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Constraints on low-energy effective field theories from weak cosmic censorship

Baoyi Chen,¹ Feng-Li Lin,² Bo Ning,³ and Yanbei Chen¹

¹*Burke Institute of Theoretical Physics and Theoretical Astrophysics 350-17,
California Institute of Technology, Pasadena, California 91125*

²*Department of Physics, National Taiwan Normal University,
No. 88, Sec. 4, Ting-Chou Road, Taipei 11677, Taiwan*

³*College of Physics, Sichuan University, Chengdu, Sichuan 610064, China*

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We examine the weak cosmic censorship conjecture (WCCC) for the extremal charged black hole in possible generalizations of Einstein-Maxwell theory due to the higher order corrections, up to fourth-derivative terms. Our derivation is based on Wald's gedanken experiment to destroy an extremal black hole. We find that the WCCC no longer holds for all possible generalizations. Thus, the WCCC can serve as a new constraint to the higher order effective field theories. However, our constraint is independent of photon's self-interactions so that precision measurement of quantum electrodynamics cannot constrain the WCCC. For higher-dimension operators induced by the one-loop correction for the minimally coupled spinor and scalar to gravity, our constraint is satisfied.

Introduction.— Even though the curvature singularity of a black hole is hidden behind the horizon, it might still be possible to throw charged or spinning matter into a black hole in particular ways that can destroy the horizon, revealing the singularity previously hidden inside. This kind of gedanken experiments was first proposed long ago by Wald [1] to test the so-called Weak Cosmic Censorship Conjecture (WCCC) [2], which asserts that the above gedanken experiments cannot succeed in order to prevent the singularity from being visible. Although the WCCC can be checked easily for extremal black holes, it is nontrivial to prove for near-extremal black holes [1, 3] and for general forms of matter. Recently, significant progress for the general proof of the WCCC has been made by Sorce and Wald [4] who adopted a general relativistic formulation of the energy conservation which can work for general forms of matter obeying the Null Energy Condition (NEC). In this way, they were able to avoid solving the complicated dynamical problems of the in-falling matter involving the self-force effect, and succeeded to show that the WCCC holds for the black holes in Einstein-Maxwell theory, up to second order variation of the black hole's mass, charge and angular momentum. Moreover, their method of examining the WCCC also provides a systematic framework for general theories other than Einstein-Maxwell.

One compelling reason to examine the WCCC for more general theories of gravity and electromagnetism is that the standard Einstein-Maxwell theory, which can be a good approximation at low energies, may need to be corrected at higher energies. In the low-energy Effective Field Theory (EFT), these quantum corrections can leave low-energy relics in the form of higher-order derivative terms beyond Einstein-Maxwell terms, modifying the black hole solutions, as well as the relativistic laws of the energy-momentum conservation. These terms may also make the WCCC fail. If we take the WCCC as a universal physical principle, then only those higher order EFTs that admit the WCCC should be accepted. This is in a similar spirit of using the weak gravity conjecture [5, 6] which takes “gravity force is the weakest in nature” as a

new physical principle to constrain the higher order EFTs [6].

By dimensional counting, the leading order correction to Einstein-Maxwell theory is photon's quartic self-interaction through a virtual scalar or spinor loop, which is a pure effect of quantum electrodynamics without involving gravity. Its Lagrangian density takes the form [15]:

$$L \propto c_7 F_{\mu\nu} F^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} + c_8 F_{\mu\nu} F^{\nu\rho} F_{\rho\sigma} F^{\sigma\mu}, \quad (1)$$

and the coefficients c_7 and c_8 can be well-measured by experiments [7]. The next leading order corrections to the Einstein-Maxwell background is given by the graviton-photon-photon interaction with a scalar or spinor loop. For the minimally-coupled case, the one-loop effective actions for the Einstein-Maxwell background induced by spinors and scalars are given by [8, 9]

$$L_{\text{spinor}} \propto 5RF^2 - 26R_{\mu\nu}F^{\mu\rho}F^\nu{}_\rho + 2R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}, \quad (2)$$

$$L_{\text{scalar}} \propto -\frac{5}{2}RF^2 - 2R_{\mu\nu}F^{\mu\rho}F^\nu{}_\rho - 2R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}, \quad (3)$$

where we have neglected terms proportional to $\nabla^\mu F_{\mu\rho}\nabla_\nu F^{\nu\rho}$, as they do not appear in the metric equation of motion, thus having no effects on the black hole metric or our parameter bound. We will specifically check whether the two theories violate the WCCC later.

EFTs, Black-Hole Solutions, and Extremality Condition.— To demonstrate the power of the WCCC as a constraint to the EFTs, in this work we consider the most general quartic order corrections to Einstein-Maxwell theory, which is given by the following EFT action:

$$I = \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \Delta L \right), \quad (4)$$

where [16]

$$\begin{aligned} \Delta L = & c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \\ & + c_4 R F_{\mu\nu} F^{\mu\nu} + c_5 R_{\mu\nu} F^{\mu\rho} F^\nu{}_\rho + c_6 R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \\ & + c_7 F_{\mu\nu} F^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} + c_8 F_{\mu\nu} F^{\nu\rho} F_{\rho\sigma} F^{\sigma\mu}. \end{aligned} \quad (5)$$

We will assume c_i 's are small and restrict our consideration to $O(c_i)$. The aforementioned self-interactions of four photons are the terms with coupling coefficient c_7 and c_8 respectively.

For simplicity, we will consider only the charged non-spinning black holes. The perturbative procedure of solving such black hole solutions has been outlined in [10], leading to a *family* of solutions parametrized by the mass and the charge (M, Q). Here we list some partial results relevant for our considerations [17], namely the Maxwell gauge field

$$A_t = -\frac{q}{r} + \frac{2q^3}{5r^5} \left[c_5\kappa + c_6\kappa \left(6 - \frac{5mr}{q^2} \right) + 8c_7 + 4c_8 \right], \quad (6)$$

and the tt -component of the metric [18]

$$\begin{aligned} -g_{tt} = & 1 - \frac{\kappa m}{r} + \frac{\kappa q^2}{2r^2} + c_2 \left(\frac{\kappa^3 m q^2}{r^5} - \frac{\kappa^3 q^4}{5r^6} - \frac{2\kappa^2 q^2}{r^4} \right) \\ & + c_3 \left(\frac{4\kappa^3 m q^2}{r^5} - \frac{4\kappa^3 q^4}{5r^6} - \frac{8\kappa^2 q^2}{r^4} \right) \\ & + c_4 \left(-\frac{6\kappa^2 m q^2}{r^5} + \frac{4\kappa^2 q^4}{r^6} + \frac{4\kappa q^2}{r^4} \right) \\ & + c_5 \left(\frac{4\kappa^2 q^4}{5r^6} - \frac{\kappa^2 m q^2}{r^5} \right) \\ & + c_6 \left(\frac{\kappa^2 m q^2}{r^5} - \frac{\kappa^2 q^4}{5r^6} - \frac{2\kappa q^2}{r^4} \right) \\ & + c_7 \left(-\frac{4\kappa q^4}{5r^6} \right) + c_8 \left(-\frac{2\kappa q^4}{5r^6} \right) + O(c_i^2). \end{aligned} \quad (7)$$

Here we define the reduced mass $m \equiv M/4\pi$, the reduced charge $q \equiv Q/4\pi$ and $\kappa = 8\pi G_N$, where G_N is the gravitational constant. Note that in (7) there is no $O(c_1)$ correction.

As shown by Ref. [10], as long as

$$m \geq \sqrt{\frac{2}{\kappa}} |q| \left(1 - \frac{4}{5q^2} c_0 \right), \quad (8)$$

the singularity of the space-time will be hidden by a horizon; more precisely, the *outer horizon* located at the outer most solution of $g_{tt}(r_H) = 0$. Here

$$c_0 \equiv c_2 + 4c_3 + \frac{c_5}{\kappa} + \frac{c_6}{\kappa} + \frac{4c_7}{\kappa^2} + \frac{2c_8}{\kappa^2}, \quad (9)$$

and $c_0 \rightarrow 0$ recovers the Reissner-Nordstrom solution of Einstein-Maxwell. For a fixed m , as q increases to, and then exceeds, the critical value at which equality holds in (8), two horizons will merge and subsequently disappear, revealing the singularity. In this way, the *extremal solution* is defined by imposing equality in (8). This implicitly defines a function $q_{ext}(m)$ for the extremal solution. For each m , the horizon radius of the extremal solution is given by

$$r_H^{ext} = \frac{m\kappa}{2} + \frac{4}{5m} \left(c_2 + 4c_3 + \frac{10c_4 + c_5 + c_6}{\kappa} - \frac{16c_7 + 8c_8}{\kappa^2} \right). \quad (10)$$

On this extremal horizon, the electrostatic potential is

$$\Phi_H^{ext} = -(\xi^a A_a)|_H = \sqrt{\frac{2}{\kappa}} \left(1 + \frac{4c'_0}{5q^2} \right), \quad (11)$$

where $\xi = \vec{\partial}_t$ is the time-like Killing vector of the space-time, and

$$c'_0 = -\frac{10c_4}{\kappa} - \frac{2c_5}{\kappa} - \frac{2c_6}{\kappa} + \frac{4c_7}{\kappa^2} + \frac{2c_8}{\kappa^2}. \quad (12)$$

Henceforth we refer to (m, q) solutions that strictly satisfy the inequality (8) as *regular solutions*, those that take equality as *extremal solutions*, and those that violate the inequality as *singular solutions*. We may still refer to them as “black holes”, even though the horizon may or may not be destroyed.

Gedanken Experiment to Destroy the Horizon.— In gedanken experiments that attempt to destroy the horizon, e.g., as set up by Wald [1, 4], we shall always (if tacitly) assume stability of our family of solutions. That is, starting off with a regular solution (m, q) , as we “throw matter into” it, the final space-time geometry and field configuration will settle down to another solution *in our family*. If our “way of throwing matter”, for example described by the on-shell metric perturbations, field perturbations and matter stress-energy tensor in the initial slice, is parameterized by w , then the final solution should be given by $(m(w), q(w))$.

In this language, the WCCC dictates that a starting regular solution (m, q) long before “throwing matter” will only lead to $(m(w), q(w))$ that are still regular. As a special case, let us now consider a starting extremal solution $(m, q_{ext}(m))$, and a particular approach of throwing matter, we can write

$$m(w) = m + w\delta m + O(w^2), \quad q(w) = q_{ext}(m) + w\delta q + O(w^2). \quad (13)$$

The condition for the starting extremal solution to not become singular, at first order in w , is given by

$$\delta m - \sqrt{\frac{2}{\kappa}} \left(1 + \frac{4c_0}{5q^2} \right) \delta q \geq 0. \quad (14)$$

We therefore need to find out whether physical laws in our modified theory imposes that (14) must hold for all infalling matter — or to find a particular way of throwing matter that violate (14). The advantage of starting-off at the extremal solution is: once Eq. (14) is violated, then any infinitesimal w will lead to destruction of the horizon, and we can restrict ourselves to linear perturbation.

By contrast, starting from a non-extremal black hole with $(m, q_{ext}(m) - \epsilon)$, a *finite step size* for w must be made to surpass the extremality contour, and in this case the higher derivatives of $m(w)$ and $q(w)$ may become important, requiring the computation of higher-order variations. This was indeed the situation encountered by Hubeny [3], which was latter addressed by Sorce and Wald [4] by considering the second order variations. Intuitively, one would expect the sub-extremal black holes will obey the WCCC if the extremal ones do, but the second order variations are needed for a rigorous examination

on the sub-extremal case. In this paper we shall restrict ourselves to the extremal black holes.

As it turns out, condition (14) coincides with the requirement that the horizon area must increase as matter fall into extremal black holes [19]. More specifically, if we denote by $\mathcal{A}(m, q)$ the area of the horizon, then one can show that

$$\partial_m \mathcal{A}(m, q) / \partial_q \mathcal{A}(m, q)|_{q=q_{\text{ext}}(m)} = dq_{\text{ext}}(m) / dm, \quad (15)$$

and that $d\mathcal{A}(m + wdm, q_{\text{ext}}(m) + wdq) / dw = 0$ is equivalent to the equality in Eq. (14). In this way, the violation of condition (14), or the *destruction of the extremal horizon*, relies on the possibility of *area decrease* at linear order. This can be possible for the theories we consider even when the NEC is satisfied, because Raychaudhuri equation is now modified, and the NEC does not always lead to attractive gravity.

Test Particle.— For a regular solution (m, q) , consider a test particle with reduced mass δm_0 and reduced charge δq_0 , falling in from infinity. Using the *minimally coupled* action of

$$S_p = 4\pi \int d\tau (\delta m_0 - \delta q_0 \vec{u} \cdot \vec{A}), \quad (16)$$

the reduced canonical momentum of the particle, $\vec{p} = \delta m_0 \vec{u} - \delta q_0 \vec{A}$, satisfies $\vec{\xi} \cdot \vec{p} = \text{const}$ along the particle's trajectory; at linear order in δm_0 and δq_0 , we do not have to consider the radiation reaction. Applying this to the particle at infinity and on the horizon, we obtain

$$\delta m_0 (\vec{u}^H \cdot \vec{\xi}) - \Phi_H^c \delta q_0 = \delta m_0 (\vec{u}^\infty \cdot \vec{\xi}) = -\delta E_\infty, \quad (17)$$

where $\vec{u}^\infty$ and $\vec{u}^H$ are the 4-velocities of the particle at infinity and on the horizon, and we have used the fact that A_t does not depend on t , hence $\vec{\xi} \cdot \vec{A}$ vanishes at infinity.

For the final space-time, assuming that it still belongs to the same family, with $(m + \delta m, q + \delta q)$. We can argue from the charge conservation that $\delta q = \delta q_0$, and, from the conservation of ADM mass, as well as the fact that the energy of gravitational radiation emitted by the in-fall process is $\mathcal{O}(\delta m^2)$, that $\delta m = \delta E_\infty$: basically, *the charge and the energy of the particle are added to those of the black hole*. We will soon give a more rigorous justification, but with this in hand we can write

$$\delta m - \Phi_H^c \delta q = -\delta m_0 (\vec{u}^H \cdot \vec{\xi}) \geq 0. \quad (18)$$

The latter inequality is because $\vec{u}^H \cdot \vec{\xi} \leq 0$: the 4-velocity of the particle must be pointed toward the future as the particle crosses the horizon. This can be saturated if the particle is able to “rest right on top of the horizon”. Inserting Eq. (11) into Eq. (18), we obtain the relation between δq and δm in this in-falling test particle situation:

$$\delta m \geq \sqrt{\frac{2}{\kappa}} \left(1 + \frac{4c'_0}{5q^2} \right) \delta q. \quad (19)$$

This is clearly different from Eq. (14). However, before discussing its consequences, we shall introduce the framework by Sorce and Wald, which provides more rigorous treatment

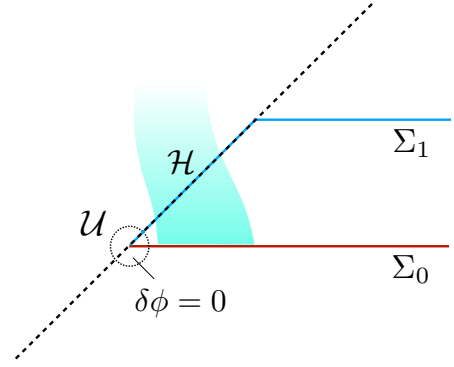


FIG. 1: The gedanken experiment to destroy an extremal black hole. Charged matter, occupying the shaded region, crosses the $\mathcal{H}$ portion of the extremal horizon.

of the energy conservation, and is able to treat more general infalling matter.

Sorce-Wald method for generic matter.— We now sketch the method of Sorce and Wald developed in [4, 12]. We follow the notation of Wald, and denote by $\phi = (g_{ab}, A_a)$ the metric and field degrees of freedom. We start off with an extremal black hole, with $(m, q_{\text{ext}}(m))$, and define a Cauchy surface Σ_0 at early time, and a hypersurface Σ_1 which starts at sufficiently late time when the matter all fall in, and terminates at null infinity. We denote by $\mathcal{H}$ the portion of the extremal horizon between Σ_0 and Σ_1 (see Fig. 1). We then apply perturbation $\delta\phi$, as well as matter, with stress-energy tensor δT_{ab} and electric current δj_a , also a form of perturbation, in an open neighborhood of Σ_0 . We will setup our initial value problem in such a way that $\delta\phi$, δT_{ab} and δj_a all vanish in an open neighborhood $\mathcal{U}$ surrounding the intersection of $\mathcal{H}$ and Σ_0 . In principle, $\delta\phi$ and δT_{ab} , δj_a should be evolved jointly into the future, but here we assume stability of our family of solutions, and therefore can impose that space-time geometry in an open neighborhood of Σ_1 is that of $(m + \delta m, q + \delta q)$ [20].

A general Noether method to derive the law of energy conservation for such an in-falling process is developed by Iyer and Wald [13], which we will briefly sketch as follows. Given a theory Lagrangian $L(\phi)$ of gravity and matter, we can introduce the Lagrangian 4-form $\mathbf{L} = L\epsilon$, where ϵ is the volume form associated with the metric. Then, variation of $\mathbf{L}$ yields

$$\delta \mathbf{L} = \mathbf{E}(\phi) \delta\phi + d\mathbf{\Theta}(\phi, \delta\phi), \quad (20)$$

where $\mathbf{E}(\phi) = 0$ is Euler-Lagrangian equation, and $\mathbf{\Theta}(\phi, \delta\phi)$ is the symplectic potential 3-form. For an arbitrary vector ξ^a , one can construct the associated Noether current $\mathbf{J}_\xi = \mathbf{\Theta}(\phi, \mathcal{L}_\xi \phi) - i_\xi \mathbf{L}$, which, because $\mathbf{J}_\xi$ is conserved, i.e., $d\mathbf{J}_\xi = 0$, can be rewritten as $\mathbf{J}_\xi = d\mathbf{Q}_\xi + \xi^a \mathbf{C}_a$ with the 3-form constraint $\mathbf{C}_a = 0$ when equations of motion are satisfied. For instance, in Einstein-Maxwell theory, the 3-form constraint is given by

$$(\mathbf{C}_a)_{bcd} = \epsilon_{ebcd} (T^e_a + j^e A_a), \quad (21)$$

with $T_{ab} = \frac{1}{\kappa} (G_{ab} - \kappa T_{ab}^{\text{EM}})$ the non-electromagnetic stress energy tensor, and $j^a = \nabla_b F^{ab}$ the charge current of the Maxwell

source. Thus the on-shell condition $\mathbf{C}_a = 0$ gives the equations of motion $G_{ab} = \kappa T_{ab}^{\text{EM}}$ and $\nabla_b F^{ab} = 0$. The form (21) also holds when the higher-order derivative corrections ΔL are present. Assuming $\mathbf{E}(\phi) = 0$ and ξ^a is a Killing vector, i.e., $\mathcal{L}_\xi \phi = 0$, it is easy to show that $\delta \mathbf{J}_\xi = di_\xi \Theta(\phi, \delta\phi)$ which is then combined with $\delta \mathbf{J}_\xi = d\delta \mathbf{Q}_\xi + \xi^a \delta \mathbf{C}_a$, and is integrated over the hypersurface $\mathcal{H} \cup \Sigma_1$ to yield

$$\int_{\infty} [\delta \mathbf{Q}_\xi - i_\xi \Theta(\phi, \delta\phi)] = - \int_{\mathcal{H} \cup \Sigma_1} \xi^a \delta \mathbf{C}_a, \quad (22)$$

where we have used the Stoke's theorem to turn the 3-surface integral into the boundary integrals at the spatial infinity ∞ and at the intersection $\mathcal{H} \cap \Sigma_0$, by also imposing $\delta\phi = 0$ at $\mathcal{H} \cap \Sigma_0$.

If we assume ξ^a is the time-like Killing vector $t^a = (\partial_t)^a$ for non-spinning black holes, then we denote the change of the ADM mass as

$$\delta \mathcal{M} = \int_{\infty} [\delta \mathbf{Q}_\xi - i_\xi \Theta(\phi, \delta\phi)], \quad (23)$$

and the charge crossing the horizon as,

$$\delta Q \equiv \int_{\mathcal{H}} \epsilon_{abcd} \delta j^a, \quad (24)$$

where the electric current δj^a and the stress tensor δT^a_b can be read off from the following on-shell relation [21]

$$(\delta \mathbf{C}_a)_{bcd} = \epsilon_{ebcd} (\delta T^e_a + A_a \delta j^e). \quad (25)$$

Combine all above and require vanishing of δj^e and δT^e_a on Σ_1 as depicted in Fig. 1, we can turn (22) into the following law of energy conservation for the in-falling process of Wald's gedanken experiment,

$$\delta \mathcal{M} - \Phi_H^c \delta Q = - \int_{\mathcal{H}} \epsilon_{ebcd} \xi^a \delta T^e_a. \quad (26)$$

On horizon $\mathcal{H}$ we can relate the 4-volume form ϵ to the 3-volume form $\tilde{\epsilon}$ by the relation $\epsilon_{ebcd} = -4n_{[e}\tilde{\epsilon}_{bcd]}$ where n^e is the null vector normal to $\mathcal{H}$. Using this relation and the fact $\xi^a \propto n^a$ on $\mathcal{H}$, the R.H.S. of (26) turns into $4 \int_{\mathcal{H}} \tilde{\epsilon} \delta T_{ab} n^a n^b$, which is non-negative if matter's stress tensor obeys the NEC. Thus, the variational identity (26) becomes an inequality for matter obeying the NEC,

$$\delta \mathcal{M} - \Phi_H^c \delta Q \geq 0. \quad (27)$$

This inequality serves as a constraint on the changes of the black hole's mass and charge for the in-falling process, and will be used to check the WCCC by comparing with the condition (14).

Parameter bounds from WCCC.—The Noether method by Iyer and Wald provides a systematic way to calculate $\delta \mathcal{M}$ of (23) and δQ of (24) for general theory by evaluating Θ , $\mathbf{Q}$ and $\mathbf{C}_a$. For example, these quantities for Einstein-Maxwell theory have been derived in [13], and the results $\delta \mathcal{M} = 4\pi\delta m$

and $\delta Q = 4\pi\delta q$ are then used to show that the WCCC holds for Einstein-Maxwell theory.

Here we apply the same method for our higher-order theory (4). The derivation is tedious but straightforward, and the result is given in the supplemental materials [22], based on which we can evaluate the corresponding $\delta \mathcal{M}$ and δQ . As a result, we find that $\delta \mathcal{M} = 4\pi\delta m$ because the corrections due to higher order Lagrangian ΔL fall off too quickly to contribute asymptotically to $\delta \mathcal{M}$. Similarly, we arrive $\delta Q = 4\pi\delta q + O(c_i^2)$ after tedious calculations [23]. The results are consistent with the test particle case. Therefore, we conclude that (27), which holds for general forms of matter obeying the NEC, gives the same condition Eq. (19) as for the test particle.

Compare the energy condition (19) and the WCCC condition (14), it is not hard to see that we must have $c'_0 \geq c_0$ for the WCCC to hold for theory (4). Explicitly we have

$$c_2 + 4c_3 + \frac{10c_4}{\kappa} + \frac{3c_5}{\kappa} + \frac{3c_6}{\kappa} \leq 0. \quad (28)$$

This is our key result, which gives the parameter bounds on the low-energy EFT of quantum gravity by demanding that this low-energy theory preserves the WCCC.

Values of c_j and connections to other bounds.—With our new bound (28) from the WCCC, it is then natural to ask how this bound works in the real world. Although black holes in the real world generally have nonzero angular momentum, our bound could still serve as a necessary condition. We first notice that the non-linear EM terms contribute to c_7 and c_8 , which do not appear in (28). This implies that the quantum electrodynamics automatically bypass the WCCC constraint. Plugging the values of c_4 , c_5 and c_6 from the EFT (2) or (3) into the bound (28), we find the inequality also holds. An important implication is then, the WCCC not only holds for Einstein-Maxwell theory, but may also hold at one-loop level. This could possibly mean the correctness of the conjecture in the real world!

When the non-minimal coupling between matter and gravity is present, however, the bound (28) may subject to change under different situations. This is consistent with the fact that the combination of c -coefficients in our bound is not invariant under the field redefinition $g_{\mu\nu} \rightarrow g_{\mu\nu} + \delta g_{\mu\nu}$ [14], where

$$\delta g_{\mu\nu} = r_1 R_{\mu\nu} + r_2 g_{\mu\nu} R + r_3 F_{\mu\rho} F_{\nu}^{\rho} + r_4 g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}. \quad (29)$$

With a proper choice of the matter-gravity coupling, it is even possible that there yields no bound for the c -coefficients, and that the WCCC always holds. A further discussion is beyond the scope of this paper and we would like to explore it in the future.

It is interesting to compare our WCCC bound (28) with the bound obtained from the weak gravity conjecture (WGC), which is [10, 14]

$$c_2 + 4c_3 + \frac{c_5}{\kappa} + \frac{c_6}{\kappa} + \frac{4c_7}{\kappa^2} + \frac{2c_8}{\kappa^2} \geq 0. \quad (30)$$

Note that the WGC is a conjecture which states that gravity should be the weakest force for any consistent theory of quantum gravity. We see that the WGC bound will constrain c_7 and

c_8 in contrast to the WCCC case. Moreover, these two bounds seem orthogonal to each other as one is bound above zero and the other bound below. Due to the seemingly orthogonality, Combining the WCCC and the WGC bounds together will be a useful tool to scrutinize the theory space of the higher order EFTs.

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 - [15] We would expect that the Lagrangian density should be the linear combination of two parity-even and Lorenz-invariant terms, namely, $(F_{\mu\nu}F^{\mu\nu})^2 \propto \mathbf{E}^2 - \mathbf{B}^2$ and $(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 \propto \mathbf{E} \cdot \mathbf{B}$. Here we follow the convention of [10] to replace the $(F_{\mu\nu}\tilde{F}^{\mu\nu})^2$ term by another parity-even and Lorentz-invariant term $F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu}$.
 - [16] We have neglected terms proportional to $\nabla^\mu F_{\mu\rho}\nabla_\nu F^{\nu\rho}$, as it does not affect the black hole metric or our parameter bound. Further note that terms like $(\nabla_\mu F_{\nu\rho})(\nabla^\mu F^{\nu\rho})$ and $(\nabla_\mu F_{\nu\rho})(\nabla^\nu F^{\mu\rho})$ can be recasted (up to some constant factor) into $\nabla^\mu F_{\mu\rho}\nabla_\nu F^{\nu\rho}$ plus existing terms in ΔL and an additional boundary term, upon using Bianchi identities, Ricci identities, and integrating by parts [11].
 - [17] See Sec. I of the supplemental materials for the full explicit expressions.
 - [18] See Sec. II of the supplemental materials for the full expressions.
 - [19] See Sec. IV of the supplemental materials for details
 - [20] We note that in general Σ_1 is only a portion of a “Cauchy surface” — with the remaining portion completed by a portion of the future null infinity. A Cauchy surface that ends at spatial infinity does not approach $(m + \delta m, q + \delta q)$, fast enough; its ADM mass is not equal to $m + \delta m$ either, because it contains the energy-momentum content of gravitational waves emitted during the in-fall process, see e.g. [12]. Sorce and Wald simply assumed that the late time solution is stable and non-radioactive to bypass the above concern [4]. On the other hand, in this paper we consider only the infall of matter into an extremal black hole for which the dynamics is non-radioactive, thus the above issue does not exist in our consideration.
 - [21] To arrive (25) we have imposed the on-shell conditions for the theory (4) along with the additional minimally coupled matter of stress tensor δT^{ab} and charge current δj^a .
 - [22] See Sec. III of the supplemental materials for the explicit expressions.
 - [23] This can also be seen as follows. By the construction of source theory, $j_a = \nabla^b(F_{ab} - S_{ab})$ in which S_{ab} is given in (5) of supplemental materials and is of $\mathcal{O}(c_i)$, and using (3) of supplemental materials, $F_{ab} = F_{ab}^{(0,j)} + S_{ab} + \mathcal{O}(c_i^2)$ where the superscript $(0, j)$ means to evaluate by plugging the background Reissner-Nordström configurations and keeping up to $\mathcal{O}(c_i)$ terms. We then arrive $j_a = \nabla^b F_{ab}^{(0,j)} + \mathcal{O}(c_i^2)$, and use the Gauss’s law the integral $Q = \int_{\mathcal{H}} \epsilon_{abcd} j^a = \int_B *F^{(0,j)} + \mathcal{O}(c_i^2)$. Then, $\delta Q = \int_B (*F^{(0,j)} - *F^{(0)}) + \mathcal{O}(c_i^2) = \delta Q + \mathcal{O}(c_i^2)$, where $\delta Q = 4\pi\delta q$ is the charge carried by the in-falling matter. Thus, $\delta Q = 4\pi\delta q + \mathcal{O}(c_i^2)$ is obtained.