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Synchronizing Chaos with Imperfections

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Previous research on nonlinear oscillator networks has shown that chaos synchronization is attainable for identical oscillators but deteriorates in the presence of parameter mismatches. Here, we identify regimes for which the opposite occurs and show that oscillator heterogeneity can synchronize chaos for conditions under which identical oscillators cannot. This effect is not limited to small mismatches and is observed for random oscillator heterogeneity on both homogeneous and heterogeneous network structures. The results are demonstrated experimentally using networks of Chua’s oscillators and are further supported by numerical simulations and theoretical analysis. In particular, we propose a general mechanism based on heterogeneity-induced mode mixing that provides insights into the observed phenomenon. Since individual differences are ubiquitous and often unavoidable in real systems, it follows that such imperfections can be an unexpected source of synchronization stability.

Synchronization in networks of chaotic oscillators is a remarkable phenomenon that is now well established theoretically and experimentally [1], with implications for numerous biological and technological systems [2–5]. Two conditions are generally assumed for this phenomenon to occur: (i) that the coupling strength be sufficiently large and (ii) that the oscillators be sufficiently identical. If the coupling is too weak, the oscillators evolve mostly independently from each other, and their trajectories tend to diverge due to sensitive dependence on initial conditions—a hallmark of chaos [6]. On the other hand, if the oscillators are not sufficiently identical, their trajectories tend to diverge due to sensitive dependence on parameters—another hallmark of chaos [6]—even if the initial conditions are exactly the same and the coupling strength is otherwise suitably strong.

Previous analyses of synchronization of nonidentical chaotic oscillators have focused mainly on cluster synchronization [7, 8] and phase synchronization [9–13]. For example, oscillator heterogeneity has been shown to mediate relay synchronization [14–16] and to induce frequency locking by suppressing chaos [17–20]. Global chaos synchronization of nonidentical oscillators, on the other hand, has been explored mainly for strong coupling and small parameter mismatches [21–24], with an emphasis on the extent to which synchrony persists when the oscillators are slightly different [25, 26]. These previous results consistently show that global synchronization degrades as heterogeneity is increased.

A different body of work has recently shown that, for periodic oscillators, heterogeneity can in fact facilitate synchronization [27–32]. A natural question is then whether a similar effect would be possible for chaotic oscillators despite the fact that their dynamics exhibit sensitive dependence on parameters and that a well-defined synchronization manifold no longer exists for nonidentical chaotic oscillators. This question is especially relevant in weak coupling regimes, in which synchronization is unstable for identical chaotic oscillators.

In this Letter, we experimentally demonstrate that oscillator

heterogeneity can enable synchronization of weakly coupled chaotic oscillators that would otherwise not synchronize. This surprising result is established using Chua’s oscillators diffusively coupled through their x components, which leads to a semi-infinite stability region for identical oscillators. The robustness of the effect is confirmed by showing that it occurs consistently for *random* parameter heterogeneity and for *different* parameters respectively associated with temporal and state variable scales. The effect is demonstrated across different network structures and is supported by simulations and theoretical analysis. The role of *oscillator* heterogeneity is isolated by showing the persistence of the effect for structurally homogeneous networks of identically coupled oscillators. These results have immediate implications for real systems, where heterogeneity is ubiquitous. They also have foundational consequences for establishing an unanticipated relation between network coupling, oscillator heterogeneity, and sensitive dependence on initial conditions. In particular, although condition (i) cannot be violated in isolation without causing desynchronization, our results show that the violation of (ii) (albeit detrimental by itself) can mitigate the effect of infringing (i), and thus the synchronization of chaotic oscillators can persist if *both* (i) and (ii) are violated together.

We start by considering networks of N diffusively coupled oscillators described by

$$\tau_i \dot{\mathbf{x}}_i = \mathbf{F}(\mathbf{x}_i) - k \sum_{j=1}^N L_{ij} \mathbf{H}(\mathbf{x}_j), \quad i = 1, 2, \dots, N, \quad (1)$$

where τ_i denotes the time scale and \mathbf{x}_i is the state variable of the i th oscillator. The functions \mathbf{F} and \mathbf{H} describe the dynamics of a single oscillator and their interactions, respectively. The Laplacian matrix $\mathbf{L} = (L_{ij}) = (\delta_{ij} \sum_{\ell} A_{i\ell} - A_{ij})$, defined in terms of the adjacency matrix $\mathbf{A} = (A_{ij})$, represents the network structure. The parameter k controls the coupling strength.

The oscillators and coupling are implemented in our experiment using electrical circuits, as shown in Fig. 1. The oscil-

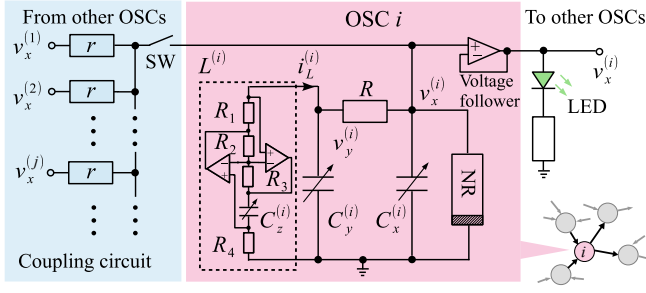


FIG. 1. Circuit diagram of coupled Chua's oscillators in our experiment. The individual oscillators are coupled through their voltage $v_x^{(i)}$, and an LED is attached to each oscillator to visualize the voltage oscillations. The capacitors are tunable and control the heterogeneity across the oscillators.

lators consist of x -coupled Chua's circuits [33] modeled as

$$\mathbf{F}(\mathbf{x}) = \begin{bmatrix} \eta\{y - x - g(x)\} \\ x - y + z \\ -y/\gamma \end{bmatrix}, \quad \mathbf{H}(\mathbf{x}) = \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix}, \quad (2)$$

$$g(x) = bx + \frac{1}{2}(b-a)(|x-1| - |x+1|), \quad (3)$$

where x , y , and z are the state variables and η , γ , a , and b are parameters. The variables correspond respectively to the voltages v_x and v_y across the capacitors C_x and C_y and the current i_L through the inductor L (which is implemented using a generalized impedance converter circuit). The parameters a and b are determined by a nonlinear resistor (NR) with a piecewise linear characteristic made from op-amps (TL084) and resistors [34]. The tunable parameters of the oscillators are controlled through tunable capacitors. The oscillators are coupled through the voltage v_x , where the directionality of the coupling is implemented using voltage followers. A light-emitting diode (LED) is attached to each oscillator so as to monitor the oscillation visually, with the diode turning on for $v_x > 0$ and off for $v_x < 0$. The voltage $v_x^{(i)}$ for each oscillator is recorded in a computer through an analog-to-digital converter (ADC) attached to the circuit.

The circuit parameters and variables are associated with the dimensionless quantities in Eqs. (1)-(3) as follows:

$$\tau_i = \frac{C_y^{(i)}}{C_y}, \quad k = \eta \frac{R}{r}, \quad \eta = \frac{C_y^{(i)}}{C_x^{(i)}}, \quad \gamma = \frac{C_z^{(i)} R_1 R_3 R_4}{C_y^{(i)} R^2 R_2},$$

$$a = m_1 R, \quad b = m_0 R, \quad x^{(i)} = \frac{v_x^{(i)}}{B_p}, \quad y^{(i)} = \frac{v_y^{(i)}}{B_p}, \quad z^{(i)} = \frac{i_L^{(i)} R}{B_p},$$

where $\bar{C}_y = \frac{1}{N} \sum_{i=1}^N C_y^{(i)}$, m_1 and m_0 are determined by the NR, and B_p depends on both the saturation voltage of the op-amps and the resistors connected to them [34]. The dimensionless time used in Eq. (1) is defined as $t' = t/(R\bar{C}_y)$ and, without loss of generality, it follows that the mean time scale is $\bar{\tau} = \frac{1}{N} \sum_{i=1}^N \tau_i \equiv 1$ (this condition is also imposed in our simulations and analysis). Unless noted otherwise, the oscillator parameters are fixed at $\eta = 10$, $\gamma = 0.056$, $a = -1.44$,

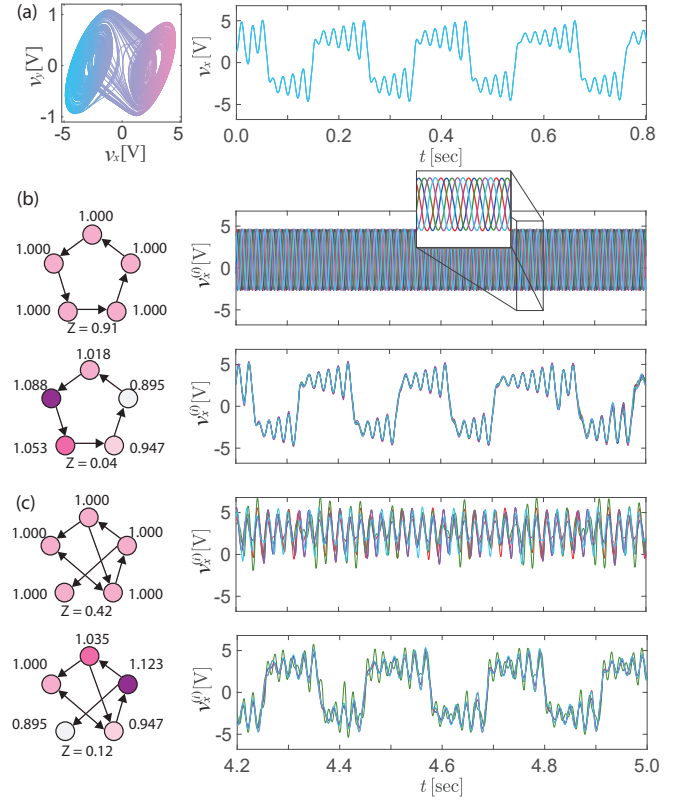


FIG. 2. (a) Double-scroll chaotic attractor constructed from the experimental time series of the voltages v_x and v_y of a single uncoupled oscillator, and the time series for v_x shown separately. (b, c) Corresponding experimental time series for oscillators in a directed ring for $k = 8.18$ (b) and in a random network for $k = 5$ (c). Left: network structures, where each node is labeled with the time scale τ_i , and the corresponding synchronization errors Z . Right: time series after the initial transient colored by oscillator for initial conditions close to the synchronous state, showing that chaos synchronization is stable in the heterogeneous system but not in the homogeneous one. In particular, the heterogeneous systems preserve both synchronization and qualitative properties of the original chaotic attractor. Videos of the time series in (b), tracked by LEDs attached to the oscillators, are included in the Supplemental Material [35].

and $b = -0.72$, which gives rise to a double-scroll chaotic attractor in the absence of coupling [36]. These parameters are realized in the experiment by setting $R_1 = R_2 = R_3 = 1\text{k}\Omega$, $R = R_4 = 1.8\text{k}\Omega$, $\bar{C}_y = 5.7\mu\text{F}$, $B_p = 1.3\text{V}$, $m_0 = -0.4\text{m}\Omega^{-1}$, and $m_1 = -0.8\text{m}\Omega^{-1}$, and keeping the capacitance ratios as

$$C_y^{(i)} = 10C_x^{(i)}, \quad C_z^{(i)} = C_x^{(i)}, \quad (4)$$

which ensure the same η and γ values across all oscillators. The capacitors $C_y^{(i)}$, which control the time scales τ_i , are tuned to vary the heterogeneity among the oscillators, and the resistors r are changed to modify the coupling strength k (see Supplemental Material [35] for details).

We first analyze in Fig. 2(b) the experimental time series of $v_x^{(i)}$ for a directed ring network of five oscillators, where the coupling strength is below the synchronization transition threshold for the identical oscillators. The upper panel con-

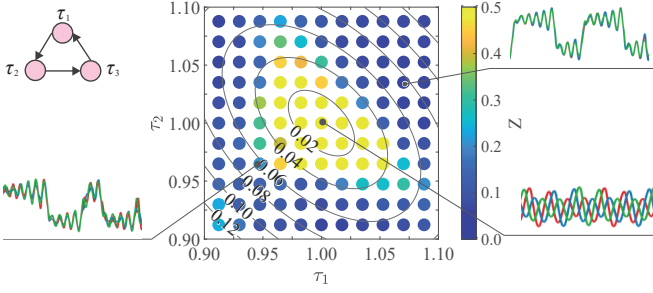


FIG. 3. Experimental results for a directed ring network of three oscillators. Synchronization error Z from the experiments, where the contour lines indicate the standard deviation of τ_i . The insets show representative examples of the time series of $v_x^{(i)}$ for the corresponding parameters. Each data point is determined from 10 experimental trials, each corresponding to a random initial condition. The coupling strength is set to $k = 5$.

firmly that, for homogeneous time scales, the trajectories of the oscillators diverge from each other and the system moves away from the synchronous chaotic state. In the lower panel, we introduce a random perturbation to the time scales, as indicated on the network image. Although the synchronization manifold $\mathbf{x}_1 = \dots = \mathbf{x}_N$ is no longer invariant, the heterogeneous system remains closely synchronized for the duration of the experiment and, collectively, exhibits double-scroll chaotic dynamics comparable to those of the uncoupled oscillators [Fig. 2(a)]. Figure 2(c) shows similar result for a random network with nonuniform indegrees. Once again, for the subcritical coupling strength considered, the synchronization is lost in the homogeneous system but persists in the heterogeneous system.

The degree of synchronization in our experiment is further quantified by calculating the synchronization error Z . We first define $e_{v_x} = \left\langle \sqrt{\frac{1}{N} \sum_{i=1}^N [v_x^{(i)}(t) - \bar{v}_x(t)]^2} \right\rangle$, where $\bar{v}_x(t)$ is the average of $v_x^{(i)}(t)$ over the oscillators at time t , and $\langle \cdot \rangle$ denotes the time average over a period of 5 seconds after the initial transient. To facilitate comparison between different conditions, we normalize e_{v_x} by the standard deviation of v_x calculated over the 5-second trajectory segment of all oscillators. The synchronization error Z is then calculated as the average of the normalized e_{v_x} over the experimental trials (taken to be 5 in Fig. 2). In Fig. 2(b), for example, the synchronization error for the homogeneous system is $Z = 0.91$, whereas for the heterogeneous system it is $Z = 0.04$.

To explore the effect of random heterogeneity more systematically, we focus on a minimal system of three circuit oscillators. Figure 3 shows the synchronization error Z in the (τ_1, τ_2) parameter space, for τ_3 satisfying $\tau_3 = 3 - \tau_1 - \tau_2$. The center of the image $(\tau_1, \tau_2, \tau_3) = (1, 1, 1)$ corresponds to the homogeneous system, which is characterized by a large synchronization error. As we move away from the center, Z eventually decreases to a value closer to zero in all directions. The contours in the figure show the standard deviation σ among the τ_i . The synchronization error Z decreases abruptly when σ

becomes larger than approximately 0.05, which indicates that the oscillator heterogeneity consistently promotes chaos synchronization in our system. Our numerical simulations for the same network and parameters using the model in Eqs. (1)-(3) show qualitatively similar transitions to synchronization as the heterogeneity is increased, further supporting the experimental findings. These results are presented in the Supplemental Material [35], where we also show that heterogeneity in the parameter γ_i stabilizes synchronization equally well, and that the phenomenon is also observed experimentally for Chua's circuits exhibiting single-scroll chaotic dynamics.

Having shown experimentally that random parameter heterogeneity facilitates synchronization, we now present a theory identifying the general mechanism behind this phenomenon. For a network of identical oscillators, the variational equation governing the synchronization stability has the following form:

$$\delta\dot{\mathbf{X}} = [\mathbf{I}_N \otimes \mathbf{DF}(\mathbf{x}) - k\mathbf{L} \otimes \mathbf{DH}(\mathbf{x})] \delta\mathbf{X}, \quad (5)$$

where $\delta\mathbf{X} = (\mathbf{x}_1 - \mathbf{x}, \dots, \mathbf{x}_N - \mathbf{x})^\top$ is the perturbation vector, \mathbf{I}_N is the $N \times N$ identity matrix, \mathbf{x} is the synchronization trajectory, and \otimes is the Kronecker product. For simplicity, we assume the Laplacian matrix \mathbf{L} is diagonalizable and, consequently, Eq. (5) can be decoupled by applying a coordinate transformation \mathbf{Q} for which $\mathbf{Q}^{-1}\mathbf{L}\mathbf{Q} = \boldsymbol{\lambda} = \text{diag}(\lambda_1, \dots, \lambda_N)$ [37]. This gives rise to N lower-dimensional equations of the same form, each corresponding to an independent perturbation mode ξ_i :

$$\dot{\xi}_i = [\mathbf{DF}(\mathbf{x}) - k\lambda_i\mathbf{DH}(\mathbf{x})] \xi_i, \quad i = 1, 2, \dots, N. \quad (6)$$

When oscillators become nonidentical, \mathbf{DF} (and \mathbf{DH} in the case of heterogeneous time scales τ_i) becomes different for each oscillator.

For small heterogeneity, assuming the change in the synchronization trajectory is negligible, the mismatches in \mathbf{DF} and \mathbf{DH} introduce a perturbation matrix $\boldsymbol{\Delta}(\mathbf{x})$ in Eq. (5):

$$\delta\dot{\mathbf{X}} = [\mathbf{I}_N \otimes \mathbf{DF}(\mathbf{x}) - k\mathbf{L} \otimes \mathbf{DH}(\mathbf{x}) + \boldsymbol{\Delta}(\mathbf{x})] \delta\mathbf{X}. \quad (7)$$

Now, when we apply the transformation matrix \mathbf{Q} to Eq. (7), we get

$$\dot{\boldsymbol{\xi}} = [\mathbf{I}_N \otimes \mathbf{DF}(\mathbf{x}) - k\boldsymbol{\lambda} \otimes \mathbf{DH}(\mathbf{x}) + \tilde{\boldsymbol{\Delta}}(\mathbf{x})] \boldsymbol{\xi}, \quad (8)$$

where $\boldsymbol{\xi} = (\xi_1, \xi_2, \dots, \xi_N)^\top$ and $\tilde{\boldsymbol{\Delta}}$ is $\boldsymbol{\Delta}$ under the new coordinates. Dividing $\tilde{\boldsymbol{\Delta}}$ into $N \times N$ blocks of equal size,

$$\tilde{\boldsymbol{\Delta}} = \begin{pmatrix} \tilde{\boldsymbol{\Delta}}_{11} & \tilde{\boldsymbol{\Delta}}_{12} & \cdots & \tilde{\boldsymbol{\Delta}}_{1N} \\ \tilde{\boldsymbol{\Delta}}_{21} & \tilde{\boldsymbol{\Delta}}_{22} & \cdots & \tilde{\boldsymbol{\Delta}}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\boldsymbol{\Delta}}_{N1} & \tilde{\boldsymbol{\Delta}}_{N2} & \cdots & \tilde{\boldsymbol{\Delta}}_{NN} \end{pmatrix}, \quad (9)$$

Eq. (8) can be written as

$$\dot{\xi}_i = [\mathbf{DF}(\mathbf{x}) - k\lambda_i\mathbf{DH}(\mathbf{x})] \xi_i + \sum_j \tilde{\boldsymbol{\Delta}}_{ij}(\mathbf{x}) \xi_j. \quad (10)$$

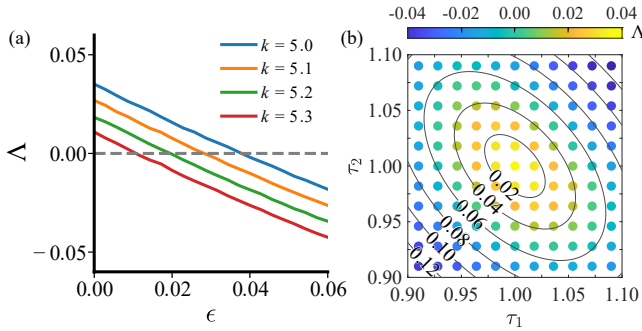


FIG. 4. Theoretical predictions based on heterogeneity-induced mode mixing for the system considered in Fig. 3. (a) Dependence of the largest Lyapunov exponent Λ on the mixing coefficient ϵ for a range of values of coupling strength k in Eq. (11). (b) Dependence of Λ on τ_1 predicted using Eq. (10) under the approximation of an invariant synchronization manifold. The theoretical prediction matches well the experimental results in Fig. 3.

Crucially, $\tilde{\Lambda}$ is generally non-block-diagonal in these coordinates. Comparing Eq. (10) with Eq. (6), it is clear that one of the main effects of parameter mismatches is the introduction of interdependence among the (originally independent) perturbation modes due to off-diagonal blocks in matrix $\tilde{\Lambda}$.

Consequently, in the presence of random oscillator heterogeneity, the perturbation modes ξ_i are no longer mutually independent and start to “mix” with each other. This effect is often ignored in previous attempts to analyze synchronization in networks of heterogeneous oscillators. For example, the extended master stability function (MSF) assumes that, under small parameter mismatch, the variational equation can still be decoupled into N independent equations, each corresponds to one of the original perturbation modes ξ_i [25]. A simple analysis shows that, within the framework of the extended MSF, the Lyapunov exponents of each perturbation mode do not depend on oscillator heterogeneity (see Supplemental Material [35] for details). This motivates us to go beyond the extended MSF in our theory, and it also justifies our choice to focus on the effect of heterogeneity-induced mode mixing.

To intuitively understand how the interdependence among perturbation modes affects stability, consider a network of identical oscillators that is weakly unstable, which typically has one unstable and $N - 2$ stable transverse perturbation modes. After introducing oscillator heterogeneity, the invariant subspaces of the variational equations are destroyed and the perturbation modes become interdependent. A perturbation vector initially aligned with the weakly unstable direction will spend time in other (more stable) directions, which will lead to an averaging effect that stabilizes all possible perturbations.

To heuristically model this “mixing” effect, we introduce a mixing coefficient ϵ that controls the mixing rate between different transverse perturbation modes ξ_i . To keep the formalism simple, we assume that ϵ is a constant and that heterogeneity causes each transverse mode to “leak” into all other transverse directions at the same rate ϵ . The model can be

described as

$$\dot{\xi}_i = [DF(x) - k\lambda_i DH(x)] \xi_i + \epsilon \sum_{j=2}^N (\xi_j - \xi_i), \quad i = 2, \dots, N, \quad (11)$$

where we used that $\epsilon \sum_{j=2}^N (\xi_j - \xi_i) = -\epsilon(N-1)\xi_i + \epsilon \sum_{j=2}^N \xi_j$. Here, $-\epsilon(N-1)\xi_i$ represents mode i “leaking” out of direction i and $\epsilon \sum_{j=2}^N \xi_j$ represents the other transverse modes “leaking” into direction i . In general, one can expect that the mixing coefficient ϵ will grow with the magnitude of parameter mismatches in the system. To eliminate confounding factors, we made the approximation that the synchronization manifold remains invariant and use the same synchronization trajectory as in Eq. (6) for calculating the largest Lyapunov exponent Λ . Thus, the index i in Eq. (11) runs from 2 to N in order to include only modes transverse to the synchronization manifold.

Figure 4(a) shows Λ as a function of ϵ for different values of k in Eq. (11) and for λ_i corresponding to the directed ring network used in Fig. 3. As we increase the mixing coefficient ϵ , Λ decreases and eventually becomes negative for all relevant k . In Supplemental Material [35], we further support our theory by providing an exact correspondence between the parameter mismatches in τ_i and the mixing matrix $\tilde{\Lambda}$. This enables us to explicitly compare the theoretical prediction [Fig. 4(b)] with experimental results, which show good agreement. Thus, mode mixing is the dominant contributor to the improved synchronization observed in our experiments and constitutes a general mechanism through which parameter mismatches in coupled oscillators can facilitate synchronization.

It is natural to ask: in which classes of systems can we expect to observe heterogeneity-induced synchronization? The answer to this question lies on the balance between two competing effects of heterogeneity on synchronization. On the one hand, we have shown that heterogeneity tends to improve stability by introducing mixing among the perturbation modes. On the other hand, the synchronization manifold often becomes “fuzzy” (i.e., the trajectory deviates from identical synchronization) when oscillators are nonidentical, and the permissible synchronization state deteriorates as heterogeneity is increased. For example, Ref. [25] shows that when synchronization is stable, the synchronization error increases linearly with oscillator heterogeneity. Thus, whether coherence in a network increases or decreases with heterogeneity depends on which of the two competing effects dominates, rather than on the details of the node dynamics and network structure.

As a result, the phenomenon of heterogeneity-induced synchronization is expected to apply to a broad class of networked dynamical systems. To further support this claim, in the Supplemental Material [35] we show that oscillator heterogeneity can also lead to dramatic improvement of synchronization in non-autonomous systems describing driven pendulum arrays. As in the networks of Chua’s oscillators, in this case too the synchronization trajectory remains qualitatively similar to the otherwise unstable synchronization trajectory of the homogeneous system. Future theoretical work will have the opportunity to further elucidate the effect of heterogeneity in the

non-perturbative regime, where the approximate invariance of the synchronization manifold breaks down and a new notion of “cohesiveness” among oscillators needs to be developed.

Our demonstration that parameter heterogeneity can enable rather than halt synchronization has several implications. In particular, it completes a full circle in revealing the interplay between chaos and coupling interactions. Early work on synchronization between coupled oscillators showed that sufficiently strong coupling can mitigate sensitive dependence on initial conditions. By demonstrating that oscillator heterogeneity can enable synchronization below the synchronization transition of identical oscillators, this work shows that, despite the sensitive dependence on oscillator parameters, parameter heterogeneity can consistently reduce the effective coupling threshold for synchronization. In man-made systems, this suggests that experimental imperfections may become an unexpected source of synchronization stability. In natural systems that rely on synchronization, it also suggests the possibility of observed mismatches being a result of evolutionary pressure that favors synchronization.

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