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Direct determination of supermassive black hole properties with gravitational-wave radiation from surrounding stellar-mass black hole binaries

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A significant number of stellar-mass black-hole (BH) binaries may merge in galactic nuclei or in the surrounding gas disks. With purposed space-borne gravitational-wave observatories, we may use such a binary as a signal carrier to probe modulations induced by a central supermassive BH (SMBH), which further allows us to place constraints on the SMBH’s properties. We show in particular the de Sitter precession of the inner stellar-mass binary’s orbital angular momentum (AM) around the AM of the outer orbit will be detectable if the precession period is comparable to the duration of observation, typically a few years. Once detected, the precession can be combined with the Doppler shift arising from the outer orbital motion to determine the mass of the SMBH and the outer orbital separation individually and each with percent-level accuracy. If we further assume a joint detection by space-borne and ground-based detectors, the detectability threshold could be extended to a precession period of ~ 100 yr.

Introduction – A significant number of stellar-mass binary black holes (BH) detectable by LIGO [1] and Virgo [2] may merge in the vicinity of supermassive BHs (SMBHs) due to both dynamical interactions [3–9] and gaseous effects if accretion disks are present [10–17]. This possibility is strengthened as the Zwicky Transient Facility [18, 19] detected a potential electromagnetic counterpart [20] to the LIGO-Virgo event GW190521 [21, 22], consistent with a binary BH merger in the accretion disk of an active galactic nucleus (AGN).

Beyond ground-based detectors, multiple space-borne gravitational-wave (GW) observatories have been planned/conceived for the coming decades, including LISA [23], TianQin [24], Taiji [25], B-DECIGO [26, 27], Deci-hertz Observatories [28], and TianGO [29]. Their sensitivities cover the 0.001-1 Hz band where a typical stellar-mass BH binary stays in band for years. It thus opens up the possibility of using a stellar-mass BH binary as a carrier to probe modulations induced by a tertiary perturber which, as argued above, can be an SMBH in many cases. This is in analog to how pulsars are used to test the strong-field relativity [30] and it offers a complementary way to probe SMBH properties to extreme and very extreme mass-ratio inspirals (EMRI and X-MRI) [31–34].

The leading-order modulation is a Doppler shift due to the inner binary’s orbital motion around the SMBH [35], creating frequency sidebands at $\Omega_o = (M_3/a_o^3)^{1/2}$ with M_3 the mass of the SMBH and a_o the semi-major axis of the outer orbit. The extra dephasing of this effect can be determined up to $a_o \simeq 1$ pc [36]. When $2\pi/\Omega_o \sim T_{\text{obs}}$ with T_{obs} the duration of observation, Ω_o can be further resolved to constrain the mass density enclosed by the outer orbit [37].

In the Letter, we extend the field by including higher-order effects. The most significant one is that the inner orbital angular momentum (AM) \mathbf{L}_i will experience a de Sitter-like (dS) precession around the outer AM \mathbf{L}_o

whose secular effect is [38–40]

$$\frac{d\hat{\mathbf{L}}_i}{dt} = \Omega_{\text{dS}} \hat{\mathbf{L}}_o \times \hat{\mathbf{L}}_i = \frac{3}{2} \frac{M_3}{a_o(1-e_o^2)} \Omega_o \hat{\mathbf{L}}_o \times \hat{\mathbf{L}}_i, \quad (1)$$

where e_o is the eccentricity of the outer orbit. Here we have used the hat symbol to indicate unity vectors. As the binary precesses, the waveform undergoes both amplitude and phase modulations, thereby allowing the extraction of the precession signatures.

We illustrate the periods of the dS precession in the (M_3, a_o) space in Fig. 1 with brown traces. The upper panel assumes a circular outer binary and the lower one has $e_o = 0.9$. The solid (dashed) traces correspond to $P_{\text{dS}} = 2\pi/\Omega_{\text{dS}} = 100$ (10) yr. As we will see later, these periods are the approximate detectability thresholds assuming a detection of a source 1 Gpc away performed jointly by space-borne and ground-based detectors and by a TianGO-like detector alone. Also shown are the periods of the outer orbit (grey traces) and sub-leading corrections due to the Lense-Thirring precession (olive traces) and the Lidov-Kozai effect (i.e., the Newtonian tidal effect; cyan traces). The explicit expressions are provided in the Supplemental Material.

To connect to astrophysical formation mechanisms of the inner binary, we indicate in dotted-orange lines the expected locations of migration traps in accretion disks [42] where massive objects are likely to accumulate and binaries may frequently merge. We find $P_{\text{dS}} < 100$ yr (10 yr) at the migration trap at $\simeq 600 M_3$ if $M_3 \lesssim 2 \times 10^7 M_\odot$ ($2 \times 10^6 M_\odot$) and the outer orbit is circular. When the outer orbit is eccentric, the $P_{\text{dS}} < 100$ yr boundary could be extended to further include $M_3 \simeq 10^8$ yr.

For bare nuclei, binaries can also be produced by various dynamical processes. Studies suggested a detection rate of $\mathcal{O}(10 - 100) \text{ yr}^{-1}$ BH binaries produced in the $a_o \lesssim 0.1$ pc region by the interaction channel [3, 5, 6, 43]. Assuming a density profile $\propto a_o^{-2}$ [3], it

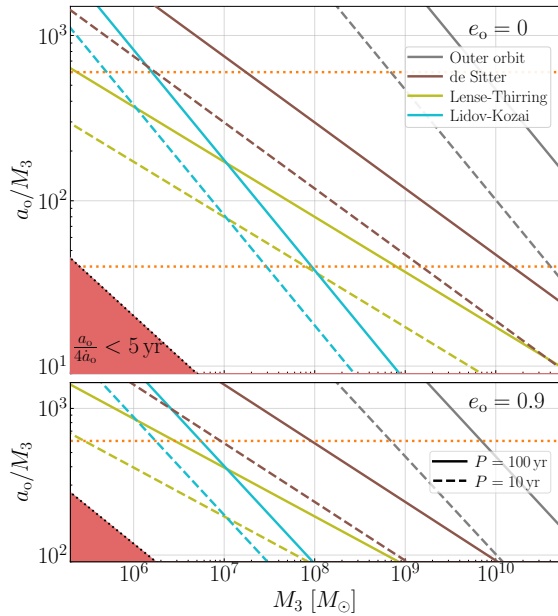


FIG. 1. Periods of various dynamical processes. Approximately, below the solid trace an effect is detectable with joint detection by space-borne and ground-based detectors. Below the dashed trace, the effect can be constrained with a single TianGO-like detector. We set the lower y-limit in each panel to $a_o(1 - e_o) = 9M_3$. The decay of the outer orbit and hence the shaded region with $a_o/4\dot{a}_o < 5 \text{ yr}$ is discarded. The triple stability [41] is always satisfied in the upper panel and its boundary is similar to the shaded region in the lower one.

indicates $\mathcal{O}(0.1 - 1)$ detection per year in the central 0.001 pc region ($\simeq 200 M_3$ for $M_3 = 10^8 M_\odot$) where the dS precession could be significant. In fact, binary formed in this channel may be launched to an outer orbit with significant eccentricity that reduces P_{dS} by a factor $(1 - e_o^2)$ and allows binary formed at greater a_o to also experience significant precession (see the bottom panel of Fig. 1).

Once observed, the dS precession allows a direct determination of properties of the SMBH and the outer orbit. Note its rate is $\Omega_{\text{dS}}/\Omega_o = M_3/[a_o(1 - e_o^2)]$. When combined with the outer orbit's Doppler shift which tells us $\Omega_o = \sqrt{M_3/a_o^3}$ (and e_o for elliptical orbits as we illustrate in the Supplemental Material which includes Ref. [44]), we can therefore infer the values of M_3 , a_o , and e_o individually.

Before this method, there are two common approaches to *directly* determine the mass of an SMBH with a typical accuracy of tens of percent, either through directly observing the dynamics of star or gas around the SMBH, or through reverberation mapping of the continuum emission of AGNs [45]. The former is limited to nearby ($\lesssim 100 \text{ Mpc}$) SMBH and the later is applicable only to Type I (broad emission-line) AGNs, a trace of the population [45]. LISA could also constrain SMBH masses via equal-mass inspirals and EMRIs. However, it is only sensitive to mergers with masses $\lesssim 10^7 M_\odot$ [46–48]. Our

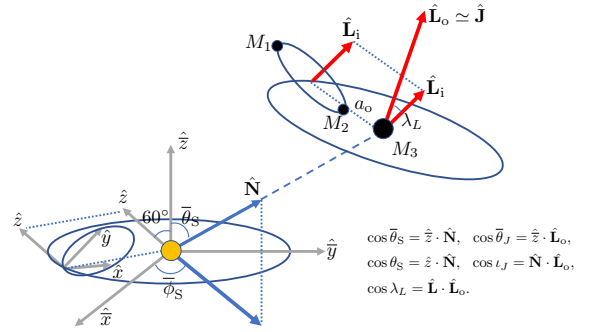


FIG. 2. Cartoon illustrating the geometry of the problem. Note the amplitudes of vectors is chosen only for visualization purpose.

approach, on the other hand, probes SMBHs across almost the entire mass range to a distance of a few Gpc and applies independent of the SMBH being active or quiescent. It is thus an invaluable complimentary to the existing methods. Furthermore, it also determines the outer orbit via measuring a_o and e_o that are hard to be extracted otherwise at $\mathcal{O}(\text{Gpc})$ distances, thereby constraining the nuclei dynamics which currently has considerable theoretical uncertainties.

Hereafter, we will focus on the dS precession and how we can utilize it to measure M_3 and a_o . We neglect the sub-leading Lense-Thirring precession and Lidov-Kozai oscillations for simplicity (but see Ref. [49]) and treat both the inner and outer orbits to be circular (we will discuss the effects of eccentricities at the end of the Letter). Gaseous frictions [11, 50–54] and encounters with background objects [5, 55] have characteristic timescales ranging from thousands to millions of years and therefore can be ignored over an observation over $T_{\text{obs}} \simeq 5 \text{ yr}$ (see the Supplemental Material for details). All the parameters in the Letter correspond to their inferred values in the detector frame [56]. We use geometrical units $G = c = 1$.

Waveforms – In Fig. 2 we demonstrate the geometry of the problem. We construct two reference frames. The (x, y, z) frame is centered on the corner detector with \hat{x} and \hat{y} pointing along two arms of TianGO [29] (for LISA, this frame is constructed as in Ref. [57]). As the detector frame changes in both location and orientation, we also construct a fixed solar frame $(\bar{x}, \bar{y}, \bar{z})$ with \hat{z} perpendicular to the ecliptic. In the solar frame, the source's sky location \hat{N} and the total AM \hat{J} with $\mathbf{J} \equiv \mathbf{L}_i + \mathbf{L}_o \simeq \mathbf{L}_o$ are labeled with polar coordinates $(\bar{\theta}_S, \bar{\phi}_S)$ and $(\bar{\theta}_J, \bar{\phi}_J)$, respectively. We further define ι_J as the angle between \hat{N} and \hat{L}_o and λ_L the angle between \hat{L}_i and \hat{L}_o . The problem now becomes projecting the GW radiation characterized by a time-varying orientation $\hat{L}_i(t)$ onto an antenna with also time-varying coordinates $(\hat{x}, \hat{y}, \hat{z})$.

To obtain the response, we follow Refs. [57, 58]. The explicit expressions for various quantities could be found

in the Supplemental Material. The frequency-domain waveform under the stationary-phase approximation is

$$\tilde{h}(f) = \Lambda(f)\tilde{h}_C(f) = [A_+^2(t)F_+^2(t) + A_\times^2(t)F_\times^2(t)]^{1/2} \times \exp\{-i[\Phi_p(t) + 2\Phi_T(t) + \Phi_D(t)]\}\tilde{h}_C(f), \quad (2)$$

where Λ characterizes the modulation due to antenna response and \tilde{h}_C is the antenna-independent “carrier”. We approximate \tilde{h}_C with the quadrupole formula, including four *intrinsic* parameters, $(\mathcal{M}, D_L, t_c, \phi_c)$, corresponding to the chirp mass, luminosity distance, and time and phase of coalescence. The antenna pattern depends on time which is further a function of frequency, $t(f) = t_c - 5(8\pi f)^{-8/3}\mathcal{M}^{-5/3}$.

The changing orientations affects the amplitude both via $A_+ = 1 + (\hat{\mathbf{L}}_i \cdot \hat{\mathbf{N}})^2$ and $A_\times = -2\hat{\mathbf{L}}_i \cdot \hat{\mathbf{N}}$, and via $F_{+(\times)}(\theta_S, \phi_S, \psi_S)$, where (θ_S, ϕ_S) are the polar coordinates of $\hat{\mathbf{N}}$ in the (x, y, z) frame and ψ_S is the polarization angle.

Besides amplitude modulations, there are also extra phase terms. The Φ_p term characterizes the polarization phase, and the precession of $\hat{\mathbf{L}}_i$ further gives rise to a Thomas precession term Φ_T [58]. Lastly, Φ_D describes a Doppler phase due to motions of both the outer orbit and the detector orbiting around the Sun.

To this point the expressions are generic. A waveform is specified when one supplies information about the orbits (for Φ_D) and the orientations $\hat{\mathbf{L}}_i$ and $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$.

We model the Doppler phase as [59]

$$\Phi_D = 2\pi f \left[a_o \sin \iota_J \cos(\Omega_o t - \phi^{(0)}) + \text{AU} \sin \bar{\theta}_S \cos(2\pi t/\text{yr} - \bar{\phi}_S) \right], \quad (3)$$

where $\phi^{(0)}$ characterizes an initial phase for the outer orbit. The dS precession of $\hat{\mathbf{L}}_i$ around $\hat{\mathbf{L}}_o$ can be written in terms of three additional parameters $(P_{\text{dS}}, \lambda_L, \alpha_0)$ with α_0 an initial phase characterizing the initial orientation of $\hat{\mathbf{L}}_i$. The detector’s orientation for both LISA and TianGO is described in Ref. [60].

We compare in Fig. 3 sample waveforms with sensitivities of various space-borne detectors. The initial GW frequencies $f^{(0)}$ is chosen such that the inner binary merges in 5 years, the fiducial value of T_{obs} . For a stellar-mass inner binary (solid traces), various missions have similar sensitivities to the precession-induced modulation with the decihertz observatories having a greater total signal-to-noise ratio (SNR). With TianGO’s sensitivity, the system corresponding to the purple-solid trace has a total SNR of 80, and an SNR of 13 if we use only the data at least 0.1 yr prior to the merger (i.e., integrating from the initial frequency to the dot markers). While the SNR from the final 0.1 yr does not directly constrain the precession, it nonetheless reduces the uncertainties on other parameters that are partially degenerate with the precession signatures and is thus critical as well. Similarly, a

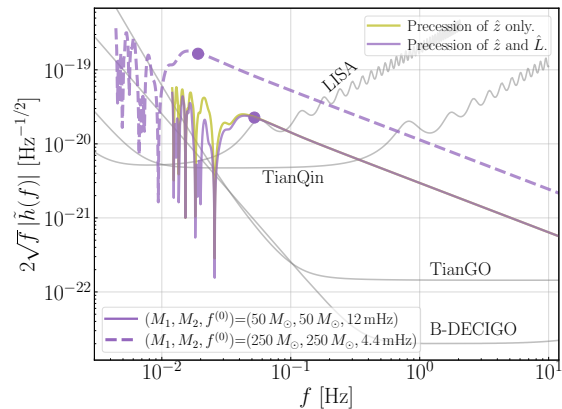


FIG. 3. Sample waveforms shown in $2\sqrt{f}|\tilde{h}|$. The olive trace includes variation in the detector’s orientation only while the purple ones further include the dS precession. The initial frequency $f^{(0)}$ is chosen such that the binary merges in $T_{\text{obs}} = 5$ yr and the dot symbols indicate the instant 0.1 yr prior to the merger. We assumed $(D_L, \theta_S, \bar{\phi}_S, \theta_J, \bar{\phi}_J) = (1 \text{ Gpc}, 33^\circ, 147^\circ, 75^\circ, 150^\circ)$ and $(P_{\text{dS}}, \lambda_L) = (2.7 \text{ yr}, 45^\circ)$.

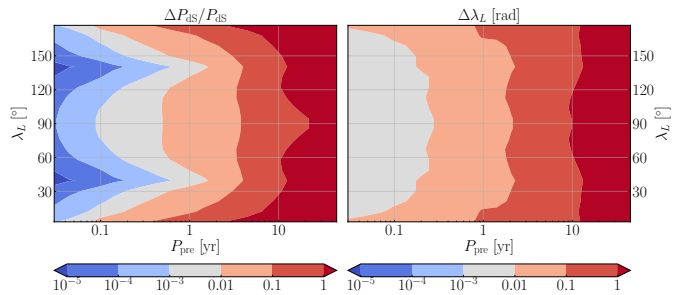


FIG. 4. PE results assuming a simple-precession problem (no Doppler phase of the outer orbit). We fix the inner binary to have $(M_1, M_2, f^{(0)}) = (50 M_\odot, 50 M_\odot, 12 \text{ mHz})$. The source’s sky location in the solar frame is $(D_L, \bar{\theta}_S, \bar{\phi}_S) = (1 \text{ Gpc}, 33^\circ, 147^\circ)$ and the orientation of the outer orbit is $(\bar{\theta}_J, \bar{\phi}_J) = (75^\circ, 150^\circ)$. Note the dS precession is detectable if $P_{\text{dS}} \gtrsim 10$ yr if the source is detected by TianGO alone.

joint detection of the source with ground-based detectors enhances the sensitivity further. If the inner binary consists of intermediate-mass BHs [the dashed trace; it has a total (early-stage) SNR of 36 (26) in LISA after combining two detectors’ responses], then LISA alone would be able to detect the modulations.

Results – We adopt the Fisher matrix formalism [57] to quantify the detectability.[61] We start by considering the parameter-estimation (PE) accuracy of a simple-precession problem (i.e., dropping the Doppler phase due to the outer orbit) and parameterize the modulation in terms of $(P_{\text{dS}}, \lambda_L, \alpha_0)$. Our aim is to establish the detectability thresholds for P_{dS} and λ_L . The results are summarized in Fig. 4 (we have randomized α_0 and plotted the median values). Throughout this Section we assume the source is detected by TianGO [29] alone.

As expected, the accuracy in both P_{dS} and λ_L improves

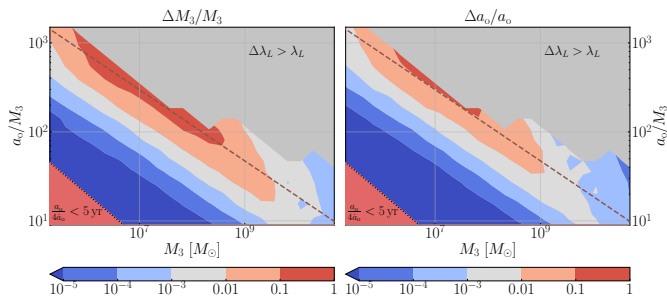


FIG. 5. PE results combining both the dS precession and the Doppler phase shift. We assumed an opening angle between the inner and outer orbits of $\lambda_L = 45^\circ$ and other parameters are the same as in Fig. 4. We also plotted the line of $P_{\text{dS}} = 10$ yr. The grey regions have $\Delta\lambda_L > \lambda_L$ and therefore are excluded. Note M_3 can be constrained to 10% if $P_{\text{dS}} \simeq 10$ yr with TianGO alone. The error decreases as M_3 increases thanks to the additional information provided by the amplitude of the Doppler phase.

as P_{dS} decreases, and at $P_{\text{dS}} \simeq 2T_{\text{obs}} = 10$ yr we have approximately $\Delta P_{\text{dS}}/P_{\text{dS}} < 1$ and $\Delta\lambda_L < 1$ rad, marking the boundary of detectability.

Note that at $P_{\text{dS}} \gtrsim 3$ yr, the error ΔP_{dS} is smallest when $\lambda_L \simeq 90^\circ$ as it maximizes the variation in the orientation. At smaller P_{dS} , the optimal detectability is achieved at $\lambda_L \simeq 40^\circ \simeq \iota_J$ (and also at 140°). This is thanks to the Thomas phase Φ_T . As shown in Ref. [58], when $\hat{\mathbf{N}}$ is inside the precession cone ($|\hat{\mathbf{L}}_o \cdot \hat{\mathbf{L}}| < |\hat{\mathbf{L}}_o \cdot \hat{\mathbf{N}}|$), each precession cycle the Thomas term contributes approximately $(-2\pi \cos \lambda_L)$ to the phase. When $\hat{\mathbf{L}}_o \cdot \hat{\mathbf{L}} > |\hat{\mathbf{L}}_o \cdot \hat{\mathbf{N}}|$, however, the contribution per cycle changes sharply to about $2\pi(-\cos \lambda_L + 1)$ [62]. Consequently, when $\lambda_L \simeq \iota_J$ (or $\pi - \iota_J$), Φ_T can be determined with high accuracy. Since the total Φ_T is proportional to the total number of precession cycles, it thus leads to good constraints on P_{dS} .

As we know the detector’s orbit, we do not see it significantly interfering with the results when $P_{\text{dS}} \simeq 1$ yr. Moreover, the Thomas phase is associated with the precession of $\hat{\mathbf{L}}_i$ only [57], further breaking the potential degeneracy between a changing $\hat{\mathbf{L}}$ and a changing $\hat{\mathbf{z}}$. It is nonetheless crucial to include the detector’s motion to constrain $\hat{\mathbf{N}}$ [29, 57].

We now combine the dS precession with the Doppler shift to study the constraints on the SMBH properties. We use M_3 and a_o as free parameters and write Ω_o and P_{dS} in terms of M_3 and a_o . The initial phase $\phi^{(0)}$ is included and randomized over.

The result is shown in Fig. 5. We only include regions where $\Delta\lambda_L \leq \lambda_L$ so that the signature of precession is unambiguously detected. Note the boundary of $\Delta\lambda_L = \lambda_L$ is broadly consistent with the line of $P_{\text{dS}} = 10$ yr, agreeing with the results we obtained in the simple-precession analysis. Along the line of $P_{\text{dS}} = 10$ yr, the fractional error in the SMBH mass is constrained to

$\Delta M_3/M_3 \sim 10\%$, demonstrating a direct determination of the SMBH property is indeed possible. We further find that $\Delta \log a_o \simeq \Delta \log M_3/3$ for most of the parameter spaces because Ω_o is determined with the highest accuracy among all the parameters describing the modulations.

Along the line of constant P_{dS} , the error decreases with increasing M_3 . This is because the “modulation depth” on the Doppler phase [Eq. (3)] increases with M_3 . With the Doppler shift alone, we cannot utilize the modulation depth due to the unknown $\sin \iota_J$. Once the precession is included, however, $\hat{\mathbf{L}}_o$ serves as the precession axis of $\hat{\mathbf{L}}_i$, allowing the outer orbit’s inclination to be inferred. Once we know $\sin \iota_J$, the modulation depth provides another measurement of a_o , enhancing the sensitivity further.

Summary and Discussion. – Our analysis so far considered detections by TianGO alone. As ground-based detectors are more sensitive to stellar-mass BHs [29], they could constrain intrinsic parameters with much higher accuracy. We thus estimate the joint-detection effect by still computing the Fisher matrix using a spaceborne detector’s sensitivity but treating (M, ϕ_c, t_c) as known parameters. For a system with $(M_3, a_o) = (10^8 M_\odot, 100 M_3)$ and the rest the same as in Fig. 5, the errors in (M_3, a_o) can be dramatically improved to $\Delta \log M_3 = 1.7 \times 10^{-4} (5.2 \times 10^{-2}) \simeq 3 \log a_o$ assuming the sensitivity of TianGO (LISA). The uncertainty in λ_L is also reduced by about a factor of 6 to $\Delta\lambda_L = 0.02$ rad for both TianGO and LISA. If a source instead has $a_o = 300 M_3$ with $P_{\text{pre}} \simeq 100$ yr, we find a median error $\Delta\lambda_L = 0.72$ rad $< \lambda_L$ with LISA’s sensitivity after randomizing initial phases, indicating the precession would still be detectable. Knowing the source’s distance and sky location further improves the accuracy in λ_L by a factor of a few. For $a_o = 300 M_3$ and LISA’s sensitivity, we find $\Delta\lambda_L = 0.16$ rad in this case.

We assumed both circular inner and outer orbits. In reality, finite eccentricities are expected especially if the inner binary is formed via dynamical channels. One plausible scenario is that both e_i and e_o follow a thermal distribution, with $e_{i,o}^2$ uniform in $[0, 1)$ [4, 63].

An elliptic outer orbit enhances the detectability. Note e_o does not affect the inference accuracy of Ω_o and itself can be well constrained from the Doppler shift (as demonstrated in the Supplemental Material). Although the instantaneous precession rate [64] should be used for waveform modeling, the secular version [Eq. (1)] nonetheless indicates the qualitative effect of e_o , which is to make the rate greater by a factor of $1/(1 - e_o^2)$. Thus at a fixed a_o the waveform is modulated by more precession cycles, making its signature more prominent. It also allows a system at greater a_o to potentially experience a significant modulation (lower panel of Fig. 1).

The eccentricity of the inner orbit e_i modifies only the carrier \tilde{h}_c . Therefore, it affects the results mostly through affecting the overall SNR. Following Ref. [65], for mild

eccentricities ($e_i \leq 0.7$ at $a_i = 1.4 \times 10^{-3}$ AU), we find both the total SNR and that from the early stage (≥ 0.1 yr prior to merger) in fact increase for TianGO, and decreases by a small amount (factor of 3) for LISA. A more extreme eccentricity would make inner orbit decay too quickly if we fix the initial $a_i^{(0)}$. Nonetheless, such a system can merge within T_{obs} starting at much greater initial separations of $\mathcal{O}(0.1)$ AU. From the evolution from 0.1 AU to 10^{-3} AU we can still obtain an integration time of more than a year and an SNR of about 5 (with the sensitivity of TianGO). Therefore, our results should not change qualitatively by the inner eccentricity (detailed calculations presented in the Supplemental Material).

We did not include the precessions of $\hat{\mathbf{L}}_i$ due to the spins of $M_{1(2)}$. Nevertheless, this should be well distinguishable from the precessions around $\hat{\mathbf{L}}_o$ thanks to the separation in scales. The spin-induced opening angle is $\lesssim M_1^2/L_i \sim 1^\circ$ when $f \sim 0.01$ Hz, in general much smaller than Λ_L which distributes approximately uniformly between 0° and 180° (e.g., Ref. [66]). Moreover, the spin-induced precession rate is $\sim L_i/a_i^3$ [58], corresponding to a period of 10 days when $f = 0.01$ Hz, and the period decreases further as the inner binary decays. In contrast, the dS precession around L_o has a constant and much longer period.

Whereas we used the quadrupole formula for the carrier, our formalism can be readily extended to incorporated more complicated dynamics of the inner binary (higher-order relativistic corrections as well as environmental effects due to gas [53] and/or gravitational lensing [67, 68] that alter the observed chirp mass [69]) by replacing the carrier part with the appropriate $\tilde{h}_C(f)$. Similar to the inner eccentricity, changing the carrier affects the detectability of extrinsic modulations mostly through changing the overall SNR.

To conclude, we demonstrated that the dS precession of $\hat{\mathbf{L}}_i$ around $\hat{\mathbf{L}}_o$ is detectable. The detectability threshold is $P_{\text{dS}} \simeq 10$ yr with space-borne detectors alone and $P_{\text{dS}} \simeq 100$ yr if the source is jointly detected by ground-based detectors. This effect allows a direct determination of the SMBH mass to better than 10% at Gpc distances and applies to both active and quiescent SMBHs. It also constrains the dynamics in galactic nuclei by pinpointing the outer orbit. Future studies incorporating the orbital eccentricities and sub-leading effects, as well as extending the PE to a more rigorous Bayesian framework would be of great value.

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