Comment on “Spin-Momentum-Locked Edge Mode for Topological Vortex Lasing”

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Phys. Rev. Lett. **127**, 209401 — Published 12 November 2021

DOI: 10.1103/PhysRevLett.127.209401

In the Letter [1], Yang et al. reported on a topological vortex laser and proposed that the near-field spin and OAM of the topological edge mode lasing have a one-to-one far-field radiation correspondence [1]. Unfortunately, their frequency dispersion relations in Fig. 1(c) (see also Fig. S6) and Eqs. (5.3) and (5.4) in Supplemental Material of the Letter [1] are wrong, resulting in mistaken mode assignment, which is directly manifested in the far-field radiation pattern. This spoils the one-to-one correspondence claimed by the Letter.

From the Dirac-type Hamiltonian common for the Letter and those in literatures [1-6], the dispersion relations of the interfacial cavity between a central trivial photonic crystal cladded by a topological one with honeycomb structure [1] are derived analytically as

$$|l,+\rangle = e^{il\phi}\left(|p_+\rangle + e^{-i\phi}|d_+\rangle\right); \qquad \omega = \omega_0 - l\delta\omega \tag{1}$$

$$|l,-\rangle = e^{il\phi}\left(|p_-\rangle + e^{i\phi}|d_-\rangle\right); \qquad \omega = \omega_0 + l\delta\omega \tag{2}$$

with the same definition of cavity mode in Eqs. (5.3) and (5.4) of the Letter [1], and $\omega_0 \gg \delta\omega > 0$.

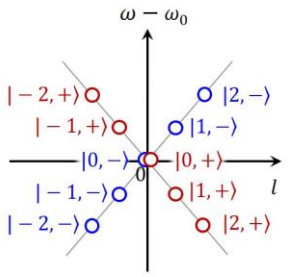

Fig. 1: Dispersion diagram for Eqs. (1) and (2), where the two modes $|0,\pm\rangle$ are shifted slightly in the horizontal direction for clarity.

We have numerically obtained by COMSOL [7] cavity modes $|2,+\rangle$ at frequency $f = 161.58$THz, $|1,+\rangle$ at $f = 163.49$ THz, $|0,+\rangle$ at $f = 165.52$ THz, $|-1,+\rangle$ at $f = 161.58$ THz, and $|-2,+\rangle$ at $f = 168.32$THz, and those for pseudospin-down modes by time-reversal symmetry. Namely, for pseudospin-up (-down) modes the eigen frequency increases (decreases) with decreasing l (see Fig. 1) agreeing with Eqs. (1) and (2), which are clearly opposite to those in the Letter. We also point out that the frequency dispersion in Fig. 1 (c) (see also Fig. S6) of the Letter is unphysical, where pseudospin-up and -down modes of the same OAM, such as ($|0,+\rangle$, $|0,-\rangle$) and ($|1,+\rangle$, $|1,-\rangle$) etc., do not take the same horizontal coordinates [1].

In Fig. 2(a,b,c,d), we show the wavefunction for mode $|-2,+\rangle$ (max and min grid size: 1/5 and 1/20 of the side length of airhole; relative tolerance of solution: 10^{-6}) [7] and the associated far-field radiation pattern calculated by Lumeircal [8], which are both missing in the Letter. It is clear that the far-field pattern in Fig. 2(d) is completely different from Fig. 3(a) of the Letter [1]. In contrast, the far-field pattern in Fig. 2(h) for mode $|2,+\rangle$ is close to Fig. 3(a) of the Letter, regarding both the scale

associated with the numerical aperture and detailed structures (except for the center part often influenced by incident stimulating light beam). Therefore, the cavity mode associated with Fig. 3(a) of the Letter should be $|2,+\rangle$, rather than $|-2,+\rangle$ as given in the Letter, a wrong mode identification caused by the mistaken dispersion relations (see Fig. S6 in Supplemental Material) [1]. Also we notice that the two far-field patterns in Figs. 3(a,b) of the Letter do not match each other [1].

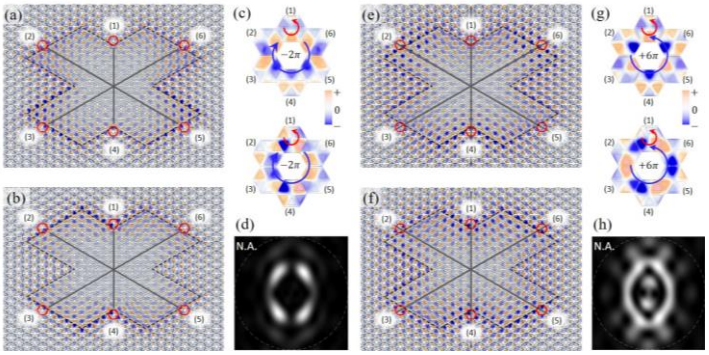

Fig. 2: (a,b) Real and imaginary parts, (c) illustration for mode identification, and (d) far-field radiation pattern for mode $|-2,+\rangle$. (e,f,g,h) same as (a,b,c,d) except for mode $|2,+\rangle$.

This work is partially supported by CREST, JST (Core Research for Evolutionary Science and Technology, Japan Science and Technology Agency) under the grant number JPMJCR18T4.

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