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Eavesdropping on the Decohering Environment: Quantum Darwinism, Amplification, and the Origin of Objective Classical Reality

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"How much information about a system $\mathcal{S}$ can one extract from a fragment $\mathcal{F}$ of the environment $\mathcal{E}$ that decohered it?" is the central question of Quantum Darwinism. To date, most answers relied on the quantum mutual information of $\mathcal{SF}$, or on the data extracted by measuring $\mathcal{S}$ directly. These are reasonable upper bounds on what is really needed but much harder to calculate – the channel capacity of the fragment $\mathcal{F}$ for the information about $\mathcal{S}$. We consider a model based on imperfect `c-not` gates where all the above can be computed, and discuss its implications for the emergence of objective classical reality. We find that all relevant quantities, such as the quantum mutual information as well as the channel capacity exhibit similar behavior. In the regime relevant for the emergence of objective classical reality this includes scaling independent from the quality of the imperfect `c-not` gates or the size of $\mathcal{E}$, and even nearly independent of the initial state of $\mathcal{S}$.

Quantum Darwinism [1–5] explains the emergence of objective classical reality in our quantum Universe: The decohering environment $\mathcal{E}$ is a "witness who monitors and can reveal the state of the system $\mathcal{S}$. Agents like us never measure systems of interest directly. Rather, we accesses fragments $\mathcal{F}$ of $\mathcal{E}$ that carry information about them. Since its inception [1], Quantum Darwinism has advanced on both theory [6–25] and experimental fronts [26–29].

Quantum mutual information $I(\mathcal{S}:\mathcal{F})$ between an environment fragment and the system yields an upper bound on what $\mathcal{F}$ can reveal about $\mathcal{S}$. It has been used to estimate the capacity of the environment as a communication channel. We analyze a solvable model based on imperfect tunable `c-not` (or `c-maybe`) gates that couple $\mathcal{S}$ to the subsystems of $\mathcal{E}$. We compute the mutual information $I(\mathcal{S}:\mathcal{E})$ as well as the Holevo $\chi(\mathcal{S}:\mathcal{F})$ [30, 31] – that characterize the channel capacity in our `c-maybe` - based model. We also compute the quantum discord [1, 32–36] – the difference of $I(\mathcal{S}:\mathcal{F})$ and $\chi(\mathcal{S}:\mathcal{F})$ that quantifies the genuinely quantum correlations between $\mathcal{S}$ and $\mathcal{F}$ [37–40].

We find that $I(\mathcal{S}:\mathcal{F})$ and $\chi(\mathcal{S},\mathcal{F})$ exhibit strikingly similar dependence on the size of $\mathcal{F}$, with the initial steep rise followed by the classical plateau where – at the level set by the entropy $H_\mathcal{S}$ of the system, the information $\mathcal{F}$ has about $\mathcal{S}$ saturates: Enlarging $\mathcal{F}$ only confirms what is already known. This behavior is universal and nearly independent of the initial state of $\mathcal{S}$ and the size of $\mathcal{E}$.

The model. The system $\mathcal{S}$ is a qubit coupled to N independent non-interacting qubits of the environment $\mathcal{E}$ via a `c-maybe` gate,

$$U_\oslash = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & s & c \\ 0 & 0 & c & -s \end{pmatrix}. \tag{1}$$

The parameters $c = \cos(a)$ and $s = \sin(a)$ (where a is the angle associated with the action by which the target qubit is rotated) quantify the imperfection.

Our Quantum Universe $\mathcal{SE}$ starts in a pure state:

$$|\Psi^0_{\mathcal{SE}}\rangle = (\sqrt{p}\,|0_\mathcal{S}\rangle + \sqrt{q}\,|1_\mathcal{S}\rangle)\bigotimes_{i=1}^{N}|0^i\rangle, \tag{2}$$

where $p+q=1$. The unitary $U_\oslash$ correlates each qubit in $\mathcal{E}$ with $\mathcal{S}$, and we obtain a branching state [41],

$$|\Psi^\oslash_{\mathcal{SE}}\rangle = \sqrt{p}\,|0_\mathcal{S}\rangle\bigotimes_{i=1}^{N}|0_{\mathcal{E}_i}\rangle + \sqrt{q}\,|1_\mathcal{S}\rangle\bigotimes_{i=1}^{N}|1_{\mathcal{E}_i}\rangle\,. \tag{3}$$

By construction $|0_\mathcal{S}\rangle$ and $|1_\mathcal{S}\rangle$ are the pointer states [42, 43]. They are orthogonal and immune to decoherence. The corresponding record states of $\mathcal{E}$ are

$$|0_{\mathcal{E}_i}\rangle \equiv |0^i\rangle \quad \text{and} \quad |1_{\mathcal{E}_i}\rangle \equiv s|0^i\rangle + c|1^i\rangle\,, \tag{4}$$

in terms of the orthogonal basis $|0^i\rangle$ and $|1^i\rangle$ of the ith qubit that defines $U_\oslash$, so that $\langle 0_{\mathcal{E}_i}|1_{\mathcal{E}_i}\rangle = s$.

We will be interested in the correlations between the fragment $\mathcal{F}$ and $\mathcal{S}$. The marginal states of $\mathcal{S}$, an m-qubit fragment $\mathcal{F}_m$, and a bipartition $\mathcal{SF}_m$ are rank-two density matrices [44]:

$$\rho_\mathcal{S} \equiv \mathrm{tr}_\mathcal{E}\left\{|\Psi^\oslash_{\mathcal{SE}}\rangle\langle\Psi^\oslash_{\mathcal{SE}}|\right\} = \begin{pmatrix} p & s^N\sqrt{pq} \\ s^N\sqrt{pq} & q \end{pmatrix}, \tag{5}$$

$$\rho_{\mathcal{F}_m} = \begin{pmatrix} p & s^m\sqrt{pq} \\ s^m\sqrt{pq} & q \end{pmatrix}, \tag{6}$$

$$\rho_{\mathcal{SF}_m} = \begin{pmatrix} p & s^{N-m}\sqrt{pq} \\ s^{N-m}\sqrt{pq} & q \end{pmatrix}. \tag{7}$$

Symmetric quantum mutual information is often used to estimate the channel capacity of $\mathcal{F}$ in Quantum Darwinism

[2–4, 11–13, 45–47]. It is defined using the von Neumann entropy, $H(\rho) = -\mathrm{tr}\,\{\rho \log_2(\rho)\}$ as;

$$I(\mathcal{S}:\mathcal{F}_m) = H_{\mathcal{S}} + H_{\mathcal{F}_m} - H_{\mathcal{S},\mathcal{F}_m}. \quad (8)$$

Joint entropy $H_{\mathcal{S},\mathcal{F}_m}$ quantifies the ignorance about the state of $\mathcal{S}\mathcal{F}_m$ in the tensor product of the Hilbert spaces of $\mathcal{S}$ and $\mathcal{F}$.

In our model $I(\mathcal{S}:\mathcal{F}_m)$ can be computed exactly [48];

$$I(\mathcal{S}:\mathcal{F}_m) = h(\lambda^{+}_{N,p}) + h(\lambda^{+}_{m,p}) - h(\lambda^{+}_{N-m,p}), \quad (9)$$

where $h(x) = -x\log_2(x) - (1-x)\log_2(1-x)$ and $\lambda^{\pm}_{k,p}$ are the eigenvalues of the density matrices (5) – (7),

$$\lambda^{\pm}_{k,p} = \frac{1}{2}\left(1 \pm \sqrt{(q-p)^2 + 4s^{2k}pq}\right). \quad (10)$$

We thus have a closed expression for the mutual information $I(\mathcal{S}:\mathcal{F}_m)$.

As seen in Fig. 1, symmetric mutual information $I(\mathcal{S}:\mathcal{F}_m)$ exhibits a steep initial rise with increasing fragment size m, as a larger $\mathcal{F}_m$ provide more data about $\mathcal{S}$. This initial rise is followed by a long *classical plateau*, where the additional information imprinted on the environment is redundant.

Note that, when $\mathcal{S}\mathcal{E}$ is in a pure state, the entropy of a fragment $\mathcal{F}$ is equal to $H_{\mathcal{S}d\mathcal{F}}$, that is the entropy $\mathcal{S}$ would have if it was decohered only by the fragment $\mathcal{F}$. When we further assume *good decoherence* [41, 48] – i.e., that the off-diagonal terms of $\rho_{\mathcal{S}}$ and $\rho_{\mathcal{S}\mathcal{F}_m}$ are negligible (which in our model corresponds to $s^{N-m} \ll s^m$) – we obtain an approximate equality;

$$I(\mathcal{S}:\mathcal{F}_m) = H_{\mathcal{F}_m} = H_{\mathcal{S}d\mathcal{F}_m}, \quad (11)$$

since $H_{\mathcal{S}} = H_{\mathcal{S}\mathcal{F}_m}$ cancel one another in Eq. (8). Furthermore, when the environment fragments are typical [49] (and in our model all fragments of the same size are identical – hence, each is typical) the plot of $I(\mathcal{S}:\mathcal{F}_m)$ is antisymmetric around $I(\mathcal{S}:\mathcal{F}_m) = H_{\mathcal{S}}$ and $m = N/2$ [41].

We will see that the behavior of $I(\mathcal{S}:\mathcal{F}_m)$ is approximately universal. This means that after suitable re-scaling its functional form is nearly independent of the size of the environment N, of the quality of the `c-maybe` gate $U_{\oslash}$, and almost independent of the initial state of $\mathcal{S}$.

Agents generally do not insist on knowing the state of $\mathcal{S}$ completely, but tolerate a finite *information deficit* δ. When $I(\mathcal{S}:\mathcal{F}_{m_\delta}) \geq (1-\delta)H_{\mathcal{S}}$ is attained already for a fragment with $m_\delta \ll N$ subsystems, a fraction $f_\delta = m_\delta/N$ of the environment, then there are many $(1/f_\delta)$ such fragments. We define redundancy of the information about $\mathcal{S}$ in $\mathcal{E}$ via:

$$\mathcal{R}_\delta \equiv N/m_\delta \quad \text{with} \quad I(\mathcal{S}:\mathcal{F}_{m_\delta}) = (1-\delta)H_{\mathcal{S}}. \quad (12)$$

Redundancy $\mathcal{R}_\delta$ is the length of the classical plateau in the units set by m_δ, see Fig. (1). The beginning of the plateau is determined by the smallest m_δ such that $I(\mathcal{S}:\mathcal{F}_{m_\delta}) \geq (1-\delta)H_{\mathcal{S}}$.

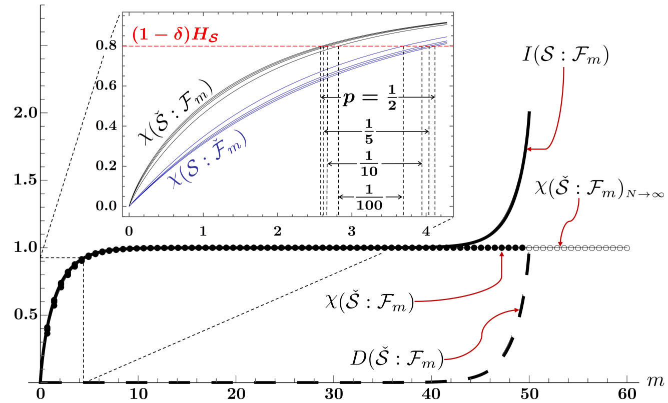

FIG. 1. *Approximate universality of mutual information:* Symmetric $I(\mathcal{S}:\mathcal{F}_m)$ and Holevo bound $\chi(\check{\mathcal{S}}:\mathcal{F}_m)$ coincide until the fragment $\mathcal{F}_m$ becomes almost as large as $\mathcal{E}$. Renormalized $I(\mathcal{S}:\mathcal{F})/H_{\mathcal{S}}$ and $\chi(\check{\mathcal{S}}:\mathcal{F})/H_{\mathcal{S}}$ depend only weakly on the probabilities of the outcomes (see inset). Their difference – quantum discord $D(\check{\mathcal{S}}:\mathcal{F})$ – vanishes until $\mathcal{F}_m$ begins to encompass almost all of $\mathcal{E}$, $m \sim N/R_\delta$. The inset also compares Holevo bounds $\chi(\check{\mathcal{S}}:\mathcal{F})/H_{\mathcal{S}}$ and the actual channel capacity $\chi(\mathcal{S}:\check{\mathcal{F}})/H_{\mathcal{S}}$ computed for several probabilities p of the pointer state $|0_{\mathcal{S}}\rangle$ in Eq. (3). Note that the fragment sizes m_δ that supply 80% of information about $\mathcal{S}$ are only modestly affected by p and quite similar for these two different information measures.

In realistic models $I(\mathcal{S}:\mathcal{F}) = H_{\mathcal{S}}$ only when $f = 1/2$ (see [41]). Thus, significant redundancy appears only when the requirement of completeness of the information about $\mathcal{S}$ that can be extracted from $\mathcal{F}$ is relaxed. Moreover, Eq. (12) is an overestimate since $I(\mathcal{S}:\mathcal{F}_{m_\delta})$ is only an upper bound of what can be found out about $\mathcal{S}$ from $\mathcal{F}$ [50].

We will now consider better estimates: Inset in Fig. 1 compares $I(\mathcal{S}:\mathcal{F}_{m_\delta})$ with the two Holevo - like χ's we are about to discuss and illustrates resulting fragment sizes (hence, redundancies) they imply.

Asymmetric mutual information is defined using conditional entropy. We mark the system whose states are used for such conditioning by an inverted "hat", so when it is $\check{\mathcal{S}}$ we consider the asymmetric mutual information:

$$J(\check{\mathcal{S}}:\mathcal{F}_m)_{\{|s_k\rangle\}} = H_{\mathcal{F}} - H_{\mathcal{F}|\check{\mathcal{S}}_{\{|s_k\rangle\}}}. \quad (13)$$

Above, $H_{\mathcal{F}_m|\check{\mathcal{S}}_{\{|s_k\rangle\}}}$ is the conditional entropy [31] that quantifies the missing information about $\mathcal{F}$ remaining after the observable with the eigenstates $\{|s_k\rangle\}$ was measured. Accordingly, the joint entropy in Eq. (8) is replaced by;

$$H_{\mathcal{F}_m,\check{\mathcal{S}}_{\{|s_k\rangle\}}} = H_{\mathcal{F}_m|\check{\mathcal{S}}_{\{|s_k\rangle\}}} + H_{\check{\mathcal{S}}_{\{|s_k\rangle\}}}. \quad (14)$$

The asymmetric joint entropy depends on whether $\mathcal{S}$ or $\mathcal{F}$ are measured and on the measurements that are used. The entropy increase associated with the wavepacket reduction means that the asymmetric entropy (14) is typically larger than the symmetric version $H_{\mathcal{S},\mathcal{F}_m}$ in Eq. (8): Local measurements cannot extract information encoded in the quantum correlations between $\mathcal{E}$ and $\mathcal{F}_m$, which is why the asymmetric $J(\check{\mathcal{S}}:\mathcal{F}_m)$ is needed, [31]; see also [51].

For optimal measurements the asymmetric $J(\check{\mathcal{S}}:\mathcal{F}_m)$ defines the Holevo bound [30],

$$J(\check{\mathcal{S}}:\mathcal{F}_m)=\max_{\{|s_k\rangle\}} J(\check{\mathcal{S}}:\mathcal{F}_m)_{\{|s_k\rangle\}}\equiv\chi(\check{\mathcal{S}}:\mathcal{F}_m). \tag{15}$$

In our model measurement of the pointer observable of $\mathcal{S}$ is optimal [48]. Indeed, Eq. (3) shows that in the pointer basis $\{|0_\mathcal{S}\rangle,|1_\mathcal{S}\rangle\}$ the conditional entropy disappears, $H_{\mathcal{F}_m|\check{\mathcal{S}}}=0$, as states of $\mathcal{F}_m$ correlated with pointer states of $\mathcal{S}$ are pure.

The limit of large $\mathcal{E}$ ($N\geq N-m\gg m$) reflects typical situation of agents (who do not even know the size of $\mathcal{E}$, and only access "their $\mathcal{F}_m$", with $m\ll N$). This is good decoherence, $s^N\leq s^{N-m}\ll s^m$, and equations simplify: Using $H_{\mathcal{S},\mathcal{F}_m}=H_\mathcal{S}$ and Eq. (11) we can thus write,

$$I(\mathcal{S}:\mathcal{F}_m)\approx H_{\mathcal{F}_m}=h(\lambda^+_{m,p})=\chi(\check{\mathcal{S}}:\mathcal{F}_m). \tag{16}$$

An immediate important consequence is that $H_{\mathcal{F}_m}$ determines both the symmetric $I(\mathcal{S}:\mathcal{F}_m)$ (except for the final rise) as well as the asymmetric (optimal) $J(\check{\mathcal{S}}:\mathcal{F}_m)=\chi(\check{\mathcal{S}}:\mathcal{F}_m)$. We have:

$$H_{\mathcal{F}_m}=-\frac{1}{2}\log_2\left(pq\left(1-s^{2m}\right)\right)-\sqrt{1-4pq\left(1-s^{2m}\right)}\ \text{Arctanh}_2\left(\sqrt{1-4pq\left(1-s^{2m}\right)}\right), \tag{17}$$

where "Arctanh_2" denotes $\text{Arctanh}\,/\ln(2)$.

Fig. 1 compares $\chi(\check{\mathcal{S}}:\mathcal{F}_m)$ with $I(\mathcal{S}:\mathcal{F}_m)$ for finite and infinite N and for different values of s and p. As it shows, Eq. (17) matches $I(\mathcal{S}:\mathcal{F}_m)$ until the far end ($N-m\ll m$) of the classical plateau. This is a consequence of two scalings: (i) "vertically", the plateau appears at $H_\mathcal{S}=-p\log_2(p)-q\log_2(q)$, and it is easy to see that for $s^N\ll s^m\ll 1$ we have $\chi(\check{\mathcal{S}}:\mathcal{F}_m)=H_\mathcal{S}$ in Eq. (17); (ii) "horizontally", $H_{\mathcal{F}_m}$ depends on s^m, so weakly entangling gates can be compensated by using more of them – larger m. What is surprising is how insensitive are these plots to p, the probability of the outcome.

This remarkably universal behavior is a consequence of *good decoherence* [48]. Both, $\rho_\mathcal{S}$ and $\rho_{\mathcal{S}\mathcal{F}_m}$, Eqs. (5) and (7), become diagonal in the pointer basis. Moreover, the quality of $U_\oslash$ (set by c and s) determines the " information flow rate" from $\mathcal{S}$ to $\mathcal{F}$. Thus, when (at a fixed p) one demands the same $H_{\mathcal{F}_m}$, this translates into identical $\rho_{\mathcal{F}_m}$ when $s_1^{m_1}\simeq s_2^{m_2}$. Therefore, less efficiently entangling gates can be compensated by relying on more of them – on a suitably enlarged $\mathcal{F}$, with $m_2=m_1\,\log(s_1)/\log(s_2)$.

Environment as a communication channel. While the mutual information $I(\mathcal{S}:\mathcal{F}_m)$ is easier to compute and a safe upper bound on the channel capacity of $\mathcal{F}_m$, it is important to verify it is also a reasonable estimate of that channel capacity (as generally assumed in much of the quantum Darwinism literature). The asymmetric mutual information extracted by optimal measurements on the environment fragment $\mathcal{F}_m$ is:

$$J(\mathcal{S}:\check{\mathcal{F}}_m)=H_\mathcal{S}-H_{\mathcal{S}|\check{\mathcal{F}}_m}=\chi(\mathcal{S}:\check{\mathcal{F}}_m)\,. \tag{18}$$

The joint entropy given in terms of the conditional entropy $H_{\mathcal{S}|\check{\mathcal{F}}_m}$ now becomes,

$$H_{\mathcal{S},\check{\mathcal{F}}_m}=H_{\mathcal{S}|\check{\mathcal{F}}_m}+H_{\check{\mathcal{F}}_m}\,. \tag{19}$$

As in Eq. (14) above, all terms in Eq. (19) depend on how $\mathcal{F}$ is measured. However, while measuring $\mathcal{S}$ in the pointer basis simplified the analysis (since e.g. $H_{\mathcal{F}|\check{\mathcal{S}}_{\{|s_k\rangle\}}}=0$) this is no longer the case when $\mathcal{F}_m$ is measured.

To compute $\chi(\mathcal{S}:\check{\mathcal{F}}_m)$ we rely on the Koashi-Winter monogamy relation [52]. Details of that calculation are relegated to the supplementary material [53].

We focus again on the limit of large $\mathcal{E}$ ($N\geq N-m\gg m$): Agents only access "their $\mathcal{F}_m$", a small fraction of $\mathcal{E}$ with $m\ll N$. Assuming good decoherence we obtain

$$\chi(\mathcal{S}:\check{\mathcal{F}}_m)=H_\mathcal{S}+\frac{1}{2}\log_2\left(pqs^{2m}\right)+\sqrt{1-4pqs^{2m}}\ \text{Arctanh}_2\left(\sqrt{1-4pqs^{2m}}\right). \tag{20}$$

Equation (20) constitutes our main result. We have decomposed the Holevo-like quantity $\chi(\mathcal{S}:\check{\mathcal{F}}_m)$ into the plateau entropy $H_\mathcal{S}$ and $H_{\mathcal{S}|\check{\mathcal{F}}_m}$ – the ignorance about $\mathcal{S}$ remaining in spite of the optimal measurement on $\mathcal{F}_m$ [54]. Rather remarkably, $H_{\mathcal{S}|\check{\mathcal{F}}_m}=H_\mathcal{S}-\chi(\mathcal{S}:\check{\mathcal{F}}_m)$ – the conditional entropy – scales *exactly* with pqs^{2m}. What remains to do is to quantify the differences of $I(\mathcal{S}:\mathcal{F}_m)$ and $\chi(\check{\mathcal{S}}:\mathcal{F}_m)$ with $\chi(\mathcal{S}:\check{\mathcal{F}}_m)$. In Fig. 2 we compare it with these other, easier to compute, quantities.

Redundancy of the information about $\mathcal{S}$ in the channel $\mathcal{F}_m$ can be now computed using $\chi(\mathcal{S}:\check{\mathcal{F}}_m)$, Eq. (20), and compared with the estimates based on $I(\mathcal{S}:\mathcal{F}_m)$. The fragment $\mathcal{F}_{m_\delta}$ can carry all but the deficit δ of the classical information about the pointer state of $\mathcal{S}$ when

$\chi(\mathcal{S}:\check{\mathcal{F}}_{m_\delta}) \geq (1-\delta)H_{\mathcal{S}}$. This leads to a transcendental equation for m_δ that we solve numerically. $R_\delta = N/m_\delta$, where N is the number of subsystems in $\mathcal{E}$.

The inset in Fig. 1 shows that – while m_δ deduced using $I(\mathcal{S}:\mathcal{F}_m) \approx \chi(\check{\mathcal{S}}:\mathcal{F}_m)$ do not coincide with those obtained using $\chi(\mathcal{S}:\check{\mathcal{F}}_m)$ – the difference is modest, unlikely to materially affect conclusions about the emergence of objective classical reality. Indeed, in the supplementary materials we estimate that the redundancy estimates based on $I(\mathcal{S}:\mathcal{F}_m)$ and $\chi(\mathcal{S}:\check{\mathcal{F}}_m)$ differ at most by $\sim 37\%$ for $\delta \leq 0.2$, and by much less in the regime where $\delta \to 0$.

In situations relevant for observers who rely on photons, $R_{\delta=0.1} \simeq 10^8$ is amassed when sunlight illuminates a $1\mu m$ dust grain in a superposition with a $1\mu m$ spatial separation for $1\mu s$ [46, 47]. It may seem like we are stretching the applicability of our `c-maybe` model too far, but the equations for $I(\mathcal{S}:\mathcal{F}_m)$ and $\chi(\check{\mathcal{S}}:\mathcal{F}_m)$ derived for photon scattering *coincide* with our Eq. (17), see supplement [53]. Thus, it appears that the information transfer from $\mathcal{S}$ to $\mathcal{E}$ leading to the buildup of redundancy has universal features captured by our model.

Quantum discord is the difference between symmetric (9) and asymmetric quantum mutual information [32–40]. The *systemic discord* is defined as;

$$D(\check{\mathcal{S}}:\mathcal{F}_m) = I(\mathcal{S}:\mathcal{F}_m) - \chi(\check{\mathcal{S}}:\mathcal{F}_m)\,. \qquad (21)$$

The measurements on pointer observables of $\mathcal{S}$ are optimal.

Mutual information for pure decoherence induced by non-interacting subsystems of $\mathcal{E}$ can be written as [48, 55]:

$$I(\mathcal{S}:\mathcal{F}) = \overbrace{\big(H_{\mathcal{F}} - H_{\mathcal{F}}(0)\big)}^{local/classical} + \overbrace{\big(H_{\mathcal{S}d\mathcal{E}} - H_{\mathcal{S}d\mathcal{E}_{\setminus\mathcal{F}}}\big)}^{global/quantum}\,. \qquad (22)$$

As $\mathcal{SE}$ is a pure product state, the initial entropy of $\mathcal{F}$ is zero, $H_{\mathcal{F}}(0) = 0$. Assuming good decoherence and conditioning on the pointer basis (hence, $\chi(\check{\mathcal{S}}:\mathcal{F}_m) = H_{\mathcal{F}_m}$), Eq. (16) we have

$$I(\mathcal{S}:\mathcal{F}_m) - J(\check{\mathcal{S}}:\mathcal{F}_m) = H_{\mathcal{S}d\mathcal{E}} - H_{\mathcal{S}d\mathcal{E}_{\setminus\mathcal{F}_m}}, \qquad (23)$$

where $H_{\mathcal{S}d\mathcal{E}}$ ($H_{\mathcal{S}d\mathcal{E}_{\setminus\mathcal{F}}}$) is the entropy of the system *decohered* by $\mathcal{E}$ (or just by $\mathcal{E}_{\setminus\mathcal{F}}$ – i.e., $\mathcal{E}$ less the fragment $\mathcal{F}$).

The global/quantum term represents quantum discord in the pointer basis of $\mathcal{S}$ [48]. Good decoherence implies $H_{\mathcal{S}d\mathcal{E}} \approx H_{\mathcal{S}d\mathcal{E}_{\setminus\mathcal{F}}}$, so $D(\check{\mathcal{S}}:\mathcal{F}_m) \approx 0$. As long as $\mathcal{E}_{\setminus\mathcal{F}}$ is large enough to induce good decoherence, Eq. (16) holds, and, hence, the systemic discord (21) vanishes [56].

Systemic quantum discord can become large again when $\mathcal{F}_m$ encompasses almost all $\mathcal{E}$, as in this case $H_{\mathcal{S}\mathcal{F}_m}$ approaches $H_{\mathcal{SE}} = 0$ (given our assumption of a pure $\mathcal{SE}$). In this (unphysical) limit $I(\mathcal{S}:\mathcal{F}_m)$ climbs to $H_{\mathcal{F}_m} + H_{\mathcal{S}} = 2H_{\mathcal{S}}$, while $\chi(\check{\mathcal{S}}:\mathcal{F}_m) \leq H_{\mathcal{F}_m}$. As good decoherence implies $\chi(\check{\mathcal{S}}:\mathcal{F}_m) \approx H_{\mathcal{F}_m}$, $D(\check{\mathcal{S}}:\mathcal{F}_m)$ can reach $H_{\mathcal{F}_m}$. Indeed, when $\mathcal{SE}$ is pure, $\chi(\check{\mathcal{S}}:\mathcal{F}_m)$ and $D(\check{\mathcal{S}}:\mathcal{F}_m)$ – classical and quantum content of the correlation – are complementary [36], see Fig. 1.

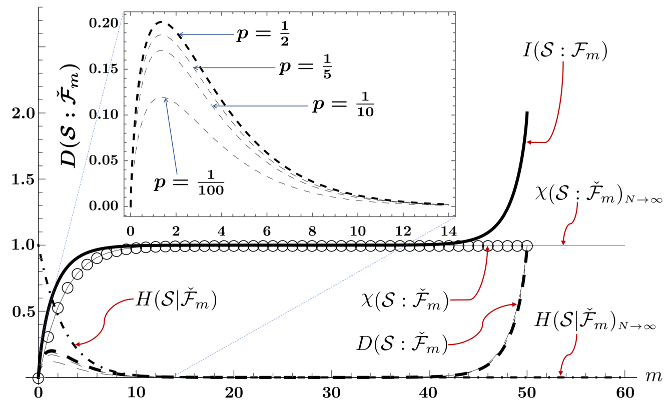

FIG. 2. *Channel capacity of the environment fragment.* Fragment $\mathcal{F}_m$ carries at most $\chi(\mathcal{S}:\check{\mathcal{F}})$ of classical information about the system it helped decohere. As seen above, this Holevo - like quantity is less than the symmetric mutual information $I(\mathcal{S}:\mathcal{F})$ or the Holevo bound $\chi(\check{\mathcal{S}}:\mathcal{F}_m)$. Their difference (quantum discord $D(\mathcal{S}:\check{\mathcal{F}})$) is significant already early on (in contrast to $D(\check{\mathcal{S}}:\mathcal{F})$), but disappears as the plateau is reached. It reappears again (as did $D(\check{\mathcal{S}}:\mathcal{F})$) when $\mathcal{F}_m$ begins to encompass almost all of $\mathcal{E}$.

The *fragmentary discord* is the difference between the mutual information $I(\mathcal{S}:\mathcal{F})$ and what can be extracted from $\mathcal{SF}$ by measuring only the fragment $\mathcal{F}$:

$$I(\mathcal{S}:\mathcal{F}) - \chi(\mathcal{S}:\check{\mathcal{F}}) \approx \chi(\check{\mathcal{S}}:\mathcal{F}) - \chi(\mathcal{S}:\check{\mathcal{F}}). \qquad (24)$$

It can be evaluated:

$$\begin{aligned} D(\mathcal{S}:\check{\mathcal{F}}_m) &\approx H_{\check{\mathcal{F}}_m} - \big(H_{\mathcal{S}} - H_{\mathcal{S}|\check{\mathcal{F}}_m}\big)\,, \\ &= \big(H_{\mathcal{S}|\check{\mathcal{F}}_m} + H_{\check{\mathcal{F}}_m}\big) - H_{\mathcal{S}}\,. \end{aligned} \qquad (25)$$

The bracketed terms in the last two expressions represent different quantities. The difference between the symmetric and asymmetric mutual information $H_{\check{\mathcal{F}}_m} - \big(H_{\mathcal{S}} - H_{\mathcal{S}|\check{\mathcal{F}}_m}\big)$ is the original definition of discord.

Note that initially decoherence does not suppress fragmentary discord $D(\mathcal{S}:\check{\mathcal{F}}_m)$. This is because the states of $\mathcal{F}_m$ that are correlated with the pointer states of $\mathcal{S}$ are not orthogonal: The scalar product of the branch fragments $\mathcal{F}_m$ corresponding to $|0_{\mathcal{S}}\rangle$ and $|1_{\mathcal{S}}\rangle$ is s^m. Thus, while the symmetric mutual information increases with m, orthogonality is approached gradually, also as m increases. Perfect distinguishability, i.e., orthogonality of record states of $\mathcal{F}$ is needed to pass on all the information about $\mathcal{S}$ [57–59]. See again Fig. 2 for an illustration of these findings.

Concluding remarks. We found that in the pre-plateau regime relevant for emergence of objective reality (where $I(\mathcal{S}:\mathcal{F})$ increases with the size of $\mathcal{F}$) the mutual information as well as the Holevo bound $\chi(\check{\mathcal{S}}:\mathcal{F})$ coincide and exhibit universal scaling behaviors independent of the size of $\mathcal{E}$, of how imperfect are the `c-maybe`'s, and only weakly dependent on the probabilities of pointer states. The corresponding Holevo $\chi(\check{\mathcal{S}}:\mathcal{F})$ and $I(\mathcal{S}:\mathcal{F})$ coincide until $\mathcal{F}$ encompasses almost all of $\mathcal{E}$.

However, the channel capacity $\chi(\mathcal{S}:\check{\mathcal{F}})$ of the environment fragments $\mathcal{F}$ differs somewhat from $I(\mathcal{S}:\mathcal{F})$ in the pre-plateau region. This difference tends to be small compared to, e.g., the level of the plateau, and disappears as the plateau is reached for larger fragments. This behavior – generic when many copies of the information about $\mathcal{S}$ are deposited in the environment – facilitates estimates of the redundancy of the information about the system in the environment, as the differences between $I(\mathcal{S}:\mathcal{F}) \approx \chi(\check{\mathcal{S}}:\mathcal{F})$ or $\chi(\mathcal{S}:\check{\mathcal{F}})$ are noticeable but inconsequential.

To sum up, sensible measures of information flow lead to compatible conclusions about R_δ. The differences in the estimates of redundancy based on these quantities are insignificant for the emergence of objective classical reality – the overarching goal of quantum Darwinism. The functional dependence of the symmetric mutual information in the photon scattering model [46, 47] is the same as in our model. Thus, the universality we noted in scaling with s and p (approximate for $I(\mathcal{S}:\mathcal{F}) = \chi(\check{\mathcal{S}}:\mathcal{F}_m)$ exact for $\chi(\mathcal{S}:\check{\mathcal{F}}_m)$) may be a common attribute of the information that reaches us, human observers.

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