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## Triplet pairing mechanisms from Hund's-Kondo models: applications to  $UTe_2$  and  $CeRh<sub>2</sub>As<sub>2</sub>$

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Observing that several U and Ce based candidate triplet superconductors share a common structural motif, with pairs of magnetic atoms separated by an inversion center, we hypothesize a triplet pairing mechanism based on an interplay of Hund's and Kondo interactions that is unique to this structure. In the presence of Hund's interactions, valence fluctuations generate a triplet superexchange between electrons and local moments. The offset from the center of symmetry allows spintriplet pairs formed by the resulting Kondo effect to delocalize onto the Fermi surface, precipitating superconductivity. We demonstrate this mechanism within a minimal two-channel Kondo lattice model and present support for this pairing mechanism from existing experiments.

Hund's coupling is a direct manifestation of the Coulomb interaction in multi-electron atoms. One wellknown consequence is the development of large moments in d and f-shell materials. The effects of Hund's interactions on valence fluctuations and the Kondo effect in such high-spin systems are less well understood.

A possible role of Hund's coupling in pairing mechanisms for triplet superconductivity has been discussed by Anderson [\[1\]](#page-5-1), Norman [\[2\]](#page-5-2), Hotta and Ueda [\[3,](#page-5-3) [4\]](#page-5-4) and more recently in the context of triplet resonating valence bonds (tRVBs) in various settings [\[5,](#page-5-5) [6\]](#page-5-6). Here we discuss the implication of these ideas for triplet-paired heavy fermion superconductors in a two-channel Kondo lattice model [\[7–](#page-5-7)[11\]](#page-5-8), with two key observations. First, we highlight a common structural motif unique to candidate triplet-paired heavy fermion materials (Table [I](#page-1-0) and Figure [1\)](#page-1-1) - an even number of magnetic ions separated by a center of symmetry. This sublattice structure enables the triplet paired correlations induced by Hund's coupling to develop coherence on the Fermi surface, inducing a Cooper instability. Second, we show how Hund's coupling modifies the structure of the Kondo interaction, inducing triplet correlations between scattered electrons and local moments.

Our observations are built on the tRVB concept [\[5,](#page-5-5) [6\]](#page-5-6), whereby valence fluctuations into entangled, high-spin

							$[\mathrm{UTe}_2]\mathrm{UCe}_2]$ UCoGe $[\mathrm{URhGe}]\mathrm{UBe}_{13}]$ UPt <sub>3</sub> $[\mathrm{CeRh}_2\mathrm{As}_2]$ CeSb <sub>2</sub>	
Ref	[12]	$\left 13\right $	$[13]$	[13]	[14]	[15]	[16]	[17]
$n_M$	2	2		4	2	$\mathcal{D}$	9	$\Omega$
$T_c$	2K	0.8K	0.8K	$0.25K$   0.95K   0.5K			0.26K	0.22K
$H_P$	3.7T	1.5T	1.5T	0.5T	1.8T	0.9T	0.5T	0.4T
$H_{c2}$	60T	3T	18T	13T	14T	2.8T	14T	3T
$T_M$	$\sim$	52K	2.7K	9.5K				

<span id="page-1-0"></span>Table I. Candidate heavy fermion triplet superconductors have either  $n_M = 2$  or 4 magnetic U/Ce atoms in the conventional unit cell separated by an inversion center. Maximum superconducting  $T_c$ , Pauli limiting critical field  $H_P$ , highest measured upper critical field  $H_{c2}$ , Curie temperature  $T_M$  for ferromagnetic superconductors.



<span id="page-1-1"></span>Figure 1. Two-sublattice structure of  $UPt_3$ ,  $UBe_{13}$  and UTe<sub>2</sub>, shown in the conventional unit cell. The inversion center is shown in red, and the two distinct sublattices of U in blue and green.

configurations (Figure [2\)](#page-2-0) can delocalize into a coherent triplet RVB superconducting state. Symmetry plays a central role in this process, for triplet-paired configurations in the excited state of a magnetic atom are even under inversion about the atom, while triplet pairing on a Fermi surface is necessarily odd parity. Anderson [\[1\]](#page-5-1) recognized that if local moments are situated at distinct sublattices away from the inversion centers, the onsite triplet pairs could acquire an inversion-odd sublattice form factor, allowing them to coherently couple to triplet Cooper pairs on the Fermi surface [\[2–](#page-5-2)[4\]](#page-5-4). Remarkably, the structural motif identified by Anderson forty years ago characterizes every candidate tripletpaired heavy fermion superconductor we know today, with only two exceptions  $[18]$ ,  $[19]$ . (see Table [I](#page-1-0) and Figure [1\)](#page-1-1). Recently,  $CeRh<sub>2</sub>As<sub>2</sub>$  and  $CeSb<sub>2</sub>$  have also emerged as new members  $[16, 17]$  $[16, 17]$  $[16, 17]$  of this class of compounds [\[20\]](#page-5-17). The near-universal correlation suggests the possibility of another family of triplet superconductors in  $PrTr<sub>2</sub>Al<sub>20</sub>$  (Tr=Ti,V) [\[21,](#page-5-18) [22\]](#page-5-19), which also has this structural motif [\[23\]](#page-5-20).

To illustrate the interplay between Hund's coupling



<span id="page-2-0"></span>Figure 2. Delocalization of Hund's coupled triplet **pairs:** (a) Hund's coupling  $(J_H)$  and crystal field  $(\Delta_{CF})$  entangle two spins in an  $\bar{f}^2$  local moment into an  $S_z = 0$  triplet state. (b) Triplet valence bond representation of  $S_z = 0$ triplet state. (c) Hybridization causes a transfer of triplet pairs (red) into the conduction (orange) sea.

and valence fluctuations, consider a magnetic ion in which the f-electrons exist in three valence states, taken for simplicity to be  $f^0$ ,  $f^1$  and  $f^2$ . Suppose that in the f 2 configuration, Hund's coupling and crystal-field splitting, enabled by spin-orbit coupling, stabilize a state in which two f-orbitals of symmetry  $\Gamma_1$  and  $\Gamma_2$  are entangled into an  $S = 1, S_z = 0$  state (Fig. 2a), forming a triplet valence bond between the two orbitals (Fig. 2b). When the ion hybridizes with the conduction sea, valence fluctuations will now allow the entangled triplet pair to escape into the conduction sea (Fig. 2c).

To understand how Hund's interactions affect superexchange, suppose the  $f^1 \Gamma_1$  Kramer's configuration is the most stable, forming a local moment. Integrating out the virtual valence fluctuations (Fig. [3\)](#page-2-1),  $f^1 \rightleftharpoons f^0 + e^-$  and  $e^- + f^1 \rightleftharpoons f^2$  via a Schrieffer-Wolff transformation [\[24,](#page-6-0) [25\]](#page-6-1) generates a second-order perturbation in the energy of conduction electrons scattering in the  $\Gamma_{1,2}$  channel

$$
H_K = -\frac{|V_{\Gamma_1}|^2}{E_0} P_S^{\Gamma_1} - \frac{|V_{\Gamma_2}|^2}{E_2} P_T^{\Gamma_2}
$$
 (1)

where  $V_{\Gamma_{1,2}}$  are the hybridization matrix elements in the two channels, while  $E_0$  and  $E_2$  are the corresponding excitation energies. The operators  $P_S^{\Gamma_1} = \frac{1}{2} - \vec{\sigma}^{\Gamma_1} \cdot \vec{S}$ and  $P_T^{\Gamma_2} = \sigma_z P_S^{\Gamma_2} \sigma_z$  project the incoming quasiparticles



<span id="page-2-1"></span>Figure 3. Low energy Hilbert space of the local moment, showing associated Hubbard operators:  $\Gamma_1$  is the irreducible representation of the spatial wavefunction of the ground-state  $f^1$  doublet. The  $f^2$  excited stated is a entangled spin-triplet state with  $S_z = 0$ , formed from electrons in the  $\Gamma_1$  and  $\Gamma_2$  orbitals.

into the singlet and triplet states of the excited  $f^0$  and  $f^2$ states, respectively. Here  $\vec{\sigma}^\Gamma$  is the spin of the conduction electron in channel Γ. By noting that a  $S_z = 0$  triplet  $|\Downarrow \uparrow + \Uparrow \downarrow \rangle = \sigma_z |\Downarrow \uparrow - \Uparrow \downarrow \rangle$  is obtained from a singlet by rotating the conduction electron spin through 180◦ about the z–axis, we have written  $P_T^{\Gamma_2} = \sigma_z P_S^{\Gamma_2} \sigma_z$  as a unitary transform of the singlet operator  $P_S^{\Gamma_2}$ .

Omitting potential scattering terms, it follows that Hund's interactions cause the Kondo interaction to develop a triplet-superexchange with XXZ anisotropy,

$$
H_K = J_1 \mathbf{S} \cdot \boldsymbol{\sigma}^{\Gamma_1} + J_2 \mathbf{S} \cdot (\sigma_z \boldsymbol{\sigma}^{\Gamma_2} \sigma_z) \tag{2}
$$

where  $J_1 = |V_{\Gamma_1}|^2/(2E_0)$  and  $J_2 = |V_{\Gamma_2}|^2/(2E_2)$  [\[26\]](#page-6-2). The Hund's-Kondo term can alteratively be written  $H_{K2} = J_2[S_z \sigma_z^{\Gamma_2} - S_x \sigma_x^{\Gamma_2} - S_y \sigma_y^{\Gamma_2}].$  Acting in isolation (*i.e.* if  $J_1 = 0$ ) the Hund's-Kondo coupling  $J_2$ flows to strong-coupling like its antiferromagnetic coun-terpart [\[27\]](#page-6-3), but forms a "screened" triplet state ( $|\Uparrow\downarrow$  $+ \Downarrow \uparrow$ ). We are interested in the interplay of the two terms in the lattice.

Although we have chosen an  $S_z = 0$  orientation of the Hund's triplet to illustrate this physics, in practice, the crystal fields will determine the orientation of the dvector of the triplet  $f^2$  excited state. Moreover, spin-orbit coupling will generically introduce additional rotations of the electron spin-quantization axis into the hybridization matrix elements. These two effects mean that pre-formed triplet pairs of any odd-parity irreducible representation allowed by the crystal structure can delocalize via the Kondo hybridization.

We now incorporate the above effects into a two channel Kondo lattice model  $H = H_c + H_{K1} + H_{K2}$ , where

$$
H_c = -\sum_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} \left[ (t_0 \gamma_{\mathbf{k}} + \mu) + t_1 \gamma_{\mathbf{k}} \alpha^x \right] c_{\mathbf{k}},
$$
  
\n
$$
H_{K1} = J_1 \sum_{j\alpha} \psi_{1\alpha}^{\dagger}(j) \sigma \psi_{1\alpha}(j) \cdot \mathbf{S}_{j\alpha},
$$
  
\n
$$
H_{K2} = J_2 \sum_{j\alpha} \psi_{2\alpha}^{\dagger}(j) \sigma_z \sigma \sigma_z \psi_{2\alpha}(j) \cdot \mathbf{S}_{j\alpha}.
$$
 (3)

Here  $H_c$  describes electron hopping on a two-sublattice body-centered cubic lattice, reminiscent of  $UT_{2}$  [\[28\]](#page-6-4), where  $c_{\mathbf{k}}^{\dagger} \equiv c_{\mathbf{k}\alpha\sigma}^{\dagger}$  creates an electron of wavevector **k** on sublattice  $\alpha = \pm 1$  with spin component  $\sigma^z = \sigma$ ,  $t_0$  and  $t_1$  are the intra- and inter-sublattice hopping integrals, respectively,  $(\alpha^x, \alpha^y, \alpha^z)$  are the sublattice Pauli matrices and  $\gamma_{\bf k} = 8 \cos \frac{k_x}{2} \cos \frac{k_y}{2} \cos \frac{k_z}{2}$  is the nearest neighbor form-factor that is invariant under the  $D_{2h}$  point group.  $H_{K1}$  and  $H_{K2}$  are the Kondo interaction in channels  $\Gamma = (\Gamma_1, \Gamma_2)$ , where

$$
\psi_{\Gamma\alpha}^{\dagger}(j) = \frac{1}{\sqrt{N_s}} \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} \Phi_{\Gamma\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{R}_j}
$$
(4)

create electrons in Wannier states of symmetry Γ coupled to spins  $\mathbf{S}_{j\alpha}$  at site  $j, \alpha$  ( $N_s$  is the number of unit cells). We choose  $\Phi_{1k} = 1$  and

$$
\Phi_{2\mathbf{k}} = i\sigma_x(\sqrt{1-\zeta^2} + i\zeta\alpha^z p_\mathbf{k}^z),\tag{5}
$$

for the two Kondo channels, consistent with timereversal and inversion symmetry.  $\frac{z}{k}$  =  $\cos(\frac{k_x}{2})\cos(\frac{k_y}{2})\sin(\frac{k_z}{2})$  is an odd-parity crystal harmonic transforming under the  $B_{1u}$  representation. The coefficient  $\zeta$  is finite when the two magnetic atoms are displaced from their common center of symmetry, activating an antisymmetric spin-orbit [\[29\]](#page-6-5) mediated coupling between tRVBs (Fig. [2b](#page-2-0)) and triplet Cooper pairs. The factor  $i\sigma_x$  in the hybridization  $\Phi_{2k}$  captures the spinorbit coupling between f-states and a conduction state with different orbital content. The term proportional to ζ describes a coupling between spin and momentum that is odd in the sublattice index [\[26\]](#page-6-2), similar to the staggered Rashba coupling [\[28,](#page-6-4) [30\]](#page-6-6) discussed in the context of layered materials like CeRh<sub>2</sub>As<sub>2</sub>.

The key physics of this model involves a co-operative action of the Kondo effect in the two channels [\[9–](#page-5-21)[11\]](#page-5-8). At high temperatures in the lattice, the Kondo coupling in both channels renormalizes to strong-coupling according the scaling equation  $\partial (J_{\Gamma}\rho)/\partial \ln \Lambda = -(J_{\Gamma}\rho)^2$  [\[31\]](#page-6-7), where  $\rho$  is the conduction electron density of states, and  $\Lambda$  the energy cut-off. Suppose channel one is the strongest channel with the largest Kondo temperature  $T_{K1} \sim De^{-1/J_1 \rho}$ , where D is the bandwidth. Then the logarithmic renormalization in channel two is interrupted at a scale  $\Lambda = T_{K1}$ , with a renormalized Kondo coupling constant given by

<span id="page-3-1"></span>
$$
\frac{1}{J_2^*} = \frac{1}{J_2^0} - \rho \ln\left(\frac{D}{T_{K1}}\right) \tag{6}
$$

At temperatures  $T \leq T_{K1}$  the local moments fractionalize into deconfined heavy fermions  $\mathbf{S}_j \to f_j^{\dagger} \left( \frac{\sigma}{2} \right) f_j$ : in an impurity model, a Kondo resonance forms in the strongest channel and nothing further happens.

However, in the Kondo lattice, the heavy fermions hybridize with the conduction electrons to form a large Fermi surface [\[32,](#page-6-8) [33\]](#page-6-9). Moreover, in the presence of a finite  $\zeta$  the residual interaction  $J_2$  created by valence fluctuations into Hund's-coupled spin-triplet states, couples triplet Cooper pairs on the Fermi surface, which reactivates its scaling, causing it to resume its upward logarithmic renormalization, ultimately diverging at  $T_c$  to form a Hund's driven triplet superconductor.

To examine this process, we note that action of  $H_{K2}$ on the deconfined fermions is given by

<span id="page-3-0"></span>
$$
H_{K2} = -J_2^* \sum_j (\psi_{2j}^\dagger \sigma_z(-i\sigma_y) f_j^\dagger)(f_j(i\sigma_y) \sigma_z \psi_{2j}) \tag{7}
$$

where the sublattice indices have been suppressed. Here we have used the particle-hole symmetry of the fractionalized spin operator to replace  $f_j \to -i\sigma_y f_j^{\dagger}$  in the usual hybridized form of the Kondo interaction. Now the hybridized quasiparticle operators formed in channel one have the form  $a_{\mathbf{k}\sigma} = u_{\mathbf{k}}f_{\mathbf{k}\sigma} + v_{\mathbf{k}}\Phi_{1\mathbf{k}}c_{\mathbf{k}\sigma}$ , where  $u_{\mathbf{k}} \sim 1$ , while  $v_{\mathbf{k}}^2 \sim \frac{m}{m^*} \ll 1$  is set by the inverse mass renormalization of the heavy fermions [\[34\]](#page-6-10). If we now decompose  $H_{K2}$  as a pair scattering potential acting on the quasiparticle pairs at the Fermi level, we obtain

$$
H^* = -J_2^* |u_{\mathbf{k}} v_{\mathbf{k}}|_{\text{FS}}^2 \sum_{\mathbf{k}, \mathbf{k'}} \Lambda_{\mathbf{k}}^\dagger \Lambda_{\mathbf{k'}} \tag{8}
$$

where  $|u_{\mathbf{k}}v_{\mathbf{k}}|_{\text{FS}}^2$  denotes the Fermi surface average of the coherence factors and  $\Lambda_{\mathbf{k}} = a_{-\mathbf{k}}(i\sigma_y)(\mathbf{d}_{\mathbf{k}}^A \cdot \sigma)a_{\mathbf{k}}$  is the projection of the triplet pair operators in [\(7\)](#page-3-0) onto the Fermi surface, where  $\mathbf{d}_{\mathbf{k}}^A = (\mathbf{d}_{\mathbf{k}} - \mathbf{d}_{-\mathbf{k}})/2$  is the oddparity component of the form factor  $\mathbf{d}_{\mathbf{k}} \cdot \boldsymbol{\sigma} = \alpha_z \sigma_z \Phi_{2\mathbf{k}} \Phi_{1\mathbf{k}},$ projected into the band eigenstates [\[26\]](#page-6-2). This antisymmetric term is absent if the magnetic ions lie at a center of symmetry, both  $\Phi_{1k}$  and  $\Phi_{2k}$  have the same parity under  $k \to -k$ , so  $\mathbf{d}_{\mathbf{k}}^A = 0$  and the triplet pseudopotential vanishes. However, in the presence of an offset center of symmetry,  $(\zeta > 0)$  it becomes finite. In our model calculation,  $\mathbf{d}_{\mathbf{k}}^A \sim i\zeta p_{\mathbf{k}}^z \hat{\mathbf{y}}$ , distinct from the d-vector of the localized f-electrons in the moment (Fig. [2\)](#page-2-0), due to the spin-orbit coupling [\[26\]](#page-6-2).

From these arguments, we see that the triplet coupling constant induced by the Hund's Kondo effect is now  $g_t \sim J_2^* \rho^*(uv)_{FS}^2 \approx J_2^* \rho^* v^2$  because  $u_{\mathbf{k}} \sim 1$ . Now at first sight, the small size of  $v^2 \sim m/m^* \ll 1$  is cause for concern, but it is compensated by the large density of states of the heavy Fermi liquid  $\rho^* \sim 1/T_{K1}$ . In fact the product  $v^2 \rho^* = dN/d\mu$  is recognized as the charge susceptibility of the heavy Fermi liquid, and since the f-component of the heavy Fermi liquid is incompressible, this quantity is equal to the unrenormalized density of states of the conduction fluid,  $dN/d\mu \sim \rho$ , so that  $g_t \sim \zeta J_2^* \rho$  is essentially unaffected by the large mass renormalization of the heavy electrons. A Cooper instability will develop when  $\frac{1}{J_2^*} - \rho d^2 \ln \left( \frac{T_{K1}}{T_c} \right) = 0$ , where we have denoted the average d-vector magnitude by  $d^2 = \langle |\mathbf{d}_{\mathbf{k}}^A|^2 \rangle_{FS}$ . Combining with  $(6)$ , a superconducting instability will take place when

<span id="page-3-2"></span>
$$
\frac{1}{J_2^0} - \rho \ln\left(\frac{D}{T_{K1}}\right) - \rho d^2 \ln\left(\frac{T_{K1}}{T_c}\right) = 0 \tag{9}
$$

Remarkably, this expression contains two logs - a Kondo log that renormalizes  $J_2$  from the band-width down to  $T_{K1}$ , followed by a further Cooper renormalization of the coupling constant below  $T_{K1}$ . Thus the second channel Kondo effect plays a vital co-operative role in enhancing the pairing process. Solving [\(9\)](#page-3-2), we find that

$$
T_c = (T_{K2})^{\frac{1}{d^2}} (T_{K1})^{1 - \frac{1}{d^2}} \tag{10}
$$

is a weighted geometric mean of the Kondo temperatures  $T_{KT} = De^{-1/J_{\Gamma}\rho}$  in the two channels, emphasizing the co-operative nature of the Hund's-Kondo effect.

To quantify this effect in greater detail, we employ an  $SU(2)$  gauge theory [\[35,](#page-6-11) [36\]](#page-6-12) description of the Kondo effect. The local moment on each sublattice may be represented in terms of Abrikosov pseudo-fermions,  $S =$  $\frac{1}{2}(f^{\dagger} \boldsymbol{\sigma} f)$ . In the Nambu basis  $F = (f, (i\sigma_y)^{\dagger} f^{\dagger})^T$ , this corresponds to  $\mathbf{S} = \frac{1}{4} F^{\dagger}(\boldsymbol{\sigma} \otimes \tau_0) F$ , a representation that is invariant under SU(2) particle-hole transformations of the spinons  $F \to e^{i\mathbf{n} \cdot \boldsymbol{\tau}} F$ , where  $\tau_i$  are Pauli matrices in Nambu space. In this representation, the Kondo couplings become four-fermion interactions which are then decoupled using a hybridization mean-field  $V_{\Gamma_{\alpha}}$  and a 'pairing' field  $\Delta_{\Gamma_{\alpha}}$  in each channel and sublattice to get  $H = H_c + H_K$ , where

$$
H_{K} = \sum_{\Gamma \mathbf{k}\alpha} \left( V_{\Gamma\alpha} \tilde{\psi}_{\Gamma \mathbf{k}\alpha}^{\dagger} f_{\mathbf{k}\alpha} + \Delta_{\Gamma\alpha} \tilde{\psi}_{\Gamma \mathbf{k}\alpha}^{\dagger} (i\sigma_{y})^{\dagger} f_{-\mathbf{k}\alpha}^{\dagger} + \text{H.c.} \right) + N_{s} \sum_{\Gamma\alpha} \frac{|V_{\Gamma\alpha}|^{2} + |\Delta_{\Gamma\alpha}|^{2}}{J_{\Gamma}}.
$$
 (11)

where  $\tilde{\psi}_{\Gamma_1} = \psi_{\Gamma_1}, \tilde{\psi}_{\Gamma_2} = \sigma_z \psi_{\Gamma_2}$ . With a suitable SU(2) gauge transformation on the spinons  $F \to e^{i \mathbf{n} \cdot \boldsymbol{\tau}} F$ , one can choose a gauge in which  $\Delta_{1\alpha} = 0$ , and the hybridization  $V_{1\alpha}$  is real. This hybridization in the first (singlet) channel then locks the  $U(1)$  gauge of the spinons to the electrons, so that they have the same charge.

We shall focus on the situation in which  $J_2$  is smaller than  $J_1$  and seek superconducting solutions in which the normal state  $V_{1\alpha} = V$  is uniform and  $\Delta_{2\alpha}$  are non-zero, leading to the gap equation

$$
\frac{\Delta_{2\alpha}}{J_2} = -\frac{1}{N_s} \sum_{\mathbf{k}} \left\langle f_{-\mathbf{k}\alpha}(i\sigma_y)\sigma_y \left(\sqrt{1-\zeta^2} + i\zeta \alpha p_{\mathbf{k}}^z\right) c_{\mathbf{k}\alpha} \right\rangle
$$

with the constraint  $n_{f\alpha} = \frac{1}{N_s} \sum_{\mathbf{k}} \left\langle f_{\mathbf{k}\alpha}^{\dagger} f_{\mathbf{k}\alpha} \right\rangle = 1$ . We see that there are two types of superconducting solution:

- 1. a sublattice-even solution where  $\Delta_{2\alpha} = \Delta_2$  and the triplet pairing term  $\langle f_{-\mathbf{k}\alpha}\sqrt{1-\zeta^2}c_{\mathbf{k}\alpha}\rangle$  is an even function of **k**. This solution is then necessarily antisymmetric in the band indices - an inter-band gap function without a Cooper instability, that will only develop for  $J_2$  above a critical value.
- 2. a staggered gap where  $\Delta_{2\alpha} \propto \alpha \Delta_2$  is odd on the two sublattices and  $\langle f_{-\mathbf{k}\alpha} \zeta p_{\mathbf{k}}^z c_{\mathbf{k}\alpha} \rangle$  is an odd function of momentum. This solution has a Cooper instability.

Below the superconducting  $T_c$  (Figure [\(4\)](#page-4-0)), we find a sublattice-odd mean-field ground state  $V_{1A}$  =  $V_{1B}, \Delta_{2A} = -\Delta_{2B}, V_{2\alpha} = 0$  corresponding to an oddparity triplet superconductor. The product of the order parameters in the two channels corresponds to a composite pairing operator[\[9\]](#page-5-21)

$$
\Psi = \langle V_{1\alpha} \Delta_{2\alpha} \rangle \propto \sum_{\mathbf{k},j,\alpha} \langle \psi_{2,-\mathbf{k}\alpha} \sigma_z(\boldsymbol{\sigma} i\sigma_y) \psi_{1\mathbf{k}\alpha} \cdot \mathbf{S}_{j\alpha} \rangle,
$$



<span id="page-4-0"></span>Figure 4. Temperature dependence of order parameters. Calculations were made with parameters  $J_{K1} = 2.67t_0, J_{K2} =$  $6.67t_0, \mu = 2t_0, t_1 = 0.67t_0, \zeta = 0.995.$ 

which transforms as the  $S_y = 0$  component of a spintriplet due to the non-trivial form-factor between the two channels  $\sum_{\alpha} \alpha \langle \psi_{2,-\mathbf{k}\alpha} \sigma_z (i \sigma_y) \psi_{1\mathbf{k}\alpha} \rangle \sim p_{\mathbf{k}}^z \sigma_y.$ 

In the Hund's-Kondo mechanism we present, the pairing symmetry is determined by the symmetry of the strongly-correlated atomic states. In this demonstration, the pair potential transforms according to the  $B_{3u}$  representation of  $D_{2h}$ , consistent with the irrep of the preformed pairs in the Hund's-coupled moment  $|f^2\rangle\langle f^0| \sim$  $\Phi_1^{\dagger} \sigma_z \Phi_2^{\dagger} \sim \sigma_z \sigma_x p_{\mathbf{k}}^z$ . This provides generic constraints on the pairing symmetries from the atomic matrix elements  $|f^{n+2}\rangle\langle f^n|$  which can be experimentally accessed by resonant inelastic X-ray scattering [\[37,](#page-6-13) [38\]](#page-6-14) and by Raman scattering [\[39\]](#page-6-15). There are two other generic features of our pairing mechanism that are experimentally verifiable. The sublattice-odd real-space form-factor of the pair wavefunction may be detected using scanning Josephson interferometry [\[5\]](#page-5-5). The continuous change in f-valence at the superconducting transition due to Kondo hybridization in a second channel is detectable using lowtemperature core-level spectroscopy and X-ray scattering.

The pair-potential in our demonstration has line-nodes corresponding to the intersection of  $p_{\mathbf{k}}^z = 0$  and the Fermi surfaces, and a pseudo-spin d-vector on the Fermi surface aligned along the y-axis. In  $UT_{2}$ , the experimental evidence for point-like gap nodes can be reconciled by noting that spin-orbit coupling induces secondary order parameters with d-vectors like  $\mathbf{d}_{\mathbf{k}} = p_{\mathbf{k}}^y \hat{z}$  in the same irreducible representation, which gap out the line nodes except where  $p_{\mathbf{k}}^y = 0$ . This results in the d-vector of the pair potential having a two-dimensional texture in momentum space, and points to one of the many ways one could engineer non-trivial topology in this system. In our simple demonstration, the gaps on the two Fermi surfaces are identical and the superconductor is topologically trivial because any time-reversal invariant  $\mathbb{Z}_2$  topological index is doubled. In  $UBe_{13}$ , a possible extension

of this model would replace  $p_{\bf k}^z$  by an f-wave form factor that transforms like  $k_x k_y k_z$ . However, the observation that superconductivity emerges directly from a local moment regime suggests that the coupling strengths in the two channels are similar  $J_1 \approx J_2$ , so that  $T_c \approx T_K$ . In the high temperature A phase of  $UPt_3$ , there is strong evidence [\[40,](#page-6-16) [41\]](#page-6-17) for an f-wave superconducting gap and  $S_z = 0$  spin-triplet pairing, readily captured by a variation of our model in which the local pairs have  $S_z = 0$ spin-structure and the orbital content differs by the fwave form factor  $\Gamma_2 \sim \Gamma_1 \times (x^2 - y^2)z$ .

There is good reason to suspect that the Kondo effect is intimately linked with the superconductivity in these materials. Tunneling experiments  $[42]$  in UTe<sub>2</sub> find a Fano lineshape in the differential conductance, that is characteristic of cotunneling into a local moment. The electrical resistivity of UTe<sub>2</sub>[\[43,](#page-6-19) [44\]](#page-6-20) and CeRh<sub>2</sub>As<sub>2</sub>[\[16\]](#page-5-13) both show clear broad maxima in the temperature dependence, putatively signaling the onset of Kondo hybridization. The superconducting  $T_c$  is well below the Kondo temperature - the local moments are not spectators to the development of triplet pairing correlations.

Our theory contrasts with many recent theoretical proposals  $[45-64]$  $[45-64]$  for the pairing symmetry of UTe<sub>2</sub> and  $CeRh<sub>2</sub>As<sub>2</sub>$  (with the noted exception of [\[62\]](#page-6-23)), in which Kondo physics does not play a role. Instead, the common theme is to consider a Kramer's doublet at each site, forming narrow dispersive bands, whose Fermi surfaces are unstable to pairing by intersite magnetic exchange interactions. The symmetry of the superconducting order parameter is then determined either by the anisotropy of ferromagnetic fluctuations [\[28\]](#page-6-4) or by the anisotropy of the band spin-orbit coupling in presence of isotropic ferromagnetic exchange [\[47\]](#page-6-24). We have alternately emphasized that the symmetric-spin pairing correlations ascribed to inter-site interactions, ferromagnetic or antiferromagnetic [\[55,](#page-6-25) [56\]](#page-6-26), are already present at the atomic level, driven directly by the largest energy scale in the system - atomic Coulomb repulsion.

The conceptual appeal of a Hund's-Kondo pairing mechanism lies in its ability to harness the coherence of Kondo hybridization to couple pre-formed Hund's triplets into a superconducting condensate. Key to this framework is the common structural motif of local moments with a sublattice degree of freedom shared by the diverse set of heavy fermion superconductors in Table [I,](#page-1-0) that allows localized triplet pairs to overlap with odd-parity Cooper pairs on the Fermi surface.

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