Symmetry of Cosmological Observables, a Mirror World Dark Sector, and the Hubble Constant

Francis-Yan Cyr-Racine, Fei Ge, and Lloyd Knox

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A Symmetry of Cosmological Observables, a Mirror World Dark Sector, and the Hubble Constant

Francis-Yan Cyr-Racine,[1] Fei Ge,[2] and Lloyd Knox[2]

[1] *Department of Physics and Astronomy, University of New Mexico, NM, USA 87106*
[2] *Department of Physics and Astronomy, University of California, Davis, CA, USA 95616*
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We find that a uniform scaling of the gravitational free-fall rates and photon-electron scattering rate leaves most dimensionless cosmological observables nearly invariant. This result opens up a new approach to reconciling cosmic microwave background and large-scale structure observations with high values of the Hubble constant, H_0: find a cosmological model in which the scaling transformation can be realized without violating any measurements of quantities not protected by the symmetry. A "mirror world" dark sector allows for effective scaling of the gravitational free-fall rates while respecting the measured mean photon density today. Further model building might bring consistency with the two constraints not yet satisfied: the inferred primordial abundances of deuterium and helium.

Introduction.—**Different methodologies for determining the current rate of expansion, the Hubble constant H_0, are leading to discrepant results. The most precise of the cosmological-model-dependent methods uses the *Planck* measurements of the cosmic microwave background (CMB). Assuming ΛCDM the result is $H_0 = (67.49 \pm 0.53)$ km/sec/Mpc [1–3]. The most precise of the more direct methods, that are relatively independent of cosmological model assumptions, comes from the SH$_0$ES team [4–7]. Using Cepheid-calibrated supernovae they find $H_0 = (73.04 \pm 1.04)$ km/sec/Mpc [8, hereafter R22], a 5σ difference with the above result.**

The preference of ΛCDM-dependent methods for a low H_0 is remarkably robust to choice of data sets and details of applications; it remains for both the "inverse distance-ladder" methods [9–15], that make minimal use of CMB data, and methods that rely on large-scale structure observations instead of CMB data [16–21].

Likewise, various averages of the more direct methodologies [22–34], including those that exclude Cepheid-calibrated supernovae, also lead to $> 4\sigma$ discrepancies with the CMB/ΛCDM-inferred value of H_0 [35]. On the other hand, there is no convergence of opinion yet among those using supernovae to measure H_0. Most notably, the Carnegie-Chicago Hubble Program [36–45] finds some inconsistencies with SH$_0$ES estimates of distances to nearby supernovae. For a review of the observational situation see Refs. [30, 46].

A lot of recent theoretical work is inspired by the possibility that the tension arises from a failure of ΛCDM. For a wide-ranging discussion of possible avenues for solution, see Ref. [47], for a summary of many of the proposed models see Ref. [48], and for a ranking of their performances with respect to a common data set see Ref. [49]. The search is difficult due to the variety of cosmological measurements, their sensitivity to details of cosmological models, their high precision, and their high degree of consistency with ΛCDM.

In this *Letter* we introduce a new insight about the structure of cosmological models to help theorists navigate this challenging environment, as they search for solutions to the Hubble tension. In particular, we **present** a transformation that leaves distance ratios and the statistical properties of fractional maps of CMB **temperature** anisotropy (i.e., $\Delta T/T$), CMB polarization and galaxy number overdensity invariant. This symmetry transformation, under which all relevant length and time scales in the problem are re-scaled by a constant scaling factor λ at all redshifts z [50], has its roots in the scale-free nature of primordial fluctuations.

A restricted version of this transformation was introduced in Ref. [51], in which only the gravitational time scales $1/\sqrt{G\rho_i}$, where ρ_i is the mean density of the $i^{\rm th}$ component, were scaled. This leads to an approximate symmetry that is severely broken at small angular scales and in polarization **on all scales**. By extending their transformation to include a scaling of the photon scattering rate, we have found a symmetry of the above-mentioned observables that is exact in the limit of equilibrium recombination **and zero neutrino mass**. Crucially, we find that **the symmetry-breaking effects of real-world departures from these limits are** mild.

We also report here partial progress toward use of this symmetry to solve the H_0 tension. Significant observational constraints prevent us from a straightforward scaling of gravitational rates by scaling of mean densities. To evade these constraints, and still reap the benefits of the scaling transformation, we are led quite naturally to the addition of a mirror world dark sector. Dark sector models that contain a mirror (or "twin") sector [52] that has exactly the same particle content and gauge interactions as the Standard Model (SM), have been extensively studied in the literature (see e.g. Refs. [53–83]) in the context of the little hierarchy problem. To scale the photon scattering rate one can alter the primordial fraction of baryonic mass in helium $Y_{\rm P}$, which is the method we use here. We show below that such models can, in principle, exploit the scaling symmetry to accommodate a higher value of H_0 with CMB observations.

A strict implementation of the scaling transformation, however, leads to conflict between measurements of $Y_{\rm P}$ and the primordial deuterium abundance with expectations from big bang nucleosynthesis (BBN) as well as the $Y_{\rm P}$ necessary for scaling up the scattering rate. Thus we have essentially re-mapped the problem**, that has proven to be quite difficult, of reconciling a high H_0 with phenomenologically-complicated CMB data, to a problem of reconciling a high H_0 with the observationally-inferred** primordial abundances of helium and deuterium. Our work motivates the search for models that can solve this re-framed problem [84]. **Finally, although we focus here on inferences from CMB observations, *all* of the cosmological-model-dependent inferences of H_0 we referred to above, including those in [9–21], would be impacted similarly; i.e., replacing ΛCDM with a model that solves our reframed problem would also reconcile these inferences with a high H_0, due to the nature of the symmetry.**

The Scaling Transformation.— Let us assume, for now, that recombination happens in equilibrium and neutrinos are massless. Then the only length scales in the linear perturbation evolution equations in the ΛCDM model, written with the scale factor $a = 1/(1+z)$ as the independent time-like variable, are the gravitational time scales of each of the $i = 1$ to N components, $1/\sqrt{G\rho_i(a)}$, and the photon mean free path between electron scatters, $1/(\sigma_T n_e(a))$. As a result, if we consider the linear evolution of a single Fourier mode with wavenumber k, any fractional perturbation, such as $\delta\rho_{\rm m}(\mathbf{k}, a)/\rho_{\rm m}(a)$ satisfying the evolution equations will also satisfy them when transformed by a uniform scaling of all relevant (inverse) length scales (including k) by some factor λ. Since the initial conditions in ΛCDM do not introduce a length scale (the spectrum of initial perturbations is a power law), the statistical properties of fractional perturbations are independent of λ except for an overall amplitude. Dependence on the amplitude can be removed [51] by extending the scaling transformation to include $A_s \to A_s/\lambda^{(n_{\rm s}-1)}$ where $A_{\rm s}$ is the amplitude of the primordial power spectrum at some fiducial value of k, and $n_{\rm s}$ is the spectral index of the power law power spectrum of initial density perturbations.

In Ref. [51] this transformation was introduced but without the photon scattering-rate scaling, **which is critically important for our purposes**. Including it, the transformation leads to an exact symmetry, in the limit of equilibrium recombination [85], massless neutrinos [86], and linearized equations [87], of distance ratios and the statistical properties of maps made in projection on the sky of quantities such as $\delta\rho/\rho$, and fractional CMB temperature and polarization anisotropies. These include galaxy clustering power spectra, shear power spectra, galaxy-shear cross correlations, fractional CMB temperature and polarization power spectra, the temperature-polarization cross spectrum, and the CMB lensing spectrum. The invariance exists for the ΛCDM model, and any other model as long as additional length scales (if any, such as **those** related to mean curvature or neutrino mass) are properly scaled as well. Absent the introduction of new length scales, the full transformation can be written as

$$\begin{aligned} &\sqrt{G\rho_i(a)} \to \lambda\sqrt{G\rho_i(a)}, \quad \sigma_{\rm T} n_e(a) \to \lambda\sigma_{\rm T} n_e(a) \\ &\quad \text{and} \quad A_{\rm s} \to A_{\rm s}/\lambda^{(n_{\rm s}-1)}. \end{aligned} \tag{1}$$

Symmetry Breaking.—The transformation given in Eq. (1) is severely constrained by observations that are sensitive to *absolute* densities of cosmological components. Most importantly, we know very precisely the mean energy density of the CMB today from measurements by FIRAS of its flux density across a broad range of wavelengths [88, 89]. By anchoring ρ_γ, this measurement severely limits our ability to exploit the scaling transformation to raise H_0 [90].

Other important effects that break the above symmetry arise from departures from thermodynamic equilibrium, as emphasized in Ref. [51]. Unlike periods of equilibrium, during which we have no sensitivity to the rates of the reactions that are maintaining equilibrium, periods in which equilibrium is lost provide us with valuable sensitivity to the relevant reaction rates. If we then assume that such microphysical rates are known, we can gain sensitivity to the expansion rate. A prime example is BBN, where sensitivity of the yield of helium **and deuterium** to nuclear reaction rates allows one to infer, from measurements of **helium and/or deuterium**, the expansion rate during BBN, and thus, through the Friedmann equation, the mass/energy density of the Universe at that time. Similarly, hydrogen recombination is an out-of-equilibrium process which is sensitive to atomic reaction rates, and thus breaks the symmetry of the Eq. (1) transformation. We will see that the impact of this latter symmetry breaking on our parameter constraints is mild.

A mirror world dark sector and free $Y_{\rm P}$.—Zahn & Zaldarriaga [51] did not provide a physical mechanism by which one could realize their transformation, other than by varying G, as their goal was only analytic understanding. We now introduce a physical mechanism that, while not allowing for the transformation as strictly written, permits a complete mimicry of its effects, so that the same invariance is achieved.

By extending the ΛCDM model to include a dark copy of the photons, baryons, and neutrinos (see e.g. Refs. [91–111]), all with the same mean density ratios as in the visible sector, we can effectively mimic the $\sqrt{G\rho_i}$ part of

Label	Data Set
1	*Planck* TT, TE, EE, lensing + BAO
2	Data Set 1 plus R21 ($H_0 = 73.2 \pm 1.3$ km/s/Mpc)

TABLE I. Definition of data sets 1 and 2. We use the Plik TT+TE+EE, $\mathrm{Lowl_T}$, $\mathrm{Lowl_E}$, and lensing *Planck* likelihoods described in Ref. [140]. BAO data sets are 6dFGS [141], SDSS MGS [142] and BOSS DR12 [143]. R21 is $\mathrm{SH_0ES}$ measurement [7], of which R22 is a recent update.

the scaling transformation while evading the constraint from FIRAS. The dark photons (which have temperature T_{D}) are a replacement for the additional visible photons that would violate the FIRAS constraint. The dark baryons (implemented as "atomic dark matter" (ADM) [112–135]) allow us to scale up the total baryon-like density without changing the well-constrained (visible sector) baryon-to-photon ratio. The dark neutrinos allow us to scale up the effective number of free-streaming neutrino species from its ΛCDM value of $N_{\mathrm{eff}}^{\mathrm{fs}} = 3.046$ [136], preserving the well-constrained ratio of free-streaming to tightly-coupled relativistic particle densities [137–139].

In our implementation, the new mirror dark sector interacts purely gravitationally with the visible sector and the CDM. Therefore, we can mimic a ΛCDM model with scaled-up densities, if the perturbations in the mirror world evolve in the same way as what they replaced in the visible sector, and thus provide the exact same source to the metric perturbations. For this to be the case the dark photons must transition from tightly coupled to freely streaming when the visible photons do. We thus ensure that the ADM recombines at approximately the same time as regular hydrogen by keeping the ratio $B_{\mathrm{D}}/T_{\mathrm{D}}$ fixed, where B_{D} is the binding energy of the ADM, and by setting the dark fine structure constant and dark proton mass equal to those in the light sector. For simplicity we assume there are no dark versions of helium or heavier nuclei. We can keep the Thomson scattering rate on the scaling trajectory by adjusting Y_{P}. At fixed baryon density we have $n_e(z) \propto x_e(z)(1 - Y_{\mathrm{P}})$ where $x_e(z)$ is the fraction of free electrons. So to scale the scattering rate appropriately, approximating $x_e(z)$ as fixed, we send $(1 - Y_{\mathrm{P}}) \to \lambda(1 - Y_{\mathrm{P}})$.

In Fig. 1 we show temperature (TT) and polarization (EE) power spectra for the best-fit ΛCDM model given data set 1 (defined below) as well as spectra for models scaled from that one with $\lambda = 1.1$, with and without the photon scattering rate scaling included. The scaled spectra with no photon scattering scaling differ significantly from the ΛCDM spectra. The small differences between the fully-scaled spectra and the ΛCDM spectra are **primarily** due to the symmetry-breaking effects of atomic reaction rates affecting hydrogen recombination.

Results.—We now explore parameter constraints in the framework just described. We define data sets 1 and 2 and model spaces A, B, and C, in Tables I and II. The results are shown in Fig. 2. In the upper left panel we see the expected results from A1: the posterior is almost flat throughout the prior region [149]. This is the numerical manifestation of the exact symmetry we have presented.

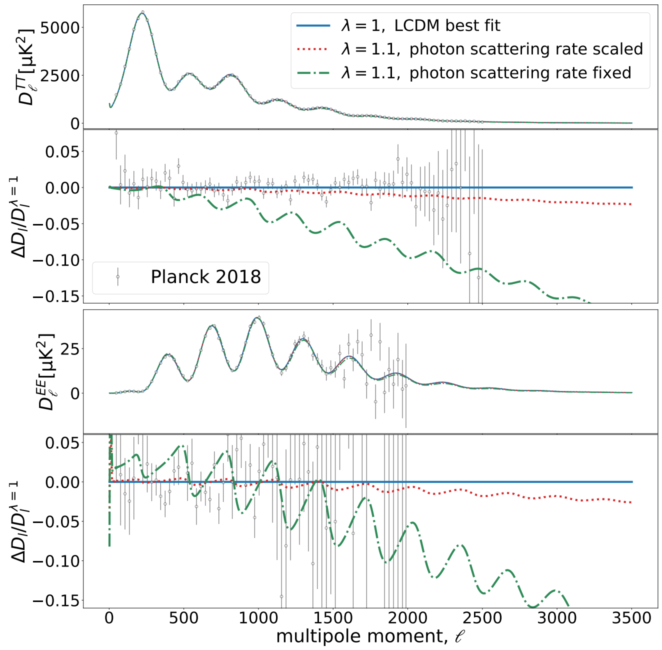

FIG. 1. CMB temperature (TT) and polarization (EE) power spectra with (red and green) and without (blue) scaling by $\lambda = 1.1$ from the best-fit ΛCDM model for data set 1. Both $\lambda = 1.1$ cases are scaled by use of the mirror world dark sector, with (red) and without (green) scaling of the photon scattering rate. Data points are from Planck 2018 [140]. Fractional differences with the best-fit ΛCDM model are shown in the second and bottom panels.

For B1, the H_0 posterior remains quite broad. The soft symmetry-breaking effects of non-equilibrium recombination, evident in Fig. 1, are sufficiently degenerate with variation of other ΛCDM parameters to avoid significant constraint on H_0. The tension with R21 has been completely eliminated. The very slight preference for $H_0 = 73.2$ km/s/Mpc in B1 over the ΛCDM value is a parameter-volume effect; the best-fit ΛCDM model has a nearly identical χ^2 value as the best-fit B model constrained to $\lambda = 1.08$, lower by $\Delta\chi^2 = 0.2$.

In the other three panels of Fig. 2 we show how well departures away from scaling are constrained by the data, if they are not prevented by fiat. The B1 contours lie over the constraints given model C; as expected, the scaling direction is preferred by the data. We also see that the region of high posterior probability density extends to parameter values far from the scaling solution, an indication of some freedom that more detailed model building could exploit. For C2, we find that $f_{\mathrm{ADM}} = 0.027 \pm 0.011$ and $T_{\mathrm{D}} = (0.68 \pm 0.06)T_\gamma$. Meanwhile, the value of

Label	Model Space
A	ΛCDM + λ (scaling enforced), $x_e(z)$ fixed
B	ΛCDM + λ (scaling enforced), $x_e(z)$ calculated
C	$\Lambda\text{CDM} + T_{\text{D}} + N_{\text{eff}}^{\text{fs}} + f_{\text{ADM}} + Y_{\text{P}}$

TABLE II. Definition of model spaces A, B & C. For model space A the one non-ΛCDM parameter is λ. We restrict the additional components, including Y_{P}, to the scaling solution and (artificially) hold $x_e(z)$ fixed to its ΛCDM best-fit value, for both the dark and light sectors. We scale Y_{P} from its BBN-consistent ΛCDM data set 1 best-fit value of 0.2454. Model space B differs only in that we calculate the visible sector $x_e(z)$ using the atomic reaction rates (and the code RecFast [144, 145]) and the dark sector ionization evolution as in Ref. [122]. Model space C only differs from B in that we allow Y_{P}, the effective number of free-streaming neutrinos $N_{\text{eff}}^{\text{fs}}$, and the fraction of mirror world (or "atomic") dark matter f_{ADM} to depart from their scaling values. For all model spaces we adopt the uniform prior $1.00001 < \lambda < 1.3$, set the ratio of dark to light photon temperatures to $T_D/T_\gamma = \left(\lambda^2 - 1\right)^{1/4}$ and, although it introduces a new length scale, we take one of the neutrino species to have a mass of 0.06 eV [146]. We modified CAMB [147] to solve the relevant Einstein-Boltzmann equations and used CosmoMC [148] to calculate parameter posterior densities.

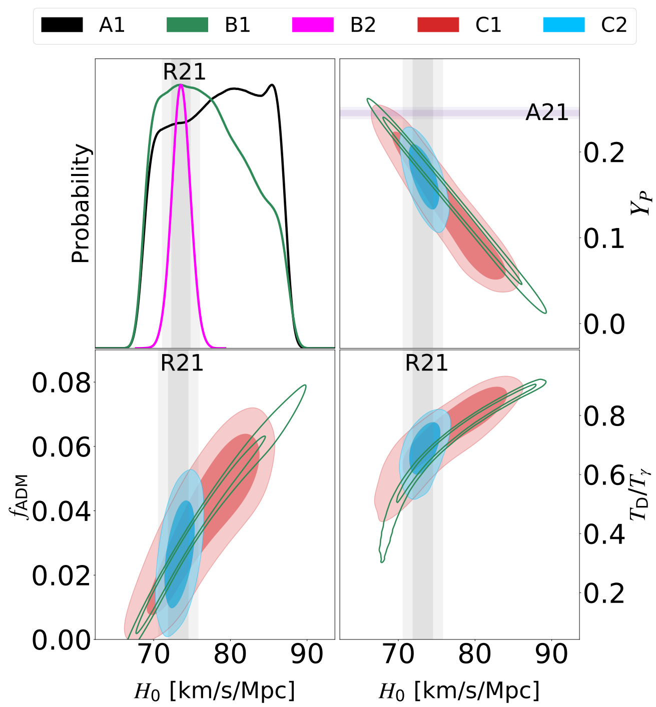

FIG. 2. Constraints on parameters from data sets 1 and 2 given models A, B and C (see Tables I and II). Top left panel: the (unnormalized) posterior probability density of H_0. Other panels: the 68% and 95% contours of equal probability density in the $H_0 - Y_{\text{P}}$, $H_0 - f_{\text{ADM}}$ and $H_0 - T_{\text{D}}/T_\gamma$ planes. The grey band 'R21' shows the 1 and 2 σ constraint on H_0 from R21. The purple band 'A21' shows the same for Y_{P} from [150, hereafter A21].

$\sigma_8 = 0.808 \pm 0.011$ is nearly unchanged (if **not** slightly lower) from its ΛCDM value, as expected.

In Fig. 2 we also see a problem: the Y_{P} values consistent with R21 are inconsistent with inferences from spectral observations of hot "metal-poor" gas such as the A21 finding of $Y_{\text{P}} = 0.2453 \pm 0.0034$. From C2 we have $Y_{\text{P}} = 0.170 \pm 0.025$, a 3.0σ difference. Compounding this trouble, the additional light relics, if we do not otherwise alter the standard thermal history, would increase the BBN-expected Y_{P} [151], in C2, to 0.2614 ± 0.0038.

Discussion.—We have found important constraints on the project of turning the scaling transformation into a solution to the H_0 tension. The first of these, from FIRAS, we have shown can be accommodated with a mirror world dark sector. Constraints from light element abundances lead us to **the** articulation of two additional targets for model building: i) a new mechanism for increasing the photon scattering rate and ii) additional model features that would bring BBN predictions for helium and deuterium abundances in line with observations.

Item (i) follows from the fact that the Y_{P} required for consistency with R21 in models B and C is 3σ too low compared to the inference from observations in A21, hence strongly suggesting that modifying Y_{P} is not a promising way to increase the photon scattering rate. Item (ii) follows since the extra relativistic species in the mirror sector required by R21 alter predictions of Y_{P} and deuterium. We have for C2 $\Delta N_{\text{eff}} = 1.3 \pm 0.34$. On the other hand BBN consistency with Y_{P} and deuterium measurements leads to $\Delta N_{\text{eff}} = -0.19 \pm 0.15$ [84].

One idea worth exploring, for boosting the photon scattering rate, is a heating of the baryons by a spectral distortion in the Wien tail, at frequencies beyond FIRAS's reach [152]. A time-varying electron mass [153, 154] is another possible solution that has met with some phenomenological success [153], and could even be well-motivated by a supersymmetric gravity sector [155, 156]. Successfully fitting the light element abundances could be achieved, for instance, by reheating the dark sector in the post-BBN era (see e.g. Refs. [157]), or by possibly introducing interactions between the light and dark sectors that could affect the predicted yields; our inferred T_D/T_γ even puts decoupling of the dark and light sectors in the right ballpark for this to occur.

Our scenario requires a mirror sector that is phenomenologically close to the SM, albeit with a lower temperature. In its simplest realization, such a sector appears tightly constrained by particle collider data (see e.g. Ref. [158]). Encouragingly however, preliminary work [159] indicates that such a mirror sector could be successfully built. More generally, our work opens the possibility of relaxing cosmological constraints (see e.g. Refs. [65, 69, 75, 160]) on such mirror scenarios, given mechanisms to adjust the photon scattering rate and light-element abundances.

We note that having $\sim 3\%$ of dark matter in ADM can lead to interesting astrophysical phenomenology such as exotic compact objects [161, 162], dark stars [163, 164], exotic gamma-bright point sources [165], and dark disks within galaxies [80, 125–128, 166–169].

Conclusions.—We have **generalized a scaling transformation so that it now** allows for large changes in the cosmological model while preserving the precisely-measured and feature-rich CMB temperature and polarization spectra, as well as many other cosmological observables. Implementing this transformation, while evading constraints from FIRAS, leads us directly to a mirror world dark sector, a type of dark sector that has been proposed before for independent reasons.

While the scaling symmetry does not, by itself, provide an end-to-end solution to the H_0 tension, we have used it to single out the photon scattering rate and light-element abundances as players with important roles in the discrepancy. We have thus provided clear model-building targets for the community to explore.

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