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1 **Riemann-encircling exceptional points for efficient asymmetric**
2 **polarization-locked devices**

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20

21 **Abstract**

22 Dynamically encircling exceptional points (EPs) has unveiled intriguing chiral
23 dynamics in photonics. However, the traditional approach based on an open manifold
24 of Hamiltonian parameter space fails to explore trajectories that pass through an
25 infinite boundary. Here, by mapping the full parameter space onto a closed manifold
26 of the Riemann sphere, we introduce a framework to describe encircling-EP loops. We
27 demonstrate that an encircling trajectory crossing the north vertex can realize
28 near-unity asymmetric transmission. An efficient gain-free, broadband asymmetric
29 polarization-locked device is realized by mapping the encircling path onto L-shaped
30 silicon waveguides.

31

32 In a non-Hermitian system obeying parity-time (PT) symmetry, two or more
33 eigenvalues and eigenstates are simultaneously degenerate at exceptional points (EPs).
34 A variety of intriguing phenomena, such as sensing enhancement [1-3], unidirectional
35 invisibility [4,5], topological light control [6,7] and asymmetric mode conversion
36 [8-15] have been demonstrated at these singularities, closely linked to their
37 topological features, and they have attracted a growing interest in optics [1,7,8,16-26],
38 acoustics [27-29], thermodynamics [30], electronics [31,32] and quantum mechanics
39 [33].

40 When a Hamiltonian adiabatically encircles an EP twice, the Berry phase is π ,
41 indicating a self-intersecting Riemann surface (Rsurf) around the EP [34]. Recent
42 studies have demonstrated that this feature enables a chiral response: by dynamically
43 encircling an EP in clockwise (CW) or anticlockwise (ACW) direction leads to
44 different final states, regardless of the starting states. Such chiral response has been
45 demonstrated by mapping the required Hamiltonian parameters onto PT symmetric
46 arrangements of waveguides [8,20,35]. Most previous studies have been applying this
47 feature to realize asymmetric conversion of modes in optical waveguides, but they
48 failed to achieve high transmission efficiencies due to path-dependent loss
49 [8,12,13,20,36,37]. The reason is that the traditional approach to EP encircling is
50 based on an open manifold in the Hamiltonian parameter space. Such approach
51 neglects a set of loops passing through the infinite boundary, enabling low
52 path-dependent losses. Due to significant challenges in practical experimental
53 implementations, high-efficiency polarization-locked device has not been achieved to
54 date [10,38].

55 In this Letter, we demonstrate that the full Hamiltonian parameter space can be
56 mapped onto the Riemann sphere (Rsph), over which infinite points lying on its
57 boundary are associated with the same eigenstates, and converge to the north vertex.
58 Benefitting from the Rsph mapping, we can encircle an EP along a continuous and
59 closed trajectory passing through the north vertex and achieve highly-efficient

60 asymmetric transmission. We present experimental results mapping this trajectory
 61 onto L-shaped waveguides, demonstrating near-unity asymmetric conversion
 62 efficiency at 1550 nm. Such a functionality corresponds to an asymmetric
 63 polarization-locked device (APLD), since the output polarization state is locked
 64 irrespective of the input polarization states [39].

65 A system state $|\psi\rangle$ propagating along z follows a Schrödinger-type equation

66 $i\partial/\partial z|\psi\rangle = H|\psi\rangle$, with second-order Hamiltonian

$$67 \quad H = \begin{bmatrix} \beta & \kappa \\ \kappa & -\beta - i\gamma \end{bmatrix} \quad (1)$$

68 where β , γ and κ represent the detuning, relative gain/loss rate and coupling
 69 strength of the system, respectively. The two eigenvalues are

70 $E_{1,2} = -i\gamma/2 \pm \sqrt{\kappa^2 + (\beta + i\gamma/2)^2}$, and the associated eigenstates are

71 $|\psi_{1,2}\rangle = \left[\sqrt{1 \pm K}, \pm \sqrt{1 - (\pm K)} \right]^T / \sqrt{2}$, with $K = (\beta + i\gamma/2) / \sqrt{\kappa^2 + (\beta + i\gamma/2)^2}$. In the

72 two-dimensional (2-D) parameter space $\zeta = (\beta/\kappa, \gamma/\kappa)$, the EPs are located at

73 $\zeta = (0, 2)$ and $\zeta = (0, -2)$. When H approaches the points at infinity in ζ , i.e.,

74 $\beta/\kappa \rightarrow \pm\infty$ and/or $\gamma/\kappa \rightarrow \pm\infty$, the two eigenstates converge to $|\psi_1\rangle = [1, 0]^T$ and

75 $|\psi_2\rangle = [0, 1]^T$, corresponding to E_1 and E_2 [$|\text{Im}(E_1)| \leq |\text{Im}(E_2)|$], respectively. The

76 energy spectra form a self-intersecting Rsurf in ζ , and the topological features can

77 be expressed as a Berry phase $\Phi = \pi$ when an EP is encircled twice with a

78 quasi-dynamic approach [40]. The Berry phase around a path \mathbf{C} can be defined as

79 $\Phi = i \int_{\mathbf{C}} \langle \chi(\zeta) | \nabla_{\zeta} \psi(\zeta) \rangle \cdot d\zeta$, where $\langle \chi(\zeta) |$ is the left eigenstate and

80 $\langle \chi(\zeta) | H = \langle \chi(\zeta) | E$, i.e., $H^{\dagger} | \chi(\zeta) \rangle = E^* | \chi(\zeta) \rangle$ [41-43]. Since H is a symmetric

81 Hamiltonian according to Eq. (1), i.e., $H^T = H$, we have $H^{\dagger} = H^*$ and

82 $| \chi(\zeta) \rangle = | \psi^*(\zeta) \rangle$. If the Hamiltonian evolves along the parameter space boundary

83 [Fig. 1(a)], $\psi(\zeta)$ is constant, i.e., $|\nabla_{\zeta} \psi(\zeta)\rangle = 0$, which makes Φ equal to zero.

84 To present the trajectory induced by the infinite boundary, we can represent the
85 full parameter space over a Rsph of radius 2, as schematically shown in Fig. 1(b). The
86 mapping procedure, called stereographic projection, is based on the following steps:
87 first, the center of the sphere is aligned with the origin of the 2-D parameter space as
88 denoted in Fig. 1(a); then, the north pole (blue point on Rsph) is connected to the
89 yellow point in the 2-D parameter space (gray plane) to form a line; last, its
90 intersecting point with the Rsph is the mapping point (black point). Here, we provide
91 some examples to show connections between loops and its mapping counterparts.
92 When the loop only encircles the origin of the parameter space, the mapping loop is in
93 the southern hemisphere [Fig. 1(c)]; when the loop expands to include two EPs, the
94 mapping loop is in the northern hemisphere [Fig. 1(d)]; and when the loop continues
95 to expand and becomes the infinite boundary (blue dashed line), the mapping loop
96 converges to the north vertex (blue point) [Fig. 1(b)]. The mapping operation can also
97 be analytically expressed: we assume that the Rsph radius is 2, and the mapping
98 relation from the 2-D parameter space to the Rsph is $\beta/\kappa = 2 \cot(\varphi_1/2) \cos(\varphi_2)$ and
99 $\gamma/\kappa = 2 \cot(\varphi_1/2) \sin(\varphi_2)$, where φ_1 is the polar angle and φ_2 is the azimuthal
100 angle as denoted in Fig. 1(b). Since all the points on the parameter space boundaries
101 with generally different loss rates are mapped onto the north vertex, the associated
102 loss rate can be an arbitrary value (Supplementary Material [44], Note 1).

103 As a function of their Berry phase, encircling loops can be divided into two
104 groups, characterized by $\Phi = 0$ and π . Encircling two EPs, as well as no EP, results
105 in a Berry phase $\Phi = 0$. Therefore, the north vertex does not contribute to the Berry
106 phase. In both cases, Rsph is split by the loop into two surfaces, one containing two
107 EPs and the other containing no EPs [Figs. 1(c) and 1(d)]. In contrast, encircling
108 trajectories including one EP splits the Rsph into two surfaces, each containing one
109 EP [Figs. 1(e) and 1(f)] associated with a Berry phase $\Phi = \pi$. The former one in Fig.
110 1(e) does not pass through the infinite boundary, and therefore the trajectories on both
111 Rsph and real parameter space are closed. In the case of the latter one in Fig. 1(f),

112 since the trajectory in real parameter space is blocked by the infinite boundary
113 (dashed blue line), it is open. By mapping the infinite boundary with the same
114 eigenstates onto the north vertex, we can achieve a closed mapping trajectory (black
115 solid line) on the Rsph. In this way, both mapping trajectories in Figs. 1(e) and 1(f)
116 show their topological equivalence, since the closed loops encircle each one of the
117 two EPs. On the Rsph, it is not necessary to distinguish which EP is encircled, since
118 the loop that encircles one EP shares the same Berry phase with the loop that encircles
119 the other EP, i.e., they are topologically equivalent (Supplementary Material [44],
120 Note 2).

121 The transmission loss accumulated along the mapping trajectory can be also
122 interpreted from the perspective of Rsph: the mapping trajectories in Figs. 1(e) and 1(f)
123 encircle each of the two EPs simultaneously, and the distance relative to each EP
124 located in the gain or loss region determines the transmission loss. The conventional
125 mapping trajectory in Fig. 1(e) is close to the loss EP [$\zeta = (0, 2)$], associated with
126 path-dependent loss along the entire path ($\gamma/\kappa > 0$), making the system prone to low
127 transmittance. The loss difference between the two eigenstates causes the occurrence
128 of nonadiabatic transitions (NATs), leading to an asymmetric response [11,12,45].
129 Instead, the mapping trajectory in Fig. 1(f), located in the middle of the two EPs, is
130 not associated with any loss except at the north vertex, and hence it enables
131 high-efficiency transmission. In Fig. 1(f), the NAT process is guaranteed by
132 introducing a non-zero loss rate at the north vertex and exerting a sufficiently long
133 evolution time to dissipate $|\psi_2\rangle$. See Supplementary Material [44], Note 3 for the
134 evolution dynamics on how the loss affects the final output states. Meanwhile,
135 through the mapping of Rsph we ensure that this loop indeed encircles each EP,
136 leading to an asymmetric response. Thus, we simultaneously achieve both high
137 efficiency and largely asymmetric transmission following this trajectory.

138 This specific trajectory can be mapped onto a suitably designed silicon
139 waveguide tailored for APLD, as schematically depicted in Fig. 2(a). The right

140 waveguide supports two orthogonal eigenmodes with different polarization directions,
 141 i.e., R1 ($|\psi_1\rangle$) and R2 ($|\psi_2\rangle$) modes with effective refractive indices n_{R1} and n_{R2}
 142 ($n_{R1} < n_{R2}$), respectively. θ represents the angle between the polarization direction
 143 of R1 and the $+45^\circ$ axis [Fig. 2(b)]. In this configuration, the eigenmode in the right
 144 waveguide can be expressed as $|\psi\rangle = [A_1, A_2]^T$, in which A_1 and A_2 indicate the
 145 projection components of the normalized electric field amplitude along $+45^\circ$ and
 146 -45° , respectively. Consequently, the Hamiltonian parameters in Eq. 1 can be
 147 expressed as $\beta = k_0(n_{R1} - n_{R2})(\sin^2 \theta - \cos^2 \theta)/2$, $\kappa = k_0(n_{R1} - n_{R2})\sin \theta \cos \theta$. γ is
 148 the loss rate exerted on the components of the electric field along -45° , and
 149 $k_0 = 2\pi/\lambda$ is the wavenumber, with λ being the wavelength (Supplementary
 150 Material [44], Note 4).

151 At the starting point A, the R1 mode is transverse-magnetic (TM) polarized with
 152 $\theta = 45^\circ$, corresponding to $\beta/\kappa = 0$ [Fig. 2(c)]. Between A and B, θ changes
 153 monotonously from 45° to 0° , reducing β/κ from 0 to $-\infty$, associated with the
 154 evolution path from A to B along the longitude on the Rsph [illustration of Fig. 2(a)].
 155 Between B and C, the right waveguide is axisymmetric along $+45^\circ$ axis, so that R1
 156 is polarized along $+45^\circ$ direction with $\theta = 0^\circ$, resulting in $\kappa = 0$ and $\beta/\kappa = \infty$
 157 [Fig. 2(c)]. The left waveguide between B and C is designed to absorb the eigenmode
 158 polarized along -45° direction in the right waveguide via adiabatic coupling,
 159 associated with $\gamma > 0$. The whole evolution path between B and C takes place at the
 160 north vertex on the Rsph. Between C and D, β/κ is varied monotonously from $+\infty$
 161 to 0 as θ is reduced from 0° to -45° , and the Hamiltonian evolves from C to D
 162 along the longitude on the Rsph. Although β , γ , and κ are finite, the Hamiltonian
 163 parameter space boundary is accessible based on the specifically tailored L-shaped
 164 waveguide.

165 Figures 3(a-d) show the evolution trajectory of the system state on R_{surf} (not
 166 R_{sph} , mapped here with $\text{atan}(E)/\kappa$ in the third dimension to cast it into a finite plot)
 167 for CW and ACW loops around the EP, where the CW and ACW loops correspond to
 168 light propagating along $+z$ and $-z$ in the right waveguide, respectively. The
 169 difference of the two eigenvalues is minimum at the EP, but both drastically change
 170 close to the EP [46]. In contrast, the difference of the two eigenvalues is maximum,
 171 but both tend to converge when approaching the infinity point. For the CW loop with
 172 $[1,1]^T$ input, associated with the TE mode, $[1,1]^T$ evolves to $[0,1]^T$ when the
 173 Hamiltonian varies slowly from A to B [Fig. 3(a)]. Meanwhile, $[1,0]^T$ is excited at B,
 174 due to the fact that rigorous adiabaticity cannot be fully guaranteed. During the NAT
 175 process from B to C, $[0,1]^T$ is dissipated and $[1,0]^T$ is preserved, making the
 176 dominant eigenstate of the system switch from $[0,1]^T$ to $[1,0]^T$. Finally, the
 177 Hamiltonian returns to the destination D with $[1,1]^T$, and the corresponding output
 178 mode is TE. For the CW loop with $[1,-1]^T$ input [Fig. 3(b)], associated with the TM
 179 mode, the dominant eigenstate during the evolution is always lossless because a NAT
 180 does not occur, and the final output state is $[1,1]^T$, TE in nature. This indicates that the
 181 output mode for the CW direction is locked to TE, irrespective of the input mode. For
 182 the ACW loop [Figs. 3(c, d)], the output state is locked to $[1,-1]^T$, associated with TM
 183 polarization. The two sheets of R_{surf} are self-intersecting, and the adiabatic evolution
 184 path between B and C is vertical, as the slope at B and C is infinite on this choice of
 185 R_{surf} . A NAT occurs between B and C in Figs. 3(a, d), during which the evolution
 186 path switches from the blue to the red branch.

187 The L-shaped silicon waveguides shown in Fig. 2(a) are carefully tailored to map
 188 this evolution path by establishing the relation between structural parameters and the
 189 Hamiltonian parameters (Supplementary Material [44], Note 5). For the CW (ACW)

190 loop with TE (TM) mode input to the right waveguide at A (D), the TE (TM) mode
 191 evolves to R2 polarized along -45° at B (C) [Fig. 4(a)]. Meanwhile, R1 polarized
 192 along $+45^\circ$ is excited because the length between A and B (D and C) is limited and the
 193 adiabatic evolution is not strictly satisfied. The device length between B and C is long
 194 enough so that R2 is fully absorbed by the left waveguide and the triggered R1 mode is
 195 retained, i.e., a NAT occurs. Finally, R1 evolves to TE (TM) from C to D (B to A) with
 196 a relatively low field intensity. For the CW (ACW) loop with TM (TE) input to the right
 197 waveguide at A (D), it evolves to R1 mode at B (C). The triggered R2 mode is absorbed
 198 between B and C, and R1 mode eventually evolves to TE (TM) mode from C to D (B to
 199 A).

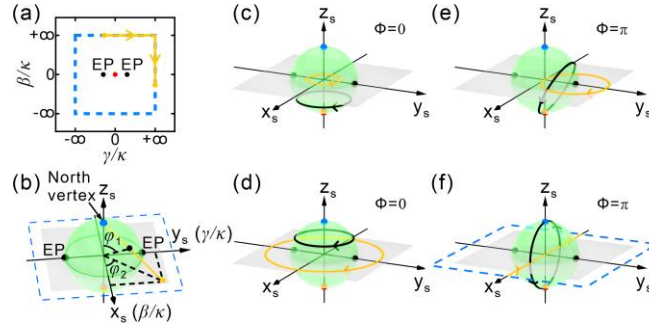
200 Figure 4(b) show the fabricated devices, with grating couplers (GCs), polarization
 201 beam splitter and rotators (PBRs) for measurement. The measured transmission
 202 efficiencies for TE and TM mode input are shown in Fig. 4(c) and 4(d), respectively
 203 (see Supplementary Material [44], Notes 6, 7 for the fabrication details and
 204 measurement scheme). $T_{P \rightarrow Q}$ ($T_{P \leftarrow Q}$) represents the transmission efficiency of Q
 205 mode from the output port for input P mode from the left (right) port (P or Q mode refer
 206 to TE or TM mode). For a left-side input, $T_{TE \rightarrow TE} \gg T_{TE \rightarrow TM}$ and $T_{TM \rightarrow TE} \gg T_{TM \rightarrow TM}$,
 207 indicating that the output mode is locked to TE. For right-side excitation,
 208 $T_{TM \leftarrow TE} \gg T_{TE \leftarrow TE}$ and $T_{TM \leftarrow TM} \gg T_{TE \leftarrow TM}$, indicating that the output mode is locked to
 209 TM. It can be observed that $T_{TM \rightarrow TE}$ and $T_{TM \leftarrow TE}$ are close to 100% around 1550 nm.
 210 The mode crosstalk is defined as the energy ratio of the undesired mode to total output
 211 [47]. The mode crosstalks $TM \rightarrow TE$, $TM \leftarrow TE$, and $TE \rightarrow TE$ are all below -20
 212 dB, while $TM \leftarrow TM$ is below -10 dB. The experimental results well validate the
 213 asymmetric polarization-locked performance. It should be noted that the APLD is
 214 sensitive to the alignment error, and therefore the alignment should be carefully
 215 controlled in fabrication (Supplementary Material [44], Note 8).

216 Here, we can also treat the APLD as a 3-section optical system (Supplementary

217 Material [44], Note 9). The AB/CD section is used as a leaky polarization controller
218 that converts the input mode into the dominant eigenmode and its orthogonal
219 eigenmode. The BC section serves as a selective optical attenuator that takes away one
220 of the eigenmodes. The approach of using a 3-section optical system can also be
221 extended to develop asymmetrical polarization conversion devices for spatial light [48].
222 These alternative schemes however are limited in terms of operational bandwidth. EP
223 encircling on the contrary ensures large working bandwidths, due to adiabatic
224 parameter evolution of the energy spectra [8,15,20]. Our simulations predict that the
225 practical bandwidth of the described operation can be as large as the entire range
226 1450-1750 nm (Supplementary Material [44], Note 10). The presented
227 Riemann-encircling strategy can be extended to study asymmetrical state-locked
228 response in a multi-state non-Hermitian system (Supplementary Material [44], Note
229 11].

230 In conclusion, we have shown that dynamically encircling an EP on R_{sph} can
231 enable asymmetric polarization conversion, and a continuous encircling trajectory
232 passing through the north vertex can realize near-unity asymmetric transmission. We
233 experimentally demonstrated nearly 100% asymmetric polarization conversion
234 between TE and TM modes with mode crosstalk below -20 dB at 1550 nm in silicon
235 waveguides. These results enable the practical realization of highly efficient gain-free,
236 broadband asymmetric polarization conversion on-chip, and hold the promise for new
237 opportunities for optical devices and applications.

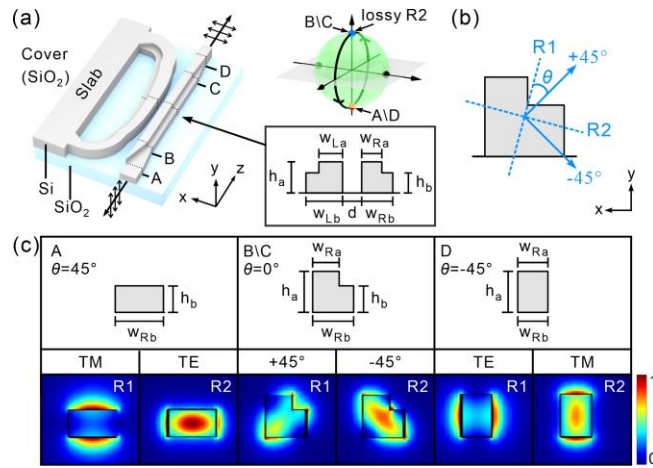
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240 **FIG. 1.** (a) Hamiltonian parameter space described by $\zeta = (\beta/\kappa, \gamma/\kappa)$. The yellow line shows an
 241 evolution path along the parameter space boundary (blue dashed line). (b) Rsph
 242 ($x_s^2 + y_s^2 + z_s^2 = 2^2$, $x_s = 2\sin\varphi_1 \cos\varphi_2$, $y_s = 2\sin\varphi_1 \sin\varphi_2$ and $z_s = 2\cos\varphi_1$) describing the
 243 Hamiltonian parameter space. (c-f) The evolution loops on Rsph (black curves) and in the 2-D
 244 parameter space (yellow curves) when encircling (c) no EP, (d) two EPs, and (e), (f) one EP. The
 245 loop in (f) locates in the center of the two EPs.

246



247

248 **FIG. 2.** (a) APLD made of silicon waveguides. The full height of the L-shaped waveguide is

249 $h_a = 340$ nm, and the height of the lower waveguide is $h_b = 220$ nm. The width of the upper

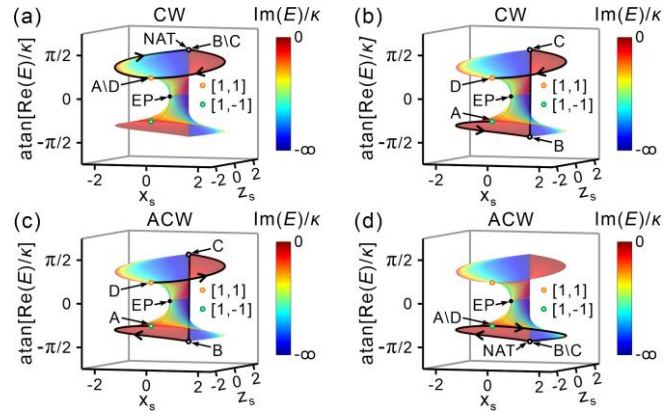
250 waveguide and the full waveguide is w_{La} (w_{Ra}) and w_{Lb} (w_{Rb}) for the left (right) waveguide,

251 respectively. The gap separation between the two waveguides is d . (b) The polarization

252 directions for R1 and R2 modes. (c) The cross sections of the right waveguide at A, B\C, and D,

253 and their associated electric field intensity distributions for the R1 and R2 eigenmodes.

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256 **FIG. 3.** (a),(b) CW loops on Rsurf formed by the real part of the eigenvalues of H when the input

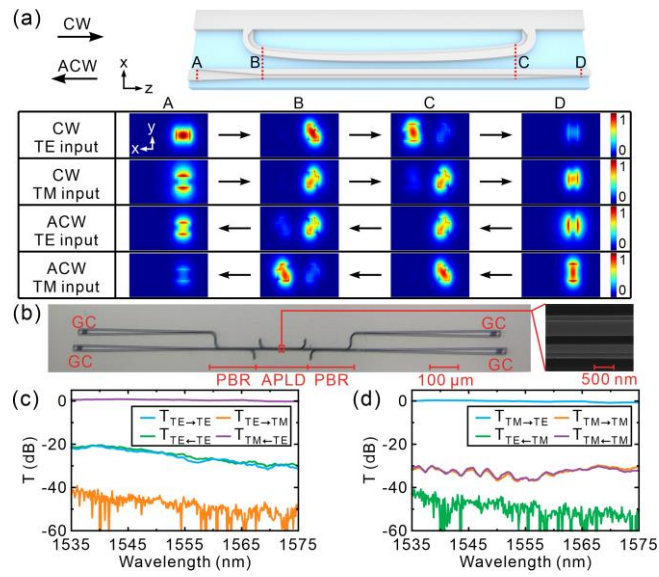
257 state is (a) $[1,1]^T$ and (b) $[1,-1]^T$. The Rsurf is extracted by casting the hemisphere with $y_s \geq 0$

258 (pure lossy case) of the Rsph onto the x_s-z_s plane. The color of the surface indicates the imaginary

259 part of the eigenvalues of H , in which a smaller imaginary part indicates a larger loss. (c), (d)

260 ACW loops when the input state is (c) $[1,1]^T$ and (d) $[1,-1]^T$.

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262

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FIG. 4. (a) The cross-sectional electric field intensity distributions at A, B, C, and D. (b) Optical

264

microscope images of the fabricated APLD, where the L-shaped waveguides are partially depicted

265

by the SEM image on the right panel. (c), (d) Measured transmittance spectra over 1535-1575 nm

266

wavelength range as (c) TE and (d) TM modes are injected, respectively.

267

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