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Bose-Einstein condensation of Efimovian triples in the unitary Bose gas

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In an atomic Bose-Einstein condensate quenched to the unitary regime, we predict the sequential formation of a significant fraction of condensed pairs and triples. At short distances, we demonstrate the two-body and Efimovian character of the condensed pairs and triples, respectively. As the system evolves, their size becomes comparable to the interparticle distance, such that many-body effects become significant. The structure of the condensed triples depends on the relative size of Efimov states to density scales. Unexpectedly, we find universal condensed triples in the limit where these scales are well-separated. Our findings provide a new framework for understanding dynamics in the unitary regime as the Bose-Einstein condensation of few-body composites.

Introduction.—Bose-Einstein condensation (BEC) gives rise to such spectacular manifestations of quantum statistics as superfluidity, superconductivity, and supersolidity [1–4]. The paradigmatic theories of Bogoliubov and Bardeen-Cooper-Schriefer (BCS) describe BECs of weakly coupled bosons and fermionic pairs, respectively and have been applied in many fields of physics [4–10]. Here, the quantum statistics of the medium alters one-body dynamics, producing quasiparticles, and two-body dynamics, producing Cooper pairs, with the latter persisting even in the absence of a two-body bound state. These guiding concepts however must be reconsidered when describing strongly-interacting systems such as liquid helium [11], ultracold gases [12], nuclear matter [13–16], and strongly-coupled polarons [17–22]. The occurrence of a richer few-body physics including three-body bound Efimov states amongst these strongly-interacting bosons and multi-component fermions [23, 24] necessitates the key shifting of many-body paradigms from two to three-body correlations. Here, the fundamental question reemerges of whether bound states in vacuum (polymers: dimers, trimers, etc.) can be bound by the medium and converted into condensed few-body composites (n-tuples: pairs, triples, etc.) possessing long-range order.

Recently, the versatility of ultracold atomic platforms was utilized to shed new light on these open problems. Despite strong three-body losses, quasi-equilibrated states were achieved in single-component Bose gases quenched to the unitary regime $n|a|^3 \gg 1$, with n the atomic density and a the s-wave scattering length [25–28]. Specifically, a macroscopic population of Efimov trimers was reported in Ref. [26], following a second sweep of interactions to weak interactions. Historically, this technique was used to measure the condensation of Cooper pairs in the BCS-BEC crossover via their conversion into weakly bound dimers [29, 30]. It is thus natural to ask whether the molecules measured in Ref. [26] reveal the existence of few-body condensates of pairs and triples in the unitary Bose gas. It is unknown whether the hypothesized universality of the medium [31], parametrized by the density (Fermi) scales $k_\mathrm{n} = (6\pi^2 n)^{1/3}$, $E_\mathrm{n} = \hbar^2 k_\mathrm{n}^2/2m$ and $t_\mathrm{n} = \hbar/E_\mathrm{n}$, produces universal pairs and triples, or conversely whether a (non-universal) sensitivity to the Efimov effect and finite-range physics is preserved at the many-body level. Answering this question in such a non-equilibrium and strongly-interacting quenched system requires a model both ergodic [32] and non-perturbative [33–37], which recovers the vacuum three-body spectrum [38–41]. Although widely used in statistical physics [42–44], the cumulant model was recently adapted to quantum gases and found to fulfill these requirements [45, 46].

In this Letter, we study a uniform BEC quenched to the unitary regime and develop a general theory of simultaneous atomic, pair, and triple condensation in strongly-interacting systems possessing the Efimov effect. Within the cumulant model, we construct generalized condensate wave functions and predict significant pair and triple condensation and associated off-diagonal long-range ordering (ODLRO) occuring between depleted atoms. We show that the Efimovian character of the triples is guaranteed at short distances, however at later times triples have a size comparable to the interparticle spacing. Remarkably even when Fermi and Efimovian scales are well-separated, medium effects lead to the persistent production of triple and pair BECs with universal populations and internal structures.

Model.—We model the system of N spinless bosons in a cubic volume V using a single-channel Hamiltonian

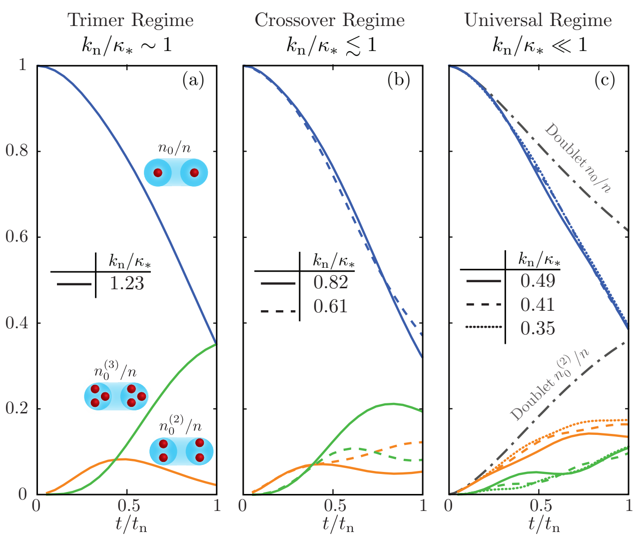

Figure 1. Atomic (blue), pair (orange), and triple (green) condensate fraction dynamics in the (a) trimer, (b) crossover, and (c) universal density regimes. The grey dot-dashed lines indicate universal results in the doublet model (Hartree-Fock Bogoliubov [47]). The comparison of Efimovian (κ_*) and Fermi (k_n) scales strongly determines the relative populations of the condensed few-body composites.

with pairwise s-wave interactions

$$\hat{H} = \int d^3r \hat{\psi}^\dagger(\mathbf{r}) \left(-\frac{\hbar^2}{2m}\Delta_\mathbf{r}\right)\hat{\psi}(\mathbf{r}) + \frac{1}{2}\int d^3r d^3r' \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r}')V(|\mathbf{r}-\mathbf{r}'|)\hat{\psi}(\mathbf{r}')\hat{\psi}(\mathbf{r}), \tag{1}$$

where $\hat{\psi}(\mathbf{r}) = (1/\sqrt{V})\sum_\mathbf{k} \hat{a}_\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}}$ are field operators and $\hat{a}_\mathbf{k}$ annihilates a boson of momentum $\hbar\mathbf{k}$. At unitarity ($|a| \to \infty$), the actual potential can be replaced by a simpler nonlocal separable potential $\hat{V} = g|\zeta\rangle\langle\zeta|$ with s-wave form factors $\langle\mathbf{k}|\zeta\rangle = \theta(\Lambda - |\mathbf{k}|)$, interaction strength $g = -\pi^3\hbar^2\bar{a}/m$ and cutoff $\Lambda = 2/\pi\bar{a}$, where $\bar{a} = 0.955 r_{\mathrm{vdW}}$ is the mean scattering length and r_{vdW} is the van der Waals length for a particular atomic species [38, 48–50]. This sets the three-body parameter $\kappa_* r_{\mathrm{vdW}} \approx 0.211$, which is the wave number of the ground-state Efimov trimer at unitarity (see [37, 38, 45, 51]).

We model the post-quench many-body dynamics of an initially pure, noninteracting atomic condensate using the method of cumulants whose hierarchical structure reflects the sequential growth of intrinsically higher-order correlations [43, 45, 46, 52–55]. Within the $U(1)$ symmetry-breaking picture, we study the dynamics of the singlet $\langle\hat{a}_\mathbf{k}\rangle = \delta_{\mathbf{k}0}\sqrt{V}\psi_0$, which describes the atomic BEC in the $k=0$ mode. In the frame rotating with the condensate phase θ_0, we study also the doublets

$$n_\mathbf{k} = \langle\hat{a}^\dagger_\mathbf{k}\hat{a}_\mathbf{k}\rangle, \quad c_\mathbf{k} = e^{-2i\theta_0}\langle\hat{a}_{-\mathbf{k}}\hat{a}_\mathbf{k}\rangle, \tag{2}$$

describing the single-particle momentum distribution and pairing, respectively, and the triplets

$$M_{\mathbf{k},\mathbf{q}} = e^{i\theta_0}\langle\hat{a}^\dagger_{\mathbf{k}-\mathbf{q}}\hat{a}^\dagger_\mathbf{q}\hat{a}_\mathbf{k}\rangle, \quad R_{\mathbf{k},\mathbf{q}} = e^{-3i\theta_0}\langle\hat{a}_{\mathbf{q}-\mathbf{k}}\hat{a}_\mathbf{k}\hat{a}_{-\mathbf{q}}\rangle. \tag{3}$$

introducing ergodic processes and the Efimov effect [32, 38, 45, 56]. Truncating the cumulant hierarchy can be justified at early times due to the sequential nature of correlation buildup [45, 51]. However the increasing importance of quadruplets, in particular for energy conservation, limits our study to times $t \lesssim t_n$.

Off-diagonal long-range ordering.—The triplet model contains anomalous averages at the one-body level (ψ_0) in the atomic condensate and at the two ($c_\mathbf{k}$) and three-body ($R_{\mathbf{k},\mathbf{q}}$) levels *within* the quantum depletion. These cumulants are intimately connected to the eigenfunctions of the reduced density matrices, signalling ODLRO and condensation [57–59]. We begin from the spectral decomposition of the one-body density matrix

$$\rho^{(1)}(\mathbf{r}_1,\mathbf{r}_2;t) = \langle\hat{\psi}^\dagger(\mathbf{r}_2)\hat{\psi}(\mathbf{r}_1)\rangle = \sum_\nu N_\nu(t)\varphi^*_\nu(\mathbf{r}_2,t)\varphi_\nu(\mathbf{r}_1,t) \tag{4}$$

where φ_ν are orthogonal one-body eigenstates. Only one eigenvalue $N_{\nu=0}$ is assumed to be macroscopic such that φ_0 is responsible for ODLRO at the one-body level

$$\lim_{|\mathbf{r}_1-\mathbf{r}_2|\to\infty} \rho^{(1)}(\mathbf{r}_1,\mathbf{r}_2;t) = N_0(t)\varphi^*_0(\mathbf{r}_2,t)\varphi_0(\mathbf{r}_1,t). \tag{5}$$

Within the cumulant approach, the long-range part of $\rho^{(1)}$ is simply $|\psi_0|^2$, such that $N_0 = V|\psi_0|^2$ coincides with the condensate population $\langle\hat{a}^\dagger_0\hat{a}_0\rangle$ and fraction $n_0 = N_0/V$.

In the presence of one-body condensation, ODLRO occurs trivially at all higher-orders [54, 59]. We isolate therefore the atomic condensate from the fluctuations $\hat{\psi}(\mathbf{r}) = \psi_0 + \delta\hat{\psi}(\mathbf{r})$, satisfying $\langle\delta\hat{\psi}(\mathbf{r})\rangle = 0$. To study intrinsically few-body ODRLO amongst fluctuations, we adapt the treatment of Yang [59] and spectrally decompose the corresponding p-body density matrices

$$\langle\delta\hat{\psi}^\dagger(\mathbf{r}'_1)\dots\delta\hat{\psi}^\dagger(\mathbf{r}'_p)\delta\hat{\psi}(\mathbf{r}_p)\dots\delta\hat{\psi}(\mathbf{r}_1)\rangle = \sum_\nu N^{(p)}_\nu(t)\varphi^{(p)*}_\nu(\mathbf{r}'_1,\dots,\mathbf{r}'_p,t)\varphi^{(p)}_\nu(\mathbf{r}_1,\dots,\mathbf{r}_p,t), \tag{6}$$

where $\varphi^{(p)}_\nu$ and $N^{(p)}_\nu$ are the orthogonal p-body eigenstates and eigenvalues, respectively. Analogous to Eq. (5), when the p-body density matrix is nonzero in the long-range limit $|\sum_{i=1}^p \mathbf{r}_i - \mathbf{r}'_i|/p \to \infty$, there exists intrinsic p-body ODLRO. In the triplet model, this limit is dominated by the anomalous contraction $\langle\delta\hat{\psi}^\dagger\dots\delta\hat{\psi}^\dagger\rangle\langle\delta\hat{\psi}\dots\delta\hat{\psi}\rangle$, such that nonzero c or R cumulants generate ODLRO. The associated normalized pair and triple wave functions are

$$\varphi^{(2)}_0(\mathbf{r},t) = \frac{c(\mathbf{r},t)}{\sqrt{N^{(2)}_0(t)}}, \quad \varphi^{(3)}_0(\mathbf{r},\boldsymbol{\rho},t) = \frac{R(\mathbf{r},\boldsymbol{\rho},t)}{\sqrt{N^{(3)}_0(t)}}, \tag{7}$$

with $N_0^{(2)} = \sum_{\mathbf{k}} |c_{\mathbf{k}}|^2$, $N_0^{(3)} = \sum_{\mathbf{k},\mathbf{q}} |R_{\mathbf{k},\mathbf{q}}|^2$, and Jacobi vectors $\mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$, $\boldsymbol{\rho} = \mathbf{r}_3 - (\mathbf{r}_1 + \mathbf{r}_2)/2$. We note that pair or triple condensation may generate trivial ODLRO in density matrices with $p \geq 4$. If one were to study *e.g.* quadruple condensation, such contributions should be removed.

Condensate fractions.—Unlike one-body condensation, the macroscopic eigenvalues $N_0^{(p)}$ cannot be directly related to condensed fractions. To understand this, we construct composite operators annihilating condensed pairs and triples

$$\hat{b}_0^{(p)} = \frac{1}{\sqrt{p!}} \left[\prod_{i=1}^{p} \int d^3 r_i \delta\hat{\psi}(\mathbf{r}_i) \right] \varphi_0^{(p)*}(\mathbf{r}_1, \ldots, \mathbf{r}_p). \quad (8)$$

Evaluating the quantum average of the commutators in the triplet model gives

$$\langle [\hat{b}_0^{(2)}, \hat{b}_0^{(2)\dagger}] \rangle = 1 + \frac{2}{N_0^{(2)}} \sum_{\mathbf{k}} |c_{\mathbf{k}}|^2 n_{\mathbf{k}}, \quad (9)$$

$$\langle [\hat{b}_0^{(3)}, \hat{b}_0^{(3)\dagger}] \rangle = 1 + \frac{3}{N_0^{(3)}} \sum_{\mathbf{k},\mathbf{q}} |R_{\mathbf{k},\mathbf{q}}|^2 n_{\mathbf{k}} (1 + n_{\mathbf{q}}), \quad (10)$$

which approximate the canonical relations only for weak excitations ($n_{\mathbf{k}} \ll 1$) or localized pairs and triples relative to the medium. In the opposite limit, composite bosons are created on top of densely populated Fourier modes, leading to Bose enhancement of atoms within the created composite and an overestimation of the condensed fraction [60]. Consequently, the rapid quantum depletion of the atomic condensate in the unitary regime yields $\hat{b}_0^{(p)}$ that are *approximately* bosonic only at $t \lesssim t_n$ (see [51]). The renormalization procedure

$$\hat{B}_0^{(p)} = \frac{\hat{b}_0^{(p)}}{\sqrt{\langle [\hat{b}_0^{(p)}, \hat{b}_0^{(p)\dagger}] \rangle}}, \quad (11)$$

effectively ensures that the bosonic canonical relations are preserved *on average* at all times, which we use to compute the pair and triple condensate fractions as $n_0^{(p)}/n = \langle \hat{B}_0^{(p)\dagger} \hat{B}_0^{(p)} \rangle / (N/p)$.

The postquench dynamics of the condensate fractions are shown in Fig. 1. Compared to the doublet model, three-body processes in the triplet model lead to an accelerated depletion of the atomic condensate, reaching $n_0/n \approx 0.4$ by $t = t_{\mathrm{n}}$. At early times, the formation of condensed triples follows sequentially the universal pair condensate growth, reflecting the hierarchical structure of the cumulant equations of motion (see [38, 51]). At later times the dynamics depend strongly on the density regime, repeating log-periodically with the density typical of the Efimov effect [39–41]. In the trimer regime (Fig. 1(a)), the ground-state Efimov trimer resonantly overlaps with the scale set by the density ($k_{\mathrm{n}}/\kappa_* \sim 1$), and triple condensation dominates clearly at later times, becoming comparable to the atomic condensate fraction. Condensed atoms can be converted to pairs and triples via low energy two and three-body scattering, respectively [56, 61, 62]. As this overlap becomes less resonant ($k_{\mathrm{n}}/\kappa_* \lesssim 1$), the system enters the crossover regime where particle-number oscillations between pair and triple BECs visible in Fig. 1(b)) are analogous to the atom-dimer coherences of Ref. [63]. In the universal regime (Fig. 1(c)), Efimovian and Fermi scales are well-separated ($k_{\mathrm{n}}/\kappa_* \ll 1$), and the oscillation becomes increasingly faster relative to t_{n}. This is the characteristic dynamical signature of the Efimov effect [38, 40, 41, 64]. At later times, pair condensation remains dominant while the non-universal oscillations fade and the condensate fractions converge universally, approaching $n_0^{(2)}/n \approx 0.2$ and $n_0^{(3)}/n \approx 0.1$ by $t = t_{\mathrm{n}}$.

Short-range expansions.—We study now how the short-range behavior of the condensate wave functions $c(\mathbf{r}, t)$ and $R(\mathbf{r}, \boldsymbol{\rho}, t)$ are dictated by few-body physics. This can be understood from the corresponding cumulant equations of motion which are identical to few-body Schrödinger equations at momenta large compared to the many-body scales [1, 38, 45, 51]. For distances larger than the short range of the potential ($r_{\mathrm{vdW}} < r < k_{\mathrm{n}}^{-1}, a$), the pair and triple condensate wave functions can be expanded in terms of the zero-energy few-body scattering wave functions

$$c(\mathbf{r}, t) \underset{r\to 0}{=} \frac{1}{4\pi} \Psi_0^{(2)}(t) \phi(\mathbf{r}), \quad (12)$$

$$R(\mathbf{r}, \boldsymbol{\rho}, t) \underset{R\to 0}{=} \frac{2^{3/2}}{3^{1/4} s_0} \Psi_0^{(3)}(t) \Phi(R, \boldsymbol{\Omega}), \quad (13)$$

which defines the macroscopic order parameters $\Psi_0^{(p)}$ (see [51]). Here $\phi(\mathbf{r}) = 1/r - 1/a$ is the zero-energy two-body scattering state, and

$$\Phi(R, \boldsymbol{\Omega}) = \frac{1}{R^2} \sin\left[s_0 \log \frac{R}{R_t} \right] \frac{\phi_{is_0}(\boldsymbol{\Omega})}{\sqrt{\langle \phi_{is_0} | \phi_{is_0} \rangle}} \quad (14)$$

is the zero-energy three-body scattering state for hyperraddius $R = \sqrt{r^2/2 + 2\rho^2/3}$ and hyperangles $\boldsymbol{\Omega} = \{\hat{\boldsymbol{\rho}}, \hat{\mathbf{r}}, \alpha = \arctan(r/\rho)\}$ [65]. Here, $s_0 \approx 1.00624$, $R_t = \sqrt{2} \exp(\mathrm{Im} \ln[\Gamma(1 + is_0)]/s_0)/\kappa_*$, and Γ is the gamma function. The hyperangular function describing s-wave pairwise scatterings is $\phi_s(\boldsymbol{\Omega}) = (1 + \hat{P}_{13} + \hat{P}_{23}) \sin(s(\pi/2 - \alpha))/\sin(2\alpha)\sqrt{4\pi}$ where $\hat{P}_{ij}$ swaps particles i and j [66]. From Eqs. (12) and (13) we see then at unitarity that the pairs have a universal behavior $\sim 1/r$ at short distances, whereas the triples have an Efimovian character, diverging as $1/R^2$ and oscillating log-periodically with the three-body parameter.

At short distances, the total probability to measure clustered pairs and triples is encoded in the two and three-body contact densities $\mathcal{C}_2$ and $\mathcal{C}_3$, respectively, central to a set of universal relations between system properties [65, 67]. In the presence of pair and triple conden-

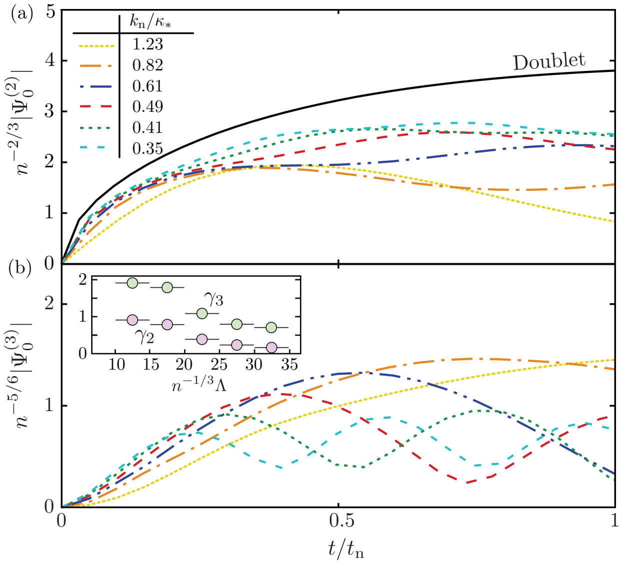

Figure 2. Macroscopic (a) pair and (b) triple order parameter dynamics over a range of densities within the triplet model. Black solid line: universal results within the doublet model. (inset) Residual exponents $\Psi_0^{(p)} \propto \Lambda^{\gamma_p}$ evaluated at fixed $t = 0.05t_{\rm n}$ converge as expected to 0 as the system becomes increasingly dilute with respect to the range of the interaction ($\Lambda/k_{\rm n} \to \infty$).

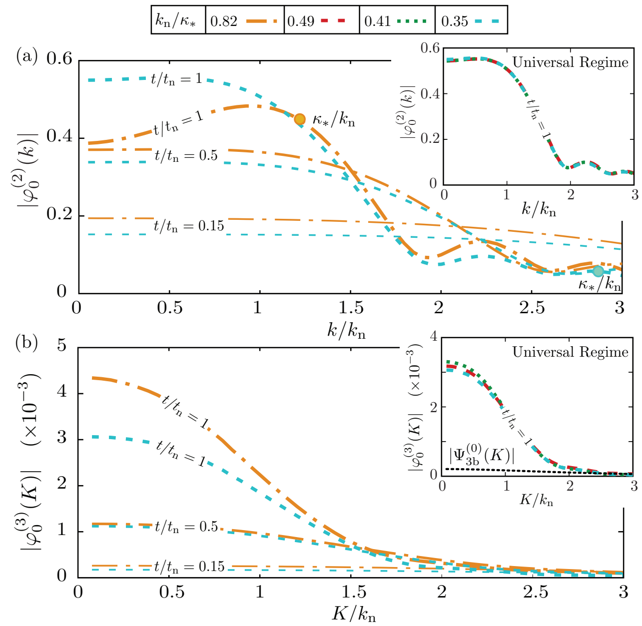

Figure 3. Internal structures of the normalized (a) pair and (b) triple condensate wave functions as the system evolves for densities within the crossover (orange) and universal (light blue) regimes. The filled circles in (a) indicate the three-body parameter. (insets) Normalized pair and triple condensate wave functions in the universal regime at $t = t_{\rm n}$ compared to the vacuum ground-state Efimov trimer $|\Psi_{\rm 3b}^{(0)}(K)|$ (black dotted) for density $k_{\rm n}/\kappa_* = 0.35$.

sation, these clusters can be divided into contributions from the order parameters and higher-order cumulants

$$\mathcal{C}_2 = \frac{m^2 g^2}{\hbar^4}\langle(\hat{\psi}^\dagger)^2\hat{\psi}^2\rangle = |\Psi_0^{(2)}|^2 + \delta\mathcal{C}_2, \tag{15}$$

$$\mathcal{C}_3 = -\frac{m^2 g^2}{2\hbar^4\Lambda^2}\left(H' + \frac{J'}{a\Lambda}\right)\langle(\hat{\psi}^\dagger)^3\hat{\psi}^3\rangle = |\Psi_0^{(3)}|^2 + \delta\mathcal{C}_3, \tag{16}$$

where H' and J' are log-periodic functions of Λ, $\hat{\psi} = \hat{\psi}(\mathbf{0})$ are local field operators, and $\delta\mathcal{C}_p$'s are contributions absent in the triplet model (see [51]). This establishes the square modulus of $\Psi_0^{(p)}$ as a probability density, analogous to ψ_0 at the one-body level. The dynamics shown in Fig. 2 can be understood from Refs. [33, 40, 41, 45]. Namely, early-time growths $|\Psi_0^{(p)}(t)| \propto t^{(p-1)/2}$ with primary ($p = 3$) and secondary ($p = 2$) visibility of non-universal trimer oscillations in the crossover regime ($k_{\rm n}/\kappa_* = 0.82$ and 0.61).

Internal structure.—We study now the longer-range internal structure of the pair and triple condensate wave functions, focusing on the interplay between Efimovian and Fermi scales. Fig. 3 shows the triplet model results for the normalized pair and triple condensate wave functions at $t/t_{\rm n} = 0.15, 0.5, 1$. To visualize the triple condensate wave function, we average $R_{\mathbf{k},\mathbf{q}}$ over internal configurations at a fixed hypermomentum $K^2 = k^2 + q^2 + kq\cos\theta$ where $\cos\theta = \hat{\mathbf{k}}\cdot\hat{\mathbf{q}}$. The corresponding $\varphi_0^{(3)}(K)$ captures variations of the coherent tripling physics with changes in the overall three-body momentum scale [51].

At early stages of evolution ($t = 0.15t_{\rm n}$), the pair and triple wave functions are relatively constant over the range of momenta considered consistent with the buildup of local correlations between nearby particles [33]. Accordingly, the small amount of clustered pairs and triples are dominated by few-body physics. This explains why their condensation dynamics shown in Fig. 1 follow the corresponding contact growth laws. We note that the local, structural origin of these laws was not recognized in Ref. [39].

At later times, both pair and triple wave functions become increasingly nonlocal. From the insets of Fig. 3 we see that as the system approaches more deeply the universal regime, both condensate wave functions acquire a universal form by $t = t_{\rm n}$. Together with the universal behavior of the triple condensed fractions in Fig. 1(c), this remarkable finding suggests the existence of a condensate composed of *universal* Efimovian triples at later times despite strongly non-universal short-distance behavior (see Fig. 2(b)). In the crossover regime results in Fig. 3(a), we find the development of a peak at momenta $k_{\rm n} \sim \kappa_*$ reminiscent of the Cooper pair in the BEC-BCS crossover [68]. The absence of this peak in simulations of the doublet model and universal regime of the triplet

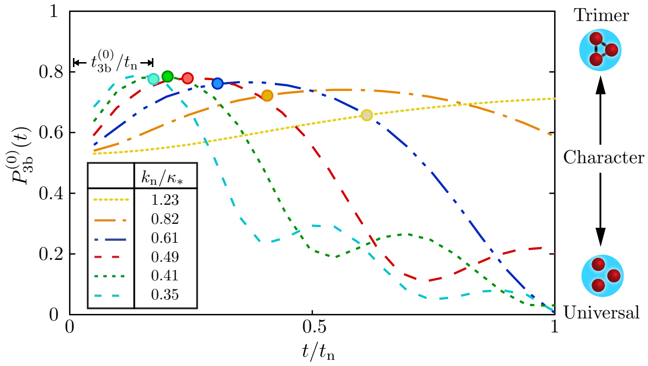

Figure 4. Evolution of the overlap (Eq. (17)) between ground-state Efimov trimer and triple condensate wave function for varying densities, with $P^{(0)}_{3b}(t=0)$ undefined. The numerical upper limit is comparable to the total probability ($\sim$ %82) projected onto the Efimov channel in the quenched three-body problem with characteristic timescale $t^{(0)}_{3b} = m/\hbar\kappa_*^2$ (filled circles) [69]. The illustrations indicate the presence or absence of a trimer character in the triple condensate.

model (Fig. 3(a)) ties it to the Efimov effect.

To study the presence and role of the ground-state Efimov trimer $|\Psi^{(0)}_{3b}\rangle$ in the triple condensate $|\varphi^{(3)}_0\rangle$, we evaluate the overlap

$$P^{(0)}_{3b}(t) = |\,\langle\varphi^{(3)}_0|\Psi^{(0)}_{3b}\rangle\,|^2, \tag{17}$$

as shown in Fig. 4 (see [51]). In the universal regime, the ground-state Efimov trimer is localized relative to the Fermi scales. At all times in this regime, $P^{(0)}_{3b}$ reflects therefore the short-range behavior of the triple condensate wave function encapsulated by $\Psi^{(3)}_0$, contributing the characteristic trimer oscillations visible in Fig. 4. After a small initial increase, the rapid decrease of $P^{(0)}_{3b}$ in this regime reflects the local to nonlocal transition of the triple condensate wave function, which bears little resemblance to the trimer as shown in the inset of Fig. 3(b). This is responsible also for the decreased visibility of the trimer oscillations in Fig. 1(c). Consequently, this local to nonlocal structural transition, generated by the medium effect of the strong quantum-depletion, is the underlying mechanism by which macroscopic observables, such as the condensate fractions in Fig. 1(c), display a universal scale invariance. This occurs despite continued local sensitivity to the three-body parameter (Eq. (13)) which becomes less relevant as the coherent physics begins to occur predominantly on the Fermi scale. In the trimer regime, we find no decrease of $P^{(0)}_{3b}$ for $t \lesssim t_n$, and the early-time increase is more gradual. From Fig. 3(b) it is clear that although the coherent physics occurs predominantly on the Fermi scale at later times as before, when one has $k_n \sim \kappa_*$ the condensed triples are increasingly dominated by the ground-state Efimov trimer. In short, the non-universal, trimer character of the triple condensate increases at later times in the trimer regime, whereas it decreases in the universal regime where one finds triples *without* a vacuum equivalent.

Conclusion.— Using the cumulant model that includes three-body correlations, we have shown that novel types of few-body condensates are generated within the quantum depletion of a quenched unitary Bose gas. Crucially, the regime of universal pair and triple condensation demonstrates a strongly-interacting many-body system behaving universally even in the presence of non-universal few-body physics such as the Efimov effect. We expect the molecular fractions produced following an interaction sweep back to weak interactions [26, 29] to reflect the few-body composites present in the unitary regime. However the highlighted difficulties of counting composite bosons extended in the medium requires a precise modeling of the projection and remains the subject of future work [37, 70]. Additionally, the tripling fluctuations discussed in this Letter raise interesting prospects for measuring non-Gaussian many-body states [71, 72].

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