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Unruh and Cherenkov radiation from a negative frequency perspective

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A ground-state atom uniformly accelerated through the Minkowski vacuum can become excited by emitting an Unruh-Minkowski photon. We show that from the perspective of an accelerated atom, the sign of the frequency of the Unruh-Minkowski photons can be positive or negative depending on the acceleration direction. The accelerated atom becomes excited by emitting an Unruh-Minkowski photon which has negative frequency in the atom's frame, and decays by emitting a positive frequency photon. This leads to interesting effects. For example, the photon emitted by accelerated ground-state atom can not be absorbed by another ground-state atom accelerating in the same direction, but it can be absorbed by an excited atom or a ground-state atom accelerated in the opposite direction. We also show that similar effects take place for Cherenkov radiation. Namely, a Cherenkov photon emitted by an atom can not be absorbed by another ground-state atom moving with the same velocity, but can be absorbed by an excited atom or a ground-state atom moving in the opposite direction.

An atom uniformly accelerated in the Minkowski vacuum can become excited by emitting a photon into the so called Unruh-Minkowski mode. This is known as Unruh acceleration radiation [1–3]. The Unruh-Minkowski photon has positive energy. The energy required to emit the photon by the accelerated atom is gained from the atom's kinetic energy.

However, atoms interact with the field locally, that is an atom feels the local value of the field at the atom's location. As a consequence, the local properties of the photon mode function determine the atom's ability to emit and absorb the photon. Here we investigate what field the accelerated atom senses when it interacts with the Unruh-Minkowski photons. We show that from the atom's perspective, a certain type of the Unruh-Minkowski modes act as if they have negative frequency (negative energy) and the atom can become excited by emitting a photon into these locally "negative frequency" modes. We find that the emitted photon can not be absorbed by another ground-state atom accelerating in the same direction, but it can be absorbed by an excited atom or a ground-state atom accelerated in the opposite direction. We also show that similar effects take place for Cherenkov radiation. Namely, a Cherenkov photon emitted by an atom can not be absorbed by another ground-state atom moving with the same velocity, but can be absorbed by an excited atom or a ground-state atom moving in the opposite direction. The possibility (or not) of the emission from relatively moving ground-state atoms to excite other atoms is the main novel finding of our paper.

Negative frequency is associated with some interesting physical effects. For example, Cherenkov radiation consists of waves that have negative frequency in the rest frame of the particle [4, 5]. Quantum friction between relatively moving dielectrics stems from the mixing of positive and negative frequency waves in the two materials [6, 7]. Hawking radiation [8] and its laboratory analogs [9, 10] originate from the change in the sign of the frequency of a wave as it crosses the event horizon. The behavior of the negative-energy quanta is essential to understanding the kinematics of amplification of waves which can explain the physics of traveling-wave-tube-type amplifiers [11], a resistive-wall amplifier [12], and amplification of ultrasound in semiconductors [13]. Cherenkov-like radiation of sound waves can be produced by a charged particle moving through a medium with a velocity exceeding the phase velocity of phonons [14]. It also occurs when an obstacle is moving through a superfluid with supersonic speed. Such motion yields Cherenkov emission of Bogoliubov's quasiparticles inside

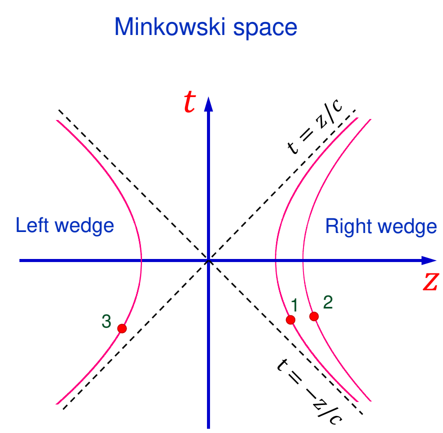

FIG. 1: Trajectory of atoms uniformly accelerated in different Rindler wedges.

the Mach cone [15].

Here we consider an electrically neutral two-level atom with a transition angular frequency ω which moves along the trajectory $t(\tau)$, $z(\tau)$ in vacuum, where τ is the proper time of the atom. For simplicity we approximate light as a scalar field described by the operator $\hat{\Phi}(t,z)$ and consider either dimension $1+1$, or dimension $3+1$, but restrict photons to have wave vector $\mathbf{k}$ parallel to the $z-$axis [16]. We will assume the following form of the interaction Hamiltonian between the atom and the scalar field

$$\hat{V}(\tau) = g\left(\hat{\sigma}e^{-i\omega\tau} + \hat{\sigma}^{\dagger}e^{i\omega\tau}\right)\frac{\partial}{\partial\tau}\hat{\Phi}(t(\tau), z(\tau)), \quad (1)$$

where g is the atom-field coupling constant, and $\hat{\sigma}$ and $\hat{\sigma}^{\dagger}$ are the atomic lowering and raising operators. Since the atom feels the local value of the field the operator $\hat{\Phi}$ is evaluated at the atom's position $t(\tau)$, $z(\tau)$. In the case of QED, $\hat{\Phi}$ is analogous to the vector potential and Hamiltonian (1) is in the dipole approximation.

The Unruh-Minkowski modes of the scalar field Φ are defined as [17]

$$F_{1\Omega}(t,z) = \frac{|t \pm z/c|^{i\Omega}}{\sqrt{2\Omega\sinh(\pi\Omega)}}\begin{cases} e^{-\frac{\pi\Omega}{2}}, & t \pm z/c > 0 \\ e^{\frac{\pi\Omega}{2}}, & t \pm z/c < 0 \end{cases}, \quad (2)$$

and

$$F_{2\Omega}(t,z) = \frac{|t \pm z/c|^{-i\Omega}}{\sqrt{2\Omega\sinh(\pi\Omega)}}\begin{cases} e^{\frac{\pi\Omega}{2}}, & t \pm z/c > 0 \\ e^{-\frac{\pi\Omega}{2}}, & t \pm z/c < 0 \end{cases}, \quad (3)$$

where $\Omega > 0$, and the $\pm$ sign corresponds to left and right propagating photons respectively. Ω is a parameter which is proportional to the photon frequency in the Rindler space [18].

The Unruh-Minkowski modes (2) and (3) are solutions of the wave equation. They form a complete set and have positive norm (defined as the Klein–Gordon inner product) and, thus, are associated with the photon annihilation operators $\hat{a}_{1\Omega}$ and $\hat{a}_{2\Omega}$. The negative-norm modes are the complex conjugates of Eqs. (2) and (3), and correspond to the photon creation operators $\hat{a}^{\dagger}_{1\Omega}$ and $\hat{a}^{\dagger}_{2\Omega}$. Expansion of the field operator in terms of the Unruh-Minkowski modes reads

$$\hat{\Phi} = \sum_{\Omega>0}\left(F_{1\Omega}\hat{a}_{1\Omega} + F^{*}_{1\Omega}\hat{a}^{\dagger}_{1\Omega} + F_{2\Omega}\hat{a}_{2\Omega} + F^{*}_{2\Omega}\hat{a}^{\dagger}_{2\Omega}\right). \quad (4)$$

The vacuum state for the Unruh-Minkowski photons is the usual Minkowski vacuum $|0_M\rangle$, that is $\hat{a}_{1\Omega}|0_M\rangle = 0$, $\hat{a}_{2\Omega}|0_M\rangle = 0$ for all Ω.

The Unruh-Minkowski modes are a convenient choice for the description of Unruh acceleration radiation. Namely, a ground-state atom with a transition frequency ω moving in the right Rindler wedge with an acceleration a (see Fig. 1) emits left-propagating waves into the single Unruh-Minkowski mode $F_{1\Omega}$ and right-propagating waves into the mode $F_{2\Omega}$, where $\Omega = \omega c/a$. To gain insight into the process of acceleration radiation we consider a uniformly accelerated atom moving along the trajectory

$$t(\tau) = \frac{c}{a}\sinh\left(\frac{a\tau}{c}\right), \quad z(\tau) = \frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right) \quad (5)$$

in the Minkowski spacetime. In Eq. (5) τ is the proper time of the atom. If $a > 0$ ($a < 0$) the atom moves in the right (left) Rindler wedge (see Fig. 1).

Along the worldline of the accelerated atom (5) the Unruh-Minkowski mode functions for the left-propagating photons have the form

$$F_{L1\Omega}(t(\tau), z(\tau)) = \frac{\left(\frac{c}{a}\right)^{i\Omega} e^{-\frac{\pi\Omega}{2}sign(a)}}{\sqrt{2\Omega\sinh(\pi\Omega)}}e^{ia\Omega\tau/c}, \quad (6)$$

$$F_{L2\Omega}(t(\tau), z(\tau)) = \frac{\left(\frac{c}{a}\right)^{-i\Omega} e^{\frac{\pi\Omega}{2}sign(a)}}{\sqrt{2\Omega\sinh(\pi\Omega)}}e^{-ia\Omega\tau/c}. \quad (7)$$

That is at the atom's location, the Unruh-Minkowski modes are harmonic function of τ, namely, $F \propto e^{-i\nu\tau}$, where $\nu = \pm a\Omega/c$ is the frequency of the wave that the atom senses. If this frequency is negative and $\nu = -\omega$ then the term in the interaction Hamiltonian

$$\hat{\sigma}^{\dagger}e^{i\omega\tau}\hat{a}^{\dagger}F^{*}(t(\tau), z(\tau)) \propto \hat{\sigma}^{\dagger}\hat{a}^{\dagger}e^{i(\nu+\omega)\tau}$$

yields resonant excitation of the atom with photon emission.

Eqs. (6) and (7) show that from the perspective of the atom accelerated in the right Rindler wedge ($a > 0$) the mode function $F_{L1\Omega}$ has negative frequency $\nu = -a\Omega/c$, while $F_{L2\Omega}$ has positive frequency $\nu = a\Omega/c$. Therefore, the ground-state atom can become excited by emitting a left-propagating photon with negative frequency into the mode $F_{L1\Omega}$ with $\Omega = \omega c/a$.

For the right-propagating Unruh-Minkowski photons the mode functions at the atom's location are

$$F_{R1\Omega}(t(\tau), z(\tau)) = \frac{\left(\frac{c}{a}\right)^{i\Omega} e^{\frac{\pi\Omega}{2}sign(a)}}{\sqrt{2\Omega\sinh(\pi\Omega)}}e^{-ia\Omega\tau/c}, \quad (8)$$

$$F_{R2\Omega}(t(\tau), z(\tau)) = \frac{\left(\frac{c}{a}\right)^{-i\Omega} e^{-\frac{\pi\Omega}{2}sign(a)}}{\sqrt{2\Omega\sinh(\pi\Omega)}}e^{ia\Omega\tau/c}. \quad (9)$$

That is, from the perspective of the atom accelerated in the right Rindler wedge ($a > 0$) the mode function $F_{R1\Omega}$ has positive frequency $a\Omega/c$, while $F_{R2\Omega}$ has negative frequency $-a\Omega/c$. Thus, the ground-state atom can become excited by emitting a right-propagating photon with negative frequency into the mode $F_{R2\Omega}$ with $\Omega = \omega c/a$.

Negative frequency of the Unruh-Minkowski photon in the atom's reference frame yields interesting observable consequences. For example, a ground-state atom 2 accelerated in the right Rindler wedge (see Fig. 1) can not become excited by absorbing a photon emitted by the atom 1 because the emitted photon has negative frequency from the perspective of atoms accelerated in the same wedge. That is, the conditional probability to find both atoms excited and the field in the initial Minkowski vacuum state is equal to zero [19]. However, such a negative-frequency photon can be absorbed by the atom 2 if atom 2 is excited. Likewise, a negative-frequency photon can not be emitted by an excited atom.

If the atom is accelerated in the left Rindler wedge ($a < 0$) then, according to Eqs. (6), (7), (8) and (9), from the perspective of such atom, the mode frequencies change to the opposite sign compared to the atom accelerated in the right wedge ($a > 0$). That is from the perspective of the atom accelerated in the left wedge the mode functions $F_{L1\Omega}$ and $F_{R2\Omega}$ have positive frequency, while $F_{R1\Omega}$ and $F_{L2\Omega}$ have negative frequency. Therefore, the ground-state atom accelerated in the left Rindler wedge can become excited by emitting a photon into the negative frequency modes $F_{R1\Omega}$ or $F_{L2\Omega}$. If the accelerated atom is initially excited then it can undergo transition into the ground state by absorbing a photon from the modes $F_{R1\Omega}$ and $F_{L2\Omega}$, or by emitting a photon into the positive-frequency modes $F_{L1\Omega}$ and $F_{R2\Omega}$.

The sign of the normal mode frequencies in the reference frame of the accelerated atom explains why the Unruh-Minkowski photon emitted by the accelerated ground-state atom can not be absorbed by another ground-state atom accelerated in the same wedge, but it can be absorbed by a ground-state atom accelerated in the opposite direction. In Supplemental Material we give a rigorous derivation of this result in $3+1$ dimension.

Unruh acceleration radiation can also be described in terms of Rindler photons which are localized in half planes bound by the lines $z = \pm ct$ [18]. However, the normal modes of Rindler photons

$$\phi_{1\Omega} = \frac{1}{\sqrt{\Omega}}(\mp z - ct)^{i\Omega}\theta(\mp z - ct), \tag{10}$$

$$\phi_{2\Omega} = \frac{1}{\sqrt{\Omega}}(ct \pm z)^{-i\Omega}\theta(ct \pm z), \tag{11}$$

from the perspective of the accelerated atom always have positive frequency. For example, along the trajectory of an atom accelerated in the right Rindler wedge the field produced by the Rindler photons is

$$\phi_{L1\Omega}(t(\tau), z(\tau)) = 0,$$

$$\phi_{L2\Omega}(t(\tau), z(\tau)) = \frac{1}{\sqrt{\Omega}}\left(\frac{c^2}{a}\right)^{-i\Omega} e^{-ia\Omega\tau/c},$$

$$\phi_{R1\Omega}(t(\tau), z(\tau)) = \frac{1}{\sqrt{\Omega}}\left(\frac{c^2}{a}\right)^{i\Omega} e^{-ia\Omega\tau/c},$$

$$\phi_{R2\Omega}(t(\tau), z(\tau)) = 0,$$

that is, the atom senses field oscillations with positive frequency $\nu = a\Omega\tau/c$. As a consequence, the description of light emission and absorption by accelerated atoms in terms of the Rindler photons is similar to the conventional description for inertial atoms. Namely, atoms become excited by absorbing a Rindler photon and deexcited by emitting a photon.

The Minkowski vacuum, however, is not a vacuum state for Rindler photons. The Minkowski vacuum is filled with Rindler photons with the average mode occupation number given by the thermal Planck factor with Unruh temperature $T_U = \hbar a/2\pi k_B c$ [1]. In the Rindler-photon picture the accelerated atom becomes excited by absorbing a positive-frequency Rindler photon out of the Minkowski vacuum.

An atom moving relative to matter with a constant velocity $\mathbf{V}$ can become excited by a mechanism similar to that of the Unruh effect [20]. Cherenkov radiation produced by a uniformly moving atom through a medium is an example [4, 5]. For description of Cherenkov radiation it is convenient to choose mode functions as plane waves which in the medium with refractive index n read (in the lab frame)

$$\varphi_{\mathbf{k}}(t, \mathbf{r}) = e^{-i\frac{ck}{n}t + i\mathbf{k}\mathbf{r}},$$

where $\mathbf{k}$ is the photon wave vector. Here we consider dimension $3+1$. At the location of the atom moving with constant velocity $\mathbf{V}$ along the trajectory

$$t(\tau) = \frac{\tau}{\sqrt{1-\frac{V^2}{c^2}}}, \quad \mathbf{r}(\tau) = \frac{\mathbf{V}\tau}{\sqrt{1-\frac{V^2}{c^2}}}$$

the mode function $\varphi_{\mathbf{k}}$ takes the form $\varphi_{\mathbf{k}}(t(\tau), \mathbf{r}(\tau)) = e^{-i\nu\tau}$, that is atom senses harmonic oscillations of the field with frequency

$$\nu = \frac{\frac{ck}{n} - \mathbf{V}\cdot\mathbf{k}}{\sqrt{1-\frac{V^2}{c^2}}}.$$

From the perspective of the moving atom if $V > c/n$, photons propagating inside the Cherenkov cone $\cos\theta > c/Vn$, where θ is the angle between $\mathbf{V}$ and $\mathbf{k}$, have negative frequency [21]. This leads to Cherenkov radiation. That is, the atom can become excited by emitting a negative-frequency photon inside the Cherenkov cone such that

$$\mathbf{k}\cdot\mathbf{V} = \frac{ck}{n} + \omega\sqrt{1-\frac{V^2}{c^2}}, \tag{12}$$

where ω is the atomic transition frequency in atom's frame.

Photons emitted outside the Cherenkov cone have positive frequency. An excited atom emits positive-frequency

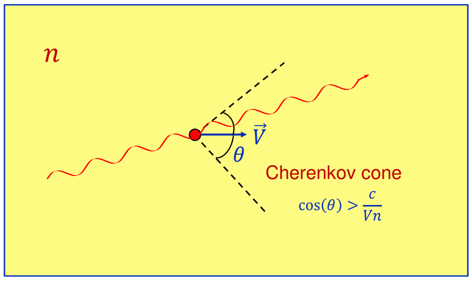

FIG. 2: A ground-state atom is moving through a medium with a constant velocity $V > c/n$. Photon propagating in the direction inside the Cherenkov cone $\cos\theta > c/Vn$ can not be absorbed by the atom.

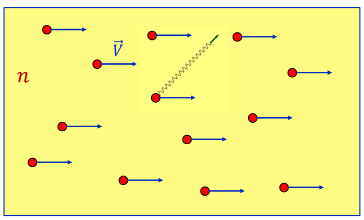

FIG. 3: Ensemble of atoms moving through a medium with an equal velocity $V > c/n$ and emitting Cherenkov radiation.

photon outside the Cherenkov cone and decays to the ground state. The excited atom can also decay to the ground state by absorbing a negative-frequency photon inside the Cherenkov cone.

Negative frequency of the Cherenkov photon in the atom's frame is analogous to the negative frequency of the Unruh-Minkowski photon in the frame of an accelerated atom. This leads to similar observable effects for the case of Cherenkov radiation. For example, light propagating inside the Cherenkov cone direction, as a ground-state atom moving through a medium with constant velocity, can not be absorbed by the atom because in the reference frame of the atom, the photons have negative frequency (see Fig. 2). The ground-state atom can only emit a photon inside the Cherenkov cone so that the total number of photons in the field increases. This is the case even if the incident light is in a coherent state produced by classical sources.

Let us now consider an ensemble of ground-state atoms moving through a medium with equal velocity $V > c/n$ (see Fig. 3). In the moving frame, the atoms are stationary and Cherenkov photons emitted by the atoms have negative frequency. As a result, the emitted photons can not be absorbed by other ground-state atoms in the ensemble. That is, the probability to find two atoms excited and the field in the vacuum state is equal to zero [19]. This is analogous to the emission/absorption of Unruh-Minkowski photons by atoms accelerated in the same direction.

An ensemble of ground-state atoms directionally moving through a medium with velocity $V > c/n$ acts as an inverted medium in a laser. Namely, light propagating inside the Cherenkov cone is not absorbed by the atoms but rather it is amplified by stimulated emission of Cherenkov photons. One should mention that light amplification can be achieved by motion of the medium in other systems. For example, steady nonuniform motion of a medium through an optical resonator can yield light amplification in the cavity at the resonator frequency [22].

If atoms in the ensemble move with different velocities, then Cherenkov photon emitted by the atom 1 can not be absorbed by the atom 2 if the photon propagates inside the atom's 2 Cherenkov cone, but it can be absorbed if the photon propagates outside the Cherenkov cone [19]. If atoms move randomly then atomic ensemble cannot amplify light because the solid angle of the Cherenkov cone is smaller than 2π sr. This agrees with the second law of thermodynamics.

In summary, we study the physics of photon emission and absorption by uniformly accelerated atoms. For this problem, it is convenient to describe the field in terms of the left and right propagating Unruh-Minkowski or Rindler modes since the accelerated two-level atom is in resonance only with one of these modes. We show that from the perspective of an accelerated atom the sign of the frequency of the Unruh-Minkowski photons can be positive or negative depending on the acceleration direction, while the frequency of the Rindler photons is always positive. As a result, the Unruh-Minkowski photons can be emitted and absorbed by the ground-state atom depending on the atom's acceleration direction, while Rindler photons can only be absorbed.

This leads to interesting effects. For example, a photon emitted by accelerated ground-state atom can not be absorbed by another ground-state atom accelerated in the same direction, but can be absorbed if the atom is excited. Moreover, an atom accelerated in the opposite direction senses the opposite sign of the Unruh-Minkowski photon frequency. As a result, a ground-state atom can absorb a photon emitted by another ground-state atom accelerated in the opposite direction.

We also show that similar effects take place for Cherenkov radiation. Namely, in $1+1$ dimension, a Cherenkov photon emitted by an atom can not be absorbed by another ground-state atom moving in the same direction with $V > c/n$, but can be absorbed if the atom is in the excited state. However, ground-state atom can absorb Cherenkov photon emitted by another ground-state atom moving in the opposite direction. In $3+1$ dimension, Cherenkov photon emitted by the atom 1 can not be absorbed by the atom 2 if the photon propagates inside the atom's 2 Cherenkov cone, but it can be absorbed if the photon propagates outside the Cherenkov cone.

Our findings can be important for understanding the residual depolarization of electrons in storage rings [23, 24]. Namely, we predict that Unruh photon emitted by a spin-flipping electron cannot flip the spin of nearby polarized electrons in the beam, but it can flip the spin of unpolarized electrons. This increases the degree of polarization of the electrons in the storage ring.

It would be interesting to look for similar effects in various analogs of the Unruh acceleration radiation [25–27] and Cherenkov radiation. For example, a ground-state atom moving above a metal surface can become excited by emitting a surface plasmon [20]. From the atom's perspective the surface plasmon has negative frequency [20]. If many atoms move with the same velocity above the metal surface, then a ground-state atom in the ensemble can not absorb surface plasmon emitted by another atom. That is probability to find atoms excited and no plasmon present is equal to zero.

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