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Quantum quench in PT-symmetric Luttinger liquid

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A Luttinger liquid (LL) describes low energy excitations of many interacting one dimensional systems, and exhibits universal response both in and out of equilibrium. We analyze its behaviour in the non-hermitian realm after quantum quenching to a PT-symmetric LL by focusing on the fermionic single particle density matrix. For short times, we demonstrate the emergence of unique phenomena, characteristic to non-hermitian systems, that correlations propagate faster than the conventional maximal speed, known as the Lieb-Robinson bound. These emergent supersonic modes travel with velocities that are multiples of the conventional light-cone velocity. This behaviour is argued to be generic for correlators in non-hermitian systems. In the long time limit, we find typical LL behaviour, extending the LL universality to the non-equilibrium non-hermitian case. Our analytical results are benchmarked numerically and indicate that the dispersal of quantum information is much faster in non-hermitian systems.

Introduction. Non-hermitian quantum mechanics has provided us with a plethora of interesting phenomena, investigated both theoretically and experimentally. These include spontaneous PT-symmetry breaking, non-unitary dynamics, encircling and manipulating exceptional points, unidirectional invisibility, complex Bloch oscillations and even topological effects[1–11], to mention a few. However, the common theme behind these studies is the underlying effective single particle picture, while excursions to the quantum many-body realm are scarce[12].

By now, hermitian quantum many-body physics in one spatial dimension is well understood thanks to the available analytical and numerical methods[13, 14], both in and out of equilibrium. In particular, many of these systems realize Luttinger liquids (LLs), including bosonic, fermionic, spin etc. models, irrespective of their statistics and microscopic details[13–15]. Therein, the Fermi-liquid description breaks down and the elementary excitations are bosonic collective modes. This fractionalization is manifested by the universal non-integer power law decays in almost all correlation functions[13]. Furthermore, when taken out of equilibrium, for example following a quantum quench, the evolution of such hermitian systems is always unitary. The changes of system parameters cause the emission of quasiparticles carrying correlations that propagate across the system with a certain velocity whose maximum value is given by the Lieb-Robinson bound[16]. The existence of a maximum speed implies a light-cone spreading of correlations[17] with only subsonic mode velocities.

Therefore, it comes as a natural question whether any of the LL universality and light-cone structure survive under non-hermitian conditions. This motivated us to explore the fate of LLs after a quantum quench to a PT-

symmetric non-hermitian system. We find that the long time limit exhibits LL behaviour with non-integer power law decays, thus extending LL universality to the non-equilibrium non-hermitian realm.

The short time behaviour, on the other hand, differs drastically from that in the hermitian realm[18]. On top of the usual light-cone[16], new supersonic modes appear and travel with velocities that are multiples of the light-cone velocity. The origin of the supersonic modes is related to an effective long range Hamiltonian, governing the time evolution, for which such supersonic modes might be expected[19]. We argue that this emergent phenomenon is characteristic to all correlation functions in non-hermitian systems. These findings are tested numerically on a non-hermitian variant of the XXZ Heisenberg model. Its density correlation function reveals three distinct light-cones, in perfect agreement with bosonization.

PT-symmetric LL. The non-hermitian LL Hamiltonian, we study, is given by

$$H = \sum_{q \neq 0} v |q| b_q^\dagger b_q + \frac{ig_2 |q| \Theta(t)}{2} [b_q b_{-q} + b_q^\dagger b_{-q}^\dagger], \quad (1)$$

with v being the bare "sound velocity", and b_q^\dagger the creation operator of a bosonic density wave. The interaction g_2 is changed from zero to a nonzero value at $t = 0$. Although the Hamiltonian is non-hermitian, its spectrum remains real[20] as $\omega_q = \tilde{v} |q|$ in the presence of imaginary interaction, with renormalized velocity $\tilde{v} = \sqrt{v^2 + g_2^2}$. This falls into the category of PT-symmetric non-hermitian systems[1–4]: Eq. (1) satisfies the antiunitary (generalized PT-) symmetry[20] as the combination of time reversal, $i \rightarrow -i$ and phase transformation $b_q \rightarrow ib_q$, $b_q^\dagger \rightarrow -ib_q^\dagger$. The Hamiltonian does not commute with the generators of each symmetry independently, but only with their product. This is to be

contrasted to the hermitian case, obtained by the replacement $ig_2 \rightarrow g_2$ in Eq. (1), in which case the sound velocity is [13] $v_- = \sqrt{v^2 - g_2^2}$. In both cases, the system is stable for $|g_2| < v$, as we show below. A distinct version of a non-hermitian LL was investigated in Ref. [21].

In the present work, we are interested in the time evolution of a LL following a sudden quantum quench. Initially, the system is prepared in the non-interacting ground state $|\phi_0\rangle$ (i.e. the vacuum for the b bosons), and time evolved with the non-hermitian Hamiltonian, Eq. (1) as $|\phi(t)\rangle = e^{-iHt} |\phi_0\rangle$. [22]. In the hermitian realm, such systems were studied exhaustively [18, 23–27].

In the fermionic realization of non-hermitian physics, the ensuing non-equilibrium dynamics can be captured by the fermionic one-particle density matrix. The original fermion field decomposes to right-going and a left-going parts [13, 14] as $\Psi(x) = e^{ik_F x} R(x) + e^{-ik_F x} L(x)$, therefore it is enough to investigate, for example, the correlator of the right-movers, defined as

$$G_r(x, t) \equiv \frac{\langle \phi(t) | R^+(x) R(0) | \phi(t) \rangle}{\langle \phi(t) | \phi(t) \rangle}, \quad (2)$$

describing excitations around the right Fermi momentum, $k \approx k_F$. The right-moving field, $R(x)$, is expressed in terms of the LL bosons as [13] $R(x) = \frac{\eta_r}{\sqrt{2\pi\alpha}} \exp(i\phi_r(x))$, where η_r denotes the Klein factor, and $\phi_r(x) = \sum_{q>0} \sqrt{2\pi/|q|} L e^{iqx - \alpha|q|/2} b_q + h.c.$ with α an ultraviolet regulator. Due to the non-unitary time evolution [12, 28], it is compulsory to treat carefully the denominator arising in Eq. (2).

The norm of the wavefunction. To warm up, let us start by evaluating the denominator in Eq. (2), which is $N(t) \equiv \langle \phi(t) | \phi(t) \rangle = \langle \phi_0 | e^{iH^+t} e^{-iHt} | \phi_0 \rangle$, which would be 1 in the hermitian case. Calculating $N(t)$ is accomplished by realizing that the operators appearing in the Hamiltonian, $K_0(q) = (b_q^+ b_q + b_{-q}^+ b_{-q})/2$, $K_+(q) = b_q^+ b_{-q}^+$ and $K_-(q) = b_q b_{-q}$ are the generators of SU(1,1) Lie algebra [29]. Exploiting a faithful 2×2 matrix representation of the SU(1,1) generators [30, 31], the product of the time evolution operators is recast as

$$e^{iH^+t} e^{-iHt} = \prod_{q>0} e^{C_+(q,t)K_+(q)} e^{C_0(q,t)K_0(q)} e^{C_-(q,t)K_-(q)}. \quad (3)$$

When taking its expectation value with the bosonic vacuum, the first and last exponentials containing $K_+(q)$ and $K_-(q)$ are Taylor expanded, and only the zeroth order term remains finite, all other terms containing powers of $b_q b_{-q}$ vanish when acting on the vacuum. Then, the expectation value of Eq. (3) reduces to $\prod_{q>0} \exp(C_0(q,t)/2)$. This is evaluated to yield $N(t) = \prod_{q>0} \tilde{v}^2 / (\tilde{v}^2 - 2g_2^2 \sin^2(\omega_q t))$, which indeed gives one for $t = 0$ [32]. Interestingly, the non-hermitian formulation remains valid only for $g_2 < v$, similarly to the hermitian

case, even though the renormalized sound velocity does not vanish for $v = |g_2|$. The norm, $N(t)$ should always be non-negative. However, with increasing g_2 , it first diverges and becomes negative for $|g_2| > v$. This happens because after the quench, the time evolved wavefunction leaves the space of normalizable wavefunction, which is signaled by the total norm becoming negative [33, 34]. This behavior can be associated with the dynamical manifestation of the equilibrium instability found in related systems [35].

The numerator of the Green's function. The two exponentials in the right-moving fields are merged using standard tricks [36] and the time evolution of this operator is then evaluated using the identity

$$\begin{aligned} \langle \phi_0 | e^{iH^+t} e^{-i(\phi_r(x) - \phi_r(0))} e^{-iHt} | \phi_0 \rangle &= \\ = \langle \phi_0 | e^{iH^+t} e^{-iHt} e^{iHt} e^{-i(\phi_r(x) - \phi_r(0))} e^{-iHt} | \phi_0 \rangle &= \\ \langle \phi_0 | e^{iH^+t} e^{-iHt} e^{-i(\phi_r(x,t) - \phi_r(0,t))} | \phi_0 \rangle. \end{aligned} \quad (4)$$

This allows us to formally define a pseudo-Heisenberg-type time evolution for the operators as $b_q(t) = e^{iHt} b_q e^{-iHt}$, though this is *not* the physical Heisenberg time evolution as that would involve the e^{iH^+t} operator to the front. The resulting pseudo-Heisenberg equation of motion is solved from $\partial_t b_q = i[H, b_q]$ and $\partial_t b_{-q}^+ = i[H, b_{-q}^+]$, which are *not* related to each other by hermitian conjugation. This equation of motion is solved as

$$b_q(t) = u_q(t) b_q + v_q(t) b_{-q}^+, \quad (5a)$$

$$b_{-q}^+(t) = u_q^*(t) b_{-q}^+ - v_q(t) b_q, \quad (5b)$$

and $|u_q(t)|^2 + |v_q(t)|^2 = 1$ [37], and the canonical commutation relation, $[b_q(t), b_q^+(t)] = 1$ is preserved. Finally, the time-dependent pseudo-Bogoliubov coefficients are

$$u_q(t) = \cos(\omega_q t) - \frac{iv}{\tilde{v}} \sin(\omega_q t), \quad v_q(t) = \frac{g_2}{\tilde{v}} \sin(\omega_q t), \quad (6)$$

which are related to the hermitian Bogoliubov coefficients [25] through the $g_2 \rightarrow ig_2$ change.

The two time evolution operators, acting on the bra vector, are then rewritten using Eq. (3) as

$$\begin{aligned} \langle \phi_0 | e^{iH^+t} e^{-iHt} &= \langle \phi_0 | \prod_{q>0} e^{C_-(q,t)K_-(q) + \frac{C_0(q,t)}{2}} = \\ &= N(t) \langle \phi_0 | \prod_{q>0} e^{C_-(q,t)K_-(q)}. \end{aligned} \quad (7)$$

Therefore, the denominator appears also in the numerator and drops out from the final expression. Using again the faithful representation of the SU(1,1) algebra, we get $C_-(q,t) = 2u_q(t)v_q(t)/(|u_q(t)|^2 - |v_q(t)|^2)$.

Supersonic modes. In order to calculate the vacuum expectation value, we normal order the pseudo-Heisenberg time evolved bosonic operators in the Green's

function. Using the pseudo-Heisenberg time evolution from Eq. (5), we obtain

$$e^{-i(\phi_r(x,t)-\phi_r(0,t))} = e^{-i\phi^+(x,t)} e^{-i\phi^-(x,t)} e^{c(x,t)}, \quad (8)$$

where $\phi^+(x,t) = \sum_{q>0} \sqrt{\frac{2\pi}{qL}} ((e^{-iqx} - 1)u_q^*(t)b_q^+ + (e^{iqx} - 1)v_q(t)b_{-q}^+)$, $\phi^-(x,t) = \sum_{q>0} \sqrt{\frac{2\pi}{qL}} ((e^{iqx} - 1)u_q(t)b_q - (e^{-iqx} - 1)v_q(t)b_{-q})$, $c(x,t) = \sum_{q>0} \frac{\pi}{qL} |e^{iqx} - 1|^2 (2|v_q^2(t)| - 1)$. In the hermitian case, the calculation would end here[24], which contains all equilibrium and quench induced correlations, since $C_-(q,t)$ would be zero. For the non-hermitian quench, by combining Eqs. (7) and (8), the three exponentials are again Taylor expanded to calculate the required vacuum expectation value. The $e^{-i\phi^-(x,t)}$ term gives one when acting on the vacuum. The expansion of the $e^{C_-(q,t)K_-(q)}$ contains the same powers of b_q and b_{-q} due to the very definition of $K_-(q)$. Therefore, in order to have a non-zero expectation value, only those terms contribute from the expansion of $e^{-i\phi^+(x,t)}$, which also contain the same powers of b_q^+ and b_{-q}^+ . This finally gives after some tedious algebra [38]

$$\frac{G_r(x,t)}{G_r^0(x)} = \exp \left(-\frac{8\pi}{L} \sum_{q>0} \frac{g_2^2 \sin^2(qx/2) \sin^2(\omega_q t)}{q(\tilde{v}^2 - 2g_2^2 \sin^2(\omega_q t))} \right) \quad (9)$$

where $G_r^0(x) = i/(2\pi(x + i\alpha))$ denotes the free fermion propagator, L the system size. This final result differs from the outcome of a hermitian quantum quench by the denominator in the exponent, but as we discuss below, it has profound consequences for the time evolution and light-cone structure.

The exponent of the Green's function is Taylor expanded in terms of $\sin^2(\omega_q t)$. Then, the various q integrals are performed and the series is resummed, yielding

$$G_r(x,t) = G_r^0(x) \exp \left(\left(1 - \frac{\tilde{v}}{v_-} \right) d(x,0) - 2 \sum_{n=1}^{\infty} \frac{\tilde{v}}{v_-} \left(\frac{-g_2^2}{v^2 + \tilde{v}v_-} \right)^n d(x,nt) \right), \quad (10)$$

where $d(x,t) = \frac{1}{4} \ln \left(\frac{(\alpha^2 + (x - 2\tilde{v}t)^2)(\alpha^2 + (x + 2\tilde{v}t)^2)}{(\alpha^2 + 4\tilde{v}^2 t^2)^2} \right)$ using the $e^{-\alpha|q|}$ cutoff in Eq. (9) [39].

For $2\tilde{v}t \gg x$, the Green's function becomes completely time-independent, similarly to the hermitian quench[18]. The characteristic non-integer power law decay of LL is observed as $G_r(x,t \rightarrow \infty) \sim |x|^{-\tilde{v}/v_-}$, and the exponent is smaller than for a hermitian quench[25] with the same interaction strength g_2 . This establishes the LL universality also in the non-equilibrium and non-hermitian case.

On the other hand, for $2\tilde{v}t \ll x$, supersonic modes that propagate faster than the sound velocity \tilde{v} , emerge and the corresponding velocities are integer multiples of $2\tilde{v}$, even though H itself is local[16]. Its origin is traced

back to the $e^{iH^+t}e^{-iHt}$ factor in Eq. (4). When merging the exponentials into a single one, a series of nested commutators arise from the Baker-Campbell-Hausdorff formula[31] due to $[H, H^+] \neq 0$, and the resulting exponent, interpreted as an effective Hamiltonian, becomes increasingly non-local and long range, therefore there is no obvious bound of the propagation speed of correlations in non-hermitian systems. This parallels to the appearance of supersonic modes in hermitian long range systems[19]. These are manifested in the denominator of Eq. (9): upon expanding it in Taylor series in $\sin^2(\omega_q t)$, the resulting expression involves terms that oscillate at frequencies $2\omega_q, 4\omega_q, 6\omega_q, \dots$, leading to the propagation velocities $2n\tilde{v}$. However, the sharpness of the supersonic light-cones at $x = 2n\tilde{v}t$ decreases with n due to the g_2^{2n} factor in Eq. (10), as shown in Fig. 1. By neglecting the $C_-(q,t)$ term, arising from $[H, H^+] \neq 0$, only a single conventional light-cone would appear.

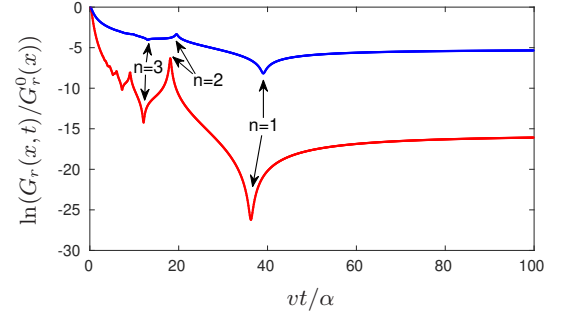


FIG. 1. The single particle density matrix is plotted from Eq. (10) for $g_2/v = 0.8$ (blue) and 0.95 (red) line with $x = 100\alpha$. The $n = 1$ light-cone from the sum in $G_r(x,t)$ corresponds to the conventional light-cone with $2\tilde{v}$ velocity, while the first two supersonic features are denoted by $n = 2$ and 3 with $4\tilde{v}$ and $6\tilde{v}$ velocity, respectively.

We argue that supersonic modes appear generically during the time evolution of any correlator of local observables \mathcal{O}_x in non-hermitian systems. Consider the correlation function $\chi(x,t) \equiv \langle \phi(t) | \mathcal{O}_x \mathcal{O}_0 | \phi(t) \rangle / N(t)$ as

$$\chi(x,t) = \langle \phi_0 | e^{iH^+t} e^{-iHt} \mathcal{O}_x(t) \mathcal{O}_0(t) | \phi_0 \rangle / N(t), \quad (11)$$

where $\mathcal{O}_x(t) = e^{iHt} \mathcal{O}_x e^{-iHt}$ is the pseudo-Heisenberg time evolved operator. Due to $e^{iH^+t}e^{-iHt}$, supersonic modes are expected from the argument below Eq. (10). Indeed, using Eq. (7), it is rewritten as

$$\chi(x,t) = \langle \phi_0 | e^{\sum_{q>0} C_-(q,t) K_-(q)} \mathcal{O}_x(t) \mathcal{O}_0(t) | \phi_0 \rangle. \quad (12)$$

The $C_-(q,t)$ function contains $1/(\tilde{v}^2 - 2g_2^2 \sin^2(\omega_q t))$, and when the expectation value is taken, this will inevitably alter the propagation velocity in $\chi(x,t)$ by even integer multiples of \tilde{v} , similarly to the single particle density matrix in Eq. (10). Supersonic modes follow from the proper Heisenberg picture using the equation of motion

method[40]. While supersonic modes were also seen in a non-hermitian non-interacting system[41], our results imply that these are expected on general ground in non-hermitian dynamics.

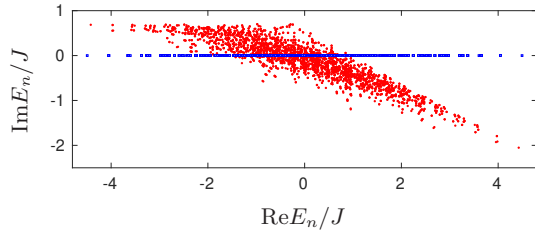


FIG. 2. The complex many-body energy spectrum, E_n of Eq. (13) for $N = 14$ at half filling is plotted for $J_z = -0.3J$ (red dots) and the non-interacting, hermitian case with $J_z = 0$ (blue squares) for comparison from exact diagonalization using periodic boundary conditions. During the time evolution with e^{-iH} , states with the largest imaginary part contribute the most, which is the upper flat part of the spectrum. Out of these, states with increasing $\text{Re} E_n$ have less influence due to their decreasing overlap with the initial state. Due to the normalization in Eq. (11), the imaginary offset of energies drops out from the expectation values.

Numerics for lattice fermions. In order to test our results, we study a simple lattice model with imaginary interactions, which is not PT-symmetric, albeit the low energy part of its spectrum can be considered effectively real, while some higher modes develop significant imaginary parts, which would only influence the long time dynamics. The Hamiltonian is

$$H = \sum_{m=1}^N \frac{J + iJ_z}{2} (c_{m+1}^\dagger c_m + \text{h.c.}) - i \frac{J_z \pi}{2} n_{m+1} n_m, \quad (13)$$

where c 's are fermionic operators and N the number of lattice sites, $n_m = c_m^\dagger c_m$ and J_z denotes the nearest neighbour interaction and the system is half filled. Its low energy excitations are sound waves with sound velocity $\tilde{v} \approx J + (\pi^2/8 - 1)J_z^2/J$ after setting the lattice constant to one, which allows us to identify $g_2 \sim -J_z/1.4$ for small J_z , while $v = J$. Let us note, that the hermitian version ($iJ_z \rightarrow J_z$ in Eq. (13)) is Bethe-Ansatz solvable[13] with sound velocity $v_- \approx J + (1 - \pi^2/8)J_z^2/J$, in perfect agreement with the bosonization discussion following Eq. (1). The main merit of introducing J_z into the hopping as well is that it eliminates the g_4 process which is only responsible for velocity renormalization[13] but does not induce non-integer power law decay of correlation functions, and makes the velocity real[13] for the non-hermitian case. Its energy spectrum at half filling obtained with exact diagonalization is shown in Fig. 2.

We consider numerically a quench dynamics, when the system is prepared initially in the non-interacting, $J_z = 0$ ground state of Hamiltonian, Eq. (13) as a Slater determi-

nant. This is determined by the density matrix renormalization group [42] approach. Then, we suddenly switch on J_z and let the system evolve according to the Hamiltonian, Eq. (13). To study the quench dynamics we use the time evolving block decimation algorithm [43] in the matrix product state representation. We have followed the time evolution of several physical quantities, such as the single particle density matrix or the density correlator, and we found that all show signs of supersonic modes. However, there is a compromise: we have to keep the ratio $|J_z/J| \lesssim 0.4$, relatively small in order to retain the flat part of the spectrum as in Fig. 2 with constant imaginary part. On the other hand, the smallness of the interaction suppresses the higher order supersonic modes, as evident from Fig. 1. In Fig. 3, we show the density correlator $\chi_{nn}(x, t)$, defined in Eq. (11) using $\mathcal{O}_x = n_x$ in a system with $N = 201$ and 101 fermions. The system is slightly away from half filling, which helps in killing the umklapp term[13]. We checked that qualitatively similar results arise exactly at half filling with $N = 200$ and 100 particles, though.

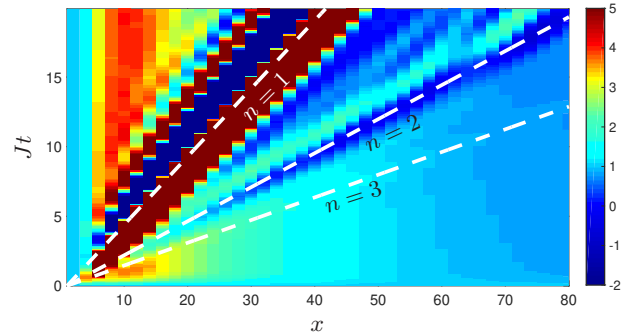


FIG. 3. Contour plot of the the density correlator, $\chi_{nn}(x, t)/\chi_{nn}(x, 0)$, where denominator cancels the initial spatial correlation in the ground state, and all features result from the non-hermitian quench dynamics with $J_z/J = -0.3$. The white dashed lines denote the $n = 1, 2$ and 3 modes by using the sound velocity, $\tilde{v} = 1.02J$ without any fitting.

Experimental relevance. Non-hermitian Hamiltonians can arise from a variety of different ways. It can arise from classical photonic waveguides, which only emulates the Schrödinger equation[6]. In this case, supersonic modes are expected to occur for arbitrary long times. The non-hermitian time evolution can also follow from a conditional Lindblad-type dynamics[35, 41], when the environment is continuously monitored in order to maintain the condition of no quantum jump[44]. The probability of having no quantum jump decreases steadily with time, rendering long time non-hermitian dynamics increasingly difficult to observe, though not impossible[45]. In this setting, Eq. (13) is realized by a dissipative lattice[46] with one-body loss, i.e. superimposing a weak resonant optical lattice (for the non-hermitian hopping) and also implementing background two-body loss[44] (for the

imaginary interactions). Finally, non-hermiticity could arise by adding the imaginary self energy (i.e. from finite lifetime quasiparticles) from diagrammatics[47] to an originally Hermitian system, and treating this as an effective non-hermitian system. In this case, however, the temporal dynamics of the system would also require to go beyond the self-energy approximation and include vertex corrections as well, and the ensuing dynamics would probably feature a single light cone, as dictated by the Lieb-Robinson bound[16].

Summary. We studied many-body non-hermitian dynamics by a quantum quench in a PT-symmetric LL. The fermionic single particle density matrix reveals displays LL *universality* in the long time limit. For short times, in contrast to unitary evolution, the Lieb-Robinson bound is violated and supersonic modes emerge due to non-unitary dynamics. They travel with velocities that are multiples of the conventional light-cone velocity. We argue and demonstrate that this emergent phenomena is characteristic to non-hermitian systems and arise from an effective long range Hamiltonian, governing the time evolution, although the physical Hamiltonian contains only short range, though non-hermitian terms. Our analytical findings are benchmarked by the numerical study of a non-hermitian short range interacting lattice fermions.

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- [1] R. El-Ganainy, K. G. Makris, M. Khajavikhan, Z. H. Musslimani, S. Rotter, and D. N. Christodoulides, *Non-hermitian physics and pt symmetry*, Nat. Phys. **14**, 11 (2018).
- [2] I. Rotter and J. P. Bird, *A review of progress in the physics of open quantum systems: theory and experiment*, Rep. Prog. Phys. **78**, 114001 (2015).
- [3] C. M. Bender and S. Boettcher, *Real spectra in non-hermitian hamiltonians having PT symmetry*, Phys. Rev. Lett. **80**, 5243 (1998).
- [4] M. V. Berry, *Physics of nonhermitian degeneracies*, Czech. J. Phys. **54**, 1039 (2004).
- [5] R. El-Ganainy, M. Khajavikhan, D. N. Christodoulides, and S. K. Ozdemir, *The dawn of non-hermitian optics*, Commun. Phys. **2**, 37 (2019).
- [6] L. Xiao, K. Wang, X. Zhan, Z. Bian, K. Kawabata, M. Ueda, W. Yi, and P. Xue, *Observation of critical phenomena in parity-time-symmetric quantum dynamics* (2018), arxiv:1812.01213.
- [7] T. Gao, E. Estrecho, K. Y. Bliokh, T. C. H. Liew, M. D. Fraser, S. Brodbeck, M. Kamp, C. Schneider, S. Höfling, Y. Yamamoto, F. Nori, Y. S. Kivshar, *et al.*, *Observation of non-hermitian degeneracies in a chaotic exciton-polariton billiard*, Nature **526**, 554 (2015).
- [8] L. Zhou, Q.-h. Wang, H. Wang, and J. Gong, *Dynamical quantum phase transitions in non-hermitian lattices*, Phys. Rev. A **98**, 022129 (2018).
- [9] T. E. Lee, *Anomalous edge state in a non-hermitian lattice*, Phys. Rev. Lett. **116**, 133903 (2016).
- [10] J. M. Zeuner, M. C. Rechtsman, Y. Plotnik, Y. Lumer, S. Nolte, M. S. Rudner, M. Segev, and A. Szameit, *Observation of a topological transition in the bulk of a non-hermitian system*, Phys. Rev. Lett. **115**, 040402 (2015).
- [11] S. Yao, F. Song, and Z. Wang, *Non-hermitian chern bands*, Phys. Rev. Lett. **121**, 136802 (2018).
- [12] Y. Ashida, S. Furukawa, and M. Ueda, *Parity-time-symmetric quantum critical phenomena*, Nat. Commun. **8**, 15791 (2017).
- [13] T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University Press, Oxford, 2004).
- [14] A. O. Gogolin, A. A. Nersisyan, and A. M. Tsvelik, *Bosonization and Strongly Correlated Systems* (Cambridge University Press, Cambridge, 1998).
- [15] M. A. Cazalilla, R. Citro, T. Giamarchi, E. Orignac, and M. Rigol, *One dimensional bosons: From condensed matter systems to ultracold gases*, Rev. Mod. Phys. **83**, 1405 (2011).
- [16] E. Lieb and D. Robinson, *The finite group velocity of quantum spin systems*, Commun. Math. Phys. **28**, 251 (1972).
- [17] P. Calabrese and J. Cardy, *Evolution of entanglement entropy in one-dimensional systems*, Journal of Statistical Mechanics: Theory and Experiment **2005**(04), P04010 (2005).
- [18] M. A. Cazalilla, *Effect of suddenly turning on interactions in the luttinger model*, Phys. Rev. Lett. **97**, 156403 (2006).
- [19] J. Eisert, M. van den Worm, S. R. Manmana, and M. Kastner, *Breakdown of quasilocality in long-range quantum lattice models*, Phys. Rev. Lett. **111**, 260401 (2013).
- [20] J. Garcia and R. Rossignoli, *Spectrum and normal modes of non-hermitian quadratic boson operators*, Phys. Rev. A **96**, 062130 (2017).
- [21] I. Affleck, W. Hofstadter, D. R. Nelson, and U. Schollwck, *Non-hermitian luttinger liquids and flux line pinning in planar superconductors*, J. Stat. Mech. **2004**, P10003 (2004).
- [22] Due to non-hermiticity, $\langle \phi(t) | = \langle \phi_0 | e^{iH^+t}$.
- [23] C. Karrasch, J. Rentrop, D. Schuricht, and V. Meden, *Luttinger-liquid universality in the time evolution after an interaction quench*, Phys. Rev. Lett. **109**, 126406 (2012).
- [24] B. Dóra, M. Haque, and G. Zaránd, *Crossover from adiabatic to sudden interaction quench in a luttinger liquid*, Phys. Rev. Lett. **106**, 156406 (2011).
- [25] A. Iucci and M. A. Cazalilla, *Quantum quench dynamics of the luttinger model*, Phys. Rev. A **80**, 063619 (2009).
- [26] J. Dziarmaga, *Dynamics of a quantum phase transition and relaxation to a steady state*, Adv. Phys. **59**, 1063 (2010).

- [27] A. Polkovnikov, K. Sengupta, A. Silva, and M. Vengalattore, *Colloquium : Nonequilibrium dynamics of closed interacting quantum systems*, Rev. Mod. Phys. **83**, 863 (2011).
- [28] H. Carmichael, *An Open Systems Approach to Quantum Optics* (Springer-Verlag, Berlin, 1993).
- [29] The generators of the SU(1,1) Lie algebra satisfy $[K_+(q), K_-(q)] = -2K_0(q)$, $[K_0(q), K_\pm(q)] = \pm K_\pm(q)$, and the operators for distinct q 's commute with each other.
- [30] A. I. Solomon, *Group theory of superfluidity*, J. Math. Phys. p. 390 (1971).
- [31] R. Gilmore, *Baker-campbell-hausdorff formulas*, J. Math. Phys. **15**, 2090 (1974).
- [32] The steady state, $t \rightarrow \infty$ limit of the overlap is evaluated analytically in the thermodynamic limit as $N(t \rightarrow \infty) = \left(\frac{1}{2} + \frac{1}{2} \frac{v_-}{v}\right)^{-L/\pi\alpha}$, being valid for $v > |g_2|$.
- [33] E.-M. Graefe, H. J. Korsch, A. Rush, and R. Schubert, *Classical and quantum dynamics in the (non-hermitian) swanson oscillator*, Journal of Physics A: Mathematical and Theoretical **48**, 055301 (2015).
- [34] R. A. Fisher, M. M. Nieto, and V. D. Sandberg, *Impossibility of naively generalizing squeezed coherent states*, Phys. Rev. D **29**, 1107 (1984).
- [35] Y. Ashida, S. Furukawa, and M. Ueda, *Quantum critical behavior influenced by measurement backaction in ultracold gases*, Phys. Rev. A **94**, 053615 (2016).
- [36] J. von Delft and H. Schoeller, *Bosonization for beginners — reformionization for experts*, Ann. Phys. (Leipzig) **7**, 225 (1998).
- [37] Note the unconventional "+" sign as opposed to the conventional "-" for the hermitian case[24].
- [38] An identical expression applies to the left-movers as well.
- [39] Note that other cutoff schemes yield distinct $d(x, t)$ functions, but its $x \gg \tilde{2}vt$ and $x \ll 2\tilde{v}t$ behaviour remains unchanged.
- [40] See Supplemental Material providing further details, which includes [48].
- [41] Y. Ashida and M. Ueda, *Full-counting many-particle dynamics: Nonlocal and chiral propagation of correlations*, Phys. Rev. Lett. **120**, 185301 (2018).
- [42] S. R. White, *Density matrix formulation for quantum renormalization groups*, Phys. Rev. Lett. **69**, 2863 (1992).
- [43] G. Vidal, *Efficient classical simulation of slightly entangled quantum computations*, Phys. Rev. Lett. **91**, 147902 (2003).
- [44] A. J. Daley, *Quantum trajectories and open many-body quantum systems*, Advances in Physics **63**, 77 (2014).
- [45] B. Jaramillo Avila, C. Ventura-Velazquez, R. de J. Leon-Montiel, Y. N. Joglekar, and B. M. Rodriguez-Lara, *Pt-symmetry from lindblad dynamics in an optomechanical system*, arXiv:1908.03240.
- [46] B. Rauer, P. Grišins, I. E. Mazets, T. Schweigler, W. Rohringer, R. Geiger, T. Langen, and J. Schmiedmayer, *Cooling of a one-dimensional bose gas*, Phys. Rev. Lett. **116**, 030402 (2016).
- [47] V. Kozii and L. Fu, *Non-hermitian topological theory of finite-lifetime quasiparticles: Prediction of bulk fermi arc due to exceptional point*, arXiv:1708.05841.
- [48] E. M. Graefe, H. J. Korsch, and A. E. Niederle, *Mean-field dynamics of a non-hermitian bose-hubbard dimer*, Phys. Rev. Lett. **101**, 150408 (2008).