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Beating the 3 dB Limit for Intracavity Squeezing and Its Application to Nondemolition Qubit Readout

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While the squeezing of a propagating field can, in principle, be made arbitrarily strong, the cavity-field squeezing is subject to the well-known 3 dB limit, and thus has limited applications. Here, we propose the use of a fully quantum degenerate parametric amplifier (DPA) to beat this squeezing limit. Specifically, we show that by *simply* applying a two-tone driving to the signal mode, the pump mode can, *counterintuitively*, be driven by the photon loss of the signal mode into a squeezed steady state with, in principle, an *arbitrarily high* degree of squeezing. Furthermore, we demonstrate that this intracavity squeezing can increase the signal-to-noise ratio of longitudinal qubit readout *exponentially* with the degree of squeezing. Correspondingly, an improvement of the measurement error by *many orders of magnitude* can be achieved even for modest parameters. In stark contrast, using intracavity squeezing of the semiclassical DPA *cannot* practically increase the signal-to-noise ratio and thus improve the measurement error. Our results extend the range of applications of DPAs and open up new opportunities for modern quantum technologies.

Introduction.—Squeezed states of light [1] form a fundamental building block in modern quantum technologies ranging from quantum metrology [2, 3] to quantum information processing [4, 5]. In particular, squeezing of a propagating field can in principle be made arbitrarily strong, due to destructive interference between the reflected input field and the transmitted cavity field; e.g., the squeezing of up to 15 dB has been experimentally achieved [6]. Such a propagating-field squeezing has been widely used for, e.g., gravitational-wave detection [7–9], mechanical cooling [10, 11], nondemolition qubit readout [12–15], and even demonstrating quantum supremacy [16, 17]. However, these applications inherently suffer from transmission and injection losses, which are a major obstacle to using extremely fragile squeezed states. To address this problem, exploiting intracavity squeezing (i.e., squeezing of a cavity field) offers a promising route.

To date, intracavity squeezing has been applied, e.g., to cool mechanical resonators [18–20], enhance light-matter interactions [21–28], improve high-precision measurements [29–31], and generate nonclassical states [32–36]. Despite such developments, the range and quality of applications of intracavity squeezing are still largely limited by the fact that quantum noise of a cavity field cannot be reduced below one half of the zero-point fluctuations in the steady state [37–39], i.e., the 3 dB limit. However, how to beat this limit has so far remained challenging, although for more complicated mechanical oscillators, the steady-state squeezing beyond 3 dB has been widely demonstrated both theoretically [40–42] and experimentally [43, 44]. The reason for the 3 dB limit of intracavity squeezing is the cavity photon loss, which is always present, destroys the essence of squeezing, i.e., two-photon correlations. In this manuscript, we show that, if such a photon loss is exploited as a resource, a strong steady-state intracavity squeezing can be achieved.

In our approach, we consider a fully quantum DPA, where both pump and signal modes are quantized. We show that a strong photon loss of the signal mode can steer the pump mode into a squeezed steady state, with a noise level reduced far beyond 3 dB. In this way, an *arbitrarily strong* steady-state squeezing of the pump mode can in principle be achieved. Note that optical experiments performed already in the 1990s (see, e.g., [45, 46]) demonstrated bright squeezing of the pump mode (i.e., the second-harmonic mode) by driving the signal mode (i.e., the fundamental mode). However, it was achieved for output squeezing only, not for intracavity squeezing.

To beat the 3 dB limit of intracavity squeezing, a theoretical approach, that requires a fast modulation of the coupling between the cavity and its environment, has been proposed [47]; and very recently, an experimental demonstration with three microwave modes coupled via a specific Josephson ring modulator was reported in Ref. [48]. In contrast, our approach relies only on common degenerate parametric amplification processes, and therefore is more compatible with current quantum technologies based on parametric amplification. More remarkably, we show that only a two-tone driving, if applied to the signal mode, can result in a strong steady-state squeezing for the pump mode. This is rather *counterintuitive*; indeed, common sense suggests that, as mentioned above, the steady-state intracavity

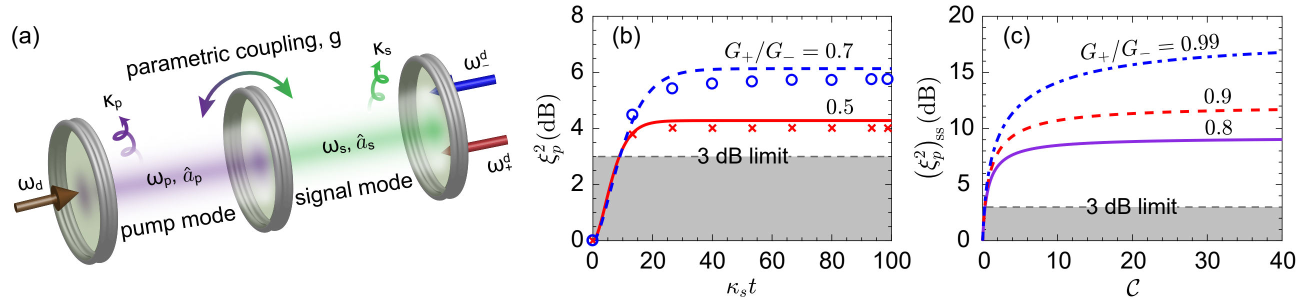

FIG. 1. (a) Schematic of our proposal with a fully quantum DPA. We use two cavities to represent the pump mode $\hat{a}_p$ (frequency ω_p, loss rate κ_p) and the signal mode $\hat{a}_s$ (frequency ω_s, loss rate κ_s). The single-photon parametric coupling between them has a strength g. A driving tone at frequency ω_d is applied to the pump mode and, simultaneously, the signal mode is driven by the other two tones at frequencies $\omega_\pm^{\rm d}$. (b) Time evolution of the squeezing parameter ξ_p^2 for $G_+/G_- = 0.5$ and 0.7. We assumed that $\Delta_s = 100g$, $\Delta_p = 0.1\Delta_s$, $\Omega_{\rm 2pd} = 0.05\Delta_s$, $\kappa_s = 100\kappa_p = 0.4g$, and $G_- = g_0$. Curves are the effective predictions, while symbols are the exact results. (c) Steady-state squeezing parameter $\left(\xi_p^2\right)_{\rm ss}$ versus the cooperativity $\mathcal{C}$ for $\kappa_s = 100\kappa_p$, and for $G_+/G_- = 0.8$, 0.9, and 0.99. In (b) and (c), the gray shaded areas refer to the regime below the 3 dB limit.

squeezing of a DPA is usually limited to 3 dB. We note that quantum intracavity noise reduction can also be realized via squeezing of photon-number fluctuations, corresponding to the sub-Poissonian photon-number statistics or photon antibunching (see, e.g., the early predictions [49, 50] and very recent demonstrations of 3 dB squeezing like in [51]).

Fast and high-fidelity nondemolition qubit readout is a prerequisite for quantum error correction [52, 53] and fault-tolerant quantum computation [54, 55]. Using squeezed light to improve such a readout is a long-standing goal [12–14, 56]. However, the simplest strategy, i.e., dispersive qubit readout [56, 57], induces a qubit-state-dependent rotation of squeezing, such that the amplified noise in the antisqueezed quadrature is introduced into the signal quadrature, ultimately limiting the improvement of the signal-to-noise ratio (SNR). Thus, related experimental demonstrations in this context have remained elusive. Until recently, an improvement, enabled by injecting squeezed light into a cavity, was realized [58] for longitudinal qubit readout [14, 56, 59–61], which can enable much shorter measurement times than the dispersive readout. However, due to transmission and injection losses, more than half of the amount of squeezing is lost, and consequently the reported SNR is increased only by $\simeq 25\%$.

Here, we propose to apply our strong intracavity squeezing to longitudinal qubit readout, thus avoiding transmission and injection losses. We demonstrate that the SNR can be increased *exponentially*, and the measurement error is improved by *many orders of magnitude* for modest parameters. In sharp contrast, intracavity squeezing of the semiclassical DPA *cannot* significantly improve the SNR during a practically feasible measurement time, even though squeezing of the output field is very strong. Our main results are summarized in Table I in [62].

Physical model.—A fully quantum DPA, as shown in Fig. 1(a), consists of a pump mode $\hat{a}_p$ and a signal mode $\hat{a}_s$, which are coupled through a single-photon parametric coupling of strength g. We assume that the pump mode is driven by a tone of frequency ω_d and amplitude $\mathcal{E}_d$, and additionally the signal mode is subject to a two-tone driving of frequencies $\omega_\pm^{\rm d}$ and amplitudes $\mathcal{E}_\pm$. The corresponding Hamiltonian in a frame rotating at ω_d is $\hat{H} = \hat{H}_0 + \hat{H}_{\rm 2td}$, with

$$\hat{H}_0 = \Delta_p\hat{a}_p^\dagger\hat{a}_p + \Delta_s\hat{a}_s^\dagger\hat{a}_s + g\left(\hat{a}_s^2\hat{a}_p^\dagger + {\rm H.c.}\right) + \left(\mathcal{E}_d\hat{a}_p^\dagger + {\rm H.c.}\right), \quad (1)$$

$$\hat{H}_{\rm 2td} = \Omega_{\rm 2td}\left(t\right)\hat{a}_s^\dagger + {\rm H.c.}, \quad (2)$$

where $\Delta_p = \omega_p - \omega_d$, $\Delta_s = \omega_s - \omega_d/2$, $\Omega_{\rm 2td}\left(t\right) = \mathcal{E}_-\exp\left(-i\omega_- t\right) + \mathcal{E}_+\exp\left(-i\omega_+ t\right)$. Here, ω_p, ω_s are the resonance frequencies of the pump and signal modes, and $\omega_\pm = \omega_\pm^{\rm d} - \omega_d/2$. We describe photon losses with the Lindblad dissipator $\mathcal{L}\left(\hat{o}\right)\hat{\rho} = \hat{o}\hat{\rho}\hat{o}^\dagger - \frac{1}{2}\left(\hat{o}^\dagger\hat{o}\hat{\rho} + \hat{\rho}\hat{o}^\dagger\hat{o}\right)$, so that the system dynamics is determined by the master equation $\dot{\hat{\rho}} = -i\left[\hat{H},\hat{\rho}\right] + \kappa_p\mathcal{L}\left(\hat{a}_p\right)\hat{\rho} + \kappa_s\mathcal{L}\left(\hat{a}_s\right)\hat{\rho}$, where κ_p and κ_s are the photon-loss rates. Upon introducing the displacement transformation $\hat{a}_p \to \hat{a}_p + \alpha_p^{\rm d}$, where $\alpha_p^{\rm d} = \mathcal{E}_d/\left(i\kappa_p/2 - \Delta_p\right)$, the Hamiltonian $\hat{H}_0$ becomes $\hat{H}_0 = \Delta_p\hat{a}_p^\dagger\hat{a}_p + \hat{H}_{\rm 2pd} + \hat{V}$. Here,

$$\hat{H}_{\rm 2pd} = \Delta_s\hat{a}_s^\dagger\hat{a}_s + \Omega_{\rm 2pd}\left(\hat{a}_s^2 + {\rm H.c.}\right), \quad (3)$$

$$\hat{V} = g\left(\hat{a}_s^2\hat{a}_p^\dagger + {\rm H.c.}\right), \quad (4)$$

where $\Omega_{\rm 2pd} = g\alpha_p^{\rm d}$ can be viewed as the strength of a two-photon driving of the mode $\hat{a}_s$. We have assumed, for simplicity, that $\alpha_p^{\rm d}$ is real.

Since the single-photon coupling g is usually weak, the most studied regime of the DPA is for $\alpha_p^{\rm d} \gg 1$. It is then standard to drop $\hat{V}$, leaving only $\hat{H}_{\rm 2pd}$. In this case, the pump mode is treated as a classical field, and the DPA is referred to as semiclassical. For such a semiclassical DPA, the signal mode cannot be squeezed above 3 dB, even with nonlinear corrections arising from the coupling $\hat{V}$ [38, 62, 63]. The reason for this moderate squeezing is the photon loss of the signal mode. That is, the leakage of single photons of some correlated photon pairs injected by the two-photon driving $\Omega_{\rm 2pd}$ causes a partial loss of two-photon correlations, and thus of intracavity squeezing. However, as demonstrated below, the photon loss of the signal mode, *when turned from a noise source into a resource via reservoir engineering*, can steer a quantized pump mode into a squeezed steady state. More importantly, *this photon loss can strongly suppress the detrimental effect of the photon loss of the pump mode on squeezing*, ultimately leading to a strong steady-state intracavity squeezing.

Squeezing far beyond 3 dB.—Recently, it has been shown experimentally that the available single-photon coupling g can range from tens of kHz to tens of MHz [64–72]. These advances allow one to consider the effect of the coupling $\hat{V}$, e.g., two-photon loss [64–66, 73–76]. We here focus on the case of $\Delta_s \neq 0$, and introduce a signal Bogoliubov mode, $\hat{\beta}_s = \hat{a}_s \cosh(r_s) + \hat{a}_s^\dagger \sinh(r_s)$, with $\tanh(2r_s) = 2\Omega_{\rm 2pd}/\Delta_s$. The Hamiltonian $\hat{H}_{\rm 2pd}$ is then diagonalized, yielding $\hat{H}_{\rm 2pd} = \Lambda_s \hat{\beta}_s^\dagger \hat{\beta}_s$, where $\Lambda_s = \sqrt{\Delta_s^2 - 4\Omega_{\rm 2pd}^2}$. Likewise, the coupling $\hat{V}$ and the two-tone driving $\hat{H}_{\rm 2td}$ become

$$\hat{V} = g_0 \hat{\beta}_s^\dagger \hat{\beta}_s \left(\hat{a}_p + \hat{a}_p^\dagger\right) + \hat{R}_1 + \hat{R}_1^\dagger, \tag{5}$$

$$\hat{H}_{\rm 2td} = \Omega_{\rm 2td}(t) \cosh(r_s) \hat{\beta}_s^\dagger + \hat{R}_2 + \text{H.c.}, \tag{6}$$

where $\hat{R}_1 = g\left[\cosh^2(r_s)\hat{\beta}_s^2 + \sinh^2(r_s)\hat{\beta}_s^{\dagger 2}\right]\hat{a}_p^\dagger$, $\hat{R}_2 = -\Omega_{\rm 2td}(t)\sinh(r_s)\hat{\beta}_s$, and $g_0 = -g\sinh(2r_s)$. We further assume the limit $\{g, \Omega_{\rm 2pd}, \Delta_p\} \ll \Delta_s$, such that $r_s \ll 1$, and both $\hat{R}_1$ and $\hat{R}_2$ can be dropped as high-frequency components (see [62]), yielding

$$\hat{V} \simeq g_0 \hat{\beta}_s^\dagger \hat{\beta}_s \left(\hat{a}_p + \hat{a}_p^\dagger\right), \tag{7}$$

$$\hat{H}_{\rm 2td} \simeq \cosh(r_s)\,\Omega_{\rm 2td}(t)\,\hat{\beta}_s^\dagger + \text{H.c.} \tag{8}$$

Equations (7, 8) are reminiscent of the two-tone driven radiation-pressure interaction in cavity optomechanics [77]. With such an interaction, the cavity photon loss can stabilize a strong squeezing of mechanical motion [40, 41, 44, 78–80]. Here, we harness a similar mechanism, and assume that $\omega_\pm = \Lambda_s \pm \Delta_p$, so that the mode $\hat{\beta}_s$ is coupled to a pump Bogoliubov mode, $\hat{\beta}_p = \hat{a}_p \cosh(r_p) + \hat{a}_p^\dagger \sinh(r_p)$, through the effective Hamiltonian [62],

$$\hat{H}_{\rm eff} = \mathcal{G}\left(\hat{\beta}_p \hat{\beta}_s^\dagger + \hat{\beta}_p^\dagger \hat{\beta}_s\right). \tag{9}$$

Here, $\tanh(r_p) = G_+/G_-$ and $\mathcal{G} = \sqrt{G_-^2 - G_+^2}$. We have defined $G_\pm = g_0 \alpha_s^\pm$, where $\alpha_s^\pm$ (given in [62]) are the field amplitudes of the mode $\hat{\beta}_s$ induced by the two-tone driving $\Omega_{\rm 2td}$, and for simplicity both have been assumed to be real.

Furthermore, we have $\mathcal{L}(\hat{a}_s)\hat{\rho} \simeq \mathcal{L}\left(\hat{\beta}_s\right)\hat{\rho}$ for $r_s \ll 1$, and the system dynamics can thus be described with the effective master equation

$$\dot{\hat{\rho}} = -i\left[\hat{H}_{\rm eff}, \hat{\rho}\right] + \kappa_p \mathcal{L}(\hat{a}_p)\hat{\rho} + \kappa_s \mathcal{L}(\hat{\beta}_s)\hat{\rho}. \tag{10}$$

It is seen that for a large κ_s, the photon loss of the mode $\hat{\beta}_s$ can cool the mode $\hat{\beta}_p$ into the ground state, corresponding to the squeezed vacuum state of the mode $\hat{a}_p$, which can theoretically have an arbitrary degree of squeezing. Such a squeezed steady state is unique, and can be reached from any state of the mode $\hat{a}_p$. The reason is that any state of the mode $\hat{a}_p$ can be expressed in terms of the ground and excited states of the mode $\hat{\beta}_p$, but of these, all the excited states are depopulated by the photon loss of the mode $\hat{\beta}_s$ in the steady state. This initial-state independence enables the detrimental effect of the photon loss of the mode $\hat{a}_p$ on squeezing to be strongly suppressed as long as $\kappa_s \gg \kappa_p$ (see [62] for more details), consequently forming a strong steady-state squeezing for the mode $\hat{a}_p$. During the formation of this squeezing, any odd photon-number state of the mode $\hat{a}_p$ is reached by two different transitions, which are induced by the two-tone driving $\Omega_{\rm 2td}$. Achieving a desired steady-state squeezing, i.e., a superposition of only even photon-number states, requires destructive interference between these two transitions to cancel out the population of all the odd photon-number states.

To quantify the degree of squeezing, we use the squeezing parameter [81],

$$\xi_p^2 = 1 + 2\left(\langle \hat{a}_p^\dagger \hat{a}_p \rangle - |\langle \hat{a}_p \hat{a}_p \rangle|\right). \tag{11}$$

Its time evolution is plotted in Fig. 1(b). Specifically, we compare the effective and exact results, and show an excellent agreement between them. Therefore, the effective master equation in Eq. (10) can be used to predict some larger squeezing by deriving the steady-state squeezing parameter,

$$\left(\xi_p^2\right)_{\rm ss} = \frac{1 + 4\mathcal{C}\exp(-2r_p)}{1 + 4\mathcal{C}}, \tag{12}$$

where $\mathcal{C} = \mathcal{G}^2/(\kappa_s \kappa_p)$ is the cooperativity of the DPA. In Fig. 1(c), $\left(\xi_p^2\right)_{\rm ss}$ is plotted versus $\mathcal{C}$. For realistic parameters of $\kappa_s = 100\kappa_p$, we find that a modest ratio G_+/G_- can keep $\left(\xi_p^2\right)_{\rm ss}$ above 3 dB even for $\mathcal{C} \simeq 0.4$. Moreover, $\left(\xi_p^2\right)_{\rm ss}$ increases as $\mathcal{C}$, and ultimately reaches its maximum value,

$$\left(\xi_p^2\right)_{\rm ss}^{\max} = \exp(-2r_p) = \frac{1 - G_+/G_-}{1 + G_+/G_-}. \tag{13}$$

For example, with $G_+/G_- = 0.99$, we predict a maximum squeezing of $\left(\xi_p^2\right)_{\rm ss}^{\max} \simeq 23$ dB. Thus by

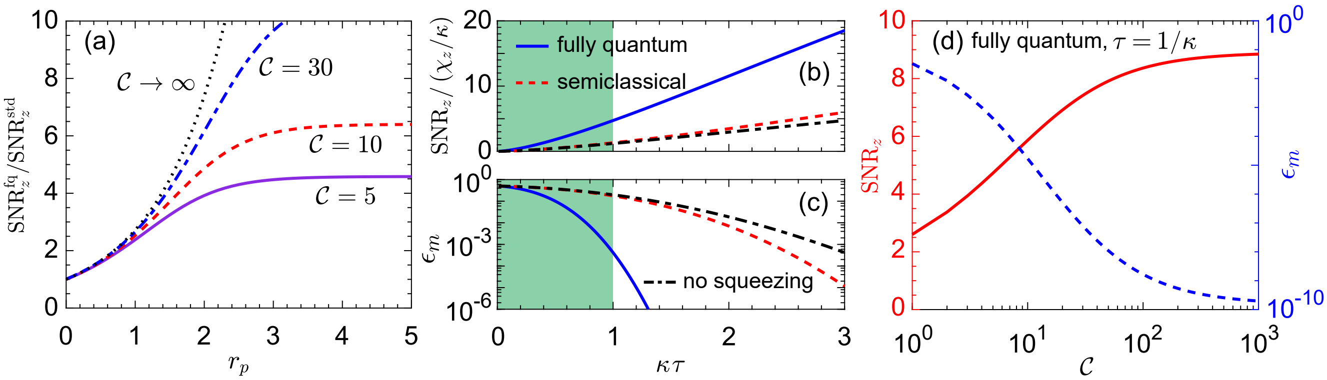

FIG. 2. (a) SNR improvement, i.e., $\mathrm{SNR}_z^{\mathrm{fq}}/\mathrm{SNR}_z^{\mathrm{std}}$, versus the degree r_p of intracavity squeezing for different values of the DPA cooperativity: $\mathcal{C}=5$, 10, 30, and ∞. An exponential improvement can be obtained for $\mathcal{C}\gg\exp(2r_p)/4$. (b) SNR and (c) measurement error versus the measurement time. The solid and dashed curves correspond to the longitudinal readout using intracavity squeezing of the fully quantum ($r_p=2$, $\mathcal{C}=5$) and semiclassical ($r_{\mathrm{out}}^{\mathrm{sc}}=2$) DPAs, respectively, while the dash-dotted curves are results of the standard longitudinal readout with no squeezing. The green shaded area represents the experimentally most interesting regime. In (b) and (c), all the parameters are the same except that $\chi_z=\kappa$ in (c). (d) SNR (left axis) and measurement error (right axis) versus $\mathcal{C}$ in the fully-quantum-DPA case for $\tau=1/\kappa$. Other parameters are the same as in (c).

increasing the ratio G_+/G_- to $\lesssim 1$, we can in principle make intracavity squeezing arbitrarily strong. This is a counterintuitive result from the usually accepted point of view: the steady-state intracavity squeezing of a DPA is fundamentally limited to 3 dB.

Enhanced longitudinal qubit readout.—As an application, we below show that our intracavity squeezing in a fully quantum DPA can *exponentially* improve the SNR of longitudinal qubit readout. In [62], we also analyze the longitudinal readout using intracavity squeezing of a semiclassical DPA. However, we demonstrate that this semiclassical-DPA intracavity squeezing *cannot* enable a practically useful increase in the SNR, even with a strong squeezing of the output field.

To begin, we consider the Hamiltonian,

$$\hat{H}_z^{\mathrm{fq}}=\hat{H}_{\mathrm{eff}}+\chi_z\hat{\sigma}_z\left(\hat{a}_p e^{-i\phi_z}+\hat{a}_p^{\dagger}e^{i\phi_z}\right),\qquad(14)$$

where $\hat{\sigma}_z$ is the Pauli matrix of the qubit. The first term is used to generate intracavity squeezing, while the second term accounts for the longitudinal qubit-field coupling of strength χ_z and phase ϕ_z. Possible experimental implementations of $\hat{H}_z^{\mathrm{fq}}$ are discussed in [62]. Since the photon loss of the mode $\hat{\beta}_s$ is strong, we adiabatically eliminate the mode $\hat{\beta}_s$ to obtain the following equation of motion for the mode $\hat{a}_p$,

$$\dot{\hat{a}}_p=-ie^{i\phi_z}\chi_z\hat{\sigma}_z-\frac{\kappa}{2}\hat{a}_p-\sqrt{\kappa}\hat{\mathcal{A}}_{\mathrm{in}}(t),\qquad(15)$$

where $\kappa=\kappa_p^{\mathrm{ad}}+\kappa_p$ is the overall photon loss rate. Here, $\kappa_p^{\mathrm{ad}}=4\mathcal{G}^2/\kappa_s$ is the rate of the adiabatic photon loss. Moreover, we have defined the overall input noise as $\hat{\mathcal{A}}_{\mathrm{in}}(t)=\left[\sqrt{\kappa_p^{\mathrm{ad}}}\hat{a}_{p,\mathrm{in}}^{\mathrm{ad}}(t)+\sqrt{\kappa_p}\hat{a}_{p,\mathrm{in}}(t)\right]/\sqrt{\kappa}$. It involves two uncorrelated noise operators, $\hat{a}_{p,\mathrm{in}}^{\mathrm{ad}}(t)$ and $\hat{a}_{p,\mathrm{in}}(t)$. The former represents the adiabatic noise arising from the photon loss of the mode $\hat{\beta}_s$, and is given by $i\hat{a}_{p,\mathrm{in}}^{\mathrm{ad}}(t)=\hat{\beta}_{s,\mathrm{in}}(t)\cosh(r_p)+\hat{\beta}_{s,\mathrm{in}}^{\dagger}(t)\sinh(r_p)$, where $\hat{\beta}_{s,\mathrm{in}}(t)$ is the noise operator of the mode $\hat{\beta}_s$. As seen in Eq. (10), $\hat{\beta}_{s,\mathrm{in}}(t)$ can be considered as the vacuum noise, and therefore $\hat{a}_{p,\mathrm{in}}^{\mathrm{ad}}(t)$ corresponds to the squeezed vacuum noise of the mode $\hat{a}_p$. Moreover, the operator $\hat{a}_{p,\mathrm{in}}(t)$ represents the vacuum noise inducing the natural photon loss of the mode $\hat{a}_p$. Note that $\hat{a}_p$ in Eq. (15) is a field operator displaced by an amount α_p^{d}, but the side effect of this displacement on the qubit readout is negligible as a high-frequency effect [62].

The longitudinal coupling maps the qubit state onto the output quadrature, $\hat{\mathcal{Z}}_{\mathrm{out}}(t)=\hat{\mathcal{A}}_{\mathrm{out}}(t)e^{-i\phi_h}+\hat{\mathcal{A}}_{\mathrm{out}}^{\dagger}(t)e^{i\phi_h}$, which is measured by a homodyne setup with a detection angle ϕ_h. Here, $\hat{\mathcal{A}}_{\mathrm{out}}(t)=\hat{\mathcal{A}}_{\mathrm{in}}(t)+\sqrt{\kappa}\hat{a}_p(t)$ is the overall output field. An essential parameter quantifying the homodyne detection is the SNR, which is evaluated using the operator $\hat{M}=\sqrt{\kappa}\int_0^{\tau}dt\,\hat{\mathcal{Z}}_{\mathrm{out}}(t)$, with τ the measurement time, and is defined as

$$\mathrm{SNR}=\left|\langle\hat{M}\rangle_{\uparrow}-\langle\hat{M}\rangle_{\downarrow}\right|\left(\langle\hat{M}_N^2\rangle_{\uparrow}+\langle\hat{M}_N^2\rangle_{\downarrow}\right)^{-1/2},\qquad(16)$$

where $\hat{M}_N=\hat{M}-\langle\hat{M}\rangle$ characterizes the measurement noise, and $\{\uparrow,\downarrow\}$ refers to the qubit state. The SNR of the readout using our fully-quantum-DPA intracavity squeezing is then given by

$$\mathrm{SNR}_z^{\mathrm{fq}}=\sqrt{\frac{1+4\mathcal{C}}{1+4\mathcal{C}\exp(-2r_p)}}\mathrm{SNR}_z^{\mathrm{std}},\qquad(17)$$

where $\mathrm{SNR}_z^{\mathrm{std}}=8\chi_z\tau\left[1-2\left(1-e^{-\kappa\tau/2}\right)/\kappa\tau\right]/\sqrt{2\kappa\tau}$ refers to the SNR of the standard longitudinal readout with no squeezing. Equation (17) shows a distinct improvement in the SNR, as in Fig. 2(a). Such an improvement increases as the cooperativity $\mathcal{C}$, which can, in principle, be made arbitrarily large. Furthermore, as long as $\mathcal{C} \gg \exp(2r_p)/4$, we have

$$\mathrm{SNR}_z^{\mathrm{fq}} \simeq \exp(r_p)\,\mathrm{SNR}_z^{\mathrm{std}}, \tag{18}$$

an exponential improvement in the SNR.

More importantly, the SNR improvement in Eqs. (17, 18) holds for *any* measurement time. The reason is that the degree of squeezing of the measurement noise equals the degree of intracavity squeezing, i.e., $\langle \hat{M}_N^2\rangle/\kappa\tau = \left(\xi_p^2\right)_{\mathrm{ss}}$, and is independent of the measurement time. This is in stark contrast to the case of using the semiclassical-DPA intracavity squeezing, where, as discussed in [62], the degree of squeezing of the measurement noise increases from the initial value zero, as the measurement time increases, and consequently a large increase in the SNR needs an extremely long measurement time. Assuming realistic parameters of $r_p = 2$ ($\simeq$ 17 dB) and $\mathcal{C} = 5$, our approach gives an approximately four-fold improvement for any measurement time, as illustrated in Fig. 2(a). However, when using the semiclassical-DPA intracavity squeezing, there is almost no improvement for the short-time measurement of most interest in experiments, even though the output-field squeezing, characterized by the parameter $r_{\mathrm{out}}^{\mathrm{sc}} = \ln\left[(\kappa_s + 4\Omega_{2\mathrm{pd}})/(\kappa_s - 4\Omega_{2\mathrm{pd}})\right]$, is strong [62].

In Figs. 2(b, c), we plot the SNR and the measurement error, $\epsilon_m = 1 - \mathcal{F}_m$, for the longitudinal readout using the fully-quantum- and semiclassical-DPA intracavity squeezing, and also for the standard longitudinal readout with no squeezing. Here, $\mathcal{F}_m = \frac{1}{2}\left[1 + \mathrm{erf}\left(\mathrm{SNR}/2\right)\right]$ is the measurement fidelity. Choosing $r_p = 2$, and $\chi_z = \kappa = 2\pi \times 3$ MHz for our approach, a short measurement time of $\tau = 1/\kappa \simeq 53$ ns gives $\mathrm{SNR}_z^{\mathrm{fq}} \simeq 4.7$ for $\mathcal{C} = 5$. This corresponds to a measurement error of $\epsilon_m \simeq 4.4 \times 10^{-4}$. When $\mathcal{C}$ increases, as in Fig. 2(d), $\mathrm{SNR}_z^{\mathrm{fq}}$ can further increase to a maximum of $\simeq 8.9$, and the measurement error rapidly decreases, reaching a minimum of $\simeq 1.5 \times 10^{-10}$. However, at the same measurement time, both the standard longitudinal readout with no squeezing and the case of using the semiclassical-DPA intracavity squeezing enable a much lower SNR, i.e., $\mathrm{SNR}_z^{\mathrm{std}} \simeq \mathrm{SNR}_z^{\mathrm{sc}} \simeq 1.1$, and correspondingly a measurement error of $\simeq 0.22$, which is many orders of magnitude larger.

Conclusions.—We have introduced a method of how to exploit a fully quantum DPA to beat the 3 dB limit of intracavity squeezing. We have demonstrated that an *arbitrary* steady-state squeezing can in principle be achieved for the pump mode, by simply applying a two-tone driving to the signal mode. This counterintuitive intracavity squeezing can *exponentially* increase the SNR of longitudinal qubit readout, and improve the measurement error by *many orders of magnitude*. In contrast, the semiclassical-DPA intracavity squeezing *cannot* enable a useful increase in the SNR, due to the impractical requirement of a long measurement time. Our proposal is valid for both microwave and optical cavities, but we believe that it is easier to implement it with microwaves in quantum circuits. The resulting intracavity squeezing is equivalent to an externally generated and injected squeezing but without transmission and injection losses. Thus, this intracavity squeezing, as a powerful alternative to that external squeezing, could find many quantum applications in addition to the qubit readout, and further excite more interest to exploit the potential of DPAs for modern quantum technologies.

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