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Astrophysical and theoretical physics implications from multimessenger neutron star observations

Hector O. Silva,[1,2] A. Miguel Holgado,[3,4,5] Alejandro Cárdenas-Avendaño,[2,6] and Nicolás Yunes[2]

[1]*Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut), Am Mühlenberg 1, D-14476 Potsdam, Germany*
[2]*Department of Physics and Illinois Center for Advanced Studies of the Universe, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*
[3]*Department of Physics and McWilliams Center for Cosmology, Carnegie Mellon University, Pittsburgh, Pennsylvania, 15213, USA*
[4]*Department of Astronomy, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*
[5]*National Center for Supercomputing Applications, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*
[6]*Programa de Matemática, Fundación Universitaria Konrad Lorenz, 110231 Bogotá, Colombia*

The *Neutron Star Interior Composition Explorer* (NICER) recently measured the mass and equatorial radius of the isolated neutron star PSR J0030+0451. We use these measurements to infer the moment of inertia, the quadrupole moment, and the surface eccentricity of an isolated neutron star for the first time, using relations between these quantities that are insensitive to the unknown equation of state of supranuclear matter. We also use these results to forecast the moment of inertia of neutron star A in the double pulsar binary J0737-3039, a quantity anticipated to be directly measured in the coming decade with radio observations. Combining this information with the measurement of the tidal Love number with LIGO/Virgo observations, we propose and implement the first theory-agnostic and equation-of-state-insensitive test of general relativity. Specializing these constraints to a particular modified theory, we find that consistency with general relativity places the most stringent constraint on gravitational parity violation to date, surpassing all other previously reported bounds by seven orders of magnitude and opens the path for future test of general relativity with multimessenger neutron star observations.

Introduction.– Neutron stars are some of the most extreme objects in Nature. Their mass (typically around $1.4\,\mathrm{M}_\odot$) combined with their small radius (between $10-14$ km) result in interior densities that can exceed nuclear saturation density ($\rho \geqslant 2.8\times10^{14}$ g/cm^3), above which exotic states of matter can arise [1]. Neutron stars are, next to black holes, the strongest gravitational field sources known, with typical gravitational potentials that are five-orders of magnitude larger than that of the Sun. These properties make neutron stars outstanding laboratories to study both matter and gravity in situations out of reach in terrestrial and Solar System experiments.

Our current poor understanding of the supranuclear equation of state translates, via the equations of stellar equilibrium, to a large variability on observable properties of neutron stars, such as their masses and radii [2]. This variability increases if one lifts the assumption that Einstein's theory of general relativity is valid in the strong-gravity environment of neutron star interiors [3]. Modifications to general relativity generically predict new equations of stellar equilibrium, which, when combined with uncertainties on the nuclear equation of state, jeopardize attempts to test Einstein's theory with isolated, neutron star observations.

One possibility to circumvent this issue is to explore whether relations between neutron-star observables that are insensitive to either (or both) the equation of state and the gravitational theory exist. Fortunately, they do. For instance, when properly nondimensionalized, the moment of inertia (I), the rotational quadrupole moment (Q) and the tidal Love number (λ) of neutron stars show a remarkable degree of equation-of-state insensitivity, at a level below 1% [4, 5]. These "I-Love-Q" relations also exist in some modified theories of gravity, although they are different from their general relativity counterparts [6].

We here combine the first measurements [7, 8] by NICER [9] of *both* the mass (M) and equatorial radius (R_e) of the isolated pulsar PSR J0030+0451 [10, 11] with known equation-of-state insensitive relations involving the compactness $\mathscr{C} = GM/(R_\mathrm{e}c^2)$ (see for instance Refs. [12–15]) to infer a number of astrophysical and theoretical physics consequences. Before doing so, let us explain how these relations are obtained.

Quasiuniversal relations.– Neutron stars can have short rotation periods of the order of milliseconds, so their surfaces are oblate instead of spherical. The inclusion of this effect is of critical importance to accurately model the thermal X-ray waveform that NICER observes, since the X-rays are produced by hotspots at the star's surface [16, 17]. The canonical approach to model relativistic rotating stars was developed in the 1970s [18, 19]. In this approach, the star's rotation is treated as a small perturbation $\varepsilon = f/f_0 \ll 1$, involving the star's rotation frequency f and its characteristic mass-shedding frequency $f_0 = (GM/R_\mathrm{e}^3)^{1/2}/(2\pi)$. Rotating stars are then found by perturbing in ε an otherwise nonrotating star, which can be obtained by solving the Tolman-Oppenheimer-Volkoff equations [20]. This slow-rotation approximation is well-justified for most neutron stars with astrophysically relevant spins. Even for a prototypical millisecond pulsar with $f = 700$ Hz, $M = 1.4\,\mathrm{M}_\odot$ and $R_\mathrm{e} = 11$ km, one has $\varepsilon = 0.37$. In the case of PSR J0030+0451, its rotation frequency is known to be $f_\star = 205.53$ Hz [10, 11], so $\varepsilon_\star = 0.14$, when one uses the best-fit M and R_e values obtained by NICER [7, 8]. Henceforth, a "$\star$" indicates observables associated with PSR J0030+0451 .

Using this technique, we numerically calculated over a thousand neutron star solutions to order ε^2 in this perturbative scheme, using a broad set of 46 different equations of state [21, 22], as detailed in the Supplemental Materials (SM) [23]. From these solutions, we then numerically computed the moment of inertia I, the rotational quadrupole moment Q, the surface eccentricity e, and the electric-type, $\ell = 2$, tidal Love number λ, which is the dominant parameter in the description of tidal effects in the late inspiral of neutron star binaries [24–26]. We nondimensionalized these quantities through division by the appropriate factors of M and dimensionless spin $\chi = (2\pi f_0)\, G\bar{I}M/c^3$, namely: $\bar{I} = c^4 I/(G^2M^3)$, $\bar{Q} = -c^4 Q/\left(G^2M^3\chi^2\right)$ and $\bar{\lambda} = c^{10}\lambda/(GM)^5$. The surface eccentricity e is dimensionless by definition, given in terms of the star's equatorial $R_{\rm e}$ and polar $R_{\rm p}$ radii as $e = [(R_{\rm e}/R_{\rm p})^2 - 1]^{1/2}$ [14]. The relations between these nondimensional quantities are strongly insensitive to the equation of state. Due to the small value of $\varepsilon_\star$ we can neglect higher-order in spin corrections in this expression.

The first step in using the approximately universal relations on NICER's first observation is to derive equation-of-state-insensitive relations between the observables $\{\bar{I}, \bar{Q}, \bar{\lambda}, e\}$, with respect to the compactness $\mathscr{C}$. Details of these "$\mathscr{C}$-relations" are given in [23]. Details of these "$\mathscr{C}$-relations" are given in the SM [23]. Our plan of attack is then clear: use the publicly available Markov-Chain Monte Carlo (MCMC) M-$R_{\rm e}$ samples [27, 28] for the best-fit model inferred by two independent analysis [7, 8] of the NICER data [29]. Although each group modeled the surface hotspots differently and used different sampling methods, their results are consistent with each other. Here we use the results for the three-hotspot model inferred by Miller et al. [8] and the favored single temperature, two-hotpot ST+PST model from Riley et al. [7] to obtain a posterior distribution for the compactness, and then use the approximately universal relations to infer other astrophysical quantities. We detail this procedure next.

Astrophysical implications.– We begin by constructing a posterior distribution $P(\mathscr{C}|\text{NICER})$ for the compactness $\mathscr{C}$ of PSR J0030+0451, using the MCMC chains [27, 28]. With this posterior in hand, we then use the $\mathscr{C}$-relations to inferred posterior distributions for $\{\bar{I}, \bar{\lambda}, \bar{Q}, e\}$.

The implementation of such an inference procedure requires a particular scheme, and we here follow a proposal that accounts for the approximately universal nature of the relations [22]. In this scheme, the maximum relative error of each fitting function defines the half-width of the 90% credible interval of a Gaussian distribution centered at each fitted value. The posterior distribution for each dimensionless quantity is then calculated using the corresponding $\mathscr{C}$-relation and the posterior distribution of the compactness, after marginalizing over the latter. From these posteriors and using the same procedure described above, we can also construct posteriors for the dimensionful versions of these quantities by a change of variables, marginalize over the nuisance variables mass M and radius $R_{\rm e}$, and then do a final rescaling of the posterior by ε $(= 0.14)$ for the surface eccentricity e and by ε^2 for the rotational quadrupole moment Q. We refer to the SM for details [23].

The resulting mean and 1σ intervals of these parameters (both the nondimensionalized and the dimensionful versions) are shown in Table I; see the SM [23] for plots of the inferred posteriors. The reported confidence intervals in all of these quantities account for both the approximate nature of the universal relations and the uncertainties in NICER's observation. These results are the *first inferences on the moment of inertia, the surface eccentricity, the Love number and the quadrupole moment of an isolated neutron star.*

Parameter	Miller et al.	Riley et al.
$\bar{I}_\star$ (10)	$1.31^{+0.13}_{-0.11}$	$1.42^{+0.26}_{-0.19}$
$\bar{\lambda}_\star$ (10^2)	$4.97^{+1.92}_{-1.28}$	$6.75^{+5.52}_{-2.69}$
$\bar{Q}_\star$	$5.92^{+0.73}_{-0.61}$	$6.50^{+1.38}_{-1.03}$
$I_\star$ (10^{45} g cm^2)	$1.71^{+0.64}_{-0.48}$	$1.42^{+0.81}_{-0.53}$
$Q_\star$ (10^{43} g cm^2)	$1.49^{+0.63}_{-0.45}$	$1.27^{+0.74}_{-0.49}$
$e_\star$ (10^{-1})	$1.56^{+0.25}_{-0.21}$	$1.58^{+0.29}_{-0.28}$

TABLE I. Inferred properties of PSR J0030+0451 using equation-of-state-insensitive relations combined with the MCMC samples by Miller et al. [27] and Riley et al. [28]. We report the values within one standard deviation from the mean, representing approximately 68% confidence intervals. These values are the first inferences of the moment of inertia, the eccentricity, the Love number and the quadrupole moment of an isolated neutron star.

We can also use NICER's observation combined with the I-C relation to estimate the moment of inertia of PSR J0737-3039A ($I_{1.3381}$), where the subscript refers to this pulsar's measured mass of $M = (1.3381 \pm 0.0007)\,{\rm M}_\odot$ [30]. The double pulsar J0737-3039 is expected to provide the first direct neutron star measurement of the moment of inertia [31]. This system is the only double-pulsar observed to date, which makes it an unique laboratory for binary stellar astrophysics [32, 33]. Moreover, an accurate measurement of $I_{1.3381}$ in combination with its known mass is expected to strongly constrain the nuclear equation of state around once and twice nuclear saturation density [34].

To predict the moment of inertia of PSR J0737-3039A from NICER's observation of PSR J0030+0451, we first need to obtain an estimate for the compactness $\mathscr{C}_{1.3381}$ of PSR J0737-3039A. This can be approximated by the substitution $\{M, R_{\rm e}\} \mapsto \{M_0 = 1.3381\,{\rm M}_\odot, R_{\rm e}\}$ at each MCMC sample [27] and then computing $\mathscr{C}_{M_0}$. This yields an approximation to the distribution of compactness for a system with mass M_0, which is assumed known and identical to PSR J0030+0451. This procedure is only justified as long as M_0 is very close to $M_\star$, as in the case of PSR J0737-3039A, whose inferred mass ($M_0 = 1.3381^{+0.0007}_{-0.0007}\,{\rm M}_\odot$) [30] is within the 1σ credible interval of NICER's mass inference ($M_\star = 1.44^{+0.15}_{-0.14}\,{\rm M}_\odot$) [8].

With an estimate of the compactness of PSR J0737-3039A, we can now obtain a prediction for PSR J0030+0451's moment of inertia repeating the procedure applied to PSR J0030+0451. Figure 1 shows our result using both NICER MCMC samples; $I^{\text{Miller et al.}}_{1.3381} = 1.64^{+0.52}_{-0.37} \times 10^{45}$ g cm^2, and $I^{\text{Riley et al.}}_{1.3381} = 1.68^{+0.53}_{-0.48} \times 10^{45}$ g cm^2, together with two other independent predictions [35, 36]. All predictions are consisten with one another. The anticipated future independent measurement of $I_{1.3381}$ from continued radio timing of PSR J0737-3039A will provide another test for nuclear theory and enable an "I-Love test" of gravity, the latter of which we define next.

Theoretical physics implications.– The combination of the

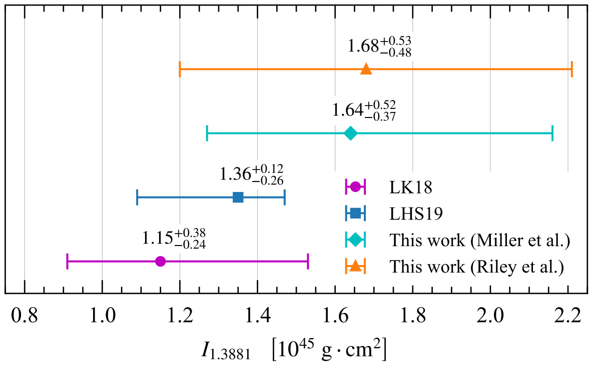

FIG. 1. Predictions for the moment of inertia of PSR J0737-3039A. We compare our predicted $I_{1.3381}$ using both the MCMC samples from Miller et al. [27] and Riley et al. [28] against: (i) Landry and Kumar [35] ("LK18"), which used binary Love [39] and I-Love relations with the tidal-deformability constraints from binary neutron-star merger GW170817 [37], and (ii) Lim et al. [36] ("LHS19") which carried out Bayesian modeling of a number of equations of state. The larger moment of inertia that we predict is due to the larger radii favored by an $M \approx 1.4\,\mathrm{M}_\odot$ neutron star by NICER's observation relative what is inferred by the two other methods, as $I \sim MR_e^2$.

inference of I with NICER data described above, and the independent measurement of λ [37] by the LIGO/Virgo collaboration from the binary neutron-star merger GW170817 [38], allows for the first implementation of an I-Love test [4]. This test would be *the first multi-messenger test of general relativity with neutron star observables*.

The idea of an I-Love test is as follows [4, 5] (see Fig. 2). Consider two independent indferences of $\bar{I}_{1.4}$ and $\bar{\lambda}_{1.4}$ for a $1.4\,\mathrm{M}_\odot$ neutron star. In the $(\bar{I}, \bar{\lambda})$-plane, these measurements yield a 90% confidence error box. If the I-Love relation in general relativity, including its small equation-of-state variability, *does not* pass through this error box, then there is evidence for a violation of Einstein's theory, regardless of the underlying equation of state. Moreover, if any theory of gravity predicts an I-Love curve that also does not pass through this error box for a given value of its coupling constants, then the I-Love test places a constraint on the couplings of this theory, which is also independent of the equation of state.

Such a test, however, requires the inference of the tidal deformability and the moment of inertia of a neutron star of the *same* mass. The LIGO/Virgo collaboration used gravitational wave data to infer the tidal deformability of a $1.4\,\mathrm{M}_\odot$ neutron star to be $\bar{\lambda}_{1.4} = 190^{+390}_{-120}$ at 90% confidence [40], obtained under the assumptions that the binary components were described by the same equation of state and were slowly-spinning. We can use NICER data to infer the moment of inertia of a $1.4\,\mathrm{M}_\odot$ neutron star with the same techniques we used to predict the moment of inertia of PSR J0737-3039A. For concreteness, we use the results from Miller et al. [8, 27], but we verified (see the SM for details [23]) that our conclusions are essentially the same had we used the results form Riley et al. [7, 28]. We find that $\mathscr{C}_{1.4} = 0.159^{+0.025}_{-0.022}$ and $\bar{I}_{1.4} = 14.6^{+4.5}_{-3.3}$ at 90% confidence. An important underlying assumption behind both inferences is that general relativity is the correct theory of gravity. The rationale behind this test is detailed in the SM [23].

Since carrying out such a test on a theory-by-theory basis would in general be complicated and time-consuming, we here develop and implement a useful *parametrization* of the I-Love test. From Newtonian gravity, we know that $\bar{I}$ scales with $\mathscr{C}^{-2}$, whereas $\bar{\lambda}$ scales with $\mathscr{C}^{-5}$. Therefore, $\bar{I} = C_{\bar{I}\bar{\lambda}}\bar{\lambda}^{2/5}$, with $C_{\bar{I}\bar{\lambda}} \approx 0.52$ a constant that depends on the equation of state very weakly [5]. This calculation can be extended, systematically, in a *post-Minkowskian expansion*, i.e., an expansion in powers of $\mathscr{C} \ll 1$ [41]. The outcome is that both $\bar{I}$ and $\bar{\lambda}$ can be written as a power series in $\mathscr{C}$ and then be combined (as just described in the Newtonian limit) to obtain $\bar{I} = \bar{I}(\bar{\lambda})$. The resulting I-Love relation has the same degree of equation-of-state independence as the original I-Love relation [4]. For our neutron star catalog, a parameterization in general relativity of the form

$$\bar{I}_{\rm GR} = \bar{\lambda}^{2/5}\left(c_0 + c_1\bar{\lambda}^{-1/5} + c_2\bar{\lambda}^{-2/5}\right), \tag{1}$$

with $c_0 = 0.584$, $c_1 = 0.980$, $c_2 = 2.695$, is sufficient to reproduce our numerical data with mean relative error $\langle\epsilon^{\bar{I}}\rangle \leqslant 2\times10^{-3}$. The prefactor $\bar{\lambda}^{2/5}$ is the Newtonian result, while the powers of $\bar{\lambda}^{-1/5}$ inside parenthesis are relativistic (post-Minkowskian) corrections because $\bar{\lambda}^{-1/5} \propto \mathscr{C} \lesssim 0.2$. Given this, we then propose a minimal deformation of the Einsteinian parameterization in Eq. (1) of the form

$$\bar{I}_{\rm p} = \bar{I}_{\rm GR} + \beta\,\bar{\lambda}^{-b/5}, \qquad \beta \in \mathbb{R}_+, \quad b \in \mathbb{Z}, \tag{2}$$

where β and b are deformation parameters that control the magnitude and type of the deviations from general relativity in the I-Love relation respectively. Such a parameterization is similar to that successfully used in gravitational-wave tests of general relativity by the LIGO/Virgo collaboration, the parameterized post-Einsteinian framework [42].

We performed such a test of general relativity through the procedure described earlier. First, we see that the I-Love relation in general relativity does indeed pass this null-test and it is consistent with the error box. Second, we considered $b \in [-2, 5]$, where the lower limit is set by requiring no deviations at the Newtonian level and the upper limit is set for simplicity. We then fixed b and calculated what the corresponding value of $\beta = \beta_{\rm crit}$ is, above which the parametrized I-Love relation (2) would be in tension with the inferred $(\bar{I}_{1.4}, \bar{\lambda}_{1.4})$ region at 90% confidence. Our results are summarized in Fig. 2, where the numbers in parenthesis correspond to $(b, \beta_{\rm crit})$. We stress that our results for $b \leqslant 0$ are of course dependent on the posterior used for $\bar{\lambda}_{1.4}$. If one treated the tidal deformabilities as independent free parameters in the waveform model [38], then the $\bar{\lambda}_{1.4}$ posterior would not have a lower limit, allowing all curves with $b \leqslant 0$ to be consistent with both observations.

With these theory-agnostic constraints in hand, we can now map them to specific theories and place constraints on their coupling parameters. As an example, let us consider dynamical Chern-Simons gravity, a theory that modifies general relativity by introducing gravitational parity-violation [43]. This

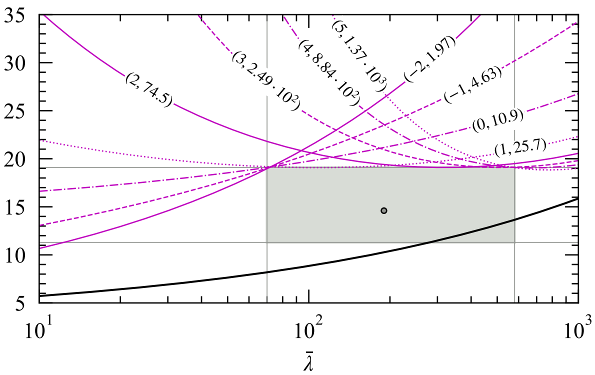

FIG. 2. Multimessenger test of general relativity using the parametrized I-Love relation. The vertical (horizontal) lines delimit the 90% confidence region (shaded) for $\bar{\lambda}_{1.4}$ [40] ($\bar{I}_{1.4}$, this work), while the circle marks the median (190, 14.6). The solid black line corresponds to the I-Love relation in general relativity [Eq. (1)] and is consistent with the inferred values of $\bar{I}_{1.4}$, $\bar{\lambda}_{1.4}$ at 90% confidence. Starting from $b = -2$ and moving clockwise, we show the parametrized I-Love curves $(b, \beta_{\rm crit})$, where $b \in [-2, 5]$ and $\beta_{\rm crit}$ is the critical value of β above which the parametrized I-Love relation [Eq. (2)] fails to pass by the 90% confidence region in the plane. Here we used the value of $\bar{I}_{1.4}$ inferred using the results by Miller et al. [8, 27]. We found similar results using the results by Riley et al. [7, 28] (See SM [23]).

theory has found applications to several open problems in cosmology, such as the matter-antimatter asymmetry and leptogenesis [44–47]. It also arises in several approaches to quantum gravity, such as string theory [48] and loop quantum gravity [49–51]. Mathematical well-posedness requires the theory to be treated as an effective field theory [52]. In this formalism, one works in a small-coupling approximation $\zeta \equiv 16\pi\alpha^2\mathscr{R}^{-4} \ll 1$, where $\mathscr{R} = [c^2 R_{\rm e}^3/(GM)]^{1/2}$ is the curvature length scale associated with a neutron star (in our case), and where α is a coupling constant with units of length squared, such that ζ is dimensionless. This theory modifies Einstein's only when gravity is strong, and thus, it passes all Solar System constraints, being only extremely-weakly constrained by Gravity Probe B and the LAGEOS satellites, and table-top experiments, to $\alpha^{1/2} \leqslant 10^8$ km [53–55]. This theory has also evaded gravitational-wave tests [56], making it a key target to test the constraining power of our new I-Love test.

Let us now map the theory-agnostic deformation of the I-Love relations in Eq. (2) to dynamical Chern-Simons gravity, though this methodology could be applied to other theories as well. As we discuss in the SM [23], The I-Love relation in this theory can be described by Eq. (2) with $b_{\rm CS} = 4$ and $\beta_{\rm CS} = 6.15\times10^{-2}\bar{\xi}$, where $\bar{\xi} = 16\pi\alpha^2/M^4$. We can now use our theory-agnostic constraints on β to place a constraint on α, the coupling constant of dynamical Chern-Simons gravity. Using the constraint on β when $b = 4$, namely $\beta_{\rm crit} \leqslant 8.84 \times 10^2$, and applying the mapping, yields $\beta_{\rm CS} = 6.15\times10^{-2}\,\bar{\xi} \leqslant 8.84\times10^2$, or simply

$$\alpha^{1/2} \leqslant 8.5\,\text{km}\,, \tag{3}$$

at 90% credibility, *if the theory is to be consistent with the observational bounds on* $\bar{I}_{1.4}$ *and* $\bar{\lambda}_{1.4}$. Using the mean value $\mathscr{C}_{1.4} = 0.159$, which implies the mean equatorial radius $R_{1.4} = 13.0$ km, we also find that $\zeta \leq 0.23$ when using Eq. (3), implying that the small-coupling approximation is indeed satisfied. This bound is *seven-orders of magnitude stronger than any previous constraints and it is unlikely to be improved upon with foreseeable gravitational-wave observations [57].*

Conclusions and outlook.– The NICER observation of PSR J0030+0451 allows the extraction of new astrophysical and theoretical physics inferences when one uses equation-of-state-insensitive relations. We have here shown the first inferences of the moment of inertia, the quadrupole moment, the surface eccentricity and the Love number of an isolated neutron star. We have also been able to perform the first theory-agnostic and equation-of-state independent test of general relativity by combining NICER and LIGO/Virgo observations. This test, in turn, was leveraged to produce the most stringent constraint on gravitational parity violation, improving previous bounds by seven orders of magnitude. This robust methodology can be applied to future multimessenger observations of neutron stars with NICER and gravitational wave observatories, with important implications to nuclear astrophysics and theoretical physics.

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