Cosmic Coincidences of Primordial-Black-Hole Dark Matter

Yi-Peng Wu, Elena Pinetti, and Joseph Silk

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Yi-Peng Wu[a], Elena Pinetti[ab], and Joseph Silk[cde]
[a]*Laboratoire de Physique Théorique et Hautes Energies (LPTHE), UMR 7589 CNRS & Sorbonne Université, 4 Place Jussieu, F-75252, Paris, France*
[b]*Dipartimento di Fisica, Universitá di Torino & INFN, Sezione di Torino, via P. Giuria 1, I-10125 Torino, Italy*
[c]*Institut dAstrophysique de Paris, UMR 7095 CNRS & Sorbonne Université, 98 bis boulevard Arago, F-75014 Paris, France*
[d]*Department of Physics and Astronomy, The Johns Hopkins University, 3400 N. Charles Street, Baltimore, MD 21218, U.S.A. and*
[e]*Beecroft Institute for Particle Astrophysics and Cosmology, University of Oxford, Keble Road, Oxford OX1 3RH, U.K.*
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If primordial black holes (PBHs) contribute more than 10 percent of the dark matter (DM) density, their energy density today is of the same order as that of the baryons. Such a cosmic coincidence might hint at a mutual origin for the formation scenario of PBHs and the baryon asymmetry of the Universe. Baryogenesis can be triggered by a sharp transition of the rolling rate of inflaton from slow-roll to (nearly) ultra-slow-roll phases that produces large curvature perturbations for PBH formation in single-field inflationary models. We show that the baryogenesis requirement drives the PBH contribution to DM, along with the inferred PBH mass range, the resulting stochastic gravitational wave background frequency window, and the associated cosmic microwave background tensor-to-scalar ratio amplitude, into potentially observable regimes.

Primordial black holes (PBHs) are one of the most interesting dark matter (DM) candidates that have been severely constrained by joint astrophysical and cosmological observations [1–14]. The asteroid-mass window $M_{\rm PBH}/M_\odot \sim 10^{-16} - 10^{-12}$ for PBHs to be all DM [15–18] will be tested in the near future by femto-lensing of gamma ray bursts [19–21], microlensing of X-ray pulsars [22], primary photons measured by next-generation MeV detectors [7, 23], neutron star disruption [24], white dwarf explosions [25], MeV photons from the Galactic Center [26] and radio emission measurable with the next-generation radio telescopes [27]. Even if PBHs only occupy a small fraction of the DM density today, their existence could be (or could have been [28]) probed by gravitational wave observations through binary mergers, either mutual or with neutron stars (for recent reviews, see [29, 30]).

If PBHs really constitute more than 10 percent of the DM density today, the PBH density up to matter-radiation equality is of the same order as that of the baryons, namely $\Omega_{\rm PBHeq}/\Omega_{\rm Beq} \sim \mathcal{O}(1)$. Such a cosmic coincidence would hint at a mutual origin for PBHs and the baryon asymmetry of the Universe. In this work, we argue that the PBH-baryon density coincidence could be a natural consequence due to baryogenesis triggered by inflation models for PBH formation. We show that the abundance of PBHs and baryons are indirectly correlated to each other via the dynamics of inflation, and, thus, the scenario is very different from previous suggestions that existing PBHs create baryon asymmetry [31–40] or account for cosmic coincidence [41–43]. (See also [44] for cosmic coincidence from asymmetric DM [45–47] collapses into PBHs.)

PBHs from Inflation. Let us focus on single-field inflation for PBH formation [48–65, 74–77]. The generic assumption is that the inflaton ϕ receives a sudden deceleration on comoving scales $k_0 \sim 10^{12} - 10^{15}$ Mpc^{-1} which leads to a sharp decrease of the first slow-roll parameter $\epsilon_H \equiv -\dot{H}/H^2$ and thus largely enhances the power spectrum of the curvature perturbation P_ζ [66]. Such an enhancement is due to the temporal dominance of the entropy mode in the curvature perturbation [64, 67, 68, 79].

The key parameter for realizing the enhancement of P_ζ is the rate-of-rolling:

$$\delta = \frac{\ddot{\phi}}{H\dot{\phi}} < -3/2, \quad t_0 < t < t_*, \tag{1}$$

where $\delta \to 0$ is the standard slow-roll inflation and $\delta \to -3$ is the so-called ultra-slow-roll (USR) limit [69–73]. With $\delta < -3$, P_ζ can exhibit a spiky peak at a desired scale for producing a nearly monochromatic distribution of PBH mass.

We consider that δ takes constant values in different phases of inflation. In terms of the e-folding numbers, $N \equiv \ln a$, where $k_0/k_* \approx a(t_0)/a(t_*) = e^{N_* - N_0}$ with $N_0 \equiv 0$, the duration N_* (for δ having a negative value) is the fundamental parameter that controls the spectral amplitude of P_ζ (and thus the PBH abundance).

The analytic structure of the USR power spectrum at the end of inflation ($N = N_{\rm end}$) has been intensively investigated [74–79]. Dilatation symmetry of the de Sitter background requires the momentum scaling at each phase with different values of δ to satisfy [77, 79]:

$$P_\zeta = \begin{cases} A_{\rm CMB} & k < k_{\rm min}, \\ A_{\rm PBH}(k/k_0)^4, & k_{\rm min} < k < k_0, \\ A_{\rm PBH}(k/k_0)^{6+2\delta}, & k_0 < k < k_{\rm end}, \end{cases} \tag{2}$$

where $A_{\rm CMB} \approx 2.2 \times 10^{-9}$ measured on CMB scales has negligible contribution to PBH formation. k_0 is the pivot scale for the enhancement and shall be fixed by the desired peak scale $M_{\rm PBH}$ in the PBH mass function. $k_{\rm min} \approx k_0 (A_{\rm CMB}/A_{\rm PBH})^{1/4}$ is the beginning scale of the k^4 growth driven by the Leach-Sasaki-Wands-Liddle mechanism [66] (sometimes also called the steepest growth [74, 80]). The amplitude $A_{\rm PBH}$ is determined by USR parameters as

$$A_{\rm PBH} \approx A_{\rm CMB} \left(\frac{k_0}{k_*}\right)^{6+4\delta} = A_{\rm CMB} e^{-N_*(6+4\delta)}. \quad (3)$$

Note that in the template (2) one should use the value of $\delta < -3$ found in the deceleration phase $N_0 < N < N_*$, since (1) must become positive for $N > N_*$ to increase ϵ_H and terminate inflation. The positive rolling rate in the final acceleration phase ($N > N_*$) is constrained by δ in the deceleration phase (with respect to the conformal symmetry due to non-violation of the adiabatic condition [79]) so that the scaling of $P_\zeta(k)$ for $k_* < k < k_{\rm end}$ is the same as $k_0 < k < k_*$.

In the case of exact USR ($\delta \to -3$), the inflaton potential $V(\phi)$ is completely flat so that ϕ is exactly massless, where quantum diffusion led by short wavelength modes well inside the horizon may have an important impact on the classical trajectory of ϕ [54, 57, 111]. A non-Gaussian tail in the high-sigma limit of the probability distribution of ζ can significantly raise the resulting PBH abundance from USR inflation [57, 109–112], indicating the real amplitude $A_{\rm PBH}$ estimated by the Gaussian spectrum (2) (based on the linear relation $\zeta = -H/\dot{\phi}\delta\phi$) should be smaller than expected. To suppress the effect of quantum diffusion, we adopt an upper bound $\delta < -3.1$, which corresponds to an effective mass $m_\phi \equiv (V_{\phi\phi})^{1/2} > H_*/2$ for the inflaton fluctuation $\delta\phi$ [92].

Baryogenesis via inflation. We now show that baryogenesis can be triggered by USR inflation. Scalar fields naturally develop large vacuum expectation values (VEVs) during inflation due to the high energy background expansion at the scale of H_* (possibly as high as 10^{13-14} GeV [81, 82]). These large VEVs provide suitable initial conditions for baryogenesis driven by the Affleck-Dine (AD) mechanism [83–87]: (1) The stochastic nature of the inflationary fluctuations always allows CP-violating VEVs arising from theories with CP invariant Lagrangian [86, 88], and (2) the post-inflationary relaxation of a B or $B-L$ violating scalar condensate is an out-of-equilibrium process.

A possible realization for the USR inflation to affect the dynamics of a charged scalar σ is given by

$$\begin{aligned}\mathcal{L} =& \mathcal{L}_\phi + |\partial\sigma|^2 + m_\sigma^2 |\sigma|^2 + \frac{c_1}{\Lambda} \left|\sigma^2\right| \Box\phi \\ &+ \frac{c_2}{\Lambda} \partial_\mu \phi \left[\sigma \partial^\mu \sigma + \sigma^* \partial^\mu \sigma^*\right] + \mathcal{O}(\Lambda^{-2}) \cdots ,\end{aligned} \quad (4)$$

where c_1 and c_2 are real constants of $\mathcal{O}(1)$. CP invariance is imposed on the Lagrangian for demonstrative purposes, yet it is not a necessary condition for AD baryogenesis. Similar couplings for enhanced charged scalar production from rolling inflaton as a chemical potential can be found in [89–91]. We ask $H_* \ll \Lambda \leq M_P$ for the cutoff Λ. Note that the c_2 term violates the conserved current $j^\mu = i(\sigma^* \partial_\mu \sigma - \sigma \partial_\mu \sigma^*)$, which is identified as a baryon number for convenience.

In terms of the mass eigenstates $\sigma_\pm$, where $\sigma \equiv (\sigma_- + i\sigma_+)/\sqrt{2}$, the charged scalar is decomposed into a pair of decoupled canonical real scalars, $\mathcal{L}_{\sigma_\pm} = \frac{1}{2}(\partial\sigma_\pm)^2 + \frac{1}{2} m_\pm^2 \sigma_\pm^2$, with asymmetric (non-degenerate) masses as

$$m_\pm^2 = m_\sigma^2 + \frac{c_1 \pm c_2}{\Lambda} \Box\phi. \quad (5)$$

One can see that the phase transition of δ for PBH formation also changes the effective masses as $\Box\phi = -\ddot{\phi} - 3H\dot{\phi} \approx -(\delta+3)\sqrt{2\epsilon_H} M_P H_*^2$. For $m_\sigma \sim H_*$ and $\Lambda > 0.1 M_P$, the sharp decrease of ϵ_H for the P_ζ enhancement usually leads to $m_\pm \approx m_\sigma$ in the ϕ-deceleration (USR) phase.

The sudden transition of $m_\pm$ from the primary slow-roll phase (with $\delta \sim 0$) to the deceleration phase with $\delta < -3$ drives the original VEVs of $\sigma_\pm$ out-of-equilibrium in their potential, triggering the coherent motion of these scalar condensates. The analytic solutions for the coherent motion of $\sigma_\pm$ are given in [92]. At the end of inflation, the VEVs of the mass eigenstates are led by

$$\sigma_\pm \sim e^{-\Delta_\pm^- N_{\rm end}}, \quad \dot{\sigma}_\pm \sim -\Delta_\pm^- e^{-\Delta_\pm^- N_{\rm end}}, \quad (6)$$

where $\Delta_\pm^- \equiv 3/2 - \sqrt{9/4 - m_\pm^2/H_*^2}$ is nothing but the negative branch of the conformal weight for a massive scalar in de Sitter [93]. The late-time approximation used in (6) applies when $m_\pm/H_* < 3/2$.

Assuming the standard reheating process driven by the coherent oscillation of ϕ, one can solve numerically the relaxation of $\sigma_\pm$ from the end of inflation to reheating completion (or radiation domination) [88, 92]. Here we consider the decay of ϕ into radiation via a perturbative channel with a decay width Γ_ϕ. The approximated time scale at the beginning of radiation domination is thus $t_r \sim 1/\Gamma_\phi$. The final baryon asymmetry in radiation domination reads

$$Y_B = \frac{n_B(t_r)}{s(t_r)} = \frac{\sigma_+(t_r)\dot{\sigma}_-(t_r) - \sigma_-(t_r)\dot{\sigma}_+(t_r)}{s(t_r)}, \quad (7)$$

where $s(t) \approx 2\pi^2 g_* T^3(t)/45$ is the entropy production and $T = (\frac{90}{\pi^2 g_*} M_p^2 H^2)^{1/4}$ is the temperature. Note that $Y_B = Y_B(\delta, N_*, N_{\rm end})$ as those parameters of USR inflation enter through the initial conditions (6).

We highlight the generic property of the final baryon asymmetry with examples of the choices $\{c_1, c_2, c_3\} = \{2, -1, 1\}$ given in Fig. 1. In general, Y_B is sensitive to $N_{\rm end}$ since initial conditions (6) are exponentially diluted by the e-fold numbers, but it approaches a constant value when $N_* \sim \mathcal{O}(1)$ depending on the value of δ. For $\delta = -3.15$, examples in Fig. 1 indicate that $Y_B \simeq$ const. when $N_* \gtrsim 2$, which corresponds to $A_{\rm PBH} \gtrsim 10^{-3}$. This

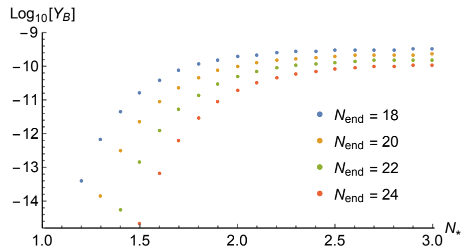

FIG. 1. The final baryon asymmetry in radiation domination with $\delta = -3.15$, $m_\sigma = H_*/2$, $\Lambda = 0.3 M_P$, where $H_* = 2.37\times10^{13}$ GeV and $\Gamma_\phi = 10^{13}$ GeV are used.

asymptotic constant behavior of Y_B is the most important property for resolving the coincidence problem for PBH DM.

PBH dark matter. As the inflaton decays during reheating, the enhanced curvature perturbation on scales $k > k_0$ is inherited by the density perturbation of the radiation. Soon after reentry into the horizon in radiation domination, PBHs are formed at high-sigma peaks of the density contrast Δ smoothed over a given comoving scale $R = 1/(aH) = 1/k$. The comoving scale R can be expressed in terms of the horizon mass parameter $M_H = \frac{4\pi}{3}H^{-3}\rho_R$, with ρ_R the energy density of the radiation dominated Universe, as

$$R(M_H) = \frac{1}{k_{\rm eq}}\left(\frac{M_H}{M_{\rm eq}}\right)^{1/2}\left(\frac{g_*}{g_{\rm eq}}\right)^{1/6}, \tag{8}$$

where $M_{\rm eq} = 2.9\times10^{17}M_\odot$ and $g_{\rm eq}\approx 3$ are the horizon mass and the number of relativistic degrees of freedom at matter-radiation equality. We use $k_{\rm eq} = 0.01{\rm Mpc}^{-1}$ and $g_* = 106.75$ for $M_H < 1.5\times10^{-7}M_\odot$ where the temperature of the Universe is higher than 300 GeV.

The mass fraction $\beta(M_{\rm PBH}, M_H)$ of a flat Universe that collapses into PBHs with mass $M_{\rm PBH}$ at a given horizon mass M_H can be obtained from the density parameter $\Omega_{\rm PBH}(R)$ of PBHs at the corresponding scale $R(M_H)$ as $\beta(M_{\rm PBH}, M_H) = d\Omega_{\rm PBH}/d\ln M_{\rm PBH}$, where $\Omega_{\rm PBH}$ is usually estimated via threshold statistics:

$$\Omega_{\rm PBH}(R) = \int\cdots\int_{\Delta_c}^{\infty}\frac{M_{\rm PBH}}{M_H} \times f_c(y_i)P(\Delta, y_i, \sigma_i)d\Delta dy_1\cdots dy_i. \tag{9}$$

Here $P(\Delta, y_i, \sigma_i)$ is the joint probability distribution of Δ and y_i are components of its first and second order spatial derivatives. $f_c(y_i)$ describes spatial constraints to ensure the selected peaks are local maxima in space [94–98]. All y_i are Gaussian random fields if Δ is Gaussian. σ_i stands for the i-th spectral moment of the Gaussian field Δ smoothed by the window function $W(kR)$. The form of σ_i is defined as

$$\sigma_i^2(R) = \int_0^\infty k^{2i}W^2(kR)P_\Delta(k)d\ln k, \tag{10}$$

where P_Δ is the dimensionless power spectrum. Δ_c is the threshold value above which the density contrast will collapse to form a PBH. The mass fraction $\beta = \beta(M_H, \Delta_c, \sigma_i)$ is therefore a general function of the smoothed scale R (or namely M_H), the threshold Δ_c, and the spectral moment σ_i.

Even for inflation close to the USR limit ($\delta\to-3$), we find that $M_{\rm PBH}\approx M_H$ can be a good approximation for resolving the PBH mass function $f(M_{\rm PBH})$ [92]. Such a monochromatic relation leads to a simple expression of the PBH density at matter-radiation equality

$$\Omega_{\rm PBHeq} = \int\beta(M_H)\left(\frac{M_{\rm eq}}{M_H}\right)^{1/2}d\ln M_H, \tag{11}$$

where $(M_{\rm eq}/M_H)^{1/2}\sim a_{\rm eq}/a$ accounts for the relative growth of PBH density during radiation domination. The PBH mass function defined from the PBH-to-DM ratio,

$$f_{\rm PBH}\equiv\frac{\Omega_{\rm PBHeq}}{\Omega_{\rm DMeq}} = \frac{1}{\Omega_{\rm DMeq}}\int f(M_H)d\ln M_H, \tag{12}$$

implies $f(M_H) = \beta(M_H)(M_{\rm eq}/M_H)^{1/2}/\Omega_{\rm DMeq}$.

The cosmic coincidence. In the standard ΛCDM Universe [100], the cold dark matter density today $\Omega_{\rm CDM0} = 0.265$ and the redshift $z_{\rm eq} = 3402$ gives $\Omega_{\rm CDMeq} = 0.42$ and $\Omega_{\rm Beq} = m_B n_{\rm Beq} = 0.08$. This shows that $\Omega_{\rm PBHeq}/\Omega_{\rm Beq}\simeq 0.5-5$ for $f_{\rm PBH} = 0.1-1$. Here $m_B = 0.938$ GeV is the averaged nucleon mass and $n_{\rm Beq} = |Y_B|s(t_{\rm eq})$ is the baryon number density at matter-radiation equality. Using $H_{\rm eq} = H_0\sqrt{\Omega_{\Lambda0}+2\Omega_{m0}(a_0/a_{\rm eq})^3}$ with $\Omega_{\Lambda0} = 1-\Omega_{m0} = 0.6847$ and $H_0 = 67.36$ km s^{-1} Mpc^{-1}, we find an expectation value $|Y_B| = 6.25\times10^{-11}$ at $t_{\rm eq}$.

An example of a parameter scan for the $|Y_B|$ given by (7) at the beginning of radiation domination is given in Fig. 2 with $\delta = -3.15$, $m_\sigma/H_* = 0.5$ and $\Lambda/M_P = 0.3$. Y_B is assumed to be a conserved quantity until matter-radiation equality. $Y_B \gtrsim 10^{-10}$ can be reached with $N_* > 1.5$ (for $N_{\rm end} < 18$), which translates to $A_{\rm PBH} > 4.4\times10^{-5}$. Changing $A_{\rm PBH}$ by one order of magnitude roughly corresponds to a 0.35 variation in N_*.

The PBH abundance is exponentially sensitive to the peak value $\nu\equiv\Delta/\sigma_0$ in all statistical methods, which means that a tiny change in $A_{\rm PBH}$ will result in a large difference to $\Omega_{\rm PBHeq}$ or $f_{\rm PBH}$. As a result, the condition $f_{\rm PBH} > 0.1$ for PBH to be an important DM contributor indeed specifies a very precise parameter space for the USR inflation.

To explore the fiducial parameter space for PBH DM, we adopt the standard Press-Schechter method [101] based on the linear density relation $P_\Delta = 16/81(kR)^4P_\zeta$ to obtain the mass fraction $\beta_{\rm PS}(M_H) = {\rm erfc}(\nu_c/\sqrt{2})$ in terms of inflation parameters $\{\delta, N_*, N_{\rm end}\}$ with a detailed analytic expression given in [92]. The mass function $f(M_H)$ based on $\beta_{\rm PS}(M_H)$ with various choices of δ is displayed in Fig. 3 (blue shadowed regions).

We compare the PBH abundance with the existing observational constraints in the literature. The Galac-

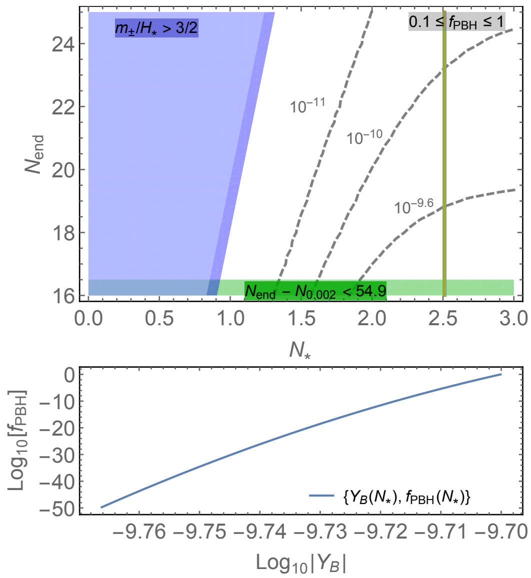

FIG. 2. (Upper panel.) Contours of the final baryon asymmetry $|Y_B|$ in radiation domination with $\delta = -3.15$, $m_\sigma = H_*/2$, $\Lambda = 0.3M_P$, where $H_* = 2.37 \times 10^{13}$ GeV and $\Gamma_\phi = 10^{13}$ GeV are used. The region $2.506 < N_* < 2.515$ corresponds to the PBH-to-DM ratio $0.1 < f_{\rm PBH} < 1$ for $\delta = -3.15$ at the pivot scale $k_0 = 9.46 \times 10^{13}$ Mpc^{-1}. (Lower panel.) $f_{\rm PBH}$ and $|Y_B|$ as functions of N_* at $N_{\rm end} = 20$ and $\delta = -3.15$ from $f_{\rm PBH} = 1$ at $N_* = 2.515$ to $f_{\rm PBH} = 10^{-50}$ at $N_* = 2.3$.

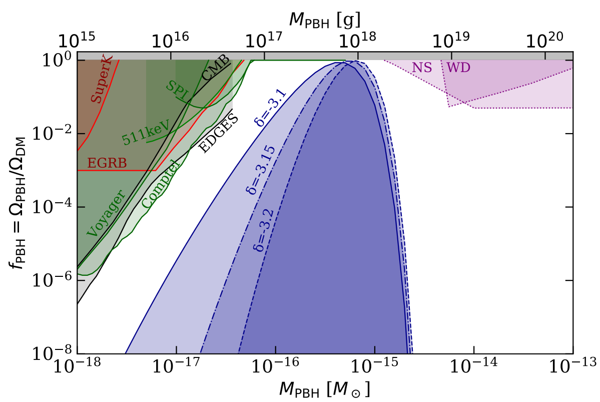

FIG. 3. The PBH mass function based on the fiducial Press-Schechter statistics at $N_{\rm end} = 20$ with various choices of δ, where N_* is fixed by the condition $f_{\rm PBH} = 1$. The pivot scale $k_0 = 9.46 \times 10^{13}$ Mpc^{-1} is used. Existing observational constraints are also shown, using the publicly available code PlotBounds.

tic constraints are displayed in green. They include the bounds from: the local flux of $e^\pm$ measured by VOYAGER1 [4], the MeV diffuse flux observed by the INTEGRAL/SPI detector [10], the 511 keV line in our Galaxy [8, 9], the primary photons detected by the COMPTEL experiment [23]. The red curves refer to the extragalactic contraints, comprising the diffuse neutrino background measured by SUPER-KAMIOKANDE [11] and the extragalactic background radiation [5]. The black lines denote the constraints from the energy injection on the cosmic microwave background at recombination [1, 2] as well as the evaporating constraints from the 21cm signal observed by EDGES [104–106]. The figure does not include the bounds from the heating of the interstellar medium in dwarf galaxies [3], recently found in the analysis of [103]. The dynamical constraints based on the destruction of white dwarfs and neutrons stars by PBHs are displayed in purple. They are shown with dash-dotted lines, since they are controversial [102, 107]. The interested reader can find a comprehensive discussion on these constraints and future prospects in [12, 30, 108].

The parameter space for $0.1 < f_{\rm PBH} < 1$ based on $\beta_{\rm PS}$ with $\delta = -3.15$ is given in Fig.1. $f_{\rm PBH}$ is invariant with respect to $N_{\rm end}$. The difference in N_* for $f_{\rm PBH} = 1$ and $f_{\rm PBH} = 0.1$ is $\mathcal{O}(10^{-2})$. The mass function computed by the peak statistics [94] with additional spatial constraints [96, 98] shows a 10^{-2} difference in N_*. The uncertainty of N_* due to non-linear effect between Δ and ζ could be as large as $\mathcal{O}(10^{-1})$ [92]. $N_{\rm end} < 16.5$ is excluded by the minimal e-fold number for the pivot scale k_0 to pass the spatial curvature constraint and the finite deviation of scale-invariant P_ζ in single-field inflation [81].

Non-perturbative contributions to the curvature perturbation ζ due to quantum diffusion of the inflaton dynamics can play an important role in the resulting PBH abundance [57, 109–112]. In general, the non-Gaussian tail of ζ in the limit of USR inflation ($\delta \to -3$) can raise the PBH abundance from the standard Gaussian prediction $\beta_{\rm PS}$ by some ten orders of magnitude so that the real N_* for $0.1 < f_{\rm PBH} < 1$ might be shifted towards a smaller value. However, for the given example in Fig. 2 with $\delta = -3.15$ and $k_0 = 9.46 \times 10^{13}$ Mpc^{-1}, we find $\beta_{\rm PS}(N_* = 2.5)/\beta_{\rm PS}(N_* = 2.2) \gg 10^{100}$ for the viable range of $N_{\rm end}$. This implies that the effect of quantum diffusion seems unlikely to shift the parameter space for $0.1 < f_{\rm PBH} < 1$ to $N_* < 2$, leaving the $\mathcal{O}(1)$ ratio $\Omega_{\rm PBHeq}/\Omega_{\rm Beq}$ nearly unchanged in the scenario.

Summary and discussion. PBHs fostered by the USR transition during inflation can contribute as a significant DM component. We have shown that such an USR transition of the inflationary background can trigger successful baryogenesis via the Affleck-Dine (AD) mechanism. The resulting baryon asymmetry is asymptotically constant towards the long USR duration limit ($N_* \gg 1$), allowing the cosmic coincidence ratio $\Omega_{\rm CDM}/\Omega_{\rm B} \sim \mathcal{O}(1)$ to be realized over large $\sim 10^{100}$ uncertainties in the PBH abundance.

The present scenario involves a plethora of observational tests, especially via many of the future gravitational wave (GW) experiments. Firstly, the stochastic GW background sourced by enhanced curvature perturbations (at second order) for PBH DM could be measured by space-based laser interferometers [113–118], where the maximal GW density associated with the onset of the USR transition at the k_0 of interest corresponds to the frequency band $f \sim 10^{-3} - 1$ Hz [64]. Secondly, the high scale inflation preferred by USR baryogenesis implies an observable tensor-to-scalar ratio for the next generation CMB measurements close to the current upper bound [81, 82]. Thirdly, one of the consequences of AD baryogenesi is that non-topological solitons (Q-balls) could have formed due to the fragmentation of scalar condensates during relaxation [119], leading to an enhanced stochastic GW background from second-order curvature perturbations via temporal Q-ball domination [120]. Last but perhaps the most important of all, PBH as a significant component of DM in the asteroid-mass window could be verified/eliminated by any of the astrophysical projects mentioned at the beginning of this work.

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