Layered Chiral Active Matter: Beyond Odd Elasticity

S. J. Kole, Gareth P. Alexander, Sriram Ramaswamy, and Ananyo Maitra

Phys. Rev. Lett. **126**, 248001 — Published 16 June 2021

DOI: 10.1103/PhysRevLett.126.248001

Layered chiral active matter: beyond odd elasticity

S. J. Kole,[1,*] Gareth P. Alexander,[2,†] Sriram Ramaswamy,[1,‡] and Ananyo Maitra[3,§]

[1] *Centre for Condensed Matter Theory, Department of Physics, Indian Institute of Science, Bangalore 560 012, India*
[2] *Department of Physics and Centre for Complexity Science, University of Warwick, Coventry CV4 7AL, United Kingdom*
[3] *Sorbonne Université and CNRS, Laboratoire Jean Perrin, F-75005, Paris, France*

In equilibrium liquid crystals, chirality leads to a variety of spectacular three-dimensional structures, but chiral and achiral phases with the same broken continuous symmetries have identical long-time, large-scale dynamics. In this paper, starting from *active model H**, the general hydrodynamics of a pseudoscalar in a momentum-conserving fluid, we demonstrate that chirality qualitatively modifies the dynamics of layered liquid crystals in active systems in both two and three dimensions due to an active "odder" elasticity. In three dimensions, we demonstrate that the hydrodynamics of active cholesterics differs fundamentally from smectic-A liquid crystals, unlike their equilibrium counterpart. This distinction can be used to engineer a columnar array of vortices, with anti-ferromagnetic vorticity alignment, that can be switched on and off by external strain. A two-dimensional chiral layered state – an array of lines on an incompressible, free-standing film of chiral active fluid with a preferred normal direction – is generically unstable. However, this instability can be tuned in easily realisable experimental settings, when the film is either on a substrate or in an ambient fluid.

Chiral molecules form a spectacular range of liquid-crystalline phases [1–3] at thermal equilibrium, of which the best known is the cholesteric, with a helical structure in which the molecular orientation, described by a headless unit vector called the director $\hat{\mathbf{n}}$, spontaneously twists at a uniform rate q_0 along the pitch axis [3]. This uniform periodic modulation does not break translational invariance: unlike in a density wave, all surfaces of constant phase are equivalent, and an arbitrary translation along the pitch axis can be compensated by a rotation about it. Nevertheless, at scales much larger than $1/q_0$, the mechanics of a cholesteric is precisely the same as that of a smectic A which has an achiral one-dimensional *density* modulation. That is, microscopic chirality leads to a one-dimensional periodic structure, but the asymptotic long-wavelength elasticity and hydrodynamics of this structure show no signature of chirality [4–6]. In this Letter we show that this equivalence does not carry over to active cholesteric and smectic A phases [7, 8] when chirality couples non-trivially to the active drive. Thanks to the presence of a mix of solid- and liquid-like directions, we predict an effect odder than odd elasticity [9], in the form of a linear elastic force even at zero strain in directions in which there is no displacement field.

Recall that active systems are materials with a sustained supply of free energy, and hence broken detailed balance, at the scale of its constituents. This microscale drive manifests itself macroscopically as nonequilibrium currents and forces [10–19]. Continuum hydrodynamic theories of fluid [20], liquid-crystalline and crystalline phases [7, 8, 14, 16, 21–23] of active matter have been constructed including extensions with chiral asymmetry [9, 24–32]. In this Letter, we construct theories of layered active chiral systems. Our hydrodynamic theory starts from an active model H* – the chiral and active variant of model H [33] that describes the coupled dynamics of a conserved scalar order parameter and a conserved momentum density field – and applies generically to active variants of any one-dimensional spatially modulated chiral state, not only cholesterics in the traditional sense. A two-dimensional active model H* also leads to the theory of an active two-dimensional chiral smectic, which could arise if three-dimensionally chiral particles were restricted to a thin film with a distinguished normal direction.

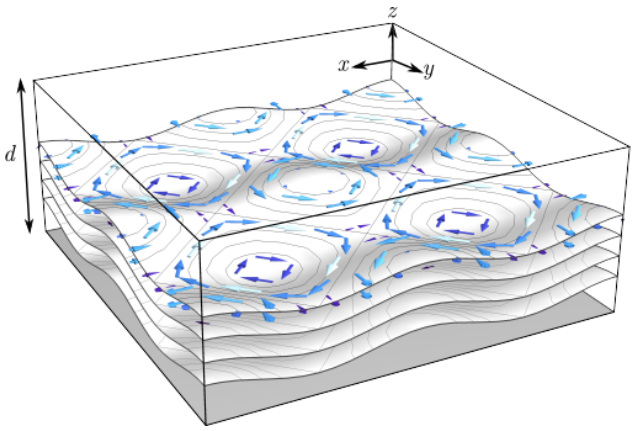

FIG. 1. The spontaneous vortex-lattice state: the chiral active force density (governed by z_c) generates counterrotating circulatory flows, with vorticity Ω_z obeying $\eta\nabla^2\Omega_z = -z_c\nabla^2\nabla_\perp^2 u$, around the undulations u of the active Helfrich-Hurault instability. Smectic layers (light grey) are shown only for the bottom half of the cell for clarity; blue arrows show the active flows only on the topmost visible layer. The bottom confining surface is shown in grey and the amplitude of the undulation varies as $\sin(\pi z/d)$, vanishing on both top and bottom surfaces.

Our central finding is that active cholesterics possess a chiral stress corresponding to a non-existent component of the strain tensor that yields a force density tangent to contours of constant mean curvature of the layers. As a result of this odder than odd elasticity, the undulational instability created by active stresses [7, 8] leads to spontaneous vortical flow arranged in a two-dimensional array with vorticity aligned along the pitch axis and alternating in sign in the plane (Fig. 1). This vortex-lattice state can be switched on or off by means of an externally imposed uniaxial stress. Lastly, a two-dimensional active cholesteric is unstable with an activity threshold that goes to zero for an infinite system. This tilted-varicose instability (Fig. 2) is however not inevitable, as we discuss later in the paper.

We now show how we obtain these results. A pattern-formation framework [34] offers a foolproof approach to the construction of the hydrodynamic equations for active cholesterics, equivalent to the traditional route [7, 8] starting with the equations of motion for an orientation field and eliminating the fast degrees of freedom, but here circumventing the need to identify and remove the non-hydrodynamic mode. Accordingly, we begin by extending [35, 36] to define *active model H**: the coupled dynamics of a pseudoscalar density ψ governed by a conservation law $\partial_t\psi = -\nabla\cdot\mathbf{J}$ and a momentum density $\rho\mathbf{v}$ whose dynamics in the Stokesian regime is governed by $\nabla\cdot\boldsymbol{\sigma} = 0$, with current $\mathbf{J} = \psi\mathbf{v} - M\nabla\mu + \mathbf{J}_a + \mathbf{J}_c$ and stress tensor $\boldsymbol{\sigma} = -\eta[\nabla\mathbf{v} + (\nabla\mathbf{v})^T] + \boldsymbol{\sigma}_\psi + p\mathbf{I} - \boldsymbol{\sigma}_c - \boldsymbol{\sigma}_a$, where subscripts a and c denote active achiral and chiral contributions respectively. Here M is a mobility, $\mu = \delta F/\delta\psi$ is a chemical potential expressed in terms of a free-energy functional $F[\psi]$, η is a viscosity, the passive force density $-\nabla\cdot\boldsymbol{\sigma}_\psi = -\psi\nabla\mu$ is the Onsager counterpart to $\psi\mathbf{v}$ [37], and the pressure p imposes overall incompressibility $\nabla\cdot\mathbf{v} = 0$. $\mathbf{J}_a = \lambda_1\psi\nabla\psi\nabla^2\psi + \lambda_2\psi\nabla(\nabla\psi)^2$ as familiar from active models B and H [35, 36, 38–41]. In what follows we ignore the chiral currents $\mathbf{J}_c$, whose effects on the dynamics of layered states arise at sub-leading order in wavenumber [42]. The achiral active stress [35, 36], in both two and three dimensions, is $\boldsymbol{\sigma}_a = \zeta_H\nabla\psi\nabla\psi$ while the chiral active stress is

$$(\sigma_c)_{ij} = \bar{z}_c\partial_l(\epsilon_{ijk}\partial_k\psi\partial_l\psi),\ d=3;$$
$$\boldsymbol{\sigma}_c = \bar{\zeta}_c\left[\boldsymbol{\varepsilon}\cdot\nabla\psi\nabla\psi - \frac{1}{2}\boldsymbol{\varepsilon}(\nabla\psi)^2\right], d=2, \quad (1)$$

where $d = 2$ corresponds to a thin film of 3D chiral material with a distinguished normal taken to be along $\mathbf{N} \equiv +\hat{\mathbf{y}}$, thus inheriting uniquely the two-dimensional antisymmetric tensor $\boldsymbol{\varepsilon}$ with components $\varepsilon_{ij} = \epsilon_{ikj}N_k$. Though here written as an antisymmetric stress $(\sigma_c)_{ij}$ can be given in an equivalent symmetric form and is allowed in momentum-conserving systems [31, 42, 49, 50].

A Swift-Hohenberg free-energy functional F [42, 51] allows model H* to describe the dynamics of spatially modulated states $\psi = \psi_0 + \psi_1$ where ψ_1, with zero spatial average, represents a modulation with wavelength $2\pi/q_s$ about a uniform background ψ_0. We consider the dynamics about a steady state with a one-dimensional spatial modulation, $\psi_1 = \psi_1^0(e^{i\phi} + e^{-i\phi})$, with $\phi = q_s(z-u)$, describing a periodic array of parallel lines or planes of constant phase, in $d = 2$ or 3 respectively, with normal along $\hat{\mathbf{z}}$, with small fluctuations $u(\mathbf{r}, t)$. We begin with $d = 3$. Defining the scaled phase-gradient vector, which is parallel to the normal of the fluctuating layers, as $\mathbf{n} = \nabla\phi/q_s = \hat{\mathbf{z}} - \nabla u(x, y, z, t)$ we obtain the dynamical equation of the displacement field of the layers from their mean positions [42]:

$$\partial_t u = \mathbf{v}\cdot\mathbf{n} + \Lambda_1\mathbf{n}\cdot\nabla E + \Lambda_2\nabla\cdot\mathbf{n}(1-2E) - \Gamma_u\frac{\delta F[u]}{\delta u}, \quad (2)$$

where $\Lambda_1 = -2{\psi_1^0}^2q_s^2(\lambda_1+\lambda_2)$ and $\Lambda_2 = 2{\psi_1^0}^2q_s^2\lambda_2$ are active, achiral permeative terms, the final term is passive permeation with $\Gamma_u = -Mq_s^2$, and $E = \partial_z u - (1/2)(\nabla u)^2$ is the covariant strain. Finally, $F[u] = (1/2)\int[BE^2 + K(\nabla^2 u)^2]$ is the rotation-invariant free energy [3, 52] which would have controlled the relaxational dynamics of the cholesteric state in the absence of activity with B being the layer-compression modulus and K being the bending rigidity of the layers which can be expressed in terms of the coefficients in the Swift-Hohenberg free energy [42]. Force balance for our system takes the form

$$\eta\nabla^2 v_i = n_i\frac{\delta F[u]}{\delta u} + \partial_i p + \partial_j[\zeta w_{ij} + z_c\partial_l(\epsilon_{ijk}w_{kl})] \quad (3)$$

where w_{ij} encodes the active stresses from (1) and preceding. To linear order in displacements, with $\perp$ denoting directions transverse to $\hat{\mathbf{z}}$, $\boldsymbol{w}_{z\perp} = \boldsymbol{w}_{\perp z} = \nabla_\perp u$, $w_{z\tilde{z}} = 2\partial_z u$ and all other components are 0, $\zeta = {\psi_1^0}^2q_s^2\zeta_H$, $z_c = {\psi_1^0}^2q_s^2\bar{z}_c$ and the pressure p enforces three-dimensional incompressibility $\nabla\cdot\mathbf{v} = 0$. The term proportional to z_c in (3) is the chiral active force density. It is the curl of the vector $\partial_l w_{kl}$ and is therefore divergence-free. Expressed as a vector this chiral active force density is $-z_c\,\hat{\mathbf{z}}\times\nabla_\perp(\nabla_\perp^2 u + \partial_z^2 u)$ and is directed (primarily) tangentially to contours of constant mean curvature of the layer undulation, driving the vortical flow shown in Fig. 1. Like odd elasticity, the z_c-term in (3) is a parity-breaking stress in response to layer displacements. In-plane gradients of layer mean curvature create responses in the perpendicular in-plane direction like an "odd" Laplace pressure gradient [53]. Unlike the odd elastic force density of two-dimensional chiral active solids [9], which arises from an antisymmetry in the linear relation between stress and strain, this cholesteric chiral force density (3) arises even when the strain $E = 0$.

It might seem that z_c does not affect the hydrodynamics of the layered state as it appears at a higher order in gradients than the achiral active force. Indeed, it does not affect the linear dynamics of the displacement field

at all: the eigenfrequency for displacement fluctuations to leading order in wavenumber, obtained by projecting (3) transverse to the wavevector and solving for the velocity field is $\omega = -(i/\eta q^4)(Bq_z^2 q_\perp^2 - \zeta q_\perp^2 q^2)$. Therefore, as noted in [7, 8], the dynamics of the displacement field of cholesterics and smectics are indeed equivalent, with the layered state having long-range order in three dimensions for $\zeta < 0$ (unlike their equilibrium counterparts which only have quasi-long-range order in three dimensions) and being unstable for $\zeta > 0$. However, the hydrodynamics of active cholesterics differs crucially from that of smectics through the effect of the chiral active force on the velocity field *in the plane of* the layers.

The vortical flow caused by the chiral active force can be used to control and create a vortex lattice state [54] with a well defined lattice constant in an active cholesteric system. This hinges on a mapping between an *externally* imposed stress and an internal and active *achiral* stress. An external stress can be imposed via a free energy term $F_{ext}[u] = \int \sigma_0 E$, which gives rise to a force $-\sigma_0 \nabla \cdot (\boldsymbol{w} + E\mathbf{l})$ [42]. As a consequence an achiral active stress acts identically to an *external* stress with $\sigma_0 = \zeta$ up to an isotropic piece which can be absorbed into the pressure in an incompressible system and the instability of an *active* layered state for $\zeta > 0$ maps onto the Helfrich-Hurault instability of a passive layered state under dilative stress [55, 56]. In the externally-stressed instability a square lattice undulated pattern $u = u_0 \cos q_p x \cos q_p y$ (an egg-crate-like structure) is realised [57, 58] and because of this mapping, the same pattern should be realised beyond the achiral active instability as well. Due to the chiral active force $\propto z_c$, the egg-crate-like undulation leads to an in-plane vorticity $\Omega_z \propto (z_c q_p^2 u_0/\eta) \cos q_p x \cos q_p y$, arising spontaneously from the active instability. This is the vortex lattice depicted in Fig. 1.

The correspondence between an external stress and the active achiral stress allows for a quantitative measurement of the activity. The critical threshold for a layered state of finite extent d in the z direction is $\zeta + \sigma_0 = (2\pi/d)\sqrt{BK}$, with the instability setting in at wavevector $q_p \approx (\pi^2 B/4d^2 K)^{1/4}$. When ζ is negative the layered state is stable to the activity and a dilatative external stress, $\sigma_0 > 0$, can be applied till the Helfrich-Hurault instability sets in [59]. This measures the active stress: $\zeta = (2\pi/d)\sqrt{BK} - \sigma_0^{cr}$. Conversely, for $\zeta > 0$ the smallest $|\sigma_0|$ that suppresses the spontaneous Helfrich-Hurault instability, σ_0^{cr} yields the active stress: $\zeta = |\sigma_0^{cr}| + (2\pi/d)\sqrt{BK}$ from which we can calculate the strength of the achiral active stress using the knowledge of the thickness of the sample, compressibility and bending modulus. The magnitude of the vorticity is $\propto z_c/\eta$. The chiral active force should scale as $z_c \sim \zeta\ell$ where ℓ is the length of an elementary active unit (both ζ and z_c are also likely to be functions of the concentration of the active units). Therefore, in principle, we can

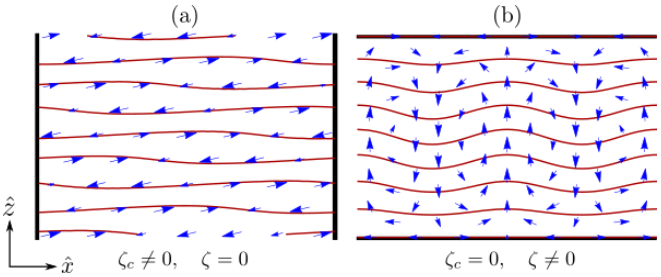

FIG. 2. Active instabilities in two-dimensional layered states. (a) Instability arising from the chiral active force (ζ_c, here positive). The layers are indicated by dark red lines, while the linear flow field at instability overlaid (blue arrows). The thick black lines indicate confining walls, which create a finite threshold for the instability. (b) Instability arising from the achiral active force (ζ) for comparison; same stylings.

estimate both the chiral and achiral active stress if an active cholesteric is prepared using living liquid crystals [60], which can be engineered, for instance, by suffusing a *passive* cholesteric with bacteria. The wavelength of the passive Helfrich-Hurault instability [55, 58] can be controlled by tuning the anchoring strength at the boundaries [58, 61, 62], which can be used to engineer a vortex array with a desired lattice constant.

We now turn to a two-dimensional layered state – an array of lines in the $x - z$ plane with normals on average along $\hat{\mathbf{z}}$ – in a chiral, internally driven fluid. As in the three-dimensional cholesteric state, we obtain the coupled dynamics of the displacement and velocity field equations to leading order in gradients: $\partial_t u = v_z$ and

$$\eta \nabla^2 \mathbf{v} = \hat{\mathbf{z}} \frac{\delta F[u]}{\delta u} + \nabla p + \nabla \cdot (\zeta \boldsymbol{w} - \zeta_c \boldsymbol{\varepsilon} \cdot \boldsymbol{w}) \qquad (4)$$

where, to linear order in u, $w_{zz} = -w_{xx} = \partial_z u$ and $w_{zx} = w_{xz} = \partial_x u$, $\zeta = (\psi_1^0 q_s)^2 \zeta_H$ and $\zeta_c = -(\psi_1^0 q_s)^2 \bar{\zeta}_c$ [42]. In (4), the chiral active force $\propto \zeta_c$ appears at the *same* order in gradients as the achiral active force, unlike in three-dimensional cholesterics. Further, again unlike in three-dimensional cholesterics, it will be shown to affect the displacement field dynamics at linear order. In fact, the term $\propto \zeta_c$ is fundamentally distinct from the force $\propto z_c$ in (3); it is not obtained by averaging a thin $x - z$ slice of a three-dimensional cholesteric. It leads to a chiral active force *along* the layers in response to both curvature and compression of the layers i.e., a pure $\hat{\mathbf{z}}$ deformation leads to a force along $\hat{\mathbf{y}}$, in a direction determined by the sign of ζ_c, which is only possible since the film has a distinguished normal, breaking three-dimensional rotation invariance, and the layered state breaks rotation (and translation) invariance in the plane of the film. This effect is related to the odd elasticity [9] of chiral active solids, but is odder still. A smectic breaks translation invariance only along one direction, so the (linearised) strain is simply $\left[\begin{smallmatrix} 0 & 0 \\ 0 & \partial_z u \end{smallmatrix}\right]$. Ordinary odd elasticity would create a stress $\propto \left[\begin{smallmatrix} 0 & \partial_z u \\ 0 & 0 \end{smallmatrix}\right]$ acting along the

layers of the smectic in a direction where there is no elastic mode. Instead, the chiral activity ζ_c produces both a *simple* shear stress $\sigma_{xz} = \sigma_{zx} = -\zeta_c \partial_z u$ in response to strain and also a *pure* shear stress $\sigma_{xx} = -\sigma_{zz} = -\zeta_c \partial_x u$ in response to tilt. The chiral active stress implies that a localised compression of the layer spacing produces a shear flow parallel to the layers.

We now demonstrate that a periodic array of lines in a two-dimensional film is generically destabilised due to the chiral active force. Eliminating the pressure using the incompressibility constraint in (4), solving for the velocity field and writing the wavevector $\mathbf{q} \equiv (q_x, q_z) = q(\sin\theta_q, \cos\theta_q)$ where θ_q is the angle between the layer normal and the wavevector, we obtain the eigenfrequency to $\mathcal{O}(q^0)$

$$\omega = -\frac{i}{4\eta}\left(B\sin^2 2\theta_q - 4\zeta\sin^2\theta_q - 2\zeta_c \sin 2\theta_q\right) + O(q^2). \tag{5}$$

This implies an instability of the layered state for wavevector direction θ_q just above (just below) zero for $\zeta_c > 0$ (< 0). This generic chiral instability for either sign of ζ_c is distinct from the spontaneous Helfrich-Hurault [3] instability of active smectics or cholesterics, which is achiral, arises for positive ζ [7, 8], and grows fastest at $\theta_q \approx \pi/2$. Eq. (5) implies that in a system confined at a scale d along $\hat{\mathbf{x}}$ so that the smallest $q_x \sim 1/d$, the minimum value of the chiral active stress for which the layered state is unstable $\sim 1/d$. Further, this instability requires both momentum conservation *and* incompressibility. It is eliminated if the film is supported on a substrate which would add a wavevector-independent damping $-\Gamma\mathbf{v}$ to (4). The eigenfrequency for the displacement fluctuations then vanishes at small q as $\mathcal{O}(q^2)$, and permeative [3] terms $\nabla\nabla u$ in the displacement equation, subdominant for a free-standing film, now enter at the same order in gradients. Of these, terms $\propto \partial_z^2 u$ are crucial, while others can be absorbed into redefinitions of ζ and ζ_c). The resulting eigenfrequency is

$$\omega = -\frac{iq^2}{4\Gamma}\left(B\sin^2 2\theta_q + \mathcal{B}\cos^2\theta_q - 4\zeta\sin^2\theta_q - 2\zeta_c\sin 2\theta_q\right) \tag{6}$$

where $\mathcal{B}$ is the coefficient of the $\partial_z^2 u$ permeative term multiplied by the friction coefficient, and the instability now occurs only if $|\zeta_c| > (B/2) + \mathcal{B}/2 - \zeta$, for directions $\theta_q \approx \pi/4$. Compressibility, as in a film bounded by bulk fluid, at large enough scales [63], is also stabilizing [64]. A detailed solution [42] of the tangent-plane velocity in this case leads to the eigenfrequency for the displacement field

$$\omega = -\frac{i|q|}{4\eta}\left[(B\cos^2\theta_q - \zeta)(1+\sin^2\theta_q) - 12\zeta_c\sin 2\theta_q\right] \tag{7}$$

Equation (7) yields an instability if $|\zeta_c| > [(B/2) - \zeta]/8$, for $\theta_q \approx \pi/4$ irrespective of the sign of ζ_c. We expand on this in [42]. While a free-standing film with a generically unstable chiral layered state may be difficult to access experimentally, films supported on a substrate or immersed in a bulk fluid can be engineered and the instability as in (6) and (7) may be observed.

In this paper we have developed the hydrodynamic theory of active chiral, layered states in two and three dimensions and demonstrated that the combination of internal drive and broken-parity qualitatively modifies the dynamics and stability of these phases unlike in their equilibrium counterparts. This difference derives from the "odder" elasticity of phases with a mixture of solid and fluid directions. By adopting the framework of a scalar field theory, active model H* offers a foolproof method for obtaining the correct hydrodynamic description[65, 67]. This general approach can be utilised beyond active cholesterics to construct the hydrodynamic theory of any spatially modulated active chiral state such as active chiral solid or columnar phases [68] and may present the simplest framework to understand the dynamics of and (possibly non-reciprocal) interactions between dislocations in these phases [69]. Further, the one-dimensional spatially modulated states discussed in this paper appear due to *micro*phase separation in active model H*; the dynamics of simple phase separation in this model [70] is also likely to be distinct from either passive model H or achiral but active model H.

We conclude with a brief discussion of proposals for experimental realisations, applications and possible extensions. A three-dimensional active or living cholesteric can be constructed by releasing swimming bacteria into passive biocompatible cholesteric liquid crystals, yielding a system which should display the vortex-lattice state we predict. Similarly, introducing *passive* chiral particles in an active but achiral fluid also leads to the chiral active forces discussed in our work, allowing for the realisation of a wide range of artificial active cholesteric materials. Furthermore, multiple biological systems display cholesteric organisation, the most spectacular of which is DNA in chromatin [72], which *in vivo* may be affected by DNA polymerases leading to chiral active forces of the form that we describe here.

In addition to free surfaces or interfaces of three-dimensional materials, there are numerous possibilities for realising a two-dimensional cholesteric phase. For instance, via the melting of a chiral version [9] of anisotropic active solids [23] along one direction in analogy with the emergence of (achiral) smectic phases due to an anisotropic dislocation-mediated melting of two-dimensional crystals in which dislocations unbind along one direction [73–78] (see [69] for a description of dislocations in chiral active solids). Two-dimensional layered states are also observed in active nematic fluids both in experiments on motor-microtubule gels [79–81] and simulations [82] and since these gels are known to be chiral [83], the physics we describe for two-dimensional, chiral layered states may be observable there. Chirality has

been shown to be important in epithelial cell layers [30] and a density modulated phase in these systems will lead to another realisation of two dimensional cholesterics. Non-mutual, two-species Cahn-Hilliard models [84, 85] also spontaneously form banded phases and chiral variants of these models would lead to two-dimensional chiral layered states. Finally, two-dimensional smectic phases in parity-broken systems are also possible in two-dimensional electron gases [86] and, when they are irradiated by microwave radiation [87], may have a dynamics equivalent to the one described here. Therefore, there are abundant possibilities for creating two- and three-dimensional active cholesteric states.

S.R. acknowledges research support from a J.C. Bose Fellowship of the SERB, India and from the Tata Education & Development Trust. We acknowledge illuminating discussions during the KITP 2020 online program on Symmetry, Thermodynamics and Topology in Active Matter. This research was supported in part by the National Science Foundation under Grant No. NSF PHY-1748958. We thank Richard Morris for beneficial discussions.

* swapnilkole@iisc.ac.in
† G.P.Alexander@warwick.ac.uk
‡ sriram@iisc.ac.in
§ nyomaitra07@gmail.com

[1] S.R. Renn and T.C. Lubensky, Abrikosov dislocation lattice in a model of the cholesteric–to–smectic-A transition, Phys. Rev. A **38**, 2132 (1988). doi: 10.1103/PhysRevA.38.2132
[2] D.C. Wright and N.D. Mermin, Crystalline liquids: the blue phases, Rev. Mod. Phys. **61**, 385 (1989). doi: 10.1103/RevModPhys.61.385
[3] P.G. de Gennes and J. Prost, The Physics of Liquid Crystals (second edition), Clarendon, Oxford (1993).
[4] Chirality is of course manifest in the bulk optical and electrical properties of cholesterics.
[5] T.C. Lubensky, Phys. Rev. A **6**, 452 (1972). doi: 10.1103/PhysRevA.6.452
[6] L. Radzihovsky and T.C. Lubensky, Nonlinear smectic elasticity of helical state in cholesteric liquid crystals and helimagnets, Phys. Rev. E **83**, 051701 (2011). doi: 10.1103/PhysRevE.83.051701
[7] C.A. Whitfield, T.C. Adhyapak, A. Tiribocchi, G.P. Alexander, D. Marenduzzo, and S. Ramaswamy, Hydrodynamic instabilities in active cholesteric liquid crystals, Eur. Phys. J. E **40**, 50 (2017). doi:10.1140/epje/i2017-11536-2
[8] T.C. Adhyapak, S. Ramaswamy, and J. Toner, Live Soap: Stability, Order, and Fluctuations in Apolar Active Smectics, Phys. Rev. Lett. **110**, 118102 (2013). doi: 10.1103/PhysRevLett.110.118102
[9] C. Scheibner, A. Souslov, D. Banerjee, P. Surówka, W.T.M. Irvine, and V. Vitelli, Odd elasticity. Nat. Phys. **16**, 475 (2020). doi:10.1038/s41567-020-0795-y
[10] S. Ramaswamy, Active Matter, J. Stat. Mech. (2017) 054002. doi:10.1088/1742-5468/aa6bc5
[11] L.P. Dadhichi, A. Maitra, and S. Ramaswamy, Origins and diagnostics of the nonequilibrium character of active systems, J. Stat. Mech. (2018) 123201. doi:10.1088/1742-5468/aae852
[12] M.C. Marchetti, J-F. Joanny, S. Ramaswamy, T.B. Liverpool, J. Prost, M. Rao, and R.A. Simha, Rev. Mod. Phys. **85**, 1143 (2013). doi:10.1103/RevModPhys.85.1143
[13] S. Ramaswamy, The Mechanics and Statistics of Active Matter, Annu. Rev. Condens. Matter Phys. **1**, 323 (2010). doi:10.1146/annurev-conmatphys-070909-104101
[14] J. Prost, F. Jülicher, and J-F. Joanny, Active gel physics. Nat. Phys. **11**, 111 (2015). doi:10.1038/nphys3224
[15] G. Salbreux and F. Jülicher, Mechanics of active surfaces, Phys. Rev. E **96**, 032404 (2017). doi: 10.1103/PhysRevE.96.032404
[16] J. Toner, Y. Tu, and S. Ramaswamy, Hydrodynamics and phases of flocks, Ann. Phys. **318**, 170 (2005). doi: 10.1016/j.aop.2005.04.011
[17] M.E. Cates and J. Tailleur, Motility-Induced Phase Separation, Annu. Rev. Condens. Matter Phys. **6**, 219 (2015). doi:10.1146/annurev-conmatphys-031214-014710
[18] K. Kruse, J-F. Joanny, F. Jülicher, J. Prost, and K. Sekimoto, Generic theory of active polar gels: a paradigm for cytoskeletal dynamics, Eur. Phys. J. E **16**, 5 (2005). doi: 10.1140/epje/e2005-00002-5
[19] D. Saintillan and M.J. Shelley, Active suspensions and their nonlinear models, Comptes Rendus Physique **14**, 497 (2013). doi:10.1016/j.crhy.2013.04.001
[20] A.P. Solon, J. Stenhammar, M.E. Cates, Y. Kafri, and J. Tailleur, Generalized thermodynamics of phase equilibria in scalar active matter, Phys. Rev. E **97**, 020602 (2018). doi:10.1103/PhysRevE.97.020602
[21] P. Romanczuk, H. Chaté, L. Chen, S. Ngo, and J. Toner, Emergent smectic order in simple active particle models, New J. Phys. **18**, 063015 (2016). doi:10.1088/1367-2630/18/6/063015
[22] L. Chen and J. Toner, Universality for Moving Stripes: A Hydrodynamic Theory of Polar Active Smectics, Phys. Rev. Lett. **111**, 088701 (2013). doi: 10.1103/PhysRevLett.111.088701
[23] A. Maitra and S. Ramaswamy, Oriented Active Solids, Phys. Rev. Lett. **123**, 238001 (2019). doi: 10.1103/PhysRevLett.123.238001
[24] B. Liebchen and D. Levis, Collective Behavior of Chiral Active Matter: Pattern Formation and Enhanced Flocking, Phys. Rev. Lett. **119**, 058002 (2017). doi: 10.1103/PhysRevLett.119.058002
[25] D. Levis and B. Liebchen, Micro-flock patterns and macro-clusters in chiral active Brownian disks, J. Phys.: Condens. Matter **30**, 084001 (2018). doi:10.1088/1361-648x/aaa5ec
[26] S. Fürthauer, M. Strempel, S.W. Grill, F. Jülicher, Active chiral fluids, Eur. Phys. J. E **35**, 89 (2012). doi: 10.1140/epje/i2012-12089-6
[27] E. Tjhung, M.E. Cates, and D. Marenduzzo, Contractile and chiral activities codetermine the helicity of swimming droplet trajectories, Proc. Natl. Acad. Sci. U.S.A. **114**, 4631 (2017). doi:10.1073/pnas.1619960114
[28] A. Maitra, M. Lenz, and R. Voituriez, Chiral Active Hexatics: Giant Number Fluctuations, Waves, and Destruction of Order, Phys. Rev. Lett. **125**, 238005 (2020). doi: 10.1103/PhysRevLett.125.238005
[29] L.A. Hoffmann, K. Schakenraad, R.M.H. Merks,

and L. Giomi, Soft Matter **16**, 764 (2020). doi: 10.1039/C9SM01851D

[30] G. Duclos, C. Blanch-Mercader, V. Yashunsky, G. Salbreux, J-F. Joanny, J. Prost, and P. Silberzan, Spontaneous shear flow in confined cellular nematics, Nat. Phys. **14**, 728 (2018). doi:10.1038/s41567-018-0099-7

[31] T. Markovich, E. Tjhung, and M.E. Cates, Chiral active matter: microscopic 'torque dipoles' have more than one hydrodynamic description, New J. Phys. **21**, 112001 (2019). doi:10.1088/1367-2630/ab54af

[32] S. Fürthauer, M. Strempel, S.W. Grill, and F. Jülicher, Active Chiral Processes in Thin Films, Phys. Rev. Lett. **110**, 048103 (2013). doi: 10.1103/PhysRevLett.110.048103

[33] P.C. Hohenberg and B.I. Halperin, Theory of dynamic critical phenomena, Rev. Mod. Phys. **49**, 435 (1977). doi: 10.1103/RevModPhys.49.435

[34] M.C. Cross and P.C. Hohenberg, Pattern formation outside of equilibrium, Rev. Mod. Phys. **65**, 851 (1993). doi: 10.1103/RevModPhys.65.851

[35] A. Tiribocchi, R. Wittkowski, D. Marenduzzo, and M.E. Cates, Active Model H: Scalar Active Matter in a Momentum-Conserving Fluid, Phys. Rev. Lett. **115**, 188302 (2015). doi:10.1103/PhysRevLett.115.188302

[36] R. Singh and M.E. Cates, Hydrodynamically Interrupted Droplet Growth in Scalar Active Matter, Phys. Rev. Lett. **123**, 148005 (2019). doi: 10.1103/PhysRevLett.123.148005

[37] S.R. De Groot and P. Mazur, Non-Equilibrium Thermodynamics, Dover Books on Physics (2011).

[38] R. Wittkowski, A. Tiribocchi, J. Stenhammar, R.J. Allen, D. Marenduzzo, and M.E. Cates, Scalar ϕ^4 Field Theory for Active-Particle Phase Separation, Nat. Commun. **5**, 4351 (2014). doi:10.1038/ncomms5351

[39] A.P. Solon, M.E. Cates, and J. Tailleur, Active brownian particles and run-and-tumble particles: A comparative study, Eur. Phys. J. Spec. Top. **224**, 1231 (2015). doi: 10.1140/epjst/e2015-02457-0

[40] C. Nardini, É. Fodor, E. Tjhung, F. v. Wijland, J. Tailleur, and M.E. Cates, Entropy Production in Field Theories without Time-Reversal Symmetry: Quantifying the Non-Equilibrium Character of Active Matter, Phys. Rev. X **7**, 021007 (2017). doi: 10.1103/PhysRevX.7.021007

[41] E. Tjhung, C. Nardini, and M.E. Cates, Cluster Phases and Bubbly Phase Separation in Active Fluids: Reversal of the Ostwald Process, Phys. Rev. X **8**, 031080 (2018). doi:10.1103/PhysRevX.8.031080

[42] Supplementary material which includes Refs [43–48]

[43] B. A. Finlayson and L. E. Scriven, Convective instability by active stress, Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences **310**, 183 (1969). doi:10.1098/rspa.1969.0071

[44] S. A. Brazovskii, Phase transition of an isotropic system to a nonuniform state, Zh. Eksp. Teor. Fiz. **68**, 175 (1975)

[45] A. V. Andreev, D. T. Son and B. Spivak, Hydrodynamics of Liquids of Chiral Molecules and Suspensions Containing Chiral Particles, Phys. Rev. Lett. **104**, 198301 (2010). doi:10.1103/PhysRevLett.104.198301

[46] R. A. Simha and S. Ramaswamy, Hydrodynamic Fluctuations and Instabilities in Ordered Suspensions of Self-Propelled Particles, Phys. Rev. Lett. **89**, 058101 (2002). doi:10.1103/PhysRevLett.89.058101

[47] F. Brochard, J.F. Lennon, Frequency spectrum of the flicker phenomenon in erythrocytes, J. Phys. France, **36**, 1035 (1975). doi:10.1051/jphys:0197500360110103500

[48] R. Chatterjee, N. Rana, A. Simha, P. Perlekar, S. Ramaswamy, Fluid flocks with inertia, arXiv:1907.03492 (2019).

[49] P.C. Martin, O. Parodi, and P.S. Pershan, Unified Hydrodynamic Theory for Crystals, Liquid Crystals, and Normal Fluids, Phys. Rev. A **6**, 2401 (1972). doi: 10.1103/PhysRevA.6.2401

[50] H. Stark and T.C. Lubensky, Poisson bracket approach to the dynamics of nematic liquid crystals: The role of spin angular momentum, Phys. Rev. E **72**, 051714 (2005). doi:10.1103/PhysRevE.72.051714

[51] J. Swift and P.C. Hohenberg, Hydrodynamic fluctuations at the convective instability, Phys. Rev. A **15**, 319 (1977). doi:10.1103/PhysRevA.15.319

[52] G. Grinstein and R.A. Pelcovits, Nonlinear elastic theory of smectic liquid crystals, Phys. Rev. A **26**, 915 (1982). doi:10.1103/PhysRevA.26.915

[53] $z \to -z$ symmetry rules out an ordinary Laplace pressure gradient $\nabla_\perp \nabla_\perp^2 u$ in active apolar materials; however this is allowed in active polar smectics [22].

[54] A. Souslov, B. van Zuiden, D. Bartolo, and V. Vitelli, Topological sound in active-liquid metamaterials. Nat. Phys. **13**, 1091 (2017).doi:/10.1038/nphys4193

[55] N.A. Clark and R.B. Meyer, Strain-induced instability of monodomain smectic A and cholesteric liquid crystals, Appl. Phys. Lett. **22**, 493 (1973). doi:10.1063/1.1654481

[56] M. Delaye, R. Ribotta, and G. Durand, Buckling instability of the layers in a smectic-A liquid crystal, Phys. Lett. A **44**, 139 (1973). doi:10.1016/0375-9601(73)90822-0

[57] J.M. Delrieu, Comparison between square, triangular, or one dimensional lattice of distortions in smectic A and cholesteric crystals for superposed strain and magnetic field of any directions, J. Chem. Phys. **60**, 1081 (1974). doi:10.1063/1.1681116

[58] B.I. Senyuk, I.I. Smalyukh, and O.D. Lavrentovich, Undulations of lamellar liquid crystals in cells with finite surface anchoring near and well above the threshold, Phys. Rev. E **74**, 011712 (2006). doi: 10.1103/PhysRevE.74.011712

[59] In practice, this external dilative stress is often imposed by the means of an external electric or magnetic field.

[60] S. Zhou, A. Sokolov, O.D. Lavrentovich, and I.S. Aranson, Living liquid crystals, Proc. Natl. Acad. Sci. U.S.A. **111**, 1265 (2014). doi:10.1073/pnas.1321926111

[61] M. O. Lavrentovich and L. Tran, Undulation instabilities in cholesteric liquid crystals induced by anchoring transitions, Phys. Rev. Research **2**, 023128 (2020). doi: 10.1103/PhysRevResearch.2.023128

[62] L. Tran, M. O. Lavrentovich, G. Durey, A. Darmon, M. F. Haase, N. Li, D. Lee, K. J. Stebe, and R. D. Kamien and T Lopez-Leon, Change in Stripes for Cholesteric Shells via Anchoring in Moderation, Phys. Rev. X **7**, 041029 (2017). doi:10.1103/PhysRevX.7.041029

[63] W. Cai and T.C. Lubensky, Hydrodynamics and dynamic fluctuations of fluid membranes, Phys. Rev. E **52**, 4251 (1995). doi:10.1103/PhysRevE.52.4251

[64] A. Maitra, Long-range uniaxial order in active fluids at two-dimensional interfaces, arXiv:2010.15044.

[65] M. te Vrugt, H. Löwen and R. Wittkowski, Classical dynamical density functional theory: from fundamentals to applications, Adv. Phys. **69**, 121 (2020).doi:

10.1080/00018732.2020.1854965
[66] H. Löwen, A phase-field-crystal model for liquid crystals, J. Phys.: Condens. Matter, **22** 364105 (2010).doi: 10.1088/0953-8984/22/36/364105
[67] R. Wittkowski, H. Löwen and H. R. Brand, Derivation of a three-dimensional phase-field-crystal model for liquid crystals from density functional theory, Phys. Rev. E **82**, 031708 (2010).doi:10.1103/PhysRevE.82.031708
[68] S. J. Kole, A. Maitra, G. P. Alexander and S. Ramaswamy, unpublished
[69] L. Braverman, C. Scheibner, and V. Vitelli, Topological defects in non-reciprocal active solids with odd elasticity, arXiv:2011.11543.
[70] S. J. Kole, A. Maitra, G. P. Alexander and S. Ramaswamy, unpublished
[71] L. Angheluta, Z. Chen, M. C. Marchetti and M. J. Bowick, The role of fluid flow in the dynamics of active nematic defects, arXiv:2012.02980.
[72] A. Leforestier and F. Livolant, Cholesteric liquid crystalline DNA; a comparative analysis of cryofixation methods, Biology of the Cell **71** 115 (1991).
[73] B.I. Halperin and D.R. Nelson, Theory of Two-Dimensional Melting, Phys. Rev. Lett. **41**, 121 (1978). doi:10.1103/PhysRevLett.41.121
[74] D.R. Nelson and B.I. Halperin, Dislocation-mediated melting in two dimensions, Phys. Rev. B **19**, 2457 (1979). doi:10.1103/PhysRevB.19.2457
[75] A.P. Young, Melting and the vector Coulomb gas in two dimensions, Phys. Rev. B **19**, 1855 (1979). doi: 10.1103/PhysRevB.19.1855
[76] S. Ostlund and B.I. Halperin, Dislocation-mediated melting of anisotropic layers, Phys. Rev. B **23**, 335 (1981). doi:10.1103/PhysRevB.23.335
[77] S. Ostlund, J. Toner, and A. Zippelius, Dynamics of 2D anisotropic solids, smectics and nematics, Ann. Phys. **144**, 345 (1982). doi:10.1016/0003-4916(82)90118-X
[78] L. Radzihovsky, Quantum smectic gauge theory, arXiv:2009.06632.
[79] T. Sanchez, D. Chen, S.J. DeCamp, M. Heymann, and Z. Dogic, Spontaneous motion in hierarchically assembled active matter, Nature **491**, 431 (2012). doi: 10.1038/nature11591
[80] A. Senoussi, S. Kashida, R. Voituriez, J.C. Galas, A. Maitra, and A. Estevez-Torres, Tunable corrugated patterns in an active nematic sheet, Proc. Natl. Acad. Sci. U.S.A. **116**, 22464 (2019). doi:10.1073/pnas.1912223116
[81] T. Strübing, A. Khosravanizadeh, A. Vilfan, E. Bodenschatz, R. Golestanian, and I. Guido, Wrinkling Instability in 3D Active Nematics, Nano Lett. **20**, 6281 (2020). doi:10.1021/acs.nanolett.0c01546
[82] A. Doostmohammadi, J. Ignes-Mullol, J.M. Yeomans, and F. Sagues, Active nematics, Nat. Commun. **9**, 3246 (2018). doi:10.1038/s41467-018-05666-8
[83] T. Yamamoto and M. Sano, Theoretical model of chirality-induced helical self-propulsion, Phys. Rev. E **97**, 012607 (2018). doi:10.1103/PhysRevE.97.012607
[84] S. Saha, J. Agudo-Canalejo, and R. Golestanian, Scalar Active Mixtures: The Nonreciprocal Cahn-Hilliard Model, Phys. Rev. X **10**, 041009 (2020). doi: 10.1103/PhysRevX.10.041009
[85] Z. You, A. Baskaran, and M.C. Marchetti, Nonreciprocity as a generic route to traveling states, Proc. Natl. Acad. Sci. U.S.A. **117**, 19767 (2020). doi: 10.1073/pnas.2010318117
[86] M.J. Lawler and E. Fradkin, Quantum Hall smectics, sliding symmetry, and the renormalization group, Phys. Rev. B **70**, 165310 (2004). doi: 10.1103/PhysRevB.70.165310
[87] J. Alicea, L. Balents, M.P.A. Fisher, A. Paramekanti, and L. Radzihovsky, Transition to zero resistance in a two-dimensional electron gas driven with microwaves, Phys. Rev. B **71**, 235322 (2005). doi: 10.1103/PhysRevB.71.235322