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Discovery of ³⁹Na

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Abstract: The new isotope ³⁹Na, the most neutron-rich sodium nucleus observed so far, was discovered at the RIKEN Nishina Center Radioactive Isotope Beam Factory using the projectile fragmentation of an intense ⁴⁸Ca beam at 345 MeV/nucleon on a beryllium target. Projectile fragments were separated and identified in-flight with the large-acceptance two-stage separator BigRIPS. Nine ³⁹Na events have been unambiguously observed in this work and clearly establish the particle stability of ³⁹Na. Furthermore, the lack of observation of ^{35,36}Ne isotopes in this experiment significantly improves the overall confidence that ³⁴Ne is the neutron dripline nucleus of neon. These results provide new key information to understand nuclear binding and nuclear structure under extremely neutron-rich conditions. The newly established stability of ³⁹Na has a significant impact on nuclear models and theories predicting the neutron dripline and also provides a key to

understanding the nuclear shell property of 39 Na at the neutron number N=28, which is normally a magic number.

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The location of the neutron dripline, the limit beyond which neutron-rich nuclei become unbound, is crucial in understanding the stability of nucleonic many-body systems with extreme neutron-to-proton ratios. However, locating the neutron dripline experimentally is challenging due to the extremely low production cross sections for the most neutron-rich isotopes. So far the dripline has only been established from hydrogen to neon (atomic number Z=10). The most recent extension of the known neutron dripline was made for fluorine (Z=9) and neon, where the dripline nuclei, *i.e.*, last bound isotopes, were determined to be 31 F and 34 Ne, respectively [1]. This result came nearly twenty years after 24 O was confirmed as the dripline nucleus of oxygen (Z=8) [2–5]. The location of the neutron dripline provides a key benchmark for advanced nuclear models and theories, as it serves as a sensitive criterion to test underlying nuclear structure and interactions. Furthermore, the neutron dripline is also important in the equation of state of asymmetric nuclear matter and thus in neutron star properties [6].

The next natural question is what are the neutron-rich limits beyond Z=10? We note that a drastic shift in the location of the neutron dripline has been seen going from Z=6 to 10, called the oxygen anomaly. The dripline nuclei with Z=6 to 8 are 22 C, 23 N, and 24 O [7], respectively, all of which have the neutron number N=16, while for Z=9 and 10 the dripline is located at 31 F with N=22 and 34 Ne with N=24, respectively. (See, for example, the nuclear chart in Fig. 1 of Ref. [1].) The addition of only one proton to oxygen induces an extra stability involving six more neutrons. The dripline nucleus 24 O is doubly-magic due to the canonical Z=8 and newly established N=16 magic numbers, and is presumed to be spherical [8]. The underlying nuclear interactions, including tensor and three-nucleon forces, create the N=16 magic number due to nuclear shell evolution, and the resultant shell gap determines the location of the neutron dripline for oxygen [9–11]. On the other hand, it is predicted [12] for neutron-rich nuclei with Z=9 to 12 that nuclear deformation develops in the region with $N \geq 20$ and plays a key role in

determining nuclear stability and the location of the neutron dripline. The nuclear deformation induces more stability to nuclear binding [12].

Such nuclear deformation is conjectured from the emergence of the socalled "Island of Inversion" (Z = 10-12, N = 20-22) [13-16], where the canonical magicity of N = 20 is lost and the ground states of the nuclei are strongly prolate-deformed. According to recent extensive experimental studies with rare isotope beams [17], the deformed region of the Island of Inversion extends towards isotopes with larger N numbers: so far, up to ³²Ne (N=22) for neon [18–22], up to ³⁵Na (N=24) for sodium (Z=11) [23–25], and up to 40 Mg (N=28) for magnesium (Z=12) [26–31]. The region also extends to a smaller N number such as ²⁹Ne (N=19) [32] and to a smaller Z number such as 29 F (N=20) [33]. Furthermore, the loss of magicity at N=28 and the occurrence of nuclear deformation were suggested not only for ⁴⁰Mg [31] but also ${}^{42}\text{Si}$ (Z = 14) [34–36]. Note that the deformation in magnesium isotopes persists all the way from N=20 to 28, indicating the merging of the magicity loss at N=20 and 28 [29]. Reviews of the evolution of nuclear structure in the Island of Inversion and its neighboring region are available in Refs. [37–39]. It is likely that nuclear stability at the limit of existence is determined reflecting details of underlying nuclear structure, such as the nuclear shell property and nuclear deformation including their evolution. As such, locating the neutron dripline provides an important key to understanding the nuclear structure under the extremely neutron-rich conditions.

This Letter presents the observation of the very neutron-rich new sodium isotope 39 Na with the mass number A = 3Z + 6, located beyond the previously known most neutron-rich isotope 37 Na [40]. This nucleus is a strong candidate to be the dripline nucleus of sodium, and establishing that it is a bound nucleus provides an important extension of the neutron dripline and a benchmark for nuclear structure calculations. We should note that 39 Na has N = 28, the canonical magic number for normal nuclei. The discovery of 39 Na would provide a key to understanding the shell property and nuclear stability at N = 28 with such an extreme neutron-to-proton ratio.

In our previous work [1], we searched for the heaviest Z=9 to 11 isotopes 32,33 F, 35,36 Ne, and 38,39 Na in order to investigate the neutron dripline at fluorine, neon, and sodium, and no events were observed for 32,33 F, 35,36 Ne, and 38 Na and only one event for 39 Na. In this Letter we present the results of a

new experiment dedicated to searching specifically for ³⁹Na during which we observed nine events, clearly establishing that it is particle-bound.

The experiment was performed at the Radioactive Isotope Beam Factory (RIBF) [41] at the RIKEN Nishina Center using an intense ⁴⁸Ca beam at 345 MeV/nucleon provided by the cascade operation of the RIBF accelerator complex. The new isotope ³⁹Na was produced by projectile fragmentation of the ⁴⁸Ca beam in a 20-mm-thick beryllium target. The beam intensity was as high as ~540 particle-nA during the experiment. The projectile fragments emerging from the target were separated and identified in-flight by the large-acceptance two-stage separator BigRIPS [42, 43].

The experimental method was essentially the same as we developed for our previous experiment [1]. The main difference here was that the magnetic rigidity settings in the BigRIPS separator were optimized solely to produce 39 Na. The same production target was used and the same wedge-shaped aluminum degraders were placed at the first and second stages of the separator for isotope separation. The mean degrader thickness was 15 and 7 mm, respectively. The momentum acceptance of the separator was set to ± 3 % as before. The detector and data acquisition systems were also essentially the same, while the slits and collimator were optimized to further reduce background events.

The particle identification was performed in the second stage of the separator, relying on the combination of the time of flight (TOF), magnetic rigidity (Bp), energy loss (ΔE) measurements, through which the Z number and the ratio of mass to atomic number A/Z were deduced for each fragment. The TOF was measured between two thin plastic scintillators installed at the intermediate and final foci of the second stage. The ΔE was measured with a stack of six identical silicon detectors installed at the final focus that were 50 mm (Horizontal) \times 50 mm (Vertical) and 1 mm thick each. The Bp value was determined from a position measurement at the intermediate focus using the plastic scintillator which also measured the TOF, see Ref. [1] for additional details.

In search for ³⁹Na, the Bρ values of the segments of the separator that were tuned for optimal transmission of ³⁹Na were 9.155 Tm from the target to the first degrader and 8.439 Tm after the first degrader. We refer to this below as the ³⁹Na setting. In addition, the production of ³⁶Ne was revisited in order to improve the confidence level of its "non-existence", given that it had been

99.3 % in our previous experiment [1]. The Bp values for optimal transmission of ³⁶Ne (if it existed) were 9.408 Tm from the target to the first degrader and 8.770 Tm after the first degrader. We refer to this as the ³⁶Ne setting. Similar to our previous experiment, these separator settings were based on detailed simulations of the reaction and transmission with the LISE++ code [44].

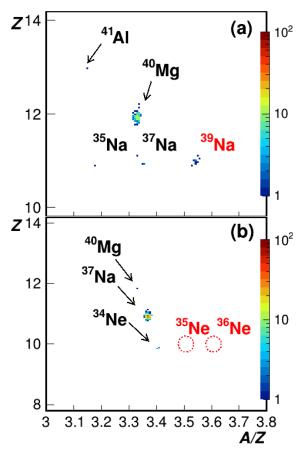


FIG. 1. Z versus A/Z particle identification plots for projectile fragments produced in the 48 Ca + Be reaction at 345 MeV/nucleon are shown for the (a) 39 Na and (b) 36 Ne settings. Nine events were observed for 39 Na in the 39 Na setting. No events were observed for 35,36 Ne in the 36 Ne setting.

Figure 1(a) shows the Z versus A/Z particle identification plot from the measurement with the 39 Na setting, during which the production target was irradiated with 5.25×10^{17} 48 Ca ions in 46.1 h. Note that the total dose is ~ 6.7 times more than that in the previous search. As indicated in the figure, we observed nine events at the position expected for the new isotope 39 Na and 143 events at the position expected for the well-established 40 Mg isotope. The projectile fragments 39 Na and 40 Mg (isotone of 39 Na) follow almost identical

trajectories through the separator. Some other events were also observed that are consistent with the following isotopes: one count for 35 Na, four counts for 37 Na, and one count for 41 Al. No events were observed that would be 36,38 Na and 39 Mg, which is consistent with our previous work establishing their unbound nature. The figure demonstrates that the present measurement had sufficient separation among the various nuclides as well as excellent background rejection. The resolution in A/Z and Z of the 40 Mg events were 0.16 % and 0.52 % (standard deviation), respectively. The observation of nine events in this work clearly demonstrates the discovery of 39 Na and establishes that it is particle-stable. The production cross section of 39 Na was estimated to be approximately 0.5 fb using the transmission efficiency simulated with the LISE++ code.

Figure 1(b) shows the particle identification plot from the measurement with the 36 Ne setting, during which the integrated beam dose and the total irradiation time were 3.09×10^{17} ions and 25.3 h, respectively. No events were observed for 35,36 Ne, being consistent with their particle instability established in our previous work, while 37 Na, the isotone of 36 Ne, was clearly observed. Some events for the dripline nucleus 34 Ne were also observed here.

As part of this work, systematic measurements of the production cross sections were performed for neutron-rich neon, sodium, and magnesium isotopes as a function of mass number by tuning the separator setting step by step for each isotope to be measured. Such cross-section systematics provide not only a consistency check of the present experiment but also the basis for extrapolation of observed cross sections for future work. Figures 2(a)-(c) show the measured cross sections along with predicted cross sections from the EPAX 2.15 systematics [45] and the Q_g systematics [46, 47]. Note that our previous measurements [1, 48] are also included in Fig. 2. The measured cross sections rely on simulations of the separator transmission with the LISE++ code.

The Q_g systematics cross sections are given by a simple exponential function $\sigma(A, \mathbb{Z}) = f(\mathbb{Z}) \exp(Q_g/T)$, where σ , Q_g , T, and f represent the cross section, the difference of mass excesses between the projectile and the fragment, an effective temperature, and a normalization coefficient, respectively. The mass values for Q_g are based on the AME2020 atomic mass evaluation [49, 50]. Note that this evaluation for neon isotopes is identical to the evaluation used in our previous work [1]. The parameters of the Q_g

systematics function for the individual neon, sodium, and magnesium isotopic chains were obtained by fitting the measured cross sections of ³⁰⁻³⁴Ne, ³¹⁻³⁹Na, and ³⁴⁻⁴⁰Mg, respectively. The resulting temperature parameters *T* obtained by the fitting process are 2.7, 2.7, and 2.9 MeV for neon, sodium, and magnesium, respectively.

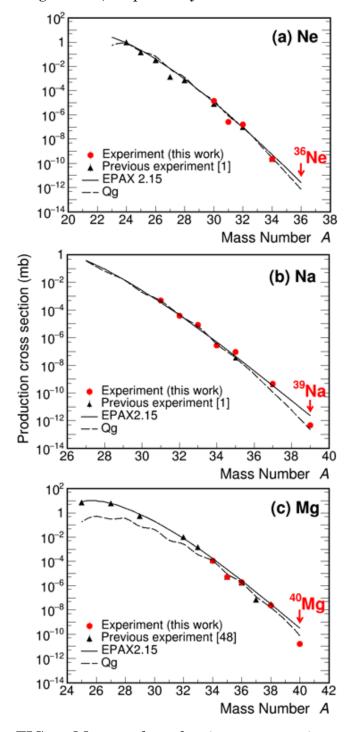


FIG. 2. Measured production cross sections are shown as a function of mass

number for neutron-rich (a) neon, (b) sodium, and (c) magnesium isotopes produced in the projectile fragmentation of a 345 MeV/nucleon ⁴⁸Ca beam on a beryllium target, along with our previous data [1, 48]. Predictions from the EPAX 2.15 systematics and the Q_g systematics are shown by solid and dashed curves, respectively.

The measured production cross sections shown in Fig. 2 on a logarithmic scale decrease fairly smoothly with increasing mass number, although some odd-even staggering is seen. This trend also supports our observation of the new isotope at the observed cross section. The predictions are in fairly good agreement with the measurements, although the Q_g systematics underestimate the cross sections for less neutron-rich magnesium isotopes and the EPAX 2.15 systematics overestimate those for ³⁹Na and ⁴⁰Mg. The measured production rates of ³⁹Na and ⁴⁰Mg were approximately 1.5x10⁻⁷ and $2.4x10^{-6}$ particles/s/particle-nA, respectively.

Based on the measurement with the 36 Ne setting, we reevaluated the confidence level (CL) that 36 Ne is particle-unbound. The CL value was calculated using the expected yield of unobserved 36 Ne from the smooth variation of cross sections assuming Poisson probability distribution. The production cross sections of 36 Ne were extrapolated to be 2.58 and 0.74±0.24 fb with the EPAX 2.15 systematics and the Q_g systematics, respectively. The smaller value from the Q_g systematics was used to evaluate the CL value. The LISE++ simulation estimated the corresponding expected yield to be 7.39±2.40 counts, giving the CL value of 99.938+0.056-0.619 %. The CL value has been significantly improved, compared with the previous value of 99.3+0.4-1.0 % [1]. The combination of the previous and present work gives 99.9996+0.0004-0.005 %, allowing the exclusion of the existence of bound 36 Ne at much higher level and hence a firm determination that 34 Ne is the dripline nucleus.

We now compare the present results with theoretical predictions and discuss the implications of the particle stability of ³⁹Na. The phenomenological mass formula with shell corrections KTUY [51] incorrectly predicts the fluorine dripline at ²⁹F, while it correctly predicts the neon dripline at ³⁴Ne, although the bound nucleus ³¹Ne is predicted to be unbound. The KTUY formula mis-locates the dripline for oxygen as ²⁶O and sodium as ³⁷Na. The finite-range droplet macroscopic model FRDM [52] correctly predicts the fluorine dripline at ³¹F and the neon dripline at ³⁴Ne, although it

incorrectly predicts the oxygen dripline at ²⁶O. The FRDM model predicts ³⁹Na to be particle-stable and the dripline nucleus of sodium. The Hartree-Fock-Bogoliubov mass model HFB-24 [53] incorrectly predicts ²⁹F and ²⁶F as the dripline and unbound nuclei, respectively, and also incorrectly predicts the oxygen dripline at ²⁸O. The HFB-24 model gives the best fit among the different versions of the model, especially for masses of neutron-rich nuclei [54]. For neon, the HFB-24 model correctly predicts the dripline at ³⁴Ne, but yet it predicts the bound nucleus ³¹Ne to be unbound. For sodium, the HFB-24 model incorrectly predicts that ³⁹Na is particle-unstable and ³⁷Na is the dripline nucleus.

Recently, Tsunoda $et\ al.$ [12] discussed the location of the neutron dripline in terms of the nuclear shell and shape evolution, using a large-scale shell model calculation in which a newly developed ab-initio effective nucleon–nucleon interaction was employed. Two major shells below and above the magic number N=20 (the sd and pf shells, respectively) were included in their calculation, and the underlying structural evolution and the mechanism to yield the neutron dripline were investigated by decomposing the calculated ground-state energies into different contributions such as those from the single particle energies, monopole interaction, pairing interaction, and quadrupole deformation.

According to their work, the oxygen dripline is determined by the N=16magic gap in single-particle shell structure as the dripline nucleus ²⁴O is predicted to be spherical and a doubly magic nucleus. The magic number N=16 at ²⁴O is due to a sizeable shell gap between the $2s_{1/2}$ and $1d_{3/2}$ orbitals partially created by the repulsive three-nucleon force. On the other hand, this situation changes drastically for the neutron-rich nuclei with Z=9 to 12 and $N \ge 20$. The calculation shows that the nuclear deformation plays a key role in nuclear binding in this region and thus in determining the location of the neutron dripline. According to the calculation, the quadrupole deformation leads to a larger binding energy that persists much beyond the Island of Inversion including N = 28, and the isotopes remain deformed up to the dripline. Such persistence of the nuclear deformation agrees with the trends obtained in the recent experiments in the relevant region with N=20 to 28 [18–31]. For instance, the deformed excited states observed in even-even neon and magnesium isotopes in this region, by the excitation energies of the first 2⁺ and 4⁺ states, are well reproduced by the calculation, including the recent

experiment for ⁴⁰Mg [31]. See Fig. 3 in Ref. [12]. ³⁹Na could be particle-bound as its ground state is deformed.

This large-scale shell model calculation correctly predicts the fluorine and neon dripline at 31 F and 34 Ne, respectively, and is consistent with our observation that 39 Na is particle-bound. The confirmation of the particle stability of 39 Na, taking into consideration the large-scale shell model results, implies that, similarly to that in 40 Mg, the quadrupole deformation in neutron-rich sodium isotopes persists all the way up to 39 Na with N=28 and that the N=28 magicity is lost at Z=11. This interpretation is supported by a different large-scale shell model calculation by E. Caurier *et al.* [55] as well. Recent Hartree-Fock-Bogoliubov calculations [56] also predict the occurrence of quadrupole deformation at N=28, including a deformed halo structure in 39 Na. Experimental studies on the nuclear structure of sodium isotopes including 39 Na will be necessary to explore this intriguing possibility.

Recently, *ab-initio* calculations, based only on realistic two-nucleon and three-nucleon forces, have reached the mass region including the present work. For instance, Stroberg *et al.* calculated ground-state energies and predicted the dripline up to Z = 26 by solving the many-body problem with the valence-space formulation of the in-medium similarity renormalization group [57]. The present results certainly provide a critical test for such state-of-the-art calculations.

In summary, we have investigated the production of the most neutron-rich sodium isotope 39 Na using the BigRIPS separator at the RIKEN RIBF. We observed nine events for 39 Na and thus established that it is particle-bound. In addition, the confidence level that 34 Ne is the neutron dripline nucleus of neon has been significantly improved by the present work. These results have provided new key information to understand the nuclear stability and nuclear structure under the extremely neutron-rich conditions. The establishment of the particle stability of 39 Na makes a significant impact on a number of modern nuclear theories and mass formulae that address the edge of the nuclear stability. In particular, the discovery of 39 Na provides a rigorous test for theories that attempt to predict nuclear shell evolution and the underlying deformation properties as nuclei approach the neutron dripline near N=28. For instance, the large-scale shell model calculation [12], which reproduced the particle stability of 39 Na, implies that the particle stability of 39 Na can be a key to understanding the N=28 shell property including the persistence of

the nuclear deformation in neutron-rich sodium isotopes. This work does not claim that ³⁹Na is the neutron dripline nucleus of sodium, since the determination that ⁴¹Na is particle-bound or not remains a challenge for future studies. However, a strong case could be made that ³⁹Na is the dripline nucleus, as the state-of-art large-scale shell model calculation predicts the sodium neutron dripline lies at ³⁹Na [12]. The heaviest known isotopes for the next three elements above sodium are presently ⁴⁰Mg, ⁴³Al [58], and ⁴⁴Si [46], respectively. According to various mass models, ⁴²Mg, ⁴⁵Al, and ⁴⁶Si are also likely particle-bound. The search for new isotopes at and close to the neutron dripline will continue to be an important challenge for new-generation rare isotope beam facilities [59].

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