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Low mechanical loss TiO₂:GeO₂ coatings for reduced thermal noise in Gravitational Wave Interferometers

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The sensitivity of current and planned gravitational wave interferometric detectors is limited, in the most critical frequency region around 100 Hz, by a combination of quantum noise and thermal noise. The latter is dominated by Brownian noise: thermal motion originating from the elastic energy dissipation in the dielectric coatings used in the interferometer mirrors. The energy dissipation is a material property characterized by the mechanical loss angle. We have identified mixtures of titanium dioxide (TiO₂) and germanium dioxide (GeO₂) that show internal dissipations at a level of 1×10^{-4} , low enough to provide almost a factor of two improvement on the level of Brownian noise with respect to the state-of-the-art materials. We show that by using a mixture of 44% TiO₂ and 56% GeO₂ in the high refractive index layers of the interferometer mirrors, it would be possible to achieve a thermal noise level in line with the design requirements. These results are a crucial step forward to produce the mirrors needed to meet the thermal noise requirements for the planned upgrades of the Advanced LIGO and Virgo detectors.

Gravitational wave (GW) detectors are highly sensitive instruments that measure the very small distance changes produced by signals of astrophysical origin [1, 2]. The current generation of GW detectors are km-scale laser interferometers [3–6] with several hundreds of kW of circulating power in the Fabry-Perot arm cavities. The test-mass mirrors are made of high-purity fused silica substrates, coated with high-reflectivity multilayer dielectric thin-film stacks [7], composed of multiple pairs of high and low refractive index metal oxide layers, making a Bragg reflector structure.

The sensitivity of the current detectors [8, 9] is limited by a combination of laser quantum noise [10] and displacement noise generated by the Brownian motion of the coatings [11, 12]. Therefore, to increase the astrophysical reach of future detectors, it is crucial to reduce coating Brownian noise. This in turn requires reducing the elastic energy dissipation in the thin film materials composing the coatings [12, 13]. The power spectral density of Brownian noise at a frequency f is a complex function of the properties of the materials used in the coatings [14, 15]. An approximate expression, assuming equal bulk and shear loss angles, is given by (see [15] and supplemental material [16]):

$$S_B(f) = \frac{2k_B T d}{\pi^2 w^2 f} \left[\left\langle \frac{Y}{1 - \nu^2 \phi} \right\rangle \frac{(1 + \nu_S)^2 (1 - 2\nu_S)^2}{Y_S^2} + \left\langle \frac{(1 + \nu)(1 - 2\nu)}{(1 - \nu)Y} \phi \right\rangle \right] \quad (1)$$

where k_B is the Boltzmann's constant, T is the ambient temperature, w is the radius of the laser beam probing

the mirror motion, d is the total thickness of the coating, Y_S and ν_S are the Young's modulus and Poisson ratio of the substrate. The angular bracket expression $\langle x \rangle$ indicates the *effective medium* average [15, 17] of the material property x through the stack, weighted by the physical thickness of the layers. The relevant properties of the coating materials are the Young's moduli Y , the Poisson ratios ν and the loss angles $\phi = \text{Im}(Y)/\text{Re}(Y)$.

The coatings used in the current Advanced LIGO mirrors are composed of alternating layers of amorphous SiO₂ of low refractive index $n_{\text{SiO}_2} = 1.45$ at 1064 nm, and TiO₂:Ta₂O₅ of high refractive index $n_{\text{TiO}_2:\text{Ta}_2\text{O}_5} = 2.10$ at 1064 nm [18, 19]. The TiO₂:Ta₂O₅ layers have a loss angle much larger than the SiO₂ layers ($3 - 4 \times 10^{-4}$ [20, 21] compared to $\sim 2 \times 10^{-5}$ [18]) and therefore they dominate in the contribution to the coating Brownian noise.

The goal for the next upgrade to the LIGO detectors, called Advanced LIGO+ [22, 23] is a reduction of the coating noise by about a factor of two, with a target Brownian noise of $S_B^{1/2} = 6.6 \times 10^{-21} \text{m}/\sqrt{\text{Hz}}$ at a frequency of 100 Hz. The SiO₂ layers can already be produced with low enough mechanical loss angle [18], so the main focus of the current research is on improving the high refractive index material. Several different approaches have been investigated, including deposition at elevated substrate temperatures [24, 25] and with assist ion bombardment [26, 27], doping and nanolayering of Ta₂O₅ [28–31], and the use of nitrides [32, 33]. Here we report results on amorphous oxide coatings based on mixtures of GeO₂ and TiO₂.

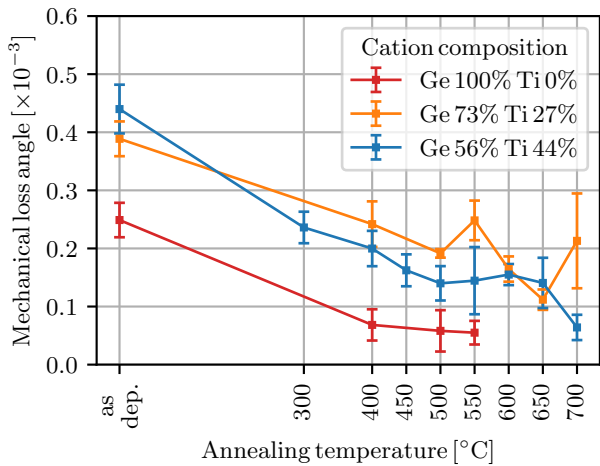


FIG. 1. Measured loss angle of $\text{TiO}_2:\text{GeO}_2$, as deposited and after 10-hours-long annealing in air, at increasing temperatures. Different color lines correspond to the cation composition listed in the legend. Only one sample for each concentration is shown here for simplicity. Other samples showed equal values within the error bars.

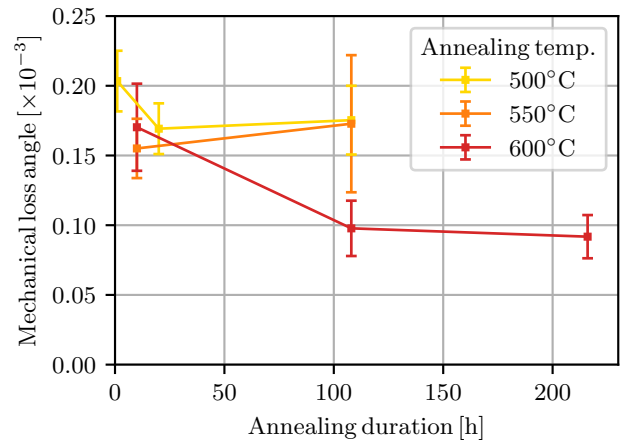


FIG. 2. Effect of the annealing duration on the measured loss angle for the 44% $\text{TiO}_2:\text{GeO}_2$ film.

82 The initial motivation to investigate coatings based on
 83 GeO_2 was the discovery of a correlation between the
 84 room-temperature mechanical loss angle and the fraction
 85 of edge-sharing versus corner-sharing polyhedra in the
 86 medium-range order, as reported in [34] for $\text{ZrO}_2:\text{Ta}_2\text{O}_5$.
 87 SiO_2 also has a prevalence of corner-sharing, and is
 88 the amorphous material that exhibits the lowest known
 89 room-temperature loss angle in the acoustic frequency
 90 range [18, 35, 36]. Additionally, the mechanical loss angle
 91 of GeO_2 at low temperatures [37] ($\lesssim 100$ K) exhibits
 92 a peak similar to the one found in SiO_2 [38, 39]. In recent
 93 experiments on GeO_2 [40], we confirmed that the atomic
 94 packing can be altered to improve medium range order by
 95 annealing and high temperature deposition. Similar correlations
 96 were found for different oxides by other groups
 97 [41–45].

98 From the optical perspective, however, GeO_2 has a refractive
 99 index $n = 1.60$ at 1064 nm that makes it unsuitable for use in a high reflector design when combined
 100 with SiO_2 , as 138 layers would be needed to achieve the
 101 required high reflectivity for the test-mass mirrors. The
 102 increase in the total thickness of a $\text{GeO}_2/\text{SiO}_2$ reflector
 103 would balance out the reduced mechanical loss, with
 104 no net improvement in the coating Brownian noise. To
 105 increase the refractive index at the laser wavelength of
 106 1064 nm, GeO_2 was co-deposited with TiO_2 with different
 107 cation concentrations.

108 Thin films of $\text{TiO}_2:\text{GeO}_2$ with Ti cation concentration of
 109 0%, 27%, and 44%, were deposited by ion beam sputtering
 110 using a biased target deposition system [46], that
 111 allowed convenient tuning of the mixture composition by
 112 adjusting the length of the pulses biasing the metallic Ti
 113 and Ge targets.

114 The cation concentration, oxygen stoichiometry and

115 atomic areal density of the films were determined by
 116 Rutherford backscattering spectrometry (RBS) [47]. The
 117 thickness and refractive index were obtained from spectroscopic
 118 ellipsometry. The mass density was computed from the RBS and
 119 ellipsometry measurements. The absorption loss at the wavelength
 120 of 1064 nm was assessed from photo-thermal common-path
 121 interferometry [48]. The thin films were annealed in air, as
 122 annealing has been shown to reduce absorption loss and
 123 room-temperature mechanical loss angle in amorphous oxides
 124 [49, 50]. Grazing incidence x-ray diffraction shows all
 125 mixture films are amorphous upon annealing at 600°C for 10
 126 and 108 hours, and show signs of crystallization when
 127 annealed at higher temperatures. The pure GeO_2 film
 128 remained amorphous up to 550°C.

129 For the Ti cation concentration of 44%, the refractive index
 130 at 1064 nm is $n_{\text{TiO}_2:\text{GeO}_2} = 1.88$. The absorption loss
 131 normalized to a quarter-wavelength (QWL) thick single
 132 layer (141 nm) is 2.3 ± 0.1 ppm after annealing at
 133 600°C. The absorption loss of pure GeO_2 after annealing
 134 at 500°C is below 1 ppm at $\lambda = 1064$ nm, showing the
 135 potential for improved absorption in the mixture. The
 136 deposition parameters are being optimized to achieve even
 137 lower optical absorption in $\text{TiO}_2:\text{GeO}_2$, to meet the
 138 advanced LIGO+ requirements of less than 1 ppm [23] for
 139 a full mirror coating. More details on the structural and
 140 optical characterizations are available in the supplemental
 141 material [16].

142 The thin films were also deposited with the same procedure
 143 on 75-mm-diameter, 1-mm-thick silica disks, to measure the
 144 material's elastic properties. The disk acts as a resonator:
 145 about 20 modes between 1 kHz and 30 kHz can be measured
 146 in a Gentle Nodal Suspension [51, 52] to obtain their precise
 147 frequency and decay time. After the thin film is deposited
 148 on the substrate, the resonant frequencies are shifted by
 149 amounts depending on the film properties, allowing an
 150 estimation of the Young's

153 modulus and Poisson ratio [18, 53]. The decay times
 154 of the modes of the coated substrates are significantly
 155 shorter than for the bare substrate, due to the elastic
 156 energy dissipation in the film. Using the measured elas-
 157 tic properties of the film material, one can compute the
 158 fraction of elastic energy in the film for each resonant
 159 mode and use it to extract the loss angle ϕ of the thin
 160 film material [54].

161 For a homogeneous amorphous material, the relation
 162 between stress and strain in the elastic regime can be
 163 described in terms of two elastic moduli, for example
 164 bulk K and shear μ moduli [55]. Similarly, the internal
 165 energy dissipation in the material should be described
 166 in terms of two loss angles $\phi_K = \text{Im}(K)/\text{Re}(K)$ and
 167 $\phi_\mu = \text{Im}(\mu)/\text{Re}(\mu)$. There is no physical reason to as-
 168 sume the two loss angles to be equal, and we shall show in
 169 the following that they are indeed significantly different
 170 for $\text{TiO}_2:\text{GeO}_2$. The layered structure of the stack im-
 171 plies that a description in terms of an equivalent isotropic
 172 material is not accurate, since the bulk and shear energy
 173 distribution in the layers is different in the case of the
 174 ring-down measurements and in the Brownian noise case.
 175 While the expression in equation 1 assumes equal loss an-
 176 gles, a more precise expression, including the distinction
 177 between bulk and shear properties for all layers, is de-
 178 scribed in the supplemental material [16], and is needed
 179 to correctly account for the multilayer structure and the
 180 different materials.

181 However, in the initial exploration of the effect of compo-
 182 sition and annealing schedule, we relied on the commonly
 183 used description with frequency independent equal bulk
 184 and shear loss angles [18, 56, 57]. The more detailed
 185 analysis of the best candidate material, described later,
 186 supports the use of this simplification for survey pur-
 187 poses, since our measurements are more sensitive to the
 188 shear than the bulk loss angle, and the former is found to
 189 be almost frequency independent. With this approach,
 190 figure 1 shows the measured loss angle for pure GeO_2
 191 and the two concentrations of TiO_2 and GeO_2 studied
 192 in detail here. The most promising results are from a
 193 mixture of 44% TiO_2 and 56% GeO_2 . The mechanical
 194 loss of amorphous oxides typically decreases with increas-
 195 ing annealing temperature and time. We observed rapid
 196 crystallization at 700°C, and therefore explored the ef-
 197 fect of annealing duration on the loss angle. We tested
 198 heat treatments of 1, 10, 20, 108 and 216 hours in to-
 199 tal, for temperatures of 500, 550 and 600°C. Figure 2
 200 shows the effect of annealing time on the loss angle of
 201 the 44% $\text{TiO}_2:\text{GeO}_2$ mixture. It was found that extended
 202 annealing at lower temperatures produces little improve-
 203 ment. Instead, after annealing at 600°C for 108 hours,
 204 the loss angle is reduced to $(0.96 \pm 0.18) \times 10^{-4}$, and the
 205 film is still amorphous. Among those tested in our work,
 206 this $\text{TiO}_2:\text{GeO}_2$ mixture is the most promising high-index
 207 material for low Brownian noise Advanced LIGO+ mir-
 208 rrors, though further characterization of mixtures with
 209 other Ti/Ge ratios in this range is planned to find the

SiO₂ property	Value
Refr. index at 1064 nm	1.45 ± 0.01
Young's modulus	73.2 ± 0.6 GPa
Poisson ratio	0.11 ± 0.07
Loss angle	$\phi_K = \phi_\mu = (2.6_{-0.6}^{+0.5}) \times 10^{-5}$
TiO₂:GeO₂ property	Value
Cation conc. Ti/(Ti+Ge)	44.6 ± 0.3 %
Refr. index at 1064 nm	1.88 ± 0.01
Optical abs. for a QWL	2.3 ± 0.1 ppm
Density	3690 ± 100 kg/m ³
Young's modulus	91.5 ± 1.8 GPa
Poisson ratio	0.25 ± 0.07
Bulk Loss angle	$a_K = (22.0_{-12.5}^{+10.6}) \times 10^{-5}$ $m_K = 1.04_{-0.36}^{+0.40}$
Shear Loss angle	$a_\mu = (8.4_{-4.0}^{+2.9}) \times 10^{-5}$ $m_\mu = -0.06_{-0.30}^{+0.15}$

TABLE I. Measured parameters for $\text{TiO}_2:\text{GeO}_2$ and SiO_2 , af-
 ter annealing at 600°C for 108 hours. The loss angle model for
 $\text{TiO}_2:\text{GeO}_2$ is $\phi(f) = a \cdot (f/10\text{kHz})^m$. Uncertainties describe
 the 90% confidence intervals.

210 optimum.

211 As a first step toward the production of a full high-
 212 reflectivity coating, and to better characterize this
 213 new material, we deposited single layers of SiO_2 and
 214 $\text{TiO}_2:\text{GeO}_2$, as well as a stack of 5 QWL layers of
 215 $\text{TiO}_2:\text{GeO}_2$ alternated with 5 layers of SiO_2 , and 20 lay-
 216 ers of $\text{TiO}_2:\text{GeO}_2$ alternated with 20 layers of SiO_2 . The
 217 depositions were performed using a commercial Spector
 218 Ion Beam Sputtering system that can produce films with
 219 better optical quality [26] than the biased target system
 220 used for the initial parameter exploration. At the laser
 221 wavelength of 1064 nm, the transmission of the 40-layer
 222 structure was 190 ppm and the optical absorption was
 223 measured to be 3.1 ppm after annealing. We also mea-
 224 sured the Young's modulus and loss angle of the stacks.
 225 However, since the multilayers structure is not isotropic,
 226 a description in terms of Y and ν is only approximate.
 227 Nevertheless, the two stacks were found to have the same
 228 Young's modulus, 78.0 ± 1.3 GPa, and the same loss angle,
 229 $(5.5 \pm 0.7) \times 10^{-5}$ after annealing at 600°C for 108 h. This
 230 is an indication that there is no evidence of any system-
 231 atic error in the measurements due to the thickness of the
 232 coatings. At an approximation level consistent with as-
 233 suming equal bulk and shear loss angles, one can compute
 234 the expected value for the stack by averaging the single
 235 material values as $\bar{\phi} = \langle Y \phi \rangle / \langle Y \rangle = (6.4 \pm 1.7) \times 10^{-5}$.
 236 Therefore there is no indication of excess loss due to in-
 237 terfaces [58].

238 A correct description of the Brownian noise in a multi-
 239 layer stack must take into account the bulk and shear
 240 moduli and loss angles of the individual materials. The
 241 resonant modes of the coated disk store different fractions

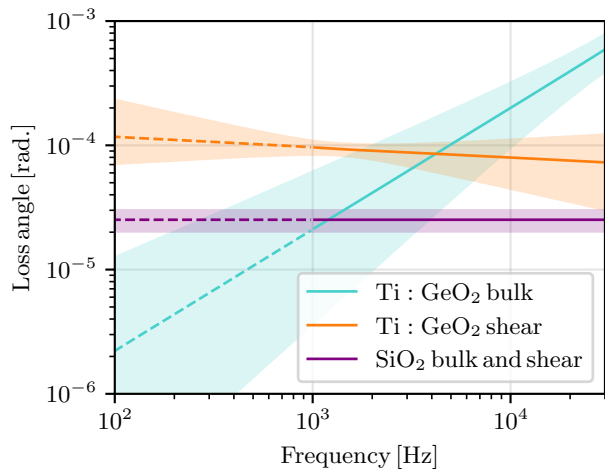


FIG. 3. Estimated bulk and shear loss angles as a function of frequency. The solid lines indicate the range of frequencies where the loss angles were measured, while the dashed lines are extrapolations to the lower frequency range. The shaded regions show the 90% confidence intervals of the estimates.

242 of bulk and shear energy in the film, and therefore it is
 243 possible to extract the bulk and shear loss angles from
 244 the measurements [53, 59]. We model a single isotropic
 245 layer with known thickness and density as measured by
 246 ellipsometry and RBS, and with Young's modulus, Pois-
 247 son ratio and bulk and shear loss angles as free param-
 248 eters. For each sample, the measurement data set con-
 249 sists of the frequency shifts due to the coating, and the
 250 reduction in the decay time, due to the energy dissipa-
 251 tion in the coating, for each of the measurable modes.
 252 We used a Markov chain Monte Carlo Bayesian Analy-
 253 sis [53, 60, 61] to find the probability distribution of the
 254 model parameters given the data. We considered either
 255 different bulk and shear loss angles or equal loss angles,
 256 and three possible frequency dependencies: constant, lin-
 257 ear or power law, for a total of six different loss models.
 258 The Bayesian analysis allows us to compute the relative
 259 likelihood of each model given the data. The best model
 260 for the $\text{TiO}_2:\text{GeO}_2$ film is a power law with different bulk
 261 and shear loss angles, while for the SiO_2 film it is a con-
 262 stant single loss angle, as shown in figure 3. It is worth
 263 noting that the bulk loss angle for the $\text{TiO}_2:\text{GeO}_2$ film
 264 shows a rather steep frequency dependence. The second
 265 most likely model for this material is the one with a lin-
 266 ear frequency dependence. The bulk loss angle does not
 267 show a frequency dependency as steep as in the power law
 268 case, but it is still predicted to be significantly smaller
 269 than the shear loss angle at low frequency. The measured
 270 value of the loss angle for SiO_2 is compatible with values
 271 reported in the literature [18]. Table I summarizes all
 272 the measured material properties. More details on the
 273 analysis and the results are in the supplemental material
 274 [16].
 275 The transmission requirements for the Advanced LIGO+
 276 test masses are similar to those for Advanced LIGO: the

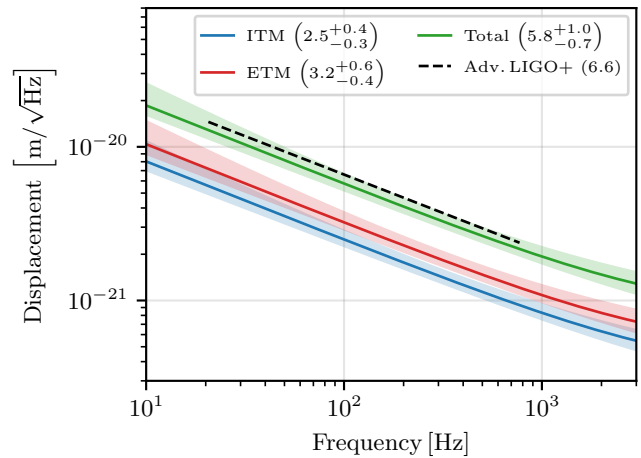


FIG. 4. Estimated Brownian noise for the Advanced LIGO+ interferometer. The red and blue traces show the contribution of a single ITM and ETM, while the green trace shows the total for all four test masses. The numbers in the legend give the Brownian noise level at 100 Hz, in units of $10^{-21}\text{m}/\sqrt{\text{Hz}}$. The shaded regions correspond to the 90% confidence intervals of the estimates. The dashed black line shows the design target for Advanced LIGO+.

277 input mirror test masses (ITM) should have a transmis-
 278 sion of 1.4% and the end test masses (ETM) of 5 ppm [3].
 279 Given the measured refractive indexes, the ITM stack is
 280 composed of 11 layers of 106 nm of $\text{TiO}_2:\text{GeO}_2$ alternated
 281 with 11 layers of 228 nm of SiO_2 , while the ETM stack
 282 is composed of 26 layers of 123 nm of $\text{TiO}_2:\text{GeO}_2$ and
 283 26 layers of 207 nm of SiO_2 . Both structures are capped
 284 with a half-wavelength-thick SiO_2 layer.

285 The Brownian noise for such mirrors can be computed
 286 using the effective medium approach described in the
 287 supplemental material [16], which has been checked to
 288 provide results within a few percent of other published
 289 formulas [14, 62]. The results are shown in figure 4. The
 290 noise is compliant with the design requirement for Ad-
 291 vanced LIGO+, reaching $(5.8^{+1.0}_{-0.7}) \times 10^{-21}\text{m}/\sqrt{\text{Hz}}$ at 100
 292 Hz. It is worth noting that this result does not depend
 293 strongly on the steep frequency dependency predicted for
 294 the bulk loss angle of $\text{TiO}_2:\text{GeO}_2$. If we use the second
 295 most probable model, with a less steep frequency depen-
 296 dency, we obtain $(6.2^{+1.5}_{-0.8}) \times 10^{-21}\text{m}/\sqrt{\text{Hz}}$ at 100 Hz,
 297 still compatible with the Advanced LIGO+ design re-
 298 quirement. Preparation of samples on disks with lower
 299 resonant frequencies to better constrain these estimates
 300 is underway.

301 In summary, we demonstrated that a mixture of 44%
 302 TiO_2 and 56% GeO_2 offers excellent optical quality and
 303 low mechanical loss angle, making it a promising material
 304 to be used as high-index layer in the test mass coatings
 305 of the Advanced LIGO+ interferometric GW detectors.
 306 We analyzed the internal energy dissipation of this novel
 307 material in terms of bulk and shear loss angles, and used
 308 the results to design multilayer high reflectivity stacks

for the Advanced LIGO+ mirrors. The Brownian noise achievable with $\text{TiO}_2:\text{GeO}_2 / \text{SiO}_2$ based mirrors reaches a level compliant with the Advanced LIGO+ design requirements.

Studies are on-going to further improve mechanical and optical absorption losses by changing deposition parameters, mixture and annealing schedule, and to characterize the scattering properties of the multilayer stacks. We are also planning to directly measure the Brownian noise of optimized high reflection mirrors designed for Advanced LIGO+ [20], to confirm the noise prediction.

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