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Visualization of Fast Ion Phase-space Flow Driven by Alfvén Instabilities

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Fast ion phase-space flow, driven by Alfvén eigenmodes (AEs), is measured by an imaging neutral particle analyzer in the DIII-D tokamak. The flow firstly appears near the minimum safety factor at the injection energy of neutral beams, and then moves radially inward and outward by gaining and losing energy, respectively. The flow trajectories in phase space align well with the intersection lines of the constant magnetic moment surfaces and constant $E - (\omega/n)P_{\zeta}$ surfaces, where E, P_{ζ} are energy and toroidal canonical momentum of ions; ω and n are angular frequencies and toroidal mode numbers of AEs. It is found that the flow is so destructive that the thermalization of fast ions is no longer observed in regions of strong interaction. The measured phase-space flow is consistent with nonlinear hybrid kinetic-magnetohydrodynamics simulation. Calculations of the relatively narrow phase-space islands reveal that fast ions must transition between different flow trajectories to experience large-scale phase-space transport.

PACS numbers:

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Wave-particle interaction is a universal phenomenon 30 of great importance in physical systems, including as-31 trophysics, laser physics, and many others. In magnet- 32 ically confined fusion devices, wave-particle resonances 33 can drive massive fast-ion transport across phase space. 34 Predicting and minimizing this transport is a key issue 35 to achieve a sustainable burning plasma in fusion reac- 36 tors. Past studies measure fast ion profiles, averaging 37 over a broad portion of phase space, to study global 38 confinement [1-7]. However, wave-particle interactions 39 occur locally in phase space and resonant fast ions mi- 40 grate along certain phase-space routes, something which 41 has not been measured before. Understanding the phase- 42 space flow formation and its evolution are important is- 43 sues that could impact operation of nuclear fusion reac- 44 tors. It is fundamental to the development of predictive modeling, control techniques to mitigate fast ion loss and advanced operation scenarios, such as utilizing the alpha channelling effect [8]. Transport by Alfvén eigenmodes (AE) is a particular concern. To address this, an imaging neutral particle analyzer (INPA) [9, 10] was developed in the DIII-D tokamak. The first ever visualization of fast ion phase space flow driven by AEs is reported here.

The INPA measures energetic neutrals, which are produced by charge-exchange reactions between confined fast ions and an active neutral beam source, as seen from Fig. 1(a). The system covers nearly all radii, and resolves the neutral energies from $\sim 30 \text{ keV}$ to $\sim 100 \text{ keV}$. The diagnostic sensitivity or 'weights' of $\sim 1\%$ of the available

INPA pixels in the radius-energy plane, depicted as the white circles in Fig. 1(b), are estimated using the synthetic diagnostic code (INPASIM) [11, 12]. Each circle corresponds to the contour lines of 30% of the maximum weights. The INPASIM also finds that fast ions with the pitch v_{\parallel}/v from ~ 0.77 to 0.84 can be collected by INPA, illustrated as the black band (v_{\parallel} refers to the fast ion velocity parallel to the magnetic field line). Note that the validation of the INPA system [9, 10] and synthetic modeling [11] in a broad range of plasma parameters were systematically reported in [11, 13].

The orbit topology over the INPA-interrogated phase space, computed by the orbit tracing code (ASCOT5) [14], is overlaid. The system interrogates the phase space occupied by well-confined fast ions on stagnation orbits

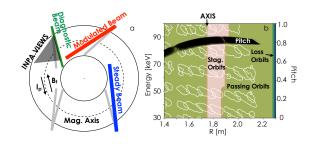


FIG. 1: (a) the neutral beam geometry, along with the INPA view. (b) the weight function of the INPA (circles) on energy-radius plane, along with the measured pitch range. The orbit topology in the INPA-interrogated phase space is overlaid.

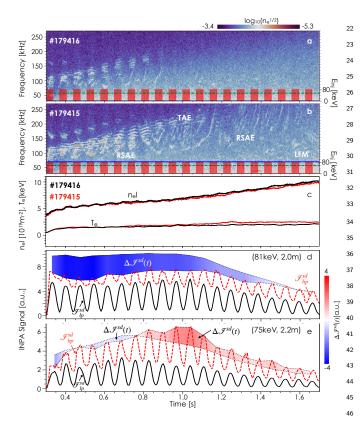


FIG. 2: Frequency spectra of the density fluctuation for the 47 low-power shot (a) and high-power shot (b), along with the 48 waveforms of the neutral beams. (c) The time evolution of 49 line-integrated n_e and T_e . The measured INPA signal in the $_{50}$ low-power shot (solid line) and high-power shot (dashed line) $_{51}$ and the signal deficit to the expectation of the neoclassical theory (colormap) for the velocity space ($E\sim81~{\rm keV}$, $R\sim100~{\rm keV}$), $R\sim100~{\rm keV}$), and $R\sim100~{\rm keV}$), $R\sim100~{\rm keV}$), $R\sim100~{\rm keV}$), $R\sim100~{\rm keV}$), $R\sim100~{\rm keV}$), and $R\sim100~{\rm keV}$), and $R\sim100~{\rm keV}$), and $R\sim100~{\rm keV}$).

near the magnetic axis and on passing orbits elsewhere. 57 Stagnation orbits are a class of orbits which are confined 58 on a given side of the magnetic axis near the device mid-59 plane which travel in a single toroidal direction. The 60 view does not cover the confined-loss boundary.

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A controlled experiment is designed to visualize the 62 phase-space flow in the plasma current ramp-up phase, 63 at a toroidal magnetic field of ~ 2.1 T and in an up- 64 per single null magnetic configuration. The strategy is 65 to vary AE activities in a pair of shots as much as pos- 66 sible, with a minimum change of the plasma parameters 67 and equilibrium. As seen from Fig. 2(a), the plasma 68 with less or no AE activity in the low-power (lp) shot 69 (#179416) is heated by two neutral beams, i.e., a steady 70 diagnostic neutral beam at 55 keV of 1 MW and a mod-71 ulated beam at 81 keV of 2.5 MW with a cycle time 72 of 70 ms and a duty cycle of 50%. The INPA detects 73 the charge-exchanged neutral flux from a time-evolving 74 slowing-down (sd) fast ion distribution. That is, the im- 75 age above 55 keV can be expressed as $\mathcal{I}_{\rm lp} \equiv \mathcal{I}_{\rm lp}^{\rm sd}(t)$.

The AE activity in Fig. 2(b) are largely enhanced af- 77

ter adding a steady (st) beam of ~ 82 keV at a modest beam power of ~ 1.7 MW. The beam is injected at the axisymmetry angle of the modulated beam. Thus, the image in the high-power (hp) shot can be expressed as, $\mathcal{I}_{\rm hp} \equiv \mathcal{I}_{\rm hp}^{\rm sd}(t) + \mathcal{I}_{\rm hp}^{\rm st}$, where $\mathcal{I}_{\rm hp}^{\rm st}$ is the image produced by the steady beam.

As seen from Fig. 2(c), the electron density n_e agrees well in the pair of shots, due to a delicate tuning of the n_e feedback control system. The electron temperature T_e also agrees well before ~ 1.0 s, and starts to deviate later. In this circumstance, neoclassical theory expects $\mathcal{I}_{\rm lp}^{\rm sd}(t) = \mathcal{I}_{\rm hp}^{\rm sd}(t)$. This is experimentally demonstrated by a quantitative agreement of $\mathcal{I}_{\rm lp}^{\rm sd}(t)$ and $\mathcal{I}_{\rm hp}^{\rm sd}(t)$, when AEs are stable, and further supported by the agreement with synthetic INPA images, using the fast ion distribution predicted by NUBEAM module of TRANSP [15]. For details, see the Supplemental Material [16].

As expected, when AEs are unstable, $\mathcal{I}_{\mathrm{lp}}^{\mathrm{sd}}(t)$ and $\mathcal{I}_{\mathrm{hp}}^{\mathrm{sd}}(t)$ deviate. Figures 3 and 4 compare images at two different modulation periods in these two discharges. In Fig. 3 at $\sim 1.0 \text{ s}$, the AE activity is absent in low-power shot and relatively weak in high-power shot (see Fig. 3(a4) and (b4)); while in Fig. 4 at ~ 0.38 s, the AE activity appears even in low-power shot and is quite strong in high-power shot (see Fig. 4(a4) and (b4)). In both cases, three snapshots at three different times in the beam modulation cycle are shown to illustrate the birth and subsequent slowing-down of ions from the modulated source. Even when the AE activity is relatively weak, noticeable differences between images in the two shots appear (Fig. 3). Although the n_e profiles match, less full-energy fast ions appear in the plasma core in the high-power shot. Moreover, in spite of a slightly higher edge T_e by $\sim 10 \%$ in the high-power shot, more INPA signal appears at the reduced energy of $\sim 75 \text{ keV}$ at R > 2.1 m, as indicated by the arrows. These features are in contrast to expectations from neoclassical theory, which would predict: (i) for the same density profile, the ionization profile of fast ions is expected to be the same; (ii) for a higher edge T_e , the slowing-down time is longer and thus less INPA signal at the lower energy range is expected for the high power shot. For convenience, the red (blue) colored region in Fig. 3(c1)-(c3) are called an inflow (outflow) region, where the amount of the confined fast ions are above (below) that in the low-power shot.

The deviation of $\mathcal{I}_{lp}^{sd}(t)$ and $\mathcal{I}_{hp}^{sd}(t)$ is even more significant, when the AE activity is strong (Fig. 4). The differences are: (i) The INPA signal of the modulated beam at the injection energy of 82 keV, indicated by the dotted lines, is mostly missing in the high-power shot, as seen from Figs. 4(b1)-(b3). It suggests a full depletion of ionized neutrals in a time scale much shorter than the camera integration time of 0.5 ms. That is, when AEs are unstable even in the low-power shot (see Fig. 4(a4)), the phase space in high-power shot exhibits strong self-organized criticality dynamics. (ii) The slowing-down of

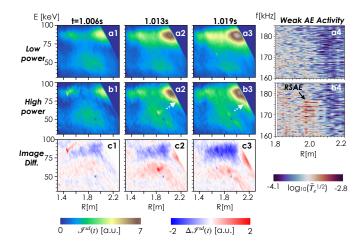


FIG. 3: INPA images of the modulated beam source during the weak AE activity (from 1.006s to 1.019s). (a1)-(a3) Low power shot 179416. (b1)-(b3) High power shot 179415. (c1)-(c3) Difference image. T_e fluctuation in low-power shot (a4) and in high-power shot (b4) at ~ 1.013 s.

fast ions is not observed across the image series in the high-power shot, which is consistent with the fact that $_{36}$ the increase of T_e is barely noticeable in the plasma core $_{37}$ after adding neutral beam power of 1.7 MW (see Fig. $_{38}$ 2(c)). (iii) As seen from the highlighted image differ- $_{39}$ ences $\Delta \mathcal{I}^{\rm sd}(t) \equiv \mathcal{I}^{\rm sd}_{\rm hp}(t) - \mathcal{I}^{\rm sd}_{\rm lp}(t)$ in Fig. 4(c1)-(c3), an $_{40}$ inflow region (red) in the plasma core emerges, as indicated by the arrow. It shows radially inward transport $_{42}$ of fast ions with energies exceeding the injection energy $_{43}$ of 82 keV . It should be mentioned that the image patern of the $\Delta \mathcal{I}^{\rm sd}(t)$ smoothly evolves, as AEs activities $_{45}$ are gradually diminishing from 0.3 s to 1.7 s.

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These changes in $\Delta T^{\rm sd}(t)$ depend on the strength of 47 the AE activity. Figures 2(d) and 2(e) show the time 48 evolution of the flows for two INPA pixels. For a pixel 49 near the radius of the strongest AE activity ($R=2.0~{\rm m}$) 50 near the injection energy of $\sim 81~{\rm keV}$, strong outflow is 51 observed that steadily decreases as the AE activity weak-52 ens in time (see Fig. 2(d)). For a pixel at lower energy 53 and larger radius, inflow occurs for intermediate levels 54 of AE activity, then ceases when the AEs become stable 55 (see Fig. 2(e)). No inflow is observed during the early 56 phase of very strong AE activity at the end of modulation 57 periods. During this phase, the strong edge toroidicity-58 induced AE (TAE) activity (see Fig. 4(b4)) prevents re-59 distribution of fast ions in the outer region of $R>2.1~{\rm m}$ 60 at reduced energy of $E<75~{\rm keV}$.

To understand the connection of the observed inflow 62 (red) and outflow (blue) areas across the phase space, 63 fast ion migration trajectories are reconstructed, referred 64 to as the streamlines below. It is known that the mag-65 netic moment μ is conserved during resonant interactions 66 with AEs as long as the AE frequency is much lower than 67 the ion cyclotron frequency. Moreover, $E' \equiv E - \omega P_{\zeta}/n$ 68 is also conserved [7], where P_{ζ} is the toroidal canoni-69

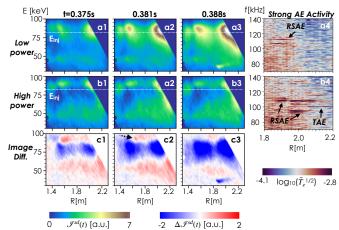


FIG. 4: INPA images of the modulated beam source during strong AE activity (from 0.375s to 0.388s). (a1)-(a3) Low power shot 179416. (b1)-(b3) High power shot 179415. (c1)-(c3) Image difference. T_e fluctuation in low-power shot (a4) and high-power shot (b4) at ~ 0.381 s.

cal momentum; ω and n are the angular frequency and toroidal mode number of the AE, respectively. To simultaneously satisfy the constraints, the streamline must follow the intersection of the curved, constant μ and E'planes in phase space. The curved μ and E' planes in the coordinate $(E, R, v_{\parallel}/v)$ are calculated by ASCOT5 and the intersection of two planes, i.e., $E' \& \mu$ streamlines, is identified. Owing to the finite pitch (v_{\parallel}/v) resolution discussed in Fig. 1(b), a majority part of the $E'\&\mu$ streamlines may intersect the phase space volume interrogated by the INPA. By scanning μ and E', and all observed ω/n , we reconstruct the streamlines in the INPA views, given as the dotted lines in Fig. 5(a). The streamline, associated with dominant reversed-shear AE (RSAE), having n=2, l=0, f=86 kHz in plasma frame, is labeled as b1; the streamlines of RSAE with n=2, l=1 and f=89 kHz for two E' constants are given as b2 and b3. (l: the number of nodes in the radial eigenmode envelope). Note that these streamlines are most relevant to the observed image pattern, since they pass through the radial positions of the minimum safety factor q_{min} at $R \sim 2.0$ m near the beam injection energy, and connect the outflow region and two inflow regions together. Besides, the majority part of streamlines, related to edge TAEs with lower ω/n , does not intersect the INPA-interrogated phase space and only a small portion appears well below the injection energy.

The phase-space flow is further investigated, using a nonlinear, kinetic-magnetohydrodynamics (MHD) hybrid code (MEGA) [17–19]. The simulation uses the measured plasma profiles, the equilibrium reconstructed by EFIT [20] and fast ion distributions ($\mathcal{F}_{\rm fi}$) from NUBEAM module of TRANSP [15] at ~ 0.44 s as the initial conditions. The $\mathcal{F}_{\rm fi}$ evolves in three separated periods in the

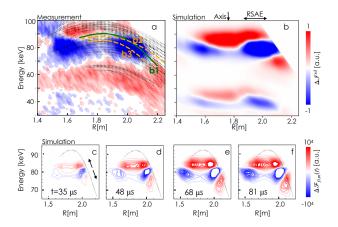


FIG. 5: (a) The measured flow images in (a), along with the streamlines; (b) the simulated flow image using MEGA-estimated fast ion distributions at INPA measured pitch of ~ 0.78 ; time evolution of the fast ion distributions in (c)-(f) by MEGA simulation.

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absence of MHD instabilities. This is to simulate the fast ion thermalization process. Following each period, the 37 kinetic-MHD hybrid phases are conducted subsequently 38 for 0.1 ms and the RSAEs of n=2 are routinely identi-39 fied. Meanwhile, the $\mathcal{F}_{\mathrm{fi,m}}$, i.e., the fast ion distribution 40 related to the modulated beam, is largely modified at 41 INPA interrogated pitch of 0.78. As one example, fig- 42 ures 5(c)-(f) present the change of $\mathcal{F}_{\rm fi,m}$ at 35, 48, 68 and 43 81 μ s after RSAE excitation in the third hybrid phase. 44 It is found that the outflow (blue) and the inflow (red) 45 region expands along the streamline, revealing formation 46 of RSAE-driven, fast ion phase space flow. Here, the 47 streamline b1, indicated by the black curve, is overlaid 48 as the reference. The synthetic INPA image in Fig 5(b) 49 is obtained by a convolution integral of the $\mathcal{F}_{\rm fi,m}$ at 81 μs $_{50}$ and the computed weight function. The result reproduces 51 the observed inflow (red) and outflow (blue) regions in 52 phase space, showing a reasonable agreement with the 53 averaged $\Delta \mathcal{I}^{\text{sd}}$ images in Fig. 5(a). It is worth pointing 54 out that the further expansion of the outflow (blue) re- 55 gion towards the boundary of INPA view (see Fig 4(c3)), 56 caused by the edge TAE modes, has not been reproduced 57 by MEGA. The discrepancy below 60 keV is also specu- 58 lated to be due to the lack of the TAE modes, which will 59 be addressed in future work.

AEs flatten the fast ion-density along $E'\&\mu$ stream-61 lines. The $\mathcal{I}_{\rm lp}^{\rm sd}$, averaged over the labeled streamlines in 62 Fig. 5(a), shows a hollow profile with a peak near the 63 $q_{\rm min}$ location at R=2.0 m in the low-power shot, as 64 seen from the black line in Fig. 6(a). This is because 65 the neutral beam nearly tangential to the magnetic field 66 line populates the magnetic axis region with a pitch of 67 ~ 0.68 , outside of the INPA-interrogated pitch [10]. In 68 contrast, the $\mathcal{I}_{\rm hp}^{\rm sd}$ in the high-power shot is significantly 69 flattened along the streamline (red line in Fig. 6(a)).

To understand the transport mechanism, the phase-71

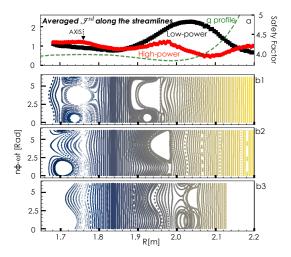


FIG. 6: (a) The averaged INPA signal along the streamlines b1-b3 in Fig. 5 in the LP shot (black) and in the HP shot (red), along with the q profile. The estimated phase-space islands along these streamlines b1-b3 in (b1)-(b3), respectively.

space islands along the labeled streamlines b1-b3 in Fig. 5(b) are studied using ASCOT5 code [14] and the results are given in Figs. 6(b1) -(b3). These island chains are generated by the wave-particle resonant interactions and visualized by the Poincaré plots of fast ions orbits in presence of RSAE modes. Fast ions with constant $\mu = 18 \text{ keV/T}$, interrogated by the INPA, are launched along the streamlines from the midplane. The radial structures of RSAE are obtained by a kinetic-MHD stability analysis code (NOVA-K) [21] and selected accordingly, based on the T_e fluctuation measured by electron cyclotron emission (ECE) [22]. The amplitude of each AE is carefully calibrated using magnetic field line tracing technique, which is able to determine the magnetic perturbation from the measured \tilde{T}_e profiles by mapping constant T_e along magnetic field lines. Note that the transient maximum amplitude of AEs are used, instead of the averaged value from Fast Fourier transform.

As seen from Figs. 6(b1)-(b3), good Kolmogorov-Arnold-Moser surfaces widely exist along each streamline. In other words, fast ions cannot flow freely along a single streamline to travel across the system. On the other hand, the observed flattening region is much broader than the widths of the phase-space islands. This contradiction suggests that fast ions cross between streamlines to achieve the large-scale phase space flow. One mechanism of streamline crossing is the natural intersection of streamlines with different values of ω/n . For example, the intersection of streamlines b1 and b2 would increase the phase-space flow traveling distance by 8 cm, as seen in Fig. 6(b1) and (b2); A second mechanism is scattering by collisions or turbulence from one streamline to a neighboring streamline of different radial extent. For example, the streamline b2 and b3 from the same RSAE are close to each other in Fig. 5(a). However, their phase space islands drift by ~ 8 cm, as seen from Fig. 6(b2) ⁴⁴ and (b3). It is hypothesized that the intermittency of fast ⁴⁵ ion avalanche transport [23] may be due to the sudden ⁴⁶ increase of the streamline crossing, either by the tran-⁴⁷ sient excitation of new instabilities or the randomized ⁴⁹ turbulence.

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In summary, the first ever visualization of fast ion 51 phase space flow driven by AEs is reported here. It is 52 found that the phase space flow is a 'two-way traffic' 53 i.e., inward transport towards higher energy and out-54 ward transport to reduced energy, determined by rela-55 tive velocity of fast ions and their initial positions to the 56 waves during resonant interactions. The result is consistent with non-linear kinetic-MHD hybrid simulations. 59 For the first time, flattening of the fast ion distribution 60 function along E'&u streamlines has been directly mea- 61 sured. When AEs are unstable even in the low-power 62 shot, the phase space along the streamline shows strong 63 self-organized criticality dynamics in high-power shot, 64 i.e., an open system with a fast relaxation mediation by $^{65}_{66}$ the threshold [24]. The transport along the streamline 67 is compared to the widths of phase-space islands, gener- 68 ated by the resonant interactions with AEs. The result 69 shows that streamline crossing, either due to the natural 70 intersection of streamlines or small pitch-angle scatter-71 ing by turbulence or collisions, plays a key role for the 72 large-scale phase-space transport.

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