



This is the accepted manuscript made available via CHORUS. The article has been published as:

# Dynamical Symmetries and Symmetry-Protected Selection Rules in Periodically Driven Quantum Systems

Georg Engelhardt and Jianshu Cao

Phys. Rev. Lett. **126**, 090601 — Published 2 March 2021

DOI: [10.1103/PhysRevLett.126.090601](https://doi.org/10.1103/PhysRevLett.126.090601)

# Dynamical symmetries and symmetry-protected selection rules in periodically driven quantum systems

Georg Engelhardt<sup>1</sup> and Jianshu Cao<sup>2\*</sup>

<sup>1</sup>*Beijing Computational Science Research Center, Beijing 100193, People's Republic of China*

<sup>2</sup>*Department of Chemistry, Massachusetts Institute of Technology,  
77 Massachusetts Avenue, Cambridge, Massachusetts 02139, USA*

(Dated: December 7, 2020)

In recent experiments, the light-matter interaction has reached the ultra-strong coupling limit, which can give rise to dynamical generalizations of spatial symmetries in periodically-driven systems. Here, we present a unified framework of dynamical-symmetry-protected selection rules based on Floquet response theory. Within this framework, we study rotational, parity, particle-hole, chiral and time-reversal symmetries and the resulting selection rules in spectroscopy, including symmetry-protected dark states (spDS), symmetry-protected dark bands (spDB), and symmetry-induced transparency (siT). Specifically, dynamical rotational and parity symmetries establish spDS and spDB conditions; a particle-hole symmetry introduces spDSs for symmetry-related Floquet states and also a siT at quasienergy crossings; chiral symmetry and time-reversal symmetry alone do not imply spDS conditions, but can be combined to define a particle-hole symmetry. These symmetry conditions arise from destructive interference due to the synchronization of symmetric quantum systems with the periodic driving. Our predictions reveal new physical phenomena when a quantum system reaches the strong light-matter coupling regime, important for superconducting qubits, atoms and molecules in optical or plasmonic fields cavities, and optomechanical systems.

*Introduction.* Over the last few decades, the light-matter interaction strength has been pushed to the ultra-strong coupling regime in opto-mechanical systems [1], quantum dots, atoms and molecules in optical or plasmonic cavities [2–6], and superconducting quantum circuits [7, 8]. As standard nonlinear perturbation theory [9] becomes unfeasible under these conditions, Floquet response theory has been developed recently, describing systems which are subject to a strong, but time-periodic driving field (of frequency  $\Omega$ ), and a weak, but arbitrary probe field [10–13]. For a monochromatic probe of frequency  $\omega_p$ , system observables generate response frequencies  $\omega_p + n\Omega$  termed *Floquet bands* [14].

Spatial symmetries give rise to appealing physical properties. Inversion symmetry results in selection rules for dipole transitions; particle-hole, chiral and time-reversal symmetries establish the so-called periodic table, a classification scheme for topological insulators [15–17]; and symmetries have an essential impact on transport properties [18–24]. For periodically-driven systems, these spatial symmetries can be generalized to dynamical symmetries, which can give rise to a generalized periodic table for topological insulators [25, 26], and new control mechanism [27–33]. A dynamical-parity symmetry can induce coherent destruction of tunneling [34], and even harmonic generation [35].

In this letter, we introduce a unified conceptual framework of selection rules based on general dynamical symmetries of periodically-driven quantum systems, as described by Floquet response theory [36]. Physically, the synchronization of symmetric quantum systems with the periodic driving gives rise to destructive interfer-

TABLE I. Overview of the spectroscopic signatures of dynamical rotational symmetry (RS), particle-hole symmetry (PHS), parity symmetry (PS), chiral symmetry (CS), and time-reversal symmetry (TRS). The signatures include symmetry-protected dark states (spDS), symmetry-protected dark band (spDB), symmetry-induced transparency (siT), and accidental dark states (aDS). The right column lists example models.

Symmetry	Effect	Example
RS	spDS spDB	benzene-ring (Fig. 1)
PS	spDS spDB	two-level sys. (Fig. 3)
PHS	spDS	dimer (Fig. 2)
$2 \times$ PHS	siT	two-level sys. (Fig. 3)
TRS	none	
CS	none	
none	aDS	all (Figs. 1,2 ,3)

ence effects in Floquet space and thus to forbidden transitions between Floquet states. This set of forbidden transitions define the symmetry-protected selection rules that are robust against symmetry-preserving parameter variations. Specifically, there are four types of forbidden transitions ordered in the increasing degree of complexity: (i) accidental dark states (aDS), appearing for a specific combination of system parameters; (ii) symmetry-protected dark states (spDS), which refers to the symmetry-protected absence of a complete transition line, similar to symmetry-protected excitations of topological band structures; (iii) symmetry-protected dark bands (spDB), which refers to the absence of a complete Floquet band, due to a combination of spDS; (iv) symmetry-induced transparency (siT), which refers to the vanishing transition intensity at the degeneracy of

\* jianshu@mit.edu

quasi-energies. While aDS is not symmetry-related, we establish symmetry-protected selection rules for important dynamical symmetries, which are classified in table I.

*Floquet response theory.* We apply a semiclassical approach based on the general Hamiltonian

$$\hat{H}(t) = \hat{H}_0(t) + \int_0^\infty d\omega \left[ \lambda \hat{V} (\hat{a}_\omega + \hat{a}_\omega^\dagger) + \omega \hat{a}_\omega^\dagger \hat{a}_\omega \right], \quad (1)$$

where  $\hat{H}_0(t) = \hat{H}_0(t + \tau)$  describes a system driven by a periodic classical electromagnetic field of frequency  $\Omega = 2\pi/\tau$ . The probe field is given by a continuum of photonic operators  $\hat{a}_\omega^\dagger$  with frequencies  $\omega$ , which are coupled via the dipole transition operator  $\hat{V}$  with strength  $\lambda$  to the driven system. The physical properties of  $\hat{H}_0(t)$  are determined by the Floquet equation

$$\left[ \hat{H}_0(t) - i \frac{d}{dt} \right] |u_\mu(t)\rangle = \epsilon_\mu |u_\mu(t)\rangle, \quad (2)$$

where  $|u_\mu(t)\rangle = |u_\mu(t + \tau)\rangle$  and  $\epsilon_\mu$  are the corresponding Floquet states and quasienergies, which generalize the concept of eigenstates and eigenenergies of time-independent systems. It is implicitly assumed that the driven system is weakly dissipative, such that for long times it approaches the stationary state

$$\rho(t) = \sum_\mu p_\mu |u_\mu(t)\rangle \langle u_\mu(t)|, \quad (3)$$

which is diagonal in the Floquet basis and thus synchronizes with the driving,  $\rho(t) = \rho(t + \tau)$ . Equation (3) is consistent with the Floquet-Redfield equation [37–40] describing periodically-driven open quantum systems. In the model calculations, we assume the special distribution  $p_\mu \propto e^{-\beta\epsilon_\mu}$ , i.e., a Floquet-Gibbs distribution, but all our predictions hold even if the Floquet-Gibbs distribution breaks down [40]. Strongly dissipative systems could be addressed by generalizing our approach to non-Hermitian Hamiltonians [17] or via the polaron transformation [41].

The interaction of  $\hat{H}_0(t)$  with the probe field is treated using the input-output formalism and a perturbation expansion for small  $\lambda$ . The input field consists of a bichromatic probe field (of frequencies  $\omega_{p,1}$  and  $\omega_{p,2} = \omega_{p,1} + n\Omega$ , integer  $n$ ). As shown separately [13], the intensity change of the output field at frequency  $\omega_{p,2}$  proportional to the coherence  $\langle \hat{a}_{\omega_{p,2}}^\dagger \hat{a}_{\omega_{p,1}} \rangle$  is given by  $\Delta I_{\text{coh}}(\omega_{p,2}) = -i\tilde{\chi}_n(\omega_{p,1}) \langle \hat{a}_{\omega_{p,2}}^\dagger \hat{a}_{\omega_{p,1}} \rangle + \text{c.c.}$ , where the susceptibility  $\tilde{\chi}_n(\omega_{p,1})$  can be evaluated using Floquet response theory and reads

$$\tilde{\chi}_n(\omega_{p,1}) = i\lambda^2 \sum_{\nu, \mu, m} \frac{V_{\nu, \mu}^{(-n-m)} V_{\mu, \nu}^{(m)} (p_\nu - p_\mu)}{\epsilon_\mu - \epsilon_\nu + m\Omega - \omega_{p,1} - i\gamma_{\nu, \mu}^{(m)}}. \quad (4)$$

The index  $n$  denotes the Floquet band, which describes

non-elastic scattering of the probe field, and the dynamical dipole matrix elements read

$$V_{\lambda, \mu}^{(n)} = \frac{1}{\tau} \int_0^\tau \langle u_\lambda(t) | \hat{V} | u_\mu(t) \rangle e^{-in\Omega t} dt. \quad (5)$$

The parameters  $\gamma_{\nu, \mu}^{(m)}$  have been added phenomenologically and denote dephasing rates.

*Unified conceptual framework of dynamical-symmetry protected selection rules.* We consider the following class of symmetry operations [25]

$$\hat{\Sigma} \left[ \hat{H}_0(t_S + \beta_S t) - i \frac{d}{dt} \right] \hat{\Sigma}^{-1} = \alpha_S \left[ \hat{H}_0(t) - i \frac{d}{dt} \right], \quad (6)$$

where  $\hat{\Sigma}$  is a time-independent spatial operator. By specifying  $\hat{\Sigma}$ ,  $t_S$ , and  $(\alpha_S, \beta_S = \pm 1)$  one can define a set of dynamical symmetries. Applying Eq. (6) to the Floquet equation Eq. (2) one can identify relations between Floquet states  $\mu$  and  $\mu'$

$$|u_{\mu'}(t)\rangle = \pi_\mu^{(S)} \hat{\Sigma} |u_\mu(t_S + \beta_S t)\rangle, \quad (7)$$

which can be used to evaluate the dynamical dipole elements in Eq. (5). Imposing an invariance condition for the transition dipole operator  $\hat{\Sigma}^\dagger \hat{V} \hat{\Sigma} = \alpha_V^{(S)} \hat{V}$  and using Eq. (7), we investigate symmetry-protected selection rules for rotational, parity, particle-hole, chiral and time-reversal symmetries.

Within the unified framework, we can establish symmetry-protected selection rules, which are robust against symmetry-conserved variations and unique for strong-light matter interactions. Among others, we investigate dark states, which are defined by the condition  $V_{\nu, \mu}^{(m)} = 0$ , such that the corresponding resonances in Eq. (4) vanish. This condition not only generalizes the dark state condition in the standard response theory to the strong-coupling regime for  $n = 0$ , but also introduces distinct dark states effects for  $n \neq 0$ . All selection rules are a consequence of destructive interference due to the synchronization of the system state with the periodic driving: (i) The dark state condition can be fulfilled by special combinations of parameters, which we denote as aDS, or (ii) as a consequence of a symmetry, which we denote as spDS. (iii) An entire Floquet band can vanish because  $\tilde{\chi}_n(\omega_p) = 0$  for specific  $n$ , which we denote as a spDB. (iv) By analyzing the susceptibility in terms of Eq. (7), we establish the condition for the siT, which is due to a destructive interference of two transitions with  $V_{\nu, \mu}^{(m)} \neq 0$ .

*Rotational symmetry.* With  $\alpha_S = \beta_S = 1$ , a unitary  $\hat{\Sigma} = \hat{R}$ , and  $t_S = t_R = \frac{\tau}{N}$  with a positive integer  $N$ , Eq. (6) defines a dynamical rotational symmetry [42], which gives rise to the eigen equation  $|u_\mu(t)\rangle = \pi_\mu^{(R)} \hat{R} |u_\mu(t + t_R)\rangle$  with eigenvalues  $\pi_\mu^{(R)} = e^{i2\pi m_\mu/N}$  and integer  $m_\mu = \{0, N-1\}$ . As shown in detail in the SI, for a dipole transition operator with  $\hat{R}^\dagger \hat{V} \hat{R} = \alpha_V^{(R)} \hat{V}$

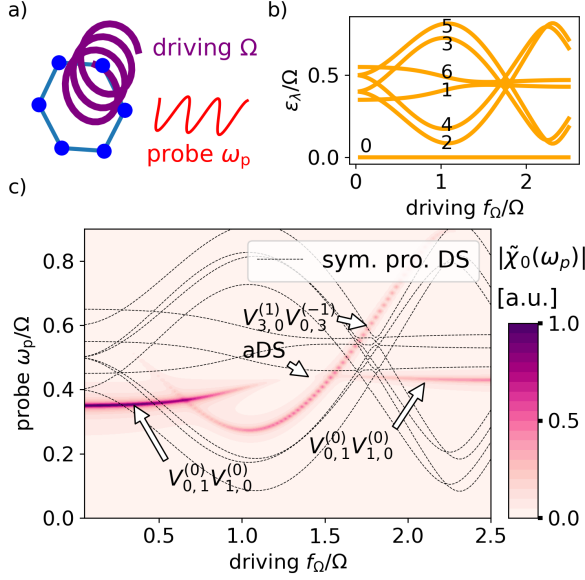


FIG. 1. (a) Benzene driven by circularly-polarized light propagating perpendicular to the ring plane. The probe field is polarized perpendicular to the plane so that it does not destroy the 6-fold dynamical rotational symmetry. (b) Quasienergies of benzene for the tunneling constant  $J_0 = 0.05\Omega$  and onsite energy  $E_0 = 0.45\Omega$ . (c) Susceptibility  $\tilde{\chi}_0(\omega_p)$  (color gradient). Rotational spDS are marked by dashed lines. One transition vanishes at the location of the aDS. The depahsing rates in all figures are  $\gamma_{\nu,\mu}^{(m)} = 0.001\Omega$ .

with  $\alpha_V^{(R)} = \pm 1$ , the dynamical rotational symmetry establishes a sufficient condition for spDS

$$\hat{V}_{\nu,\mu}^{(m)} \propto \begin{cases} 1 & \text{if } e^{i\frac{2\pi}{N}(m_\mu - m_\nu + m)} \alpha_V^{(R)} = 1, \\ 0 & \text{else.} \end{cases} \quad (8)$$

Applying Eq. (8) to evaluate the susceptibility in Eq. (4), we find

$$\tilde{\chi}_n(\omega_p) = \begin{cases} 1 & \text{if } e^{i\frac{2\pi}{N}n} = 1, \\ 0 & \text{else,} \end{cases} \quad (9)$$

which is the condition for the complete disappearance of Floquet band  $n$ , i.e., a spDB. Physically, this effect appears as the stationary state Eq. (3) synchronizes with the driving field, such that the density matrix adopts the dynamical rotational symmetry, i.e.,  $\rho(t + n/N\tau) = \hat{R}^n \rho(t) \hat{R}^{\dagger n}$ .

As an example, we consider a benzene ring driven by circularly-polarized light sketched in Fig. 1(a), which is described by a tight-binding Hamiltonian

$$\hat{H}_0(t) = \sum_{j,j'=1}^6 J_{j,j'} |e_j\rangle \langle e_{j'}| + \sum_{j=1}^6 [i f_j(t) |e_j\rangle \langle e_{j+1}| + h.c.],$$

where  $|e_j\rangle$  denotes the excitation on site  $j$  (defined modulo 6),  $J_{j,j} = E_0$  is the onsite energy,  $J_{j,j'} = \delta_{j,j'\pm 1} J_0$  is

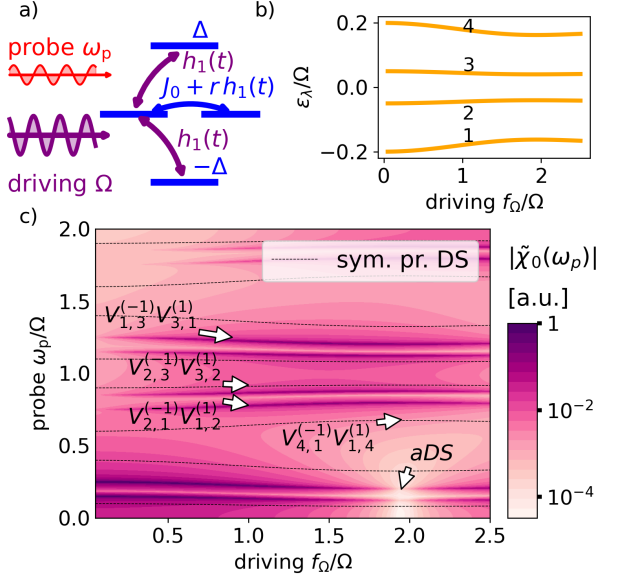


FIG. 2. (a) Sketch of the dimer model Eq. (11) with  $h_1(t) = f_\Omega \cos(\Omega t)$ . (b) Quasienergy spectrum for  $J_0/\Omega = 0.05$ ,  $r = 2$  and  $\Delta = 0.2\Omega$ . (c) The susceptibility  $|\tilde{\chi}_0(\omega_p)|$  is depicted as a color gradient. spDS (marked by dashed lines) are generated by a particle-hole symmetry.

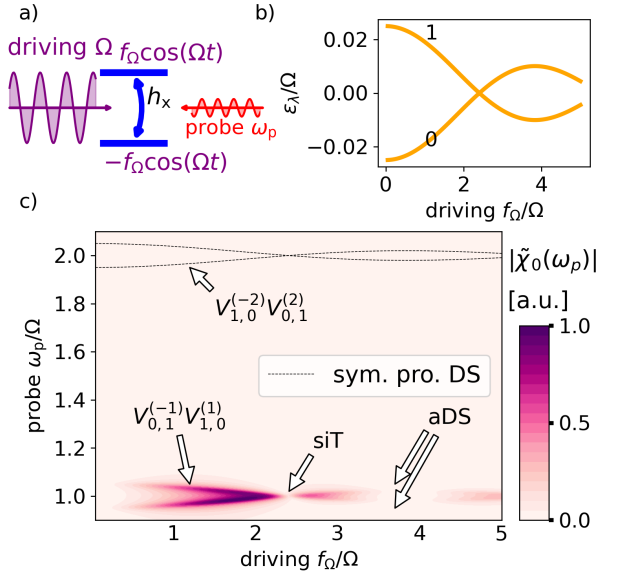


FIG. 3. (a) Sketch of the ac-driven two-level system. (b) Quasienergy spectrum for  $h_x/\Omega = 0.05$ . (c) The spectrum of the susceptibility  $\tilde{\chi}_0(\omega_p)$  exhibits a siT and spDSs. Here,  $p_0 = 0.6$  and  $p_1 = 0.4$  in Eq. (3) to highlight the siT.

the tunneling constant, and  $f_j(t) = f_\Omega \cos(\Omega t + 2\pi j/6)$  is the time-dependent tunneling strength with the driving amplitude  $f_\Omega$ . The driving terms are motivated by the Peierls substitution describing a vectorial current-gauge-field coupling  $\mathbf{j} \cdot \mathbf{A}(t)$  [43] with a circularly rotating vector potential  $\mathbf{A}(t)$ . The dipole transition operator

$\hat{V} = \sum_{j=1}^N d_0 |e_j\rangle \langle g|$  excites the ground state  $|g\rangle$  to the single-excitation manifold, whose quasienergies are depicted in Fig. 1(b). The stationary state is  $\rho_s(t) = |g\rangle \langle g|$  in agreement with Eq. (3), i.e., a Floquet-Gibbs state for low temperatures. A rotational symmetry is fulfilled for  $N = 6$  and  $\hat{R} = \sum_{j=1}^N |e_{j+1}\rangle \langle e_j|$ .

In Fig. 1(c) we depict the susceptibility  $\tilde{\chi}_0(\omega_p)$  of the benzene model. The resonances of the dark states defined by Eq. (8) are marked by dashed lines (optically invisible), and only two transitions,  $\hat{V}_{0,1}^{(0)} \hat{V}_{1,0}^{(0)}$  and  $\hat{V}_{3,0}^{(1)} \hat{V}_{0,3}^{(-1)}$ , are visible. An aDS can be found for  $\hat{V}_{0,1}^{(0)} \hat{V}_{1,0}^{(0)}$  at  $f_\Omega = 1.5\Omega$ . As a consequence of the spDB in Eq. (9), only Floquet bands  $\tilde{\chi}_n(\omega_p)$  with  $n \bmod 6 = 0$  appear.

*Parity symmetry.* A dynamical parity symmetry is a specification of the dynamical rotational symmetry with  $N = 2$  and a Hermitian operator  $R^\dagger = R$ , such that the spDS condition Eq. (8) and the spDB condition Eq. (9) are equally valid. The spDSs will be illustrated for the TLS in Eq. (13) along with the siT discussed below.

*Particle-hole symmetry.* A particle-hole symmetry is defined for  $-\alpha_S = \beta_S = 1$ ,  $t_S = t_P = \tau N_1/2N_2$  with integers  $N_1 \in \{0, 1\}$ ,  $N_2 \geq 1$ , and  $\hat{\Sigma} = \hat{P}\hat{\kappa}$  with a unitary operator  $\hat{P}$  and the complex conjugation operator  $\hat{\kappa}$ , such that  $\hat{P}\hat{H}^*(t + t_P)\hat{P} = -\hat{H}(t)$ . The particle-hole symmetry establishes a symmetry between excitation and deexcitation processes, and has its origin in fermionic systems, where adding and removing quasiparticles results in physically equivalent behaviors. Here we use the particle-hole symmetry in a general context. Using the particle-hole symmetry in Eq. (2), we find that each Floquet state  $|u_\mu(t)\rangle$  with quasienergy  $\epsilon_\mu$  has its symmetry-related partner  $|u_{\mu'}(t)\rangle = \pi_\mu^{(P)} \hat{P} |u_\mu(t + t_P)\rangle^*$  with energy  $\epsilon_{\mu'} = -\epsilon_\mu$  and a gauge-dependent  $\pi_\mu^{(P)}$ . For  $t_P = \tau/(2N_2)$  the particle-hole symmetry gives rise to a rotational symmetry defined by  $\hat{R} = \hat{P}\hat{P}$  and  $t_R = \tau/N_2$ , such that the dark state selection rules of the rotational symmetry apply. The particle-hole symmetry can give rise to a distinct dark state condition. For a dipole transition operator with  $\hat{P}^\dagger \hat{V}^* \hat{P} = \alpha_V^{(P)} \hat{V}$ ,  $\alpha_V^{(P)} = \pm 1$ ,  $t_P = 0, \tau/2$ , and  $\hat{P}^* \hat{P} = 1$ , the particle-hole symmetry results in  $V_{\mu,\mu'}^{(m)} = \alpha_V^{(P)} e^{im\Omega t_P} V_{\mu,\mu'}^{(m)}$  for symmetry related states  $\mu, \mu'$ , so that

$$\hat{V}_{\mu,\mu'}^{(m)} \propto \begin{cases} 0 & \text{if } \alpha_V^{(P)} e^{im\Omega t_P} = -1; \mu, \mu' \text{ sym. rel.} \\ 1 & \text{else,} \end{cases} \quad (10)$$

as shown in detail in the SI. In contrast to Eq. (8), where each transition can vanish for an appropriate  $m$ , only transitions between symmetry-related states are affected by Eq. (10).

To illustrate Eq. (10), we use the dimer model sketched in Fig. 2(a), with the Hamiltonian given by

$$H_0(t) = \Delta (\hat{A}_{f,f} - \hat{A}_{g,g}) + J_0 \hat{A}_{e_1,e_2}$$

$$+ h_1(t) [\hat{A}_{e_1,f} + \hat{A}_{g,e_1} + r \hat{A}_{e_1,e_2}], \quad (11)$$

where  $\hat{A}_{\alpha,\beta} \equiv |\alpha\rangle \langle \beta| + \text{h.c.}$ , and  $g, e_1, e_2$  and  $f$  label the ground state, two single-excitation states and the double excitation state, respectively.  $\Delta$  is the excitation gap,  $J_0$  is the tunneling constant, and  $h_1(t) = f_\Omega \cos(\Omega t)$  is the driving field. The  $r$  term enhances higher-order dipole elements  $V_{\mu,\mu'}^{m \neq 0}$ . The particle-hole symmetry is defined by  $\hat{P} = \hat{A}_{g,f} + \hat{A}_{e_1,e_1} - \hat{A}_{e_2,e_2}$  and  $t_P = 0$ . The quasienergy spectrum in Fig. 2(b) is symmetric with respect to  $E = 0$ . The dipole transition operator is  $\hat{V} = \hat{A}_{e_1,f} + \hat{A}_{g,e_1}$ , such that  $\hat{P}^\dagger \hat{V}^* \hat{P} = -\hat{V}$ . In Fig. 2(c), we depict the susceptibility in Eq. (4). According to the above considerations, the transitions between the particle-hole symmetry related pairs vanish, i.e.,  $V_{1,4}^{(m)} = V_{4,1}^{(m)} = V_{2,3}^{(m)} = V_{3,2}^{(m)} = 0$  for all  $m$ . These resonances are marked by dashed lines. The other transitions not affected by the symmetry constrain remain visible in Fig. 2(c).

*Symmetry-induced transparency.* The particle-hole symmetry can also give rise to a siT at the quasienergy crossing  $\epsilon_\mu = \epsilon_{\mu'} = 0$  of symmetry related Floquet states  $\mu, \mu'$ . While a spDS is generated by a vanishing dipole element  $V_{\lambda,\mu}^{(n)} = 0$ , the siT is generated by a destructive interference of two transitions with  $V_{\lambda,\mu}^{(n)} \neq 0$ . As shown in the SI in detail, for two distinct particle-hole symmetries  $\hat{P}_1 \neq \pm \hat{P}_2$ ,  $\hat{P}_i^2 = \mathbb{1}$  and  $[\hat{P}_1, \hat{P}_2] = 0$ , the siT condition reads

$$\tilde{\chi}_n(m\Omega) \propto \begin{cases} 0 & \text{if } e^{im\Omega(t_{P_1} - t_{P_2})} = 1; \epsilon_\mu = \epsilon_{\mu'} = 0 \\ 1 & \text{else,} \end{cases} \quad (12)$$

where  $t_{P_i}$  denote the reference times related to  $\hat{P}_i$ .

For illustration, we consider an ac-driven two-level system (TLS) sketched in Fig. 3(a) and described by the Hamiltonian

$$\hat{H}_0(t) = \frac{h_x}{2} \hat{\sigma}_x + \frac{f_\Omega}{2} \cos(\Omega t) \hat{\sigma}_z, \quad (13)$$

where  $\hat{\sigma}_x, \hat{\sigma}_z$  are the Pauli matrices,  $h_x$  is the tunneling amplitude, and  $f_\Omega$  the driving strength. The TLS is weakly dissipative, as in the spin-boson model, such that it reaches the stationary state in Eq. (3). The dipole transition operator in Eq. (1) is  $\hat{V} = \hat{\sigma}_x$ . For  $\hat{R} = \hat{\sigma}_x$  and  $t_R = \tau/2$ , the two-level system exhibits a dynamical parity symmetry defined above, which gives rise to coherent destruction of tunneling effect at exact quasienergy crossing, depicted in Fig. 3(b) at  $f_\Omega \approx 2.4\Omega$  [34, 44], and enables the siT in the current context. Additionally, the TLS exhibits spDSs and spDBs according to Eq. (8) and Eq. (9) as  $V_{\mu,\nu'}^{(m)} = 0$  for even  $m$  because of the dynamical parity symmetry.

For the TLS, a particle-hole symmetry is defined for  $\hat{P}_1 = \hat{\sigma}_z$  and  $t_{P_1} = \tau/2$ , for  $h_x = 0$  a second particle hole symmetry is given for  $\hat{P}_2 = \mathbb{1}$  and  $t_{P_2} = \tau/2$ . As in this case  $\epsilon_\mu = 0$  and  $\hat{P}_i \hat{\sigma}_x \hat{P}_i = (-1)^i \hat{\sigma}_x$ , siT with  $\tilde{\chi}_n(m\Omega) = 0$

appears according to Eq. (12) and the response  $\tilde{\chi}_n(\omega_p)$  is complete suppressed for all  $n$ . In Fig. 3(c) we consider  $\tilde{\chi}_0(\omega_p)$  for a finite, but small  $h_x \ll \Omega$ , such that the quasienergy degeneracy is lifted except of the crossing, and the particle-hole symmetry  $\hat{P}_2$  is slightly broken. As a consequence, the siT is not complete, but scales as  $\tilde{\chi}_n(m\Omega) \propto h_x/\Omega$  at the crossing.

*Time-reversal and chiral symmetries.* A time-reversal symmetry (chiral symmetry) is defined by Eq. (6) for  $\alpha_S = -\beta_S = 1$  ( $\alpha_S = \beta_S = -1$ ), arbitrary  $t_S$ , and  $\hat{\Sigma} = \hat{T}\hat{\kappa}$ , ( $\hat{\Sigma} = \hat{C}$ ), where  $\hat{T}$  ( $\hat{C}$ ) is a unitary operator. As shown in the SI, neither time-reversal symmetry nor chiral symmetry alone implies spDSs. However, the combination of time-reversal symmetry and chiral symmetry defines a particle-hole symmetry with  $\hat{P} = \hat{C}\hat{T}$ , and  $t_P = t_T - t_C$ . When they further fulfill  $t_T - t_C \in \{0, \tau/2\}$ ,  $\hat{C}^*\hat{C} = \mathbb{1}$ ,  $\hat{T}^*\hat{T} = \mathbb{1}$ , and  $[\hat{C}, \hat{T}] = 0$ , such that  $\hat{P}^*\hat{P} = 1$ , spDSs appear because of the particle-hole symmetry. In general, the presence of any two symmetries out of particle-hole symmetry, chiral symmetry, and time-reversal symmetry implies the existence of the third one.

*Conclusions.* Using a unified conceptional framework based on Floquet response theory, we have predicted selection rules in periodically-driven quantum systems, namely accidental dark states, symmetry-protected dark states, symmetry-protected dark bands, and symmetry-

induced transparency. The latter three effects are protected by symmetries, such that variations of symmetry preserving parameters do not destroy them. These symmetry-induced selection rules result from the destructive interference of a driven system synchronized to the periodic driving. The different effects have been illustrated in three example systems fulfilling different symmetries, demonstrating the flexibility and generality of our unified framework. The predicted selection rules are valid even for more complicated and realistic systems, as long as the corresponding dynamical symmetries are fulfilled.

Our theoretical results are experimentally observable in systems that can reach the strong-light matter coupling regime such as cold-atom experiments [2] and superconducting circuits [45–48]. For experiments with molecules, strong driving fields are necessary to generate high-order Floquet bands but in cavity QED or plasmonic fields the strong driving interaction condition can be relaxed for molecules ensembles interacting collectively with the light field [49, 50].

*Acknowledgements* G. E. gratefully acknowledges financial support from the China Postdoc Science Foundation (Grant No. 2018M640054) and the Natural Science Foundation of China (Grant No. 11950410510 and No. U1930402), J. C. acknowledges support from the NSF (Grants No. CHE 1800301 and No. CHE1836913). The authors thank Tao Wang for helpful discussions.

- 
- [1] K. Fang, J. Luo, A. Metelmann, M. H. Matheny, F. Marquardt, A. A. Clerk, and O. Painter, “Generalized nonreciprocity in an optomechanical circuit via synthetic magnetism and reservoir engineering,” *Nature Physics* **13**, 465–471 (2017).
  - [2] M.-J. Yin, T. Wang, X.-T. Lu, T. Li, Y.-B. Wang, X.-F. Zhang, W.-D. Li, A. Smerzi, and H. Chang, “Rabi spectroscopy and sensitivity of a Floquet engineered optical lattice clock,” arXiv:2007.00851 (2020).
  - [3] X. Li, D. Dreon, P. Zupancic, A. Baumgrtner, A. Morales, W. Zheng, N. R. Cooper, T. Donner, and T. Esslinger, “Measuring the dynamics of a first order structural phase transition between two configurations of a superradiant crystal,” (2020), arXiv:2004.08398 [cond-mat.quant-gas].
  - [4] J. D. Pino, J. Feist, and F. J. Garcıavalıdal, “Signatures of vibrational strong coupling in Raman scattering,” *Journal of Physical Chemistry C* **119**, 29132–29137 (2015).
  - [5] A. Shalabney, J. George, J. A. Hutchison, G. Pupillo, C. Genet, and T. W. Ebbesen, “Coherent coupling of molecular resonators with a microcavity mode,” *Nature Communications* **6**, 5981–5981 (2015).
  - [6] M. Sukharev and A. Nitzan, “Optics of exciton-plasmon nanomaterials,” *Journal of Physics: Condensed Matter* **29**, 443003 (2017).
  - [7] P. Forn-Dıaz, L. Lamata, E. Rico, J. Kono, and E. Solano, “Ultrastrong coupling regimes of light-matter interaction,” *Rev. Mod. Phys.* **91**, 025005 (2019).
  - [8] A. F. Kockum, A. Miranowicz, S. De Liberato, S. Savasta, and F. Nori, “Ultrastrong coupling between light and matter,” arXiv: Mesoscale and Nanoscale Physics **1**, 19–40 (2019).
  - [9] S. Mukamel, *Principles of nonlinear optical spectroscopy* (OXFORD UNIVERSITY PRESS, 1995).
  - [10] S. Kohler, “Dispersive readout: Universal theory beyond the rotating-wave approximation,” *Phys. Rev. A* **98**, 023849 (2018).
  - [11] B. Gu and I. Franco, “Optical absorption properties of laser-driven matter,” *Phys. Rev. A* **98**, 063412 (2018).
  - [12] G. Cabra, I. Franco, and M. Y. Galperin, “Optical properties of periodically driven open nonequilibrium quantum systems,” *Journal of Chemical Physics* **152**, 094101 (2020).
  - [13] G. Engelhardt and J. Cao, “Signatures of dynamical symmetries in Floquet response theory and spontaneous emission processes,” (), in preparation.
  - [14] A. Kumar, M. Rodriguez-Vega, T. Pereg-Barnea, and B. Seradjeh, “Linear response theory and optical conductivity of Floquet topological insulators,” *Phys. Rev. B* **101**, 174314 (2020).
  - [15] A. Altland and M. R. Zirnbauer, “Nonstandard symmetry classes in mesoscopic normal-superconducting hybrid structures,” *Phys. Rev. B* **55**, 1142–1161 (1997).
  - [16] Josias Langbehn, Yang Peng, Luka Trifunovic, Felix von Oppen, and Piet W. Brouwer, “Reflection-symmetric second-order topological insulators and superconductors,” *Phys. Rev. Lett.* **119**, 246401 (2017).
  - [17] Kohei Kawabata, Takumi Bessho, and Masatoshi Sato,

- “Classification of exceptional points and non-Hermitian topological semimetals,” *Phys. Rev. Lett.* **123**, 066405 (2019).
- [18] C. Duan, C.-Y. Hsieh, J. Liu, J. Wu, and J. Cao, “Unusual transport properties with noncommutative system-bath coupling operators,” *The Journal of Physical Chemistry Letters* **11**, 4080–4085 (2020).
- [19] J. Thingna, D. Manzano, and J. Cao, “Magnetic field induced symmetry breaking in nonequilibrium quantum networks,” *New Journal of Physics* **22**, 083026 (2020).
- [20] J. Wu, R. J. Silbey, and J. Cao, “Generic Mechanism of Optimal Energy Transfer Efficiency: A Scaling Theory of the Mean First-Passage Time in Exciton Systems,” *Phys. Rev. Lett.* **110**, 200402 (2013).
- [21] C. Chuang, C. K. Lee, J. M. Moix, J. Knoester, and J. Cao, “Quantum diffusion on molecular tubes: Universal scaling of the 1D to 2D transition,” *Phys. Rev. Lett.* **116**, 196803 (2016).
- [22] J. Thingna, D. Manzano, and J. Cao, “Dynamical signatures of molecular symmetries in nonequilibrium quantum transport,” *Scientific reports* **6**, 28027 (2016).
- [23] G. Engelhardt and J. Cao, “Tuning the Aharonov-Bohm effect with dephasing in nonequilibrium transport,” *Phys. Rev. B* **99**, 075436 (2019).
- [24] I. Fleming, *Molecular orbitals and organic chemical reactions. Student edition* (Weinheim, Germany: Wiley-VCH, 2009).
- [25] R. Roy and F. Harper, “Periodic table for Floquet topological insulators,” *Phys. Rev. B* **96**, 155118 (2017).
- [26] Yang Peng and Gil Refael, “Floquet second-order topological insulators from nonsymmorphic space-time symmetries,” *Phys. Rev. Lett.* **123**, 016806 (2019).
- [27] G. Engelhardt, M. Benito, G. Platero, and T. Brandes, “Topological instabilities in ac-driven bosonic systems,” *Phys. Rev. Lett.* **117**, 045302 (2016).
- [28] G. Engelhardt, M. Benito, G. Platero, and T. Brandes, “Topologically enforced bifurcations in superconducting circuits,” *Phys. Rev. Lett.* **118**, 197702 (2017).
- [29] A. Gómez-León and G. Platero, “Transport blocking and topological phases using ac magnetic fields,” *Phys. Rev. B* **85**, 245319 (2012).
- [30] J. Yuen-Zhou, S. K. Saikin, T. Zhu, M. C. Onbasli, C. A. Ross, V. Bulovic, and M. A. Baldo, “Plexciton Dirac points and topological modes,” *Nature communications* **7** (1), 11783 (2015).
- [31] Y. Yan, Z. Lü, J. Y. Luo, and H. Zheng, “Role of generalized parity in the symmetry of the fluorescence spectrum from two-level systems under periodic frequency modulation,” *Phys. Rev. A* **100**, 013823 (2019).
- [32] K. Chinzei and T. N. Ikeda, “Time crystals protected by Floquet dynamical symmetry in Hubbard models,” *Phys. Rev. Lett.* **125**, 060601 (2020).
- [33] B. Bua, J. Tindall, and D. Jaksch, “Non-stationary coherent quantum many-body dynamics through dissipation,” *Nature Communications* **10**, 1730 (2019).
- [34] F. Grossmann, T. Dittrich, P. Jung, and P. Hänggi, “Coherent destruction of tunneling,” *Phys. Rev. Lett.* **67**, 516–519 (1991).
- [35] R. Bavli and H. Metiu, “Properties of an electron in a quantum double well driven by a strong laser: Localization, low-frequency, and even-harmonic generation,” *Phys. Rev. A* **47**, 3299–3310 (1993).
- [36] G. Engelhardt and J. Cao, “Signatures of dynamical symmetries in floquet response theory and spontaneous emission processes,” (in preparation) ().
- [37] S. Kohler, R. Utermann, P. Hänggi, and T. Dittrich, “Coherent and incoherent chaotic tunneling near singlet-doublet crossings,” *Phys. Rev. E* **58**, 7219–7230 (1998).
- [38] R. Blattmann, P. Hänggi, and S. Kohler, “Qubit interference at avoided crossings: The role of driving shape and bath coupling,” *Phys. Rev. A* **91**, 042109 (2015).
- [39] T. Shirai, T. Mori, and S. Miyashita, “Condition for emergence of the Floquet-Gibbs state in periodically driven open systems,” *Phys. Rev. E* **91**, 030101 (2015).
- [40] G. Engelhardt, G. Platero, and J. Cao, “Discontinuities in Driven Spin-Boson Systems due to Coherent Destruction of Tunneling: Breakdown of the Floquet-Gibbs Distribution,” *Phys. Rev. Lett.* **123**, 120602 (2019).
- [41] D. Xu and J. Cao, “Non-canonical distribution and nonequilibrium transport beyond weak system-bath coupling regime: A polaron transformation approach,” *Frontiers of Physics* **11**, 110308 (2016).
- [42] One can construct a Hamiltonian fulfilling a time-rotational symmetry defined by  $\hat{H}_0(t) = \sum_j \sum_m f_j(t - m\frac{\tau}{N}) \hat{A}_j^{(m)}$ , where  $\hat{A}_j^{(m)} = \hat{R}^m \hat{A}_j^{(0)} \hat{R}^m$  for arbitrary Hermitian operators  $\hat{A}_j^{(0)}$ , and the arbitrary functions  $f_j(t + \tau) = f_j(t)$  shall be  $\tau$  periodic.
- [43] B. A. Bernevig and L. H. Taylor, *Topological Insulators and Topological Superconductors* (Princeton University Press, 2013).
- [44] J. Gong, L. Morales-Molina, and P. Hänggi, “Many-body coherent destruction of tunneling,” *Phys. Rev. Lett.* **103**, 133002 (2009).
- [45] T. Wang, Z. Zhang, L. Xiang, Z. Jia, P. Duan, Z. Zong, Z. Sun, Z. Dong, J. Wu, Y. Yin, and G. Guo, “Experimental Realization of a Fast Controlled-Z Gate via a Shortcut to Adiabaticity,” *Phys. Rev. Applied* **11**, 034030 (2019).
- [46] C. Zha, V. M. Bastidas, M. Gong, Y. Wu, H. Rong, R. Yang, Y. Ye, S. Li, Q. Zhu, S. Wang, Y. Zhao, F. Liang, J. Lin, Y. Xu, C.-Z. Peng, J. Schmiedmayer, K. Nemoto, H. Deng, W. J. Munro, X. Zhu, and J.-W. Pan, “Ergodic-localized junctions in a periodically-driven spin chain,” *arXiv:2001.09169* (2020).
- [47] L. Magazzù, P. Forndiaz, R. Belyansky, J. L. Orgiazzi, M. A. Yurtalan, M. Otto, A. Lupascu, C. Wilson, and M. Grifoni, “Probing the strongly driven spin-boson model in a superconducting quantum circuit,” *Nature Communications* **9**, 1403 (2018).
- [48] M.-B. Chen, B.-C. Wang, S. Kohler, Y. Kang, T. Lin, S.-S. Gu, H.-O. Li, G.-C. Guo, X. Hu, H.-W. Jiang, G. Cao, and G.-P. Guo, “Double resonance landau-zener-stckelburg-majorana interference in circuit qed,” (2020), *arXiv:2011.03697*.
- [49] F. Herrera and F. C. Spano, “Cavity-controlled chemistry in molecular ensembles,” *Phys. Rev. Lett.* **116**, 238301 (2016).
- [50] T. E. Li, A. Nitzan, and J. E. Subotnik, “On the origin of ground-state vacuum-field catalysis: Equilibrium consideration,” *J. Chem. Phys.* **152**, 234107 (2020).