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Junctions and superconducting symmetry in twisted bilayer graphene

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Junctions provide a wealth of information on the symmetry of the order parameter of superconductors. We analyze junctions between a scanning tunneling microscope (STM) tip and superconducting twisted bilayer graphene (TBG) and TBG Josephson junctions (JJs). We compare superconducting phases that are even or odd under valley exchange (s - or f -wave). The critical current in mixed (s - and f -) JJs strongly depends on the angle between the junction and the lattice. In STM-TBG junctions, due to Andreev reflection, f -wave leads to a prominent peak in subgap conductance, as seen in experiments.

Introduction. Graphene multilayers host a myriad of exotic correlated and topological phases [1–23]. Perhaps most interesting and enigmatic among them is superconductivity, possibly with unconventional pairing symmetries and mechanisms, observed in alternating-twist stacks of up to five layers [24–30] and in Bernal bilayers and rhombohedral trilayers [31–34]. Crucially, the observed superconductivity violates the Pauli limit for spin-singlet pairing [29, 31–35] and has been observed in settings that break time-reversal symmetry (TRS) [36], strongly suggesting a spin-triplet pairing in these materials. However, the pairing may be a mixture of singlet and triplet [37], and the exact symmetries involved (s -, p -, d - and/or f -) are still unknown despite intense theoretical and experimental efforts to uncover them.

Recently, several experiments have studied these unconventional superconducting states using transport measurements: either with a scanning tunneling microscope (STM) tip [16, 22] or with Josephson junctions (JJs) [38–42] and Superconducting Quantum Interference Devices (SQUIDs) [43]. In the former setup, by comparing the transmission between the STM tip and the superconducting surface in the weak and strong-coupling regimes, one can gain important insights about the symmetry of the order parameter. For instance, the experimental observations, such as the peak in the subgap conductance [16, 22], seem inconsistent with s -, p - and d -wave pairings [37, 44]. In the latter setups, the overlap of the superconductors’ wavefunctions at the junction’s link gives rise to a zero-frequency supercurrent whose magnitude and superconducting phase-dependence carry characteristics of the pairing symmetry [45–47].

Building on these experimental insights, we argue in this Letter that transport measurements in junctions are ideal probes of the pairing symmetry in twisted graphene superconductors, similar to the elucidation of d -wave pairing in cuprate superconductors [48, 49], and that existing STM data [16, 22] are consistent with f -wave pairing. The Fermi surface of these graphene-based systems contains two valleys. We consider superconducting order parameters that are either even or odd under valley ex-

change, which in the absence of spin-orbit coupling correspond to spin-singlet s -wave superconductivity or spin-triplet f -wave superconductivity respectively. In ‘mixed’ Josephson junctions connecting a s -wave to a f -wave superconductor, we observe that the critical current dramatically depends on the angle between the junction and the graphene lattice axis. Therefore, Josephson junctions are useful for determining whether two superconducting phases differ in their valley exchange parity.

In the STM-superconductor junction, we find that the subgap conductance shows a prominent zero-bias peak for f -wave pairing only, due to enhanced Andreev reflection. This peak has been observed in experiments on both twisted bilayer [16] and twisted trilayer graphene [22]. This result puts forward f -wave pairing as a leading candidate for the superconducting symmetry of twisted bilayer graphene, which is also consistent with previous theoretical models based on Coulomb-interaction-mediated Cooper pairing [50, 51].

STM tip-superconducting TBG junction. The model. General features of transport in normal-superconductor junctions are described in Ref. [52]. The coupling between the two electrodes is given by a scattering matrix, determined by a dimensionless transmission amplitude, T . The model has been extended in [37, 44]. As in Ref. [52], the normal metal tip and the superconducting electrode are described in terms of incoming and outgoing single channels. On the superconductor, the states in the channel are defined as suitable averages in momentum space of the quasiparticles. The momentum dependence of the gap leads to a momentum dependence of the mixing between electron and hole-like states in the superconductor, and it modifies the transmission of the junction, both in the tunneling and in the contact regimes. The Blonder-Tinkham-Klapwijk (BTK) model [52] has also been extended to strongly coupled superconductors, where the chemical potential can be below the bottom of the band [53, 54].

We describe the metal–superconductor junction as one ingoing normal channel, which represents the tip, and two outgoing superconducting channels, which represent the two valleys in TBG. The signs of the gaps in these

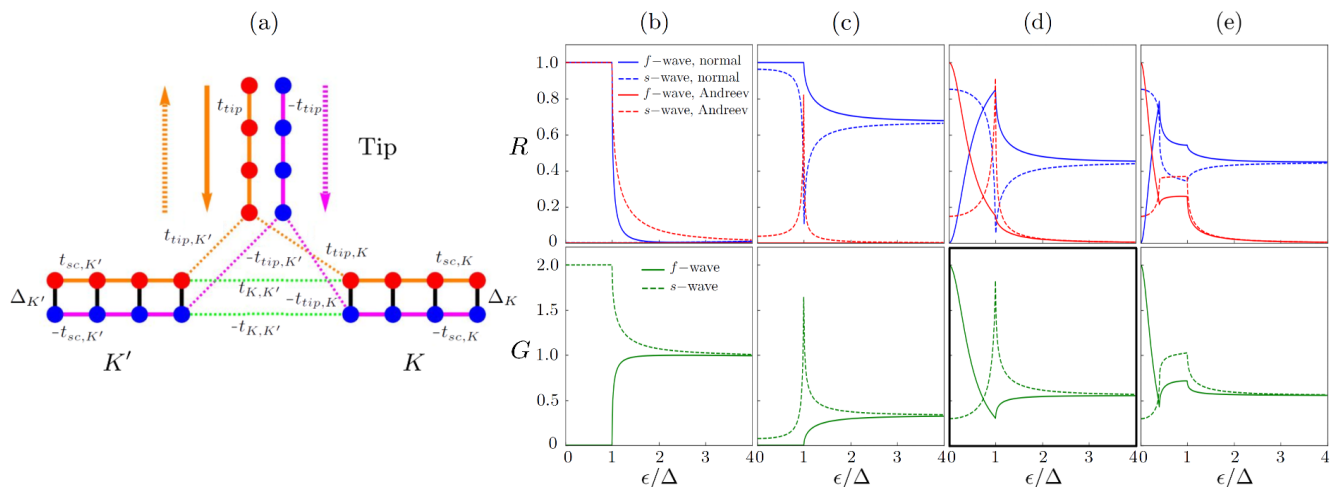


Figure 1. STM tip-superconducting TBG junction. (a) Sketch of the model for the junction used to calculate its transport properties. See text for details. (b-e): normal and Andreev reflections (top) and total conductance (bottom) for junctions with spin-singlet s -wave (dashed) and spin-triplet f -wave pairing (solid), in the perfect contact limit. (b) With equal Fermi velocities in all channels, $t_{tip} = t_{sc,K} = t_{sc,K'} = 1$, $t_{tip,K} = t_{tip,K'} = 1/\sqrt{2}$, $t_{K,K'} = 0$, $\Delta_K = 0.05$, $\Delta_{K'} = \pm 0.05$. (c) With a large Fermi velocity mismatch in the normal and superconducting channels: $t_{tip} = 10$, $t_{tip,K} = t_{tip,K'} = 10/\sqrt{2}$, others as in (b). (d) With Fermi velocity mismatch and intervalley scattering $t_{K,K'} = 1$, others as in (c). (e) With Fermi velocity mismatch, intervalley scattering and spin-orbit coupling: $\Delta_K = 0.05$, $\Delta_{K'} = \pm 0.02$, others as in (d).

channels can be equal, describing a spin-singlet s -wave superconductor, or opposite, describing a spin-triplet f -wave superconductor [55]. The model can also be applied to an Ising superconductor [34] in a system with strong spin-orbit coupling, characterized by spin-valley locked Cooper pairs of the type $|K, \uparrow; K', \downarrow\rangle$.

The three-channel model described above is discretized as a tight-binding model, see Fig. 1(a). The normal channel is described by nearest-neighbor hopping t_{tip} , which determines its Fermi velocity and density of states. The superconducting channels are described by two nearest neighbor hoppings, $t_{sc,K}$ and $t_{sc,K'}$, and two gaps, Δ_K and $\Delta_{K'}$. The coupling between the normal channel and the two superconducting channels is described by the hoppings $t_{tip,K}$ and $t_{tip,K'}$. Without loss of generality, we assume that the Fermi energy is $\epsilon_F = 0$, so that each channel has exact electron-hole symmetry. Finally, we consider that the tip is a local perturbation which can induce intervalley scattering, parametrized by another hopping, $t_{K,K'}$.

We solve the transmission of the junction by matching incoming and outgoing waves in the three channels. If the energy ϵ is within the superconducting gaps, we use evanescent waves in the superconducting channels. For each energy, there are four propagating or evanescent waves in each channel. We assume that there is an incoming wave of electron character and amplitude 1 in the tip channel. In the same channel, there can be one electron and one hole outgoing channels, describing normal and Andreev reflection, with amplitudes R_N and R_A , respectively. In each of the two superconducting channels there

can be two decaying evanescent waves, when the energy is within the gap, or two outgoing propagating waves. We describe the four amplitudes as $T_{i,j}$, where $i = K, K'$ stands for the channel, and $j = 1, 2$ stands for the wavefunction within each channel. The transport properties of the junction are determined by these six amplitudes. The conductance of the junction is $G = 1 - |R_N|^2 + |R_A|^2$. The matching conditions involve the amplitudes of the wavefunctions at the three sites which describe the junction. The equations can be found in Ref. [56].

STM tip-superconducting TBG junction. Results. When the Fermi velocities in all channels are equal, the tip channel merges smoothly into the even combination of the K and K' channels and the junction behaves as described by the BTK theory in the regime of perfect contact, see Fig. 1(b). For s -wave pairing, and at zero voltage, Andreev scattering leads to a conductance twice as large as a single normal channel [52]. For f -wave pairing, negative interference between the two hole channels cancels Andreev reflection. This cancellation can be expected whenever the order parameter has a sign change between states related by TRS [44]. At high voltages the conductance reduces to the conductance of a single channel in both cases.

The bandwidth and Fermi velocity in TBG are considerably smaller than in a normal metal. This Fermi velocity mismatch induces elastic back-scattering in the normal phase, which reduces the conductance above the gap, see Fig. 1(c). Subgap Andreev reflection for s -wave superconductivity is strongly suppressed, and it remains zero for the f -wave phase, for a detailed explanation see

Ref. [56]. The tip can also induce a perturbation on the superconductor, on scales comparable to the atomic spacing. Such a perturbation will induce intervalley scattering. Fig. 1(d) shows results obtained for an intervalley coupling comparable to the bandwidth of the superconductor. This perturbation can be considered as disorder which does not violate TRS. The presence of intervalley scattering does not change significantly the conductance of the junction in an s -wave superconductor, in agreement with Anderson's theorem [57]. On the other hand, it is a pair breaking perturbation in an f -wave superconductor, which induces subgap states, see Ref. [56]. These states allow for subgap Andreev reflection. As a result, the subgap conductance of the junction is strongly enhanced by intervalley scattering in a f -wave superconductor, leading to a zero bias peak, highlighted in Fig. 1(d), that has been seen in the experiments of Refs. [16, 22].

Recent transport experiments [33, 34] reveal that proximity induced spin-orbit coupling promotes the superconducting properties of Bernal bilayer graphene. An effect of spin-orbit coupling is to break the equivalence between the Cooper pairs $|K, \uparrow; K', \downarrow\rangle$ and $|K, \downarrow; K', \uparrow\rangle$. In the model studied here, the spin-orbit coupling makes the two channels inequivalent. Results are shown in Fig. 1(e).

Josephson junctions. The model. For the study of JJs, our setup consists of a TBG crystal, in which the electrodes are superconducting and the weak link is in a normal metal or band insulating phase, as shown in Fig. 2(c). We start from a tight-binding, non-interacting Hamiltonian \mathcal{H}_0 [58] that includes Hartree electron-electron interactions through an electrostatic potential [59, 60]. The parameters in the tight binding model are scaled, such that the central bands of a TBG with twist angle θ are approximated by the central bands of an equivalent lattice with twist angle $\lambda\theta$, with $\lambda > 1$ [61–63], see Fig. 2(a).

The critical current comes from second order perturbation theory and is the derivative of the free energy E with respect to the superconducting phase difference ϕ :

$$\mathcal{I} = \frac{e}{\hbar} \frac{\partial E}{\partial \phi}. \quad (1)$$

To obtain the energies of the TBG junction, we diagonalize the Bogoliubov-de Gennes Hamiltonian,

$$\mathcal{H}_{BdG} |\Psi\rangle = \begin{pmatrix} \mathcal{H}_0 - \epsilon_F & f(\Delta) \\ f^\dagger(\Delta) & \epsilon_F - \mathcal{H}_0 \end{pmatrix} \begin{pmatrix} \Psi_e \\ \Psi_h \end{pmatrix} = E \begin{pmatrix} \Psi_e \\ \Psi_h \end{pmatrix}, \quad (2)$$

where ϵ_F is the Fermi energy. Again, we compare s -wave pairing, which we model with a on-site attractive Hubbard term $f(\Delta) = -\Delta_S \mathbb{1}$, and f -wave pairing, which results from Haldane-like hoppings [64, 65] that allow an electron excitation to convert to a hole excitation via second nearest-neighbor imaginary intralayer hoppings, see Fig. 2(b).

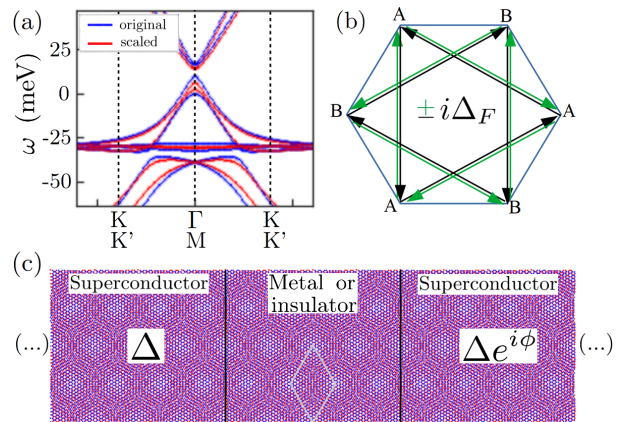


Figure 2. (a) Low-energy bandstructure of TBG at $\theta \approx 1.08^\circ$ and filling $n = -2.4$, with and without scaling. (b) Hoppings inducing f -wave superconducting pairing. (c) Central part of the lattice of the TBG Josephson junction [66, 67]. The electrodes are superconductors with a phase difference of ϕ and the link region, with a length of four moiré periods, is metallic or insulating. The rhombus is a unit cell of TBG.

Josephson junctions. Results. Figure 3(a) compares the current-phase relations (CPRs) of TBG JJs with s - and f -wave pairings in multiple configurations. CPRs can be measured with a SQUID geometry [43]. The main message of Fig. 3 is that the type of pairing, s -wave or f -wave, plays a minor role when both electrodes are equal, compare dashed and solid lines in Fig. 3(a-b).

In SNS JJs the CPR is skewed, due to high transmission of Andreev bound states, which carry over 80% of the current in these junctions and are mostly localized in AA stacking regions, see Fig. 3(c). In contrast, in SIS junctions the current comes from tunnelling states, so the CPR is sinusoidal [69]. An exception occurs when the insulating gap in the link is comparable to the superconducting gap, resulting in skewness and large currents. The current in SIS junctions exponentially depends on the similitude between both gaps, see Fig. 3(d). We note that the authors of Ref. [43] report a sinusoidal CPR in TBG, without skewness, despite having a SNS JJ. This may be due to low transmission in the junction [47]. Figure 3(b) shows the critical current for all JJs as a function of twist angle. For a comparison to experiments, see Ref. [56]. The current in SNS JJs increases with twist angle, suggesting that larger Fermi velocities compensate the reduced density of states. Electron-hole asymmetry is very notable, e.g. near $\theta = 1.1^\circ$, the current in SIS junctions with fillings $-2.4/4/-2.4$ is over two orders of magnitude larger than with $2.4/-4/2.4$ due to the asymmetry in the size of the gaps between narrow bands and electron- or hole-like remote bands.

Ref. [38] reports a significant length dependence of the critical current in JJs prepared in mixed configurations, e.g. with the electrodes doped near one superconducting dome and the link near the other. This indicates that

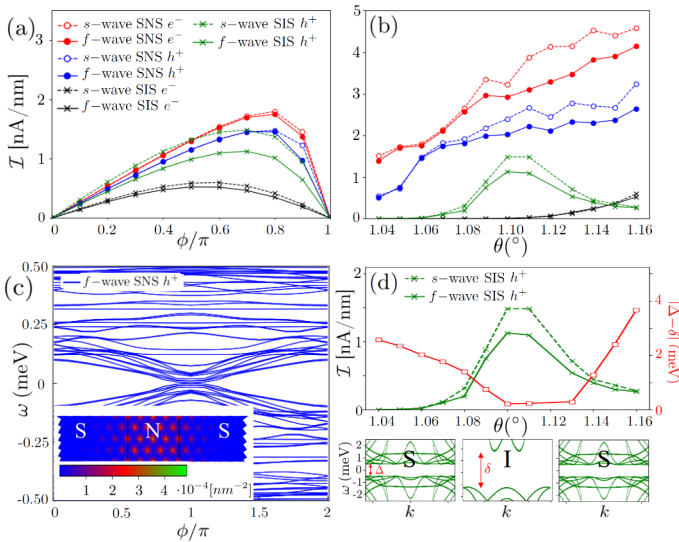


Figure 3. TBG Josephson junctions with equal electrodes. (a) Current-phase relations for near magic-angle junctions with different pairing symmetry: spin-singlet s -wave (dashed) or spin-triplet f -wave (solid), for electron and hole superconducting domes (fillings $n = \pm 2.4$), with a metallic (SNS) or insulating link (SIS) [68]. We set the superconducting gap to 1 meV [16]. $\theta = 1.06^\circ$ for SNS h^+ and 1.16° for SIS e^- . Units: nanoampere per nanometer junction width. (b) Critical current versus twist angle for all configurations. (c) Andreev spectrum at 1.06° . Inset: charge map of an Andreev bound state. (d) Critical current in SIS JJs compared to the difference between the superconducting and insulating gaps, as a function of twist angle, and a sketch of the bands in the different regions of a SIS junction.

the superconducting pairing symmetry in the electron and hole domes may differ. The results in Fig. 4 for mixed f -wave/ s -wave TBG JJs propose an experiment that could verify the hypothesis. For these JJs, the critical current dramatically depends on the angle between the junction and the lattice. A similar result in non-superconducting junctions was found in Ref. [70]. The critical current is sizeable when the junction axis is nearly parallel to the graphene armchair direction, but close to zero when parallel to the zigzag direction. As long as the perpendicular momentum is conserved, the zigzag JJ suffers destructive interference of the superconducting pockets along the green lines drawn in Fig. 4. Also, the CPRs have a period of π , half the one of standard JJs. The origin of this effect is the existence of two sets of energy levels, due to coupling of the s -wave pocket to the two f -wave pockets, which have an intrinsic phase difference of π [46, 71]. Furthermore, the CPR shows a π -junction behaviour, i.e. it is first negative [47, 72]. A requisite for these phenomena is that the triplet electrode is spin unpolarized, otherwise the current is zero due to spin conservation. The same occurs in a one-dimensional toy model [56, 73].

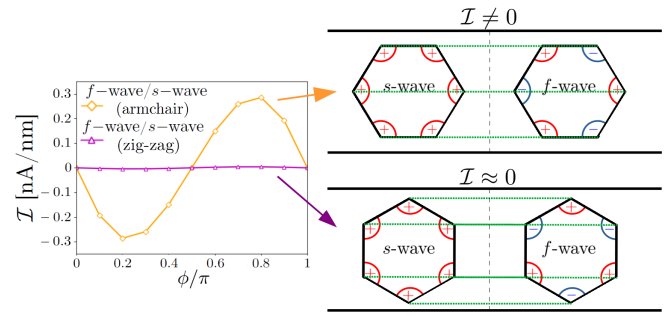


Figure 4. Current-phase relation in mixed f -wave/ s -wave TBG Josephson junctions, nearly parallel to the graphene armchair, as in Fig. 2(c), or zigzag directions. The critical current is ~ 100 times larger for armchair junctions.

Discussion. We have studied the role of the superconducting order parameter in transport through superconducting TBG junctions. We focus on s - and f -wave pairing (even and odd valley combinations), as these two choices are equally favored by long range interactions, either attractive or repulsive [74].

We have calculated the critical current, and the current-phase relation for different types of Josephson junctions. JJs in which both electrodes are either s - or f -wave superconductors show similar features (unlike the s - and p - cases considered in Ref. [75]). On the other hand, the critical current in mixed (s - and f -) junctions depends strongly on the orientation of the junction with respect to the graphene lattice axes, with maxima for armchair junctions, and zeroes for zigzag junctions. Hence, mixed junctions are useful for determining whether two superconducting phases differ in their valley exchange parity. Such junctions can exist in various setups: i) different superconducting regions in the phase diagram of TBG show different order parameters [38], ii) the superconducting state changes locally because of the spin-orbit coupling induced by a substrate [33], iii) superconducting TBG is combined with s -wave proximitized graphene [76, 77].

For a junction between a normal STM tip and superconducting TBG, we find a prominent peak in subgap conductance for an f -wave order parameter, due to Andreev states induced by the tip, in agreement with the experiments of Refs. [16, 22]. f -wave is also consistent with the U- and V-shaped densities of states measured in the weak coupling regime, as shown in Refs. [56, 78]. The agreement between the experiments [16, 22] and the results presented here puts forward f -wave pairing as a leading candidate for the pairing symmetry of twisted graphene superconductors.

We note that the results for the STM-superconductor junction apply equally well to all graphene superconductors [27–34]. Extending the Josephson junction calculations to non-twisted graphene superconductors [31–34] is a promising direction for future research.

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