Tuning Stoichiometry to Promote Formation of Binary Colloidal Superlattices

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Tuning Stoichiometry to Promote Formation of Binary Colloidal Superlattices

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The self-assembly of binary nanoparticle superlattices from colloidal mixtures is a promising method for the fabrication of complex colloidal co-crystal structures. However, binary mixtures often form amorphous or metastable phases instead of the thermodynamically stable phase. Here we show that in binary mixtures of differently sized spherical particles, an excess of the smaller component can promote—and, in some cases, may be necessary for—the self-assembly of a binary co-crystal. Using computer simulations, we identify two mechanisms responsible for this phenomenon. First, excess small particles act like plasticizers and enable systems to reach a greater supersaturation before kinetic arrest occurs. Second, they can disfavor competing structures that may interfere with the growth of the target structure. We find the phase behavior of simulated mixtures of hard spheres closely matches published experimental results. We demonstrate the generality of our findings for mixtures of particles of arbitrary shape by presenting a binary mixture of hard shapes that only self-assembles with an excess of the smaller component.

Binary colloidal mixtures are known to self-assemble into a diverse array of binary superlattices, providing a simple way to prepare colloidal co-crystals with novel combinations of properties. [1–9]. The structure of the superlattice dictates important material properties, *e.g.* photonic response [10] and catalytic activity[11]; thus much effort has focused on designing particles that self-assemble particular colloidal crystal structures [12–19]. However, the self-assembly of co-crystal phases appears particularly susceptible to kinetic limitations, as these phases frequently fail to assemble, instead forming glasses[20–23] or metastable phases[24, 25].

In this Letter, we demonstrate using computer simulation that variation of the stoichiometry can enhance the kinetics of co-crystal self-assembly in binary mixtures whose components differ in size. Self-assembly of binary crystals is usually attempted "on-stoichiometry," in which the initial fluid phase has the same stoichiometry as the target crystal[22, 26–29]. We show that going "off-stoichiometry" by adding an excess of the smaller component can dramatically improve self-assembly. We demonstrate that this enhancement can be attributed to two mechanisms, both of which we observe in our simulations. Specifically, we show that an excess of small particles (i) enables the large component to remain mobile at higher supersaturation, facilitating self-assembly of the equilibrium structure and avoiding kinetic arrest; and (ii) can disfavor competing structures that may interfere with the growth of the equilibrium structure.

We first investigate an additive binary inverse power law (IPL) system with a power of 50 and a size ratio (γ) of 0.55. The particles are characterized by purely repulsive interactions, similarly to many experimental colloids[1, 2, 30, 31]. Setting the power n to 50 makes the particles similar in softness (*i.e.*, steepness of repulsion with interparticle distance – less steep is softer) to some experimental microgels[2] but slightly softer than most PMMA beads[32]. We make them slightly soft so as to be able to use standard molecular dynamics (MD) algorithms; from our previous work[33] and the phase diagram computed here, we do not expect their phase behavior to deviate significantly from hard spheres. We used HOOMD-Blue[34–36] to conduct, freud[37] to analyze, and signac[38] to organize the MD simulations. Full simulation methodology is provided in section S1 of the supplementary material[37–46]. We describe stoichiometry throughout this work in two ways: using the number ratio N_L:N_S or the fraction of small particles $x_S = N_S/(N_L + N_S)$. We give distances in terms of the diameter σ of the large particles and energies in terms of the energy scale ϵ of the IPL.

Via free energy calculations[41, 43] (S2 of the supplementary material), we computed the thermodynamic phase diagram of the binary IPL model at $kT/\epsilon = 1$, as shown in Figure 1, plotted in terms of reduced pressure $P^* = P\sigma^3/\epsilon$ and x_S. Because of comparable experimental[1] and simulation[8] studies, we considered the following candidate phases: a face-centered cubic crystal of the large particles (FCC$_L$), a face-centered cubic crystal of the small particles (FCC$_S$), an AlB$_2$ co-crystal, a NaZn$_{13}$ co-crystal, and the fluid phase. Their stoichiometries N_L:N_S are 1:0, 0:1, 1:2, and 1:13, respectively. We assume there is no compositional disorder in the solids at equilibrium, so at any stoichiometry besides their own (*e.g,* $x_S = 2/3$ for AlB$_2$), they must coexist with another phase.

The phase diagram tells us the equilibrium phase(s) for a given set of conditions, but does not tell us whether the phases are kinetically accessible. For self-assembly to occur, the average time for another phase to nucleate and grow must be shorter than the time accessible in experiment (or simulation). Both nucleation and growth rates are strongly influenced by the degree of supersaturation. For a fluid-to-solid transition, increasing the degree of supersaturation has contrasting effects: the free energy

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barrier for nucleation decreases, favoring assembly, but the particle mobility decreases, disfavoring assembly[47]. If the mobility decreases too much before the free energy barrier becomes surmountable, the particles become kinetically arrested, inhibiting the formation of the equilibrium solid phase.

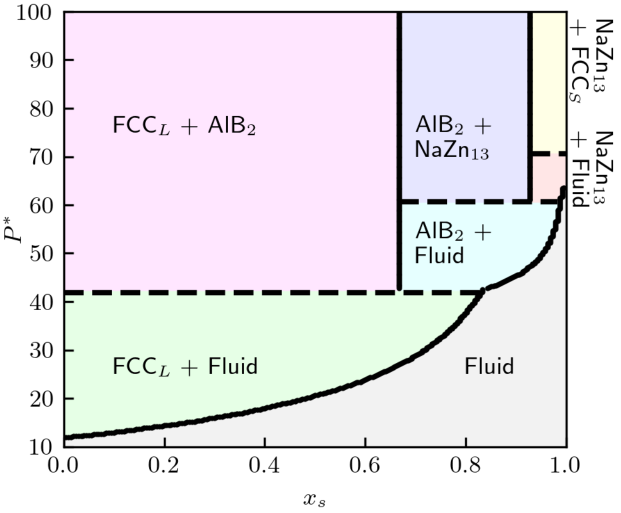

FIG. 1. Thermodynamic phase diagram for the binary inverse power law model (IPL) at $\gamma = 0.55$, $n = 50$, and $kT/\epsilon = 1$. Five phases are present: fluid, FCC_L, FCC_S, AlB_2, and $NaZn_{13}$. Dashed lines indicate three-phase coexistence.

We first investigate whether AlB_2 will homogeneously nucleate from a fluid for a variety of pressures and stoichiometries. The simulations were initialized in a fluid-like state with 27,000 particles and run in the NPT ensemble for $4{\cdot}10^5\tau$ time steps, where $\tau = \sigma(m/\epsilon)^{1/2}$ and m is particle mass. Because we observed some crystal growth at $N_L{:}N_S = 1{:}3$ and $P^* = 70$ and wanted to verify that the crystal continued to grow, we continued that simulation for an additional $4{\cdot}10^5\tau$ timesteps. In Figure 2a we show the evolution in the number of AlB_2-like particles up to 200 particles (according to our order parameter; see section S3 of the supplementary material) to observe the initial growth of the co-crystal nuclei. We note that, according to our phase diagram, pure AlB_2 is expected to form at 1:2, while at 1:3 and 1:5, AlB_2 is expected to coexist with a fluid phase or $NaZn_{13}$ (depending upon the pressure). For the set of simulations shown in Figure 2, we only observe AlB_2 coexisting with the fluid.

Figure 2a shows that N_{AlB_2} never exceeds 16 for the on-stoichiometry systems at the chosen pressures, indicating that self-assembly never occurs. In contrast, we find that N_{AlB_2} increases to 200 and beyond for systems with an excess of small particles. The results are consistent with the system snapshots shown in Figures 2c-e, where crystal grains are only apparent at 1:3 and 1:5. We note the presence of small grain sizes, which mirrors the results obtained by Bommineni et al. with particle swap moves[48] in binary mixtures of hard spheres. At the highest pressure we simulated for $N_L{:}N_S = 1{:}2$ ($P^* = 74$), particle mobility is extremely limited, as shown in Figure 2b where we plot the temporal decay of the first peak ($q*$) in the intermediate scattering function calculated for the large particles ($F_{LL}(t)$). We thus conclude that self-assembly is only possible with an excess of small particles on the time scale of our simulations. We attribute this result in part to particles being more mobile at higher x_s. For example, by fitting the decay of $F_{LL}(t)$ to a stretched exponential (indicated by the lines in Figure 2b), we computed that the structural relaxation time is around 75 times longer at a stoichiometry of 1:2 than at 1:3 (18,200τ versus 242τ) at $P^* = 70$, which indicates much slower equilibration at 1:2. We also show the decay of $F_{LL}(t)$ for a system at 1:1.5 and $P^* = 70$, which is an order of magnitude slower than at 1:2. For this reason we did not attempt self-assembly with an excess of large particles. The trend in mobility with stoichiometry stems from the higher freezing and kinetic arrest pressures of the smaller particles relative to the larger component (as can be seen by comparing the P^* at which FCC_L and FCC_S become stable in Figure 1), which is generally true in size disperse systems of purely repulsive particles.

We next analyze the growth of AlB_2 in the presence of crystalline seeds. By construction, these simulations bypass the need to form a critical nucleus and thus may allow self-assembly on shorter time scales than required for homogeneous nucleation. Each simulation was prepared by compressing a fluid around a perfect (constructed) seed of AlB_2 and then allowing the fluid and seed to evolve in an NPT simulation. The seed crystals were chosen to be small but post-critical, as evidenced by their persistence in the simulations.

Figure 3a shows the evolution of the fraction of large particles classified as AlB_2-like (N_{AlB_2}/N_{Total}). We consistently find more crystal growth off-stoichiometry at $N_L{:}N_S = 1{:}3$, with final values of N_{AlB_2}/N_{Total} ranging from 0.25 to 0.48, than on-stoichiometry, for which N_{AlB_2}/N_{Total} never rises above 0.035.

Inspection of the growing seeds at $N_L{:}N_S = 1{:}2$ revealed the accumulation of non-AlB_2 layers of particles on the seed (an example at $P^* = 55$ is shown in Figure 3b). We identified many of these layers to be two (or more) subsequent close-packed planes of large particles. This possibility seemed likely because FCC_L, which consists of close-packed planes, is metastable under the conditions we investigate, and AlB_2 has a close-packed layer of large particles in its structure onto which additional close-packed layers could grow. We call a layer of these particles a "mixed layer" and the associated coordination environment "mixed FCC-AlB_2;" we denote the number of these particles N_{mix}. In Figure 3b we illustrate their presence in dark green for a seed grown at $P^* = 55$ and $N_L{:}N_S = 1{:}2$.

We quantify the formation of the mixed layer during the seeded simulations in Figure 3c, plotting N_{mix}/N_{AlB_2} versus N_{AlB_2}. For $N_L{:}N_S = 1{:}2$ we plot only the results

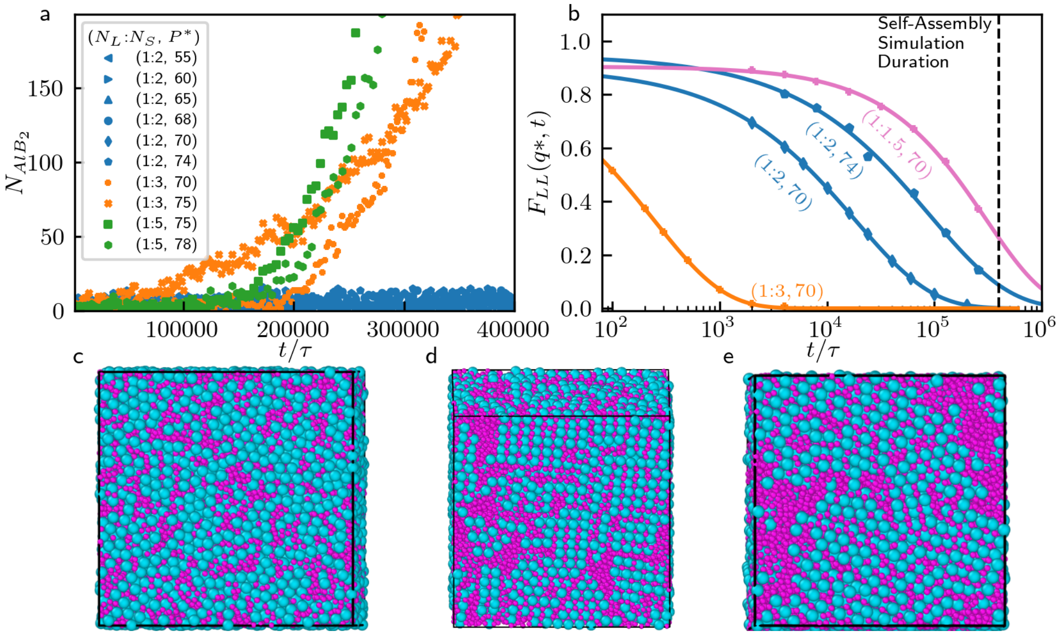

FIG. 2. Self-assembly of AlB_2. The plot in a) shows the evolution of the number of large particles identified as AlB_2 for NPT simulations at the given pressure and stoichiometry. All simulations at $N_L{:}N_S$ = 1:2 (colored blue on the plot) overlap substantially because they never exceed $N_{AlB_2} = 16$. The plot in b) shows the decay of the intermediate scattering function for certain combinations of stoichiometry and pressure. The lines are fits to the data. The dotted black line indicates the duration of simulations in a). Snapshots of the results are shown for stoichiometries and pressures of c) $N_L{:}N_S$ = 1:2, $P^* = 70$, d) $N_L{:}N_S$ = 1:3, $P^* = 70$, and e) $N_L{:}N_S$ = 1:5, $P^* = 75$.

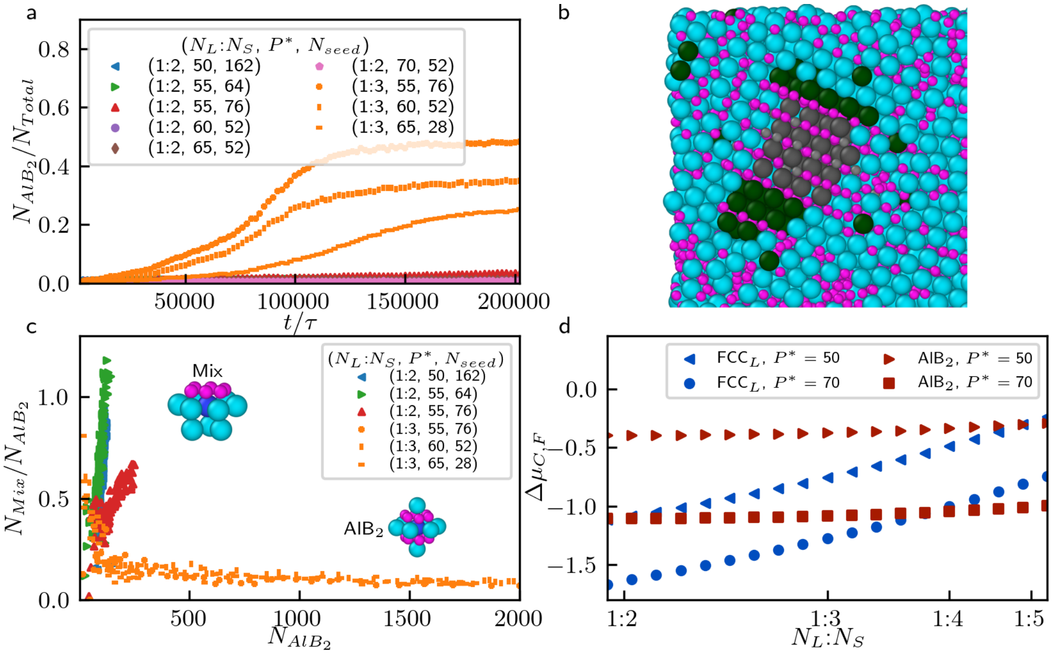

FIG. 3. Crystal growth in seeded simulations. The plot in a) shows the evolution of the number of large particles identified as AlB_2 from seeded simulations for different x_s, $P\sigma^3/\epsilon$, and initial seed size (N_{seed}). The image in b) is a snapshot of the end of the seeded simulation at $N_L{:}N_S$ = 1:2 and $P\sigma^3/\epsilon = 55$. Large and small particles belonging to the initial seed are colored dark grey and light grey, respectively; large particles classified as mixed FCC-AlB_2 are colored dark green. The plot in c) shows the number of particles classified as AlB_2 (N_{AlB_2}) versus the ratio of the number classified as mixed FCC-AlB_2 to N_{AlB_2} (N_{Mix}/N_{AlB_2}). The insets illustrate the mixed FCC-AlB_2 and AlB_2 environments. The plot in d) shows the chemical potential driving force $\Delta\mu_{C,F}$ for the FCC_L and AlB_2 as a function of pressure and stoichiometry, where $\Delta\mu_{C,F}$ is defined by Equation 1. Errors (calculated as described in S2 of the supplementary material) are smaller than the size of the points.

for $P^* \leq 55$ because at higher pressures N_{AlB_2} never exceeds 100 (*i.e.*, those seeds grow negligibly over the simulation). Off-stoichiometry at N_L:N_S = 1:3, the proportion of mixed layers decreases with crystal growth in all cases. In contrast, on-stoichiometry at 1:2 the proportion always increases, indicating that mixed layers form more frequently than AlB_2 layers. The formation of the mixed layers instead of the equilibrium AlB_2 phase is similar to previous reports of "self-poisoning" in nucleation, in which the formation of a metastable phase interferes with the growth of the equilibrium phase[24, 49].

We identify a thermodynamic reason as to why the mixed layers are more prevalent at N_L:N_S = 1:2. Because the mixed layer is essentially the formation of an FCC layer where an AlB_2 layer should have formed, its appearance likely correlates with the thermodynamic stability of the competing FCC_L phase. In Figure 3d we examine the chemical potential driving force for crystallization:

$$\Delta\mu_{C,F} = \hat{G}_C - (1 - x_C)\mu_F^L - x_C \cdot \mu_F^S. \qquad (1)$$

The quantity $\hat{G}_C$ is the specific Gibb's free energy of the crystal; x_C is the fraction of small particles in the crystal; and μ_F^L and μ_F^S are the chemical potentials of the large and small species in the fluid, respectively. More negative $\Delta\mu_{C,F}$ values indicate stronger thermodynamic driving forces for crystallization.

Figure 3d shows that the $\Delta\mu_{C,F}$ of both crystals decreases with pressure but increases with a greater proportion of small particles. However, we find that $\Delta\mu_{FCC_L,F}$ is more sensitive to stoichiometry than $\Delta\mu_{AlB_2,F}$. For example, at $P^* = 70$, changing the stoichiometry from 1:2 to 1:3 increases the $\Delta\mu_{C,F}$ of FCC_L by 0.37 kT while only increasing the $\Delta\mu_{C,F}$ of AlB_2 by 0.02 kT, resulting in a greater preference of the fluid to form AlB_2 relative to FCC_L.

To summarize these results, we find that AlB_2 does not self-assemble or even grow from a seed crystal in an on-stoichiometry fluid. We identified two reasons its formation is inhibited: slow dynamics and interference from a competing phase. Both issues are alleviated by adding an excess of small particles. Adding excess large particles reduces particle mobility and makes FCC_L even more favored relative to AlB_2 and thus will not alleviate the issues.

Our simulations should be most comparable with the experiments of Bartlett et al.[1, 50] using PMMA particles because our results are for a similar size ratio (0.55 vs. 0.58) and they explore how stoichiometry affects assembly. In Table I, we compare the binary crystals we obtain with theirs. Our results at N_L:N_S of 1:2, 1:3, 1:5, are shown in Figure 2; results for the other stoichiometries are shown in section S4 of the supplementary material. We denote any experiment not reported with "-".

TABLE I. Crystals Observed in Simulation and Experiment

N_L:N_S	x_S	Sim. Structures	Exp. Structures*
1:2	2/3	Amorphous	Amorphous
1:3	3/4	AlB_2	-
1:4	4/5	AlB_2	AlB_2
1:5	5/6	AlB_2	-
1:6	6/7	AlB_2	AlB_2
1:9	9/10	AlB_2/$NaZn_{13}$	$NaZn_{13}$
1:13	13/14	$NaZn_{13}$	-
1:14	14/15	$NaZn_{13}$	$NaZn_{13}$
1:20	20/21	$NaZn_{13}$	$NaZn_{13}$
1:30	30/31	$NaZn_{13}$	$NaZn_{13}$

*Bartlett et al.[1]

Overall, we see strong agreement between simulation and the published experimental results. We both obtain an amorphous structure at 1:2, but see AlB_2 with a slight excess of small particles. Around a stoichiometry of 1:9, we both begin to see $NaZn_{13}$ self-assemble, and continue to see it self-assemble at stoichiometries up to 1:30.

To establish whether the self-assembly of other binary crystals may be assisted by an excess of small particles, we also simulated a binary mixture of hard cuboctahedra and octahedra at a volume ratio of 5:1. Despite this mixture being capable of comprising a space-filling CsCl-type structure, previous work found that additional attractive interactions were required for self-assembly[26, 51]. In Figure 4, we present our results for self-assembly conducted at stoichiometries of 1:1 and 1:2 through slow compression in the NVT ensemble. We used 4096 particles due to the higher cost of simulating anisotropic particles.[46]

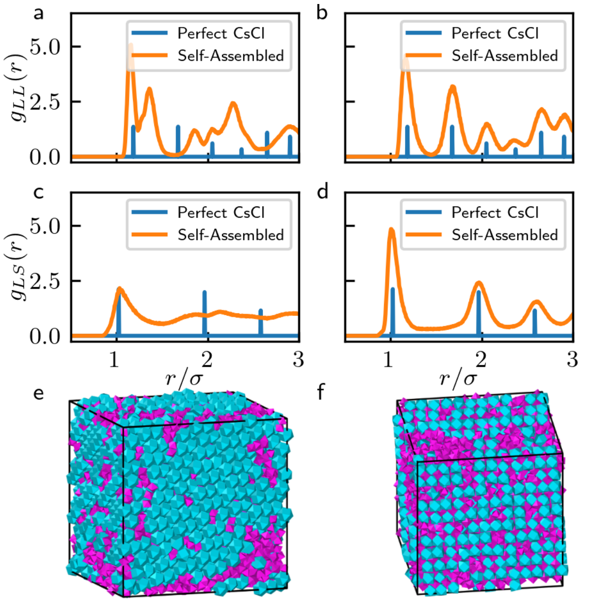

FIG. 4. Self-assembly of hard cuboctahedra and octahedra. The plots show the radial distribution functions (RDFs) averaged over the final few frames of self-assembly at stoichiometries of a,c) 1:1 and b,d) 1:2. Also shown are the RDFs for a perfect CsCl structure. We show the RDF for large particles (g_{LL}) and for large and small particles (g_{LS}); the RDF for small particles is dominated by fluid-like small particles. Snapshots of the results are shown for e) 1:1 and f) 1:2.

By comparison with the RDFs of perfect CsCl, we identified the result at 1:2 to be CsCl. At 1:1, a single-component structure composed of the large particles self-assembles, while the small particles remain fluid-like. It is thus apparent that, although particle mobility is not limited, the single-component structure (successfully) competes with CsCl when the fluid is on-stoichiometry, and an excess of small particles is necessary to observe the thermodynamically preferred binary structure[51]. We note a similar trend was recently observed in hostguest structure-forming hard particles[52].

In summary, we demonstrated that the self-assembly of binary nanoparticle superlattices can be promoted by adding an excess of the smaller component to the colloidal fluid mixture. The principles elucidated here are quite general and likely apply to other binary mixtures of size disperse particles, although it is difficult to predict *a priori* which systems require an excess of small particles to co-crystallize. We can say that the enhanced particle mobility should benefit systems which are prone to kinetic arrest, which includes many binary mixtures of purely repulsive particles[22, 23, 26, 28, 53]. The enhancement to particle mobility should also increase for smaller size ratios and be limited if the components are similar in size, although depletion will occur for extreme size disparity [54]. Here we observed enhanced self-assembly kinetics in binary mixtures with volume ratios of $0.55^3 \approx 0.166$ and 0.2, and therefore we expect similar enhancement in mixtures with similar volume ratios.

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