrum produced by `CLASS_EDE`. We assess convergence using the Gelman-Rubin statistic [48] with a tolerance of $|R - 1| < 0.03$. We report confidence limits using the credible interval defined in Section IV of [22] and implemented in `GetDist` [49][4] for all parameters, except $f_{\rm EDE}$, for which we quote the 95% confidence one-tailed upper bound. We determine maximum *a posteriori* (MAP) values using a simulated annealing approach [50] described in the Supplemental Material [43].

We vary the six ΛCDM cosmological parameters ($\Omega_b h^2$, $\Omega_c h^2$, A_s, n_s, τ, θ_s) assuming flat, uninformative priors identical to those in Section 2.1 of [2]. We adopt the following uniform priors for the EDE parameters: $f_{\rm EDE} \in [0.001, 0.5]$, $\log_{10}(z_c) \in [3, 4.3]$, and $\theta_i \in [0.1, 3.1]$. The prior on $\log_{10}(z_c)$ is chosen such that we consider only EDE models with dynamics occurring at the epoch to

resolve the Hubble tension, which has been shown in previous works (e.g., [16, 21, 33, 51, 52]) to lie in this regime. The prior on θ_i is chosen to avoid numerical instabilities that arise when initial field values are near 0 or π. For a detailed discussion of priors on EDE parameters, see [21, 51, 52]. We include all of the recommended nuisance parameters and priors to account for systematic effects in the datasets we consider. In order to be consistent with the eBOSS Lyα likelihood, we assume three species of massless neutrinos.

Results— To qualitatively demonstrate the incompatibility between the Lyα measurements and EDE cosmologies that are consistent with the baseline dataset and can resolve the Hubble tension, Fig. 1 compares the best-fit linear matter power spectrum and its derivative at the pivot redshift $z_p = 3.0$ derived from our baseline ΛCDM and EDE analyses with that from analyses of the eBOSS [30] and XQ-100 [32] Lyα forest (see the Supplemental Material [43] for the parameter values). The EDE

	$f_{\rm EDE}$	$\log_{10}(z_c)$	n_s	H_0 [km/s/Mpc]	$\Omega_c h^2$
Baseline	< 0.073 (0.072)	$3.67^{+0.24}_{-0.30}$ (3.56)	$0.9705^{+0.0046}_{-0.0066}$ (0.9791)	$68.92^{+0.55}_{-0.91}$ (70.33)	$0.1221^{+0.0013}_{-0.0031}$ (0.1267)
+SH0ES	$0.096^{+0.032}_{-0.026}$ (0.114)	$3.64^{+0.21}_{-0.16}$ (3.57)	$0.9851^{+0.0065}_{-0.0063}$ (0.9878)	$71.40^{+0.91}_{-0.91}$ (72.01)	$0.1287^{+0.0035}_{-0.0035}$ (0.1311)
+eBOSS Lyα	< 0.028 (0.021)	$3.52^{+0.78}_{-0.52}$ (3.03)	$0.9549^{+0.0039}_{-0.0035}$ (0.9510)	$67.88^{+0.43}_{-0.46}$ (67.81)	$0.1211^{+0.0011}_{-0.0011}$ (0.1213)
+eBOSS Lyα+SH0ES	< 0.039 (0.026)	$3.48^{+0.82}_{-0.48}$ (3.06)	$0.9574^{+0.0044}_{-0.0037}$ (0.9532)	$68.69^{+0.42}_{-0.41}$ (68.66)	$0.1202^{+0.0010}_{-0.0014}$ (0.1203)
+XQ-100 Lyα	< 0.041 (0.022)	$3.61^{+0.28}_{-0.42}$ (3.53)	$0.9646^{+0.0041}_{-0.0050}$ (0.9666)	$68.28^{+0.47}_{-0.66}$ (68.46)	$0.1216^{+0.0011}_{-0.0018}$ (0.1223)
+XQ-100 Lyα+SH0ES	$0.060^{+0.025}_{-0.028}$ (0.092)	$3.55^{+0.06}_{-0.15}$ (3.54)	$0.9750^{+0.0054}_{-0.0060}$ (0.9788)	$70.25^{+0.84}_{-0.84}$ (71.01)	$0.1256^{+0.0032}_{-0.0034}$ (0.1289)

TABLE II. Marginalized constraints on cosmological parameters for EDE from the datasets labeled in the first column. For each dataset, we report the posterior mean and the 68% CL upper and lower limits for all parameters that are detected at $> 2\sigma$, otherwise we report the 95% CL upper limits. MAP values are shown in parentheses. The marginalized constraints for the artificially "shifted" eBOSS Lyα analysis are included in the Supplemental Material [43].

fit to the baseline dataset prefers enhanced power relative to the ΛCDM fit at wavenumbers near the pivot scale $k_p = 0.009$ s/km.[5] The best-fit eBOSS Lyα cosmology predicts less power than the baseline ΛCDM results. Both Lyα datasets prefer a milder slope of the linear matter power spectrum near the pivot wavenumber, as shown in the sub-panel. To compensate for the enhanced eISW effect in the CMB, EDE cosmologies that can preserve the fit to CMB + BAO data and resolve the Hubble tension require an increased amplitude and slope of $P_{\rm lin}(k)$ than that in ΛCDM cosmologies [21, 29]. These requirements move precisely against the direction preferred by Lyα data.

Fig. 2 shows the main results of this *Letter*. The top right panel presents constraints on the effective Lyα parameters n_L and Δ_L^2 for our main analyses, alongside the Lyα likelihoods for the eBOSS and XQ-100 Lyα datasets. The baseline dataset is in significant tension with both Lyα analyses. This tension already exists for ΛCDM cosmologies, where it is predominantly sourced by the low values of n_s and $\Omega_c h^2$ for the eBOSS Lyα analysis [30] and the low (high) value of n_s (σ_8) for the XQ-100 analysis [32]. We discuss this tension in the Supplemental Material [43] and emphasize that the direction of the tension is exactly opposite to the parameter shifts necessary for EDE cosmologies that can increase H_0. To demonstrate that our conclusions are not a consequence of this tension, we include an analysis with a "shifted" eBOSS Lyα likelihood centered at the posterior mean of the baseline ΛCDM analysis $(\bar{\Delta}_{L^2}, \bar{n}_L) = (0.355, -2.304)$.

The remainder of Fig. 2 shows the marginalized posteriors for a subset of the EDE and ΛCDM parameters. The positive correlation between $f_{\rm EDE}$ and n_s arising from the compensation of the enhanced eISW effect produced in EDE cosmologies is visible in the baseline results. The 95% CL upper bound on $f_{\rm EDE}$ reduces from 0.07 to 0.03 (0.04) after including eBOSS (XQ-100) Lyα data, strongly excluding models with $f_{\rm EDE} \approx 0.1$, as needed to resolve

the Hubble tension [16]. Including eBOSS data leads to a bimodal z_c posterior. The samples with large values of $\log_{10}(z_c)$ represent scenarios where the EDE field decays too early to have a significant impact on CMB observables. The samples with low values of $\log_{10}(z_c)$ are associated with changes in the damping physics in EDE cosmologies where $f_{\rm EDE}$ peaks just prior to recombination, as discussed in the Supplemental Material [43].

Including the shifted eBOSS Lyα likelihood reduces the 95% CL upper bound on $f_{\rm EDE}$ to 0.04 and provides tighter constraints than XQ-100 on many parameters. These tight constraints are driven by (i) the increased precision of the eBOSS likelihood relative to the XQ-100 likelihood and (ii) the misalignment between the $\Delta_L^2 - n_L$ degeneracy axis of the eBOSS likelihood and that of the baseline analysis. This test illustrates that even if the eBOSS likelihood were not in tension with the baseline dataset, it would still significantly constrain EDE.

Table II presents marginalized constraints on the cosmological parameters. We also include results with a SH0ES-derived H_0 prior, $H_0 = 73.04 \pm 1.04$ km/s/Mpc [1]. Even with this prior, the H_0 posterior for the baseline + eBOSS analysis falls well below the SH0ES measurement. Given the role of prior-volume effects when using MCMC techniques to sample the EDE parameter space [25, 28, 53], we compare the posterior mean with the MAP [54]. Similar to previous works, we find noticeable disagreement between the posterior mean and MAP for the baseline dataset, indicating that the interpretation of our baseline constraints is sensitive to prior-volume effects. In contrast, for all analyses that include Lyα data, we find excellent agreement between the posterior mean and MAP for all parameters,[6] suggesting that our Lyα constraints are significantly less sensitive to prior-volume effects than the baseline results. We verify this explicitly in the Supplemental Material [43] using a profile likelihood.

Conclusions— In this *Letter* we used two independent measurements of the Lyα forest flux power spectrum to place the first Lyα-based constraints on axion-like

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[5] Converting between length and velocity units depends on the cosmology via a factor $H(z_p)/(1+z_p)$. We find $k_p = 1.00\,h/{\rm Mpc}$ for the MAP EDE cosmology of the baseline dataset.

[6] The only exception is $\log_{10}(z_c)$ when including eBOSS Lyα, a consequence of the aforementioned bimodality.

EDE models. Combining the eBOSS (XQ-100) Lyα data with a baseline dataset comprised of CMB and BAO measurements reduces the 95% CL upper bound on the maximum fractional contribution of EDE to the cosmic energy budget, f_{EDE}, from 0.07 to 0.03 (0.04) and constrains $H_0 = 67.9^{+0.4}_{-0.5}$ ($68.3^{+0.5}_{-0.7}$) km/s/Mpc at 68% CL. Our tight constraints are driven by the tension between the low values of n_s preferred by Lyα forest data and the high values of n_s necessary for EDE cosmologies that fit the *Planck* CMB data.

Several caveats arise when applying the compressed likelihoods considered here to constrain EDE cosmologies. First, deriving Lyα constraints on the Δ_L^2 – n_L plane within the context of EDE cosmologies would require running many hydrodynamical simulations with EDE-based initial conditions (as in, *e.g.*, [55] for warm dark matter), which is beyond the scope of this work. Second, our conclusions are sensitive to systematics in the measurements of the eBOSS and XQ-100 1D Lyα forest flux power spectra, and the simulations and emulators used to model them. Seeing as the Lyα likelihoods applied in this study are already in tension with the ΛCDM constraints from CMB and BAO data, and with each other, it is crucial to determine if this tension is a result of systematics [56].[7] If the tension is physical, the Lyα forest excludes the canonical EDE model considered in this work as a resolution to the Hubble tension.

There are several ways to extend our analysis. On the theoretical front, it would be useful to develop models that can resolve the Hubble tension without significantly increasing n_s, *e.g.*, by invoking EDE couplings to additional fields [57–60]. In terms of measurements, we made the conservative choice of using only *Planck* CMB and BAO distance measurements, but one could include additional datasets, such as a full-shape likelihood for BOSS galaxy clustering [22, 23, 27, 38]. Including probes with low values of S_8, such as the Dark Energy Survey [61] would further reduce f_{EDE} and H_0 since EDE models exacerbate the "S_8 tension" [16, 21, 22, 62]. In light of the preference for non-zero values of f_{EDE} in the recent Atacama Cosmology Telescope results [51, 52, 63], it would be interesting to repeat our analysis including ACT DR4 data [3, 64]. In the near future, this analysis could be repeated using Lyα forest measurements from the Dark Energy Spectroscopic Instrument [65–68] and WEAVE [69], which will provide the largest sample of quasar spectra to date.

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