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Superconducting cavity piezo-electromechanics: the realization of an acoustic frequency comb at microwave frequencies

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We present a nonlinear multimode superconducting electroacoustic system, where the interplay between superconducting kinetic inductance and piezoelectric strong coupling establishes an effective Kerr nonlinearity among multiple acoustic modes at 10 GHz that could hardly be achieved via intrinsic mechanical nonlinearity. By exciting this multimode Kerr system with a single microwave tone, we further demonstrate a coherent electroacoustic frequency comb and provide theoretical understanding of multimode nonlinear interaction in the superstrong coupling limit. This nonlinear superconducting electroacoustic system sheds light on the active control of multimode resonator systems and offers an enabling platform for the dynamic study of microcombs at microwave frequencies.

Introduction.— Mechanical and acoustic resonators with low dissipation have become pivotal elements in many applications such as weak mass and force detection [1–6], timing and frequency control [7, 8], and frequency conversion [9–15]. Recent investigations of the nonlinear behaviors in mechanical resonators [16–18], especially at nano-/micro-scales, have attracted great attentions for their potential in novel device functionality; by exploiting the mechanical nonlinearity, parametric amplification [19], frequency tuning [20], and frequency stabilization [21] have been realized.

For many applications, high operation frequencies above gigahertz [22–24] are highly desired for suppressing the impact of thermomechanical noise, improving the time-keeping precision, and preparing non-classical mechanical states. However, so far, most nonlinear mechanical or acoustic systems can only operate at low frequencies up to a few megahertz. The challenges arise from the preparation of high-quality (Q) factor mechanical resonators at such high frequencies, as the Q factor quickly reduces with frequency, limited by the empirical $f \cdot Q$ product [25]. At the same time, the intrinsic mechanical nonlinearity also degrades with increasing frequency, since the mechanical resonators become stiffer with their deformation amplitudes significantly reduced. As a result, the intrinsic nonlinear effects of gigahertz mechanical resonators, such as the commonly exploited Duffing [16] and electrostatic spring effects [26, 27], are typically very weak. To overcome these obstacles, it is of great importance to develop a hybrid mechanical system where nonlinearity can be substantially enhanced.

In this work, we demonstrate such a nonlinear superconducting electroacoustic system at 10 GHz where strong nonlinearity is achieved in the multimode strong coupling regime [28]. By engineering the kinetic inductance of a superconducting resonator, we establish effective Kerr nonlinear interactions [29] among multiple high- Q acoustic modes in a bulk acoustic resonator (BAR). Leveraging this nonlinear multimode system, we demonstrate a coherent electroacoustic frequency comb driven

by a single microwave parametric pump. We further study the coherence of the frequency comb by characterizing the stable relative phase of each comb line via time-domain measurement.

Phononic frequency combs have been recently investigated in micro-electromechanical systems (MEMS). These MEMS combs operate at megahertz frequencies in order to harness the intrinsic mechanical nonlinearity [30–33]; typically, only one or two mechanical modes are utilized to generate low-frequency comb lines through the nonlinear mixing between the driven mode and the parametrically excited harmonic or sub-harmonic modes. In contrast, here, our superconducting electroacoustic system features an array of high- Q acoustic modes at 10 GHz frequencies, where the desired Kerr nonlinearity is acquired through the strong electromechanical coupling with a kinetic-inductance dominated superconducting resonator. As a result, cascaded four-wave mixing (FWM) and the subsequent electroacoustic comb generation can be achieved. The hybrid nature of our system also makes the demonstrated electroacoustic comb distinct from the widely studied nanophotonic Kerr microcombs [34–39] and microwave microcombs in a long nonlinear superconducting resonator [40], providing a new platform for exploring complex nonlinear phononic-photon interactions.

Nonlinear multimode electroacoustics.— Our nonlinear hybrid system comprises a nonlinear frequency-tunable superconducting Ouroboros resonator [41] strongly coupled with a bulk acoustic resonator (BAR) through the piezoelectric effect [28, 42]. As the core nonlinear element, the Ouroboros resonator enables two functions: it not only serves as the gain medium for parametric amplification, but also provides frequency tunability to fulfill frequency matching requirement for the comb generation as discussed later. The Ouroboros is fabricated in 50-nm-thick niobium titanium nitride (NbTiN) film on 127- μ m-thick sapphire substrate. As shown in Fig. 1(a), it is formed by a planar capacitor (C) and a nonlinear inductor (L) consisting of six narrow wires with high kinetic in-

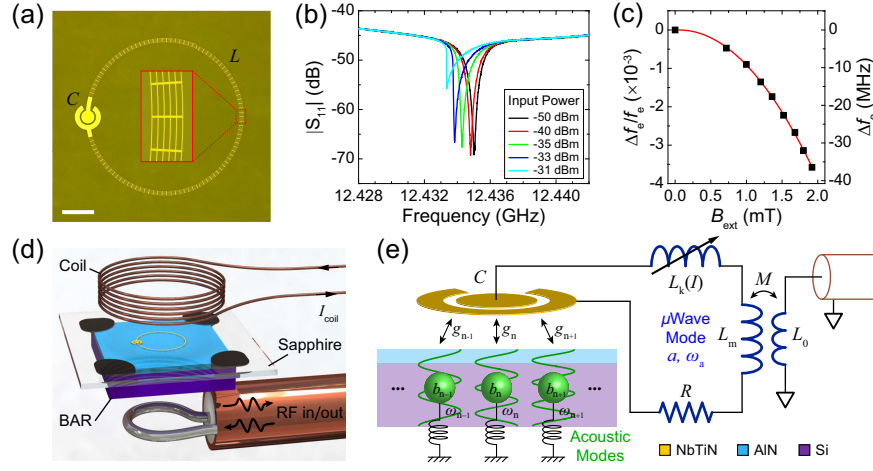


FIG. 1. (a) An optical image of the nonlinear frequency-tunable Ouroboros resonator. The scale bar is $200\ \mu\text{m}$. Inset: zoomed-in view of the nonlinear inductor (wire width: $1\ \mu\text{m}$, spacing: $4\ \mu\text{m}$). (b) Microwave reflection spectrum of the Ouroboros at different input power showing the nonlinear resonance shift. (c) Frequency tuning of the Ouroboros resonator under an external magnetic field. Red line shows a quadratic fitting. (d) A schematic of the nonlinear electroacoustic device (not to scale). The Ouroboros chip is flipped over and bonded with a piezo-bulk acoustic resonator (BAR) by cryogenic epoxy (black) at the four corners. A superconducting coil is placed above the Ouroboros to provide external magnetic field for frequency tuning. The Ouroboros is inductively coupled to a loop probe for microwave signals input and output. (e) Illustration of the nonlinear electroacoustic system with multimode strong coupling between the Ouroboros and the BAR. g_n : piezo-electromechanical coupling strength. L_k and L_m : the nonlinear kinetic inductance and the magnetic inductance of the Ouroboros. R : equivalent resistance (intrinsic dissipation) of the microwave mode.

ductance. The frequency tuning of Ouroboros is achieved by the hole-array structure in the nonlinear inductor; the application of an external magnetic field induces screening DC supercurrents surrounding the holes, which modify the kinetic inductance and hence tune the resonant frequency of the Ouroboros [41]. As shown in Fig. 1(c), a relative frequency tuning of 0.36% (corresponding to 39 MHz) is realized within 2 mT magnetic field.

The Kerr nonlinearity of the Ouroboros resonator originates from the quadratic dependence of kinetic inductance L_k on the current I , given by $L_k(I) = L_{k0}(1 + I^2/I_*^2)$ in the small-current limit [43]. Here, I_* is a characteristic current on the order of the critical current of the nanowire. This nonlinearity results in a FWM interaction term in the Hamiltonian described by $H_{\text{FWM}} = -\hbar \frac{K}{12} (\hat{a} + \hat{a}^\dagger)^4$ with $K = 3L_{k0}I_{\text{zpf}}^4/I_*^2$. Here, \hat{a} (\hat{a}^\dagger) is the annihilation (creation) operator of the Ouroboros microwave mode. $I_{\text{zpf}} \equiv \sqrt{\hbar\omega_e/2L}$ is the zero point fluctuation current in the nanowire inductor, where $\omega_e = 1/\sqrt{LC}$ is the resonant frequency, and $L = L_k + L_m$ with L_m being the magnetic (or geometric) inductance.

As a manifestation of third-order nonlinearity, the Ouroboros behaves like a Duffing resonator under a strong microwave drive. As shown in Fig. 1(b), at low input powers, a symmetric Lorentzian-shape resonance is observed in the reflection spectrum of the Ouroboros. As the input power increases, the resonance starts to distort and shift towards lower frequencies with a bifurcation

threshold power around $-33\ \text{dBm}$. Based on the fitting of the nonlinear resonances [44], we extract a Kerr coefficient of $K = 2\pi \times 0.11\ \text{MHz}$. It is worth mentioning that, the kinetic inductance nonlinearity also provides opportunity for a three-wave-mixing (TWM) interaction in presence of a DC current. In our experiment, since the pump and the hybrid modes are engineered to only fulfill the FWM condition, the TWM process will not participate.

The BAR consists of a 550-nm-thick aluminum nitride (AlN) layer on top of a 500- μm -thick high-resistivity silicon substrate. Multiple high- Q longitudinal acoustic modes with a free spectral range (FSR) around $\mathcal{F}_0 = 9.1\ \text{MHz}$ are supported in the BAR. The thickness of the AlN is chosen to match half acoustic wavelength at 10 GHz to maximize the piezoelectric coupling. To combine the multiple acoustic modes with superconducting nonlinearity, the Ouroboros is flipped over and bonded with the BAR by cryogenic epoxy at the corners (Fig. 1(d)). As a result, a strongly coupled nonlinear multimode electroacoustic system is achieved (Fig. 1(e)).

The interplay between the Kerr nonlinearity and electro-acoustic coupling can be modeled by the total Hamiltonian of the nonlinear multimode system,

$$H = \hbar\omega_e \hat{a}^\dagger \hat{a} - \frac{\hbar}{2} K \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} + \hbar \sum_n \omega_n \hat{b}_n^\dagger \hat{b}_n + \hbar \sum_n g_n (\hat{a}^\dagger \hat{b}_n + \hat{b}_n^\dagger \hat{a}), \quad (1)$$

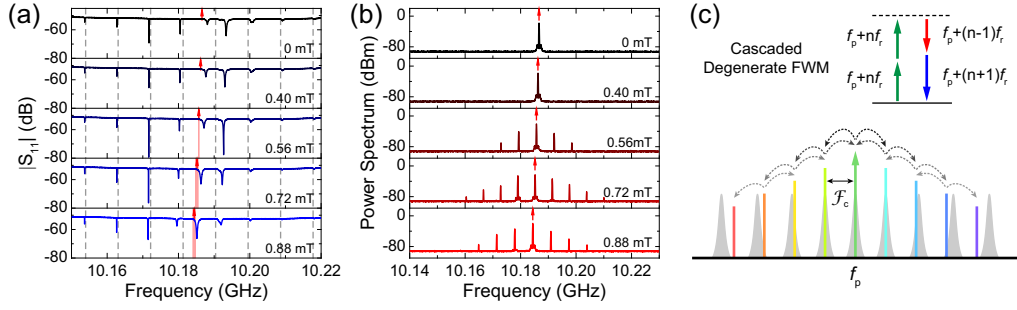


FIG. 2. (a) Microwave reflection spectrum at different magnetic field. Gray dashed lines indicate the unperturbed acoustic resonant frequencies. Red arrows mark the corresponding pump frequencies in (b). The pink shades indicate the pump frequency range where a hybrid comb can be generated. (b) Microwave power spectrum of the hybrid comb at different magnetic field under a pump power of -18 dBm. Red arrows indicated the pump frequency. (c) Principle of comb generation via cascaded degenerate FWM. Under a single pump tone at f_p , two pump photons (green arrows in the upper inset) are annihilated to create one signal and one idler photons (red and blue arrows). In presence of multiple hybrid modes (gray shades), a cascaded process is initiated to form a frequency comb with an FSR of \mathcal{F}_c .

where \hat{b}_n and ω_n are the annihilation operator and the frequency of the n -th acoustic mode, respectively. g_n is the piezo-electromechanical coupling strength between the Ouroboros microwave mode and the n -th acoustic mode. Note that we have applied the rotating wave approximation and approximate the FWM interaction by $H_{\text{FWM}} \approx -\frac{\hbar}{2} K \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a}$, which appears as the second term in Eq. (1). It is worth pointing out that the piezo-electromechanical coupling has a linear form, which is distinct from the nonlinear coupling involved in the study of internal resonances in other mechanical systems [33, 45].

To better reveal the multimode nonlinear interaction enabled by the strong coupling, it's helpful to describe the system on the hybrid-mode basis. The multimode strong coupling results in hybridization between the microwave mode and the acoustic modes, leading to a set of new hybrid phononic-photonic eigenmodes (\hat{c}_p). Mathematically, this corresponds to a unitary transformation which diagonalizes out the linear coupling terms in Eq. (1) and transforms the Hamiltonian to [44]

$$H = \hbar \sum_p \tilde{\omega}_p \hat{c}_p^\dagger \hat{c}_p - \frac{\hbar}{2} \sum_{p,q,r,s} K_{pqrs} \hat{c}_p^\dagger \hat{c}_q^\dagger \hat{c}_r \hat{c}_s, \quad (2)$$

Here, $\tilde{\omega}_p$ is the frequency of the p -th new hybrid mode. (p, q, r, s) are the dummy summation indices. The effective Kerr coefficient can be expressed as $K_{pqrs} = \frac{K}{\mathcal{N}_p \mathcal{N}_q \mathcal{N}_r \mathcal{N}_s}$, where $\mathcal{N}_p = \sqrt{1 + \sum_n \frac{g_n^2}{(\omega_n - \tilde{\omega}_p)^2 + \kappa_n^2}}$ is a normalization factor and κ_n is the dissipation rate of the n -th acoustic mode (see [44] for derivation). Equation (2) clearly shows the nonlinear interaction between hybrid modes. As expected, the Kerr nonlinearity is stronger between hybrid modes with more microwave component, i.e. when $\omega_{p,q,r,s}$ is close to ω_e , because the nonlinearity originates from the superconducting resonator. It is worth pointing out that Eq. (2) is valid even in the super-

strong coupling regime [46, 47] where the mode coupling strength becomes comparable with or even larger than the FSR. With this Kerr nonlinearity, a cascaded FWM process can be triggered to generate hybrid phononic-photonic frequency combs.

Hybrid frequency comb.— In experiment, the device is enclosed in a copper holder with a superconducting coil placed above the Ouroboros to provide external magnetic field (B_{ext}) for frequency tuning. Microwave signals are sent into and read out from the Ouroboros by using an inductively coupled loop probe (Fig. 1(d)). The device is then mounted on the Still plate of a dilution refrigerator and measured at 900 mK. The passive response of the device is characterized by the microwave reflection spectrum using a vector network analyzer. A low-power probe signal of -60 dBm is used to avoid any nonlinear distortion of the spectrum.

Figure 2(a) plots the multimode spectra at different external bias magnetic fields. A set of hybrid modes are clearly observed over a wide frequency range. Due to the dielectric loading of the BAR, the Ouroboros resonant frequency is brought down from ~ 12 to ~ 10 GHz as designed. At zero magnetic field (top panel in Fig. 2(a)), the Ouroboros mode at 10.191 GHz is near-resonant with one of the bulk acoustic modes, resulting in normal mode splitting and two most pronounced hybridized modes. The device parameters are extracted by fitting the spectrum using the coupled mode formula (see [44]). The microwave and the highest acoustic intrinsic Q factors are $Q_{e,i} \approx 3.6 \times 10^4$ and $Q_{m,i} \approx 1.6 \times 10^6$, respectively. The piezo-electromechanical coupling strength is extracted to be $\frac{g_n}{2\pi} \approx 2.8$ MHz, which is larger than the dissipation rates of the microwave and acoustic modes ($g_n > \frac{\kappa_e}{2}, \frac{\kappa_m}{2}$), confirming the multimode strong coupling nature. The FSR of the unperturbed acoustic modes is extracted to be $\mathcal{F}_0 = 9.1$ MHz.

Frequency comb generation necessitates a pump tone

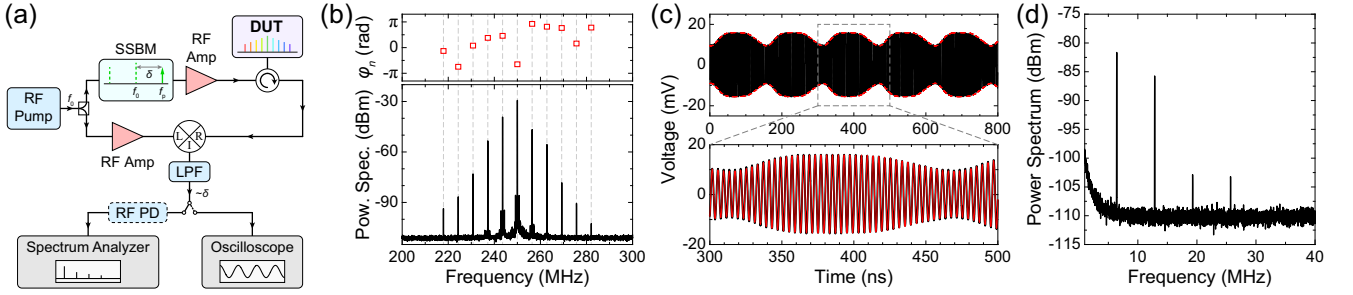


FIG. 3. (a) Measurement diagram for the comb coherence. An RF pump source ($f_0 \sim 10$ GHz) is used to generate a pump tone at $f_p = f_0 + \delta$, where $\delta = 250$ MHz, via single-sideband modulation (SSBM). The 10-GHz frequency comb from the device is then demodulated using the same RF source to down-convert the comb to low frequencies centered at δ . DUT: device under test. RF Amp: RF amplifier. LPF: low-pass filter. PD: photodiode. (b) Microwave power spectrum of the down-converted comb centered at 250 MHz. The top panel shows the relative phase (φ_n) of each comb line extracted from the fitting of the temporal oscillation in (c). (c) Time-domain oscillation of the down-converted comb. A zoomed-in section with a perfectly matched fitting (red line) is shown in the bottom panel shows. The red dashed lines in the top panel indicate the envelope of entire the fitting curve. (d) Power spectrum of the mixing signal of the down-converted comb after the RF photodiode.

to provide parametric gain. Since the nonlinearity originates from the superconducting resonator, the pump tone should be applied on a hybrid mode with large microwave component so that sufficient parametric gain can be generated with a reasonable amount of pump power. This means that the hybrid comb generation is most efficient to operate in the strong coupling band where strong hybridization between the microwave mode and the nearby acoustic modes takes place. Nevertheless, strong coupling inevitably perturbs the originally evenly-spaced acoustic resonances [28, 46, 48], leading to non-uniformly distributed hybrid modes with frequency mismatch that hinders FWM and comb generation. This challenge is overcome by the frequency tunability of the Ouroboros, which allows the microwave mode to be adjusted between neighboring bulk acoustic modes to achieve the desired frequency matching condition.

To investigate the electroacoustic comb generation, a microwave pump of -18 dBm is sent to the device. Power spectra of the output signal at different magnetic fields are recorded and shown in Fig. 2(b). In order to optimize frequency comb generation, the pump frequency (f_p) is swept across the hybrid mode that has most microwave component (the resonance around the red arrow in Fig. 2(a)), which we will refer to as the “pump mode” hereafter. When the pump is tuned into the resonance, the increase of cavity photons will induce resonance shift towards lower frequencies due to the Kerr effect. Therefore, the pump is swept slowly from higher to lower frequencies to make sure the pump can gradually follow and enter the resonance before passing it.

As shown in Fig. 2(b), at zero magnetic field, no comb lines are observed. This is because the asymmetric distribution of the hybrid modes with respect to the pump frequency cannot satisfy the energy conservation requirement for the FWM process—the sum of the signal and idler modal frequencies must equal to twice pump fre-

quency. As the magnetic field increases, the pump mode gradually moves towards lower frequencies. A hybrid comb starts to appear at 0.56 mT and reaches the maximum number of lines at 0.72 mT when the pump mode is tuned to the middle of the two adjacent acoustic resonances. The FSR of the comb at 0.72 mT is measured to be $\mathcal{F}_c = 6.4$ MHz, which matches the frequency spacing from the pump to the two nearest neighboring modes in Fig. 2(a) but differs from the FSR of the unperturbed acoustic modes ($\mathcal{F}_0 = 9.1$ MHz). This indicates that the comb lines are formed first from the two nearest neighboring modes due to the enhanced density of states at matched frequencies, then expand and get broadened through the cascaded FWM process as illustrated in Fig. 2(c). During such process, all the comb lines are generated with well-defined relative phase instead of being random, and hence the comb is expected to be coherent [49].

Coherence of the comb.—The coherence of the electroacoustic comb is first studied by characterizing the temporal oscillation. To measure the time-domain signal, the 10-GHz comb is down-converted to low frequencies centered at 250 MHz. As illustrated in Fig. 3(a), the pump tone is generated from a 10-GHz RF source via single-sideband modulation and sent to the device after amplification. The comb signal from the device is demodulated using the same RF source to obtain the down-converted comb (Fig. 3(b) lower panel). The low-frequency oscillatory signal is then directly measured by an oscilloscope to obtain the time trace. As shown in Fig. 3(c), a stable beating pattern is clearly observed with a beating period measured to be 156 ns, which agrees well with the FSR of the comb ($1/\mathcal{F}_c \approx 156$ ns). This indicates that each comb line has a stable relative phase (φ_n); and hence, the hybrid comb is coherent. By fitting the time trace as the sum of sinusoidal oscillations at each comb line frequency, $V = \sum_n A_n \cos(2\pi f_n t + \varphi_n)$ where A_n and f_n

are the amplitude and frequency of each comb line, all the relative phases φ_n are extracted and plotted in the upper panel in Fig. 3(c). More details about the fitting are provided in [44].

The coherence of the comb is also revealed by the spectrum of the mixing signal. An RF diode is used to mix all the lines of the down-converted comb. In the case of an incoherent comb, the relative phases between different comb lines will fluctuate randomly and produce noises in the spectrum. In contrast, for a coherent comb, since the comb lines have stable and well-defined relative phases, only sharp peaks at multiple FSRs are expected in the spectrum. Indeed, as shown in Fig. 3(d), equally spaced sharp peaks are clearly observed on top of a clean flat background. The frequency of the first peak, as well as the spacing between the adjacent peaks, is measured to be 6.4 MHz, which matches the comb FSR (\mathcal{F}_c), confirming that the electroacoustic comb is coherent.

Conclusion.— In summary, we have demonstrated a nonlinear multimode superconducting electroacoustic system at 10 GHz. By harnessing the kinetic inductance nonlinearity and the piezo-electromechanical strong coupling, we realize substantial nonlinear interactions in multimode mechanics at high frequencies and generate a coherent hybrid microcomb. The demonstrated nonlinear phononic-photon platform could open a new route towards the active control of multimode systems as well as advanced information processing in both classical and quantum regimes.

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