Dicke Superradiance Requires Interactions beyond Nearest Neighbors

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Dicke superradiance requires interactions beyond nearest-neighbors

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Photon-mediated interactions within an excited ensemble of emitters can result in Dicke superradiance, where the emission rate is greatly enhanced, manifesting as a high-intensity burst at short times. The superradiant burst is most commonly observed in systems with long-range interactions between the emitters, although the minimal interaction range remains unknown. Here, we put forward a new theoretical method to bound the maximum emission rate by upper-bounding the spectral radius of an auxiliary Hamiltonian. We harness this tool to prove that for an arbitrary ordered array with only nearest-neighbor interactions in all dimensions, a superradiant burst is not physically observable. We show that Dicke superradiance requires minimally the inclusion of next-nearest-neighbor interactions. For exponentially-decaying interactions, the critical coupling is found to be asymptotically independent of the number of emitters in all dimensions, thereby defining the threshold interaction range where the collective enhancement balances out the decoherence effects. Our findings provide key physical insights to the understanding of collective decay in many-body quantum systems, and the designing of superradiant emission in physical systems for applications such as energy harvesting and quantum sensing.

Introduction.— Collective spontaneous emission of N initially-inverted atoms with identical all-to-all interactions mediated by the electromagnetic vacuum results in a burst of light with intensity scaling as N^2 [1–3]. This phenomenon is commonly referred to as "Dicke superradiance" or "superradiant burst". Over the past decades, this many-body phenomenon has attracted a lot of interest in both theoretical [4–24] and experimental studies [25, 26] using a multitude of physical platforms such as trapped ions [27], molecular aggregates [28–31], solid-state emitters [32–36], cold atoms and molecules [37–40], and superconducting qubits [41–43], with wide-ranging applications including the generation of multi-photon states with improved metrological properties [18, 44–47], energy harvesting [48–50], ultrabright LEDs [51] and quantum sensing [52, 53].

The atoms in Dicke's original model were assumed to be confined within a spatial extent smaller than the emission wavelength λ. Consequently, the atoms become indistinguishable with respect to the absorption or emission of photons, such that their quantum state $|j = N/2, m\rangle$ (with $-N/2 \leq m \leq N/2$) is permutation-invariant. This permutation symmetry greatly reduces the complexity of the problem, as it constrains the dynamics to $N + 1$ states, instead of exploring the full Hilbert space (which scales as 2^N). Recently, there has been substantial research progress with extended systems where atoms are distributed over a region larger than λ, thus breaking this symmetry. Of particular interest are ordered atomic arrays [54, 55], in which the superradiant properties can be greatly affected by the geometry and dimensionality of the lattice [16, 19, 20, 22, 24, 56]. The interactions between the emitters are typically modelled by long-range dipole-dipole interactions mediated via the electromagnetic vacuum [57, 58].

A long-standing fundamental question is the *minimal* interaction range required for the occurrence of a superradiant burst. Intuitively, superradiance can be thought of as a competition between (transient) phase synchronization, which leads to the buildup of atomic correlations, and decoherence [59]. Both effects stem from the same dissipative interactions [8, 22]. Since synchronization of nonlinear classical phase oscillators has been demonstrated with nearest-neighbor (NN) coupling [60], one may expect the atomic phases to synchronize for sufficiently strong NN interactions resulting in a superradiant burst [59]. Moreover, for a fixed interaction range, higher dimensionality was reported to result in stronger superradiance due to long-range order [19, 24]. On the flip side, it could also be argued that for short-range interactions, the buildup of correlations is not strong enough to overcome decoherence, thereby preventing superradiance.

In this Letter, we prove that superradiant burst is impossible in an arbitrary D-dimensional array with only nearest-neighbor interactions, for arbitrary times and initial states. That is, we show that, in all cases, the emission rate is upper bounded by that of independent emitters, resulting in no enhancement from collective dynamics. Including next-nearest-neighbor interactions, we show that a superradiant burst can be physically observed for certain values of the interaction strengths,

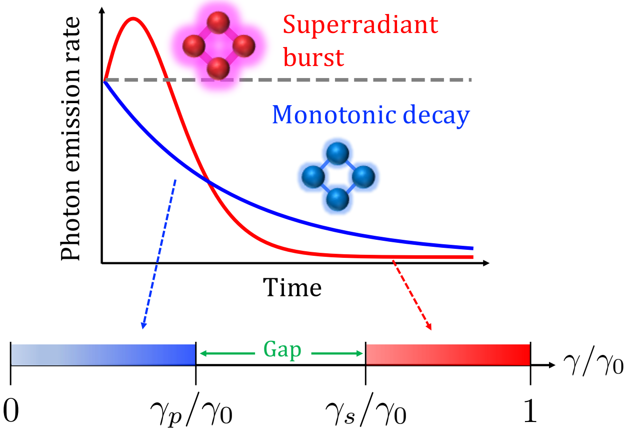

FIG. 1. Dynamics of the photon emission rate $R(t)$ for emitter arrays with only nearest-neighbor interactions of strength γ, normalized by the individual emitter decay rate γ_0. For $\gamma/\gamma_0 < \gamma_s$, $\dot{R}(0) < 0$ and the photon emission rate decays monotonically without a superradiant burst (blue). Superradiance occurs for $\gamma/\gamma_0 > \gamma_s$ (red). The physically-valid regime is defined by $0 < \gamma/\gamma_0 \leq \gamma_p$. For nearest-neighbor interactions, $\gamma_p < \gamma_s$ (with a finite gap between γ_p and γ_s) for any arbitrary emitter array in all dimensions, rendering Dicke superradiance physically impossible.

thereby defining a minimal interaction range for superradiance. Another question is the *threshold* interaction range, which we define to be such that the critical coupling required for a burst becomes independent of the number of emitters, for any D. We show that exponentially-decaying interactions lie on the threshold interaction range for which the synchronization of the dipoles arising from the emission balances the decoherence effects.

Model.—The dynamics of an undriven ensemble of N emitters can be described by the Lindblad master equation (setting $\hbar = 1$)

$$\dot{\rho} = -i \sum_{i,j=1}^{N} \left[J_{ij} \sigma_i^+ \sigma_j^-, \rho \right] + \sum_{i,j=1}^{N} \gamma_{ij} \mathcal{D}[\sigma_i^-, \sigma_j^-]\rho \equiv \mathcal{L}[\rho], \tag{1}$$

with $J_{ij} = J_{ji}^*$ and $\gamma_{ij} = \gamma_{ji}^*$ to ensure Hermiticity. The raising and lowering operators for the j^{th} emitter are denoted as $\sigma_j^+ \equiv |e_i\rangle \langle g_i|$ and $\sigma_j^- \equiv |g_i\rangle \langle e_i|$ which describe transitions between the ground state $|g_i\rangle$ and excited state $|e_i\rangle$. The first term contains the coherent Hamiltonian interactions between the emitters, while the second term captures processes such as collective and local dissipation of the emitters via the superoperator $\mathcal{D}[\sigma_i^-, \sigma_j^-]\rho = \sigma_i^- \rho \sigma_j^+ - \{\sigma_j^+ \sigma_i^-, \rho\}/2$. We assume J_{ij} and γ_{ij} to be time-independent, such that the superoperator $\mathcal{L}$ generates a dynamical semigroup describing the dynamics of a Markovian open quantum system.

For a physically valid evolution (i.e., a completely positive and trace-preserving map), the matrix $\mathbf{\Gamma}$ containing the elements γ_{ij} (which we will refer to as the *decoherence matrix*) must be positive semi-definite [61–63]. The decoherence matrix can be diagonalized to yield N decay rates $\Gamma_\nu \geq 0$, with $\nu \in \{1, \ldots, N\}$ and the corresponding collective jump operators $\hat{c}_\nu$. The total photon emission rate of the emitters, integrated over all emission directions, is defined for any state ρ as

$$R_\rho \equiv \sum_{\nu=1}^{N} \Gamma_\nu \langle \hat{c}_\nu^\dagger \hat{c}_\nu \rangle = \sum_{\nu=1}^{N} \Gamma_\nu \mathrm{Tr}(\hat{c}_\nu^\dagger \hat{c}_\nu \rho). \tag{2}$$

For independent emitters with $\gamma_{ij} = \gamma_0 \delta_{ij}$, the total emission rate has a maximum of $N\gamma_0$ (saturated by the fully-excited state), and $R(t) \equiv R_{\rho(t)}$ decays exponentially. However, interactions between the emitters can cause $R(t)$ to increase beyond its initial value. This speedup in emission is commonly referred to as the superradiant burst, first discovered by Dicke [1] (see Fig. 1). Throughout this work, we refer to superradiant burst as the increase in the total emission rate beyond $N\gamma_0$, but the peak intensity need not scale as N^2. In general, characterizing the burst at arbitrary times can be difficult, hence one typically uses

$$\dot{R}_\rho = i \sum_\nu \langle [H, \hat{c}_\nu^\dagger \hat{c}_\nu] \rangle - \sum_{\mu,\nu} \Gamma_\mu \Gamma_\nu \langle \hat{c}_\mu^\dagger [\hat{c}_\mu, \hat{c}_\nu^\dagger] \hat{c}_\nu] \rangle \tag{3}$$

evaluated at the fully-excited initial state $\rho(0)$, with $\dot{R}(0) \equiv \dot{R}_{\rho(0)} > 0$ a sufficient condition for a superradiant burst. While we consider the burst at $t = 0$, we will provide physical justification on why this is sufficient.

Here, we put forward a new (and complementary) criterion to preclude any possibility of a burst: by a simple change of basis, one can write Eq. (2) as the expectation value of an auxiliary spin Hamiltonian

$$H_\Gamma = \sum_{j,k=1}^{N} \gamma_{kj} \sigma_j^+ \sigma_k^-, \tag{4}$$

with $R_\rho = \mathrm{tr}(H_\Gamma \rho)$. The maximum photon emission rate can thus be calculated by bounding the spectral radius of the auxiliary spin Hamiltonian. If the upper bound is equal or smaller than $N\gamma_0$, no burst can occur for all times and arbitrary initial states. While finding the largest eigenvalue of H_Γ may be non-trivial, this criterion allows one to definitively prove the absence of a burst for arbitrary times, thus going beyond the condition $\dot{R}(0) \equiv \dot{R}_{\rho(0)} > 0$. Furthermore, this approach opens up the possibility of finding theoretical limits for the emission rate arising from superradiant dynamics, as we show below and in the Supplementary Information [64].

No superradiance for nearest-neighbor coupling.— Let us consider a hypercube array of N emitters with arbitrary dimension D ($N = n^D$). For the case of NN interactions, $\gamma_{ii} \equiv \gamma_0 = 1$ and $\gamma_{ij} = \gamma$ if emitters i and

j are nearest-neighbor ($\gamma_{ij} = 0$ otherwise). The coupling $\gamma \in [0, 1]$ is required for the matrix $\mathbf{\Gamma}$ to be positive semidefinite. Without loss of generality, we have assumed γ_{ij} to be real and positive. We prove that for this model, superradiant burst cannot occur for any $t > 0$, for any arbitrary initial state and for any Hamiltonian coupling J_{ij}. To determine the physically valid regime, we impose the condition that $\mathbf{\Gamma}$ is positive semidefinite. Notice that the decoherence matrix can be expressed as $\mathbf{\Gamma} = \mathbf{I}_N + \gamma\mathbf{A}$, where $\mathbf{I}_N$ is the $N \times N$ identity matrix, and $\mathbf{A}$ is the adjacency matrix of a $n \times n$ grid graph. Using the fact that the grid graph is the Cartesian product of D path graphs $P_n \square \cdots \square P_n$, it can be shown that the smallest eigenvalue of $\mathbf{\Gamma}$ is [64]

$$\Gamma_{\min} = 1 - 2D\gamma \cos\left(\frac{\pi}{N^{1/D} + 1}\right), \tag{5}$$

which gives the physically valid regime as $\gamma \leq \gamma_p$,

$$\gamma_p = \left[2D \cos\left(\frac{\pi}{N^{1/D} + 1}\right)\right]^{-1}. \tag{6}$$

This rate reduces to $\gamma_p = 1/(2D)$ in the $N \to \infty$ limit, or when imposing periodic boundary conditions for a finite N. This can be regarded as coming from the coordination number for each emitter, which approaches $2D$ in the infinite-array limit. We now state our main result.

Theorem 1 *Let $\mathbf{\Gamma}$ be the decoherence matrix for a nearest-neighbor interaction model, with $\gamma_{ij} = \delta_{ij} + \gamma\delta_{\langle ij \rangle}$, where $\gamma \in [0, 1]$, and $\delta_{\langle ij \rangle} = 1$ if the emitters indexed by i and j are nearest-neighbor on the $D-$dimensional regular lattice, and 0 otherwise. For $\gamma \leq (2D)^{-1}$, the emission rate R_ρ is maximized by the fully-excited state $|e\rangle^{\otimes N}$ with $R_\rho = N$.*

We provide a sketch of the proof here, while the details can be found in the Supplementary Information [64]. By expressing H_Γ in the product-state basis and using the Gershgorin circle theorem [65], we can upper bound $\max_t R(t) \leq N$ in the physically valid regime $\gamma < 1/(2D)$. This is saturated by N independent emitters in the fully-excited state, with eigenvalue N. Hence, Theorem 1 implies that superradiant burst is impossible at all times. To gain a deeper physical understanding, we evaluate the superradiant regime $\gamma > \gamma_s$ for the fully-excited initial state, characterized by the transition at $\dot{R}(0) = 0$, for which [64]

$$\gamma_s = \left[2D(1 - N^{-1/D})\right]^{-1/2}. \tag{7}$$

For all $2 < N^{1/D} < \infty$, it can be shown that $\gamma_p < \gamma_s^2$ and therefore $\gamma_p < \gamma_s$. Hence, the superradiant regime does not overlap with the physically valid regime. Generalization to the hyper-rectangle configuration where the number of sites along each dimension can be different is straightforward, and the same conclusion is obtained [64]. While our analysis of the NN model is valid for any initial state, we consider a fully-inverted initial state for the next two sections: the analysis of next-nearest neighbor and exponentially decaying interactions.

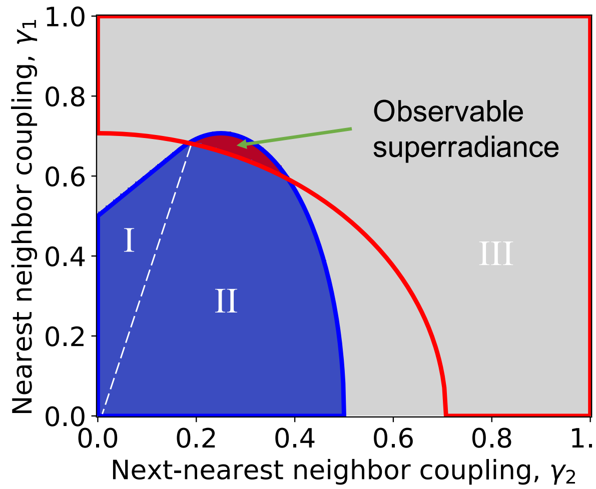

FIG. 2. Region of superradiant burst in the $\gamma_2 - \gamma_1$ plane. The physically valid (superradiant) regime is contained within the blue (red) boundary lines, with the conditions stated in the main text. Blue shaded region: Physically valid, but not superradiant. Regions I, II and III are defined in the main text. Red shaded region: Physically valid with superradiant burst. Grey shaded region: unphysical regime. The red shaded region requires a minimum of $\gamma_2 \approx 0.185$. All shaded regions here are obtained from numerical calculations for $N = 100$, which agree very well with the analytical results obtained in the infinite-array limit.

Next-nearest neighbor coupling.—Including the NNN interactions, we now show that a superradiant burst is indeed possible. For simplicity, let us consider a 1D ring of N emitters with periodic boundary conditions. In this configuration, $\mathbf{\Gamma}$ turns out to be a circulant matrix with the first column given by $(1, \gamma_1, \gamma_2, 0, \ldots, 0, \gamma_2, \gamma_1)^T$ with $0 \leq \{\gamma_1, \gamma_2\} \leq 1$. The subsequent columns are simply cyclic permutations of the first column. Diagonalizing $\mathbf{\Gamma}$ exactly yields the eigenvalues

$$\Gamma_\nu = 1 + 2\gamma_1 \cos\left(\frac{2\pi\nu}{N}\right) + 2\gamma_2 \cos\left(\frac{4\pi\nu}{N}\right) \tag{8}$$

for $\nu = 0, \ldots, N-1$. In the infinite-array limit $N \to \infty$, the eigenvalues form a continuous band in momentum space $\Gamma(k) = 1 + 2\gamma_1 \cos(k) + 2\gamma_2 \cos(2k)$, with the dimensionless wavevector $0 \leq k < 2\pi$. At the turning points where $\partial_k \Gamma = 0$, we have: $\Gamma(0) = 1 + 2(\gamma_1 + \gamma_2)$ which is always positive, $\Gamma(\pi) = 1 - 2(\gamma_1 - \gamma_2)$ and $\Gamma(k_*) = 1 - (\gamma_1^2 + 8\gamma_2^2)/4\gamma_2$ where $\cos k_* = -\gamma_1/4\gamma_2$. Demanding that $\Gamma(k) > 0$ thus produces the physically valid regimes: (I) $\gamma_1 - \gamma_2 \leq \frac{1}{2}, \gamma_1 > 4\gamma_2$ and (II) $\gamma_1^2 + 8\gamma_2^2 \leq 4\gamma_2, \gamma_1 \leq 4\gamma_2$, together with the bounds

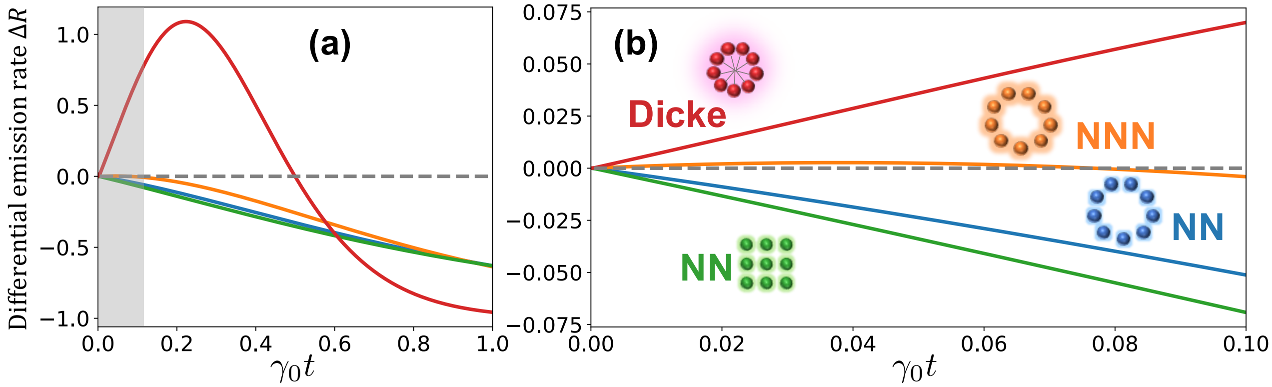

FIG. 3. Differential emission rate $\Delta R = R(t)/R(0) - 1$ against time (in units of emitter lifetime), for $N = 9$ emitters. $\Delta R > 0$ indicates superradiance. (a) Dynamical behavior of ΔR for the Dicke model (red), Next-nearest neighbor 1D ring (NNN, orange), Nearest-neighbor 1D ring (NN, blue) and Nearest-neighbor 2D square (NN, green) (see labels in (b)). The coupling parameters are chosen to maximize $g^{(2)}(0)$. (b) Short-time behavior obtained by zooming into the grey region of (a). Only the Dicke and the next-nearest neighbor models exhibit superradiance. The curve for the Dicke model is scaled down by a factor of 10 for visualization purposes.

$\gamma_1, \gamma_2 \in [0, 1]$ (blue regions in Fig. 2). The superradiant condition can be obtain from $\dot{R}(0) = 0$ as (III) $\gamma_1^2 + \gamma_2^2 > 1/2$.

There is an overlap region with the physically valid regime, as shown by the red shaded region in Fig. 2. For certain values of γ_1, γ_2, superradiant burst can occur. Moreover, the fact that this overlap region requires $\gamma_2 > (4 - \sqrt{2})/14 \approx 0.185$ is consistent with our previous conclusion of no superradiance using only NN coupling (i.e., $\gamma_2 = 0$). Superradiance is also forbidden by having only NNN coupling (i.e., $\gamma_1 = 0$). Results from numerical simulations of $N = 9$ emitters are presented in Fig. 3, which show that the NNN model has a small superradiant burst compared to the Dicke model, and no superradiance for NN models. We remark that this superradiance arises from destructive interference leading to dark decay channels with suppressed decay rates $\Gamma_\nu \approx 0$ while the dominant decay channel has a rate that does not scale with N. This mechanism is generally true for all models with a sharp interaction cutoff beyond a certain range.

Threshold interaction range for a superradiant burst.— In many previous works [16, 19, 20, 22, 24], $\mathbf{\Gamma}$ is obtained from a realistic modelling of the atomic interactions mediated by electromagnetic vacuum using the appropriate Green's function. Our goal here, however, is to shed light on the essential physics of superradiance by considering analytically tractable models that still exhibit interesting behaviors. Consider an interaction which decays exponentially with the separation r_{ij} between the emitters: $\gamma_{ij} \propto e^{-\kappa r_{ij}}$, where κ controls the decay of the interaction strength with emitter separation. We set the diagonal elements of $\mathbf{\Gamma}$ as 1, and define $\gamma \equiv e^{-\kappa d}$ with d the emitter NN separation such that $\gamma_{ij} = \gamma^{|\vec{x}_i - \vec{x}_j|}$, where $\vec{x}_i \in \mathbb{Z}^D$ is the position vector of the i^{th} lattice site. Physically, this model describes exponentially-decaying interactions

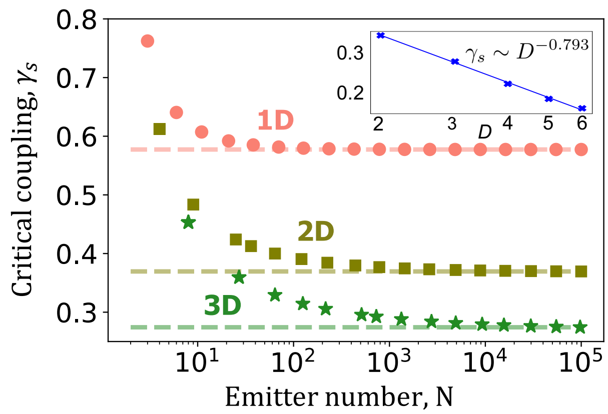

FIG. 4. Critical coupling γ_s for exponentially-decaying interactions in a 1D chain, a 2D square array and a 3D cubic array with N emitters. Superradiance occurs for $\gamma > \gamma_s$. For all dimension D, γ_s becomes independent of N for large N. (Inset) Log-log plot of γ_s against D for $N \approx 10^6$. γ_s decreases as D increases with a power-law scaling $\gamma_s \sim D^{-0.793}$.

between the atoms. For a sufficiently large N in D dimensions such that $\gamma^N \ll 1$, $\dot{R}(0)$ is approximately given by the asymptotic form

$$\dot{R}(0) \sim N\left(\frac{2D\gamma^2}{1-\gamma^2} - 1 + \frac{C}{(-\ln\gamma)^D}\right) \qquad (9)$$

for some constant C [64]. Interestingly, this suggests that the critical coupling parameter γ_s for superradiance is independent of N as $N \to \infty$ for all dimension, agreeing with the numerical results shown in Fig. 4. This is in stark contrast with previous results (primarily using long-range power-law interactions such as $\gamma_{ij} \propto 1/r_{ij}$), which predict that the critical emitter separation increases with N in 2D and 3D arrays [19, 24]. Figure 4 also shows that for large N, $\gamma_s \sim D^{-0.793}$ exhibits a power-law scaling with the spatial dimension. This is intuitive as the average coupling per emitter increases with D which in turn lowers the critical coupling required for superradiance [24]. The N-independence of γ_s for our short-range exponential model can be physically interpreted as the threshold interaction range where the synchronization effects due to collective interactions scales similarly with N as the local decoherence, such that adding more emitters do not affect the onset of the superradiant regime. For even shorter-range interactions such as the NN model, the local decoherence dominates which prevents superradiance. Longer-range models such as power-law interactions favor synchronization and thus enhance superradiance as N increases.

Scaling of the peak emission rate with number of emitters— Eq. (4) shows that the problem of calculating the emission rate is equivalent to finding the average energy of a state under the Hamiltonian H_Γ. This enables us to find upper bounds on the scaling of the peak emission rate with N, for arbitrary geometries and types of interactions. As we have shown before in Theorem 1, the maximum emission rate for arbitrary NN models is $N\gamma_0$. For 1D arrays with an exponentially-decaying interaction, the upper bound on the emission rate is found to scale as $O(N)$ for $\gamma < 1$ [64]. This bound increases to $O(N \log N)$ for 1D arrays with a power-law interaction of the form $1/r$ [64]. This latter scaling is consistent with the numerical results obtained in the literature which, in contrast to our bound, have only been obtained for relatively small systems and under certain approximations [20, 22, 24]. While finding exact bounds may be exponentially hard, one could in principle upper-bound other models, as well as tighten the currently-obtained bounds.

Discussion.—In this Letter, we addressed the fundamental problem of the minimal interaction range required for superradiance. Crucially, we proved that nearest-neighbor interactions cannot induce emitter correlations faster that the decoherence, resulting in the impossibility of superradiance. As shown, the minimal interaction range is therefore next-nearest neighbor, and longer-range interactions generally lead to stronger superradiance. We also found that the short-range exponential interaction marks the threshold interaction range in all dimensions where the emitter correlations and local decoherence scale similarly with the number of emitters such that the critical coupling required for superradiant burst becomes independent of the number of emitters, in stark contrast with previous conclusions using longer-range power-law interactions. We stress that, apart from the nearest-neighbor model, our classification of a superradiant burst is strictly speaking only valid at short times up to $\mathcal{O}((\gamma_0 t)^2)$ (if $\ddot{R}(0) < 0$ which is true for the models considered here [64]), where the dynamics of the fully-excited emitters do not depend on the Hamiltonian. This can be physically justified for later times using second-order mean field theory [64].

The techniques used in this work have broader applications in determining the theoretical bounds for the emission rate of different models, thereby exposing the ultimate limitations of superradiance beyond the NN model. Beyond providing fundamental insights to the physics of superradiance, our results can also motivate the design of atomic lattices in engineered baths such as nanophotonic crystals with engineered interactions or superconducting resonator arrays for qubits. Moreover, hypercube geometries should be within reach of state-of-the-art quantum simulators, given the recent advances in generating arbitrary networks in cavity [66] and circuit [67] quantum electrodynamics platforms.

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