Heat and Work Along Individual Trajectories of a Quantum Bit

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Heat and work along individual trajectories of a quantum bit

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We use a near quantum limited detector to experimentally track individual quantum state trajectories of a driven qubit formed by the hybridization of a waveguide cavity and a transmon circuit. For each measured quantum coherent trajectory, we separately identify energy changes of the qubit as heat and work, and verify the first law of thermodynamics for an open quantum system. We further establish the consistency of these results by comparison with the master equation approach and the two-projective-measurement scheme, both for open and closed dynamics, with the help of a quantum feedback loop that compensates for the exchanged heat and effectively isolates the qubit.

Continuous measurement of a quantum bit can be used to track individual trajectories of its state. Due to the intrinsic quantum fluctuations of a detector, measurement is an inherently stochastic process [1]. If a quantum system starts in a given state, then by accurately monitoring the fluctuations of the detector, it is possible to reconstruct single quantum trajectories, which describe the evolution of the quantum state conditioned to the measurement outcome [1]. The idea of quantum trajectories made its transition from a theoretical tool (unraveling) to simulate open quantum systems [2] to a physically accessible quantity with the experimental ability of tracking these trajectories in optical [3, 4] and more recently in solid state [5, 6] systems. Continuous monitoring of superconducting qubits has, for example, enabled continuous feedback control [7–9], the determination of weak values [10–12], and the production of deterministic entanglement [13, 14]. In view of their ability to combine quantum trajectory monitoring with external unitary driving, these superconducting devices additionally offer a unique platform to explore energy exchanges and thermodynamics along single quantum trajectories.

The laws of thermodynamics classify energy changes for macroscopic systems as work performed by external driving and heat exchanged with the environment [15]. In past decades, these principles have been successfully extended to the level of classical trajectories to account for thermal fluctuations [16]. By providing a theoretical and experimental framework for determining work and heat along individual trajectories, stochastic thermodynamics has paved the way for the study of the energetics of microscopic systems, from colloidal particles to enzymes and molecular motors [17, 18]. The further generalization of thermodynamics to include quantum fluctuations faces unique challenges, ranging from the proper identification of heat and work to the clarification of the role of coherence [19–22]. Quantum heat is commonly associated with the nonunitary part of the dynamics [23–25], carrying over the classical notion of energy exchanged with the surroundings. This definition has recently been extended to the level of single discrete quantum jumps [26–31] and to individual continuous quantum trajectories [32, 33]. Other definitions of quantum work and heat have been put forward, for instance based on the single shot approach [34, 35], resource theory [36, 37] or path integrals [38]. This diversity of theoretical approaches emphasizes the crucial importance of an experimental study.

We here report the measurement of work and heat associated with unitary and non-unitary dynamics along single quantum trajectories of a superconducting qubit. The qubit evolves under continuous unitary evolution and is only weakly coupled to the detector. As a result, information about its state may be inferred from the measured signal without projecting it into eigenstates. This system might thus generically be in coherent superpositions of energy eigenstates. We show that the measured heat and work are consistent with the first law and prove the agreement with both the two-projective-measurement (TPM) scheme [39] and the master equation approach [23–25]. We finally establish the correspondence with the TPM work in the unitary limit by employing a phase-locking quantum feedback loop that effectively compensates for the heat.

Heat and work along quantum trajectories. In macroscopic thermodynamics, work performed on a thermally isolated system is defined as the variation of internal energy, $W = \Delta U$ [15]. According to the first law, heat is given by the difference, $Q = \Delta U - W$, for systems that are not isolated [15]. Thermal isolation is thus essential to distinguish heat from work. At the quantum level, identifying heat and work is more involved, because quantum systems do not necessarily occupy definite energy states. Energy changes are usually defined in terms of transition probabilities between energy eigenstates obtained via projective measurements at the beginning and end of a process in the TPM scheme [39]. For a driven quantum system described by the Hamiltonian H_t, the distribution of the total energy variation ΔU is thus [39],

$$P(\Delta U) = \sum_{m,n} P^{\tau}_{m,n} P^0_n \delta[\Delta U - (E^{\tau}_m - E^0_n)], \quad (1)$$

where P^0_n denote the initial occupation probabilities, $P^{\tau}_{m,n}$ the transition probabilities between initial and final eigenvalues E^0_n and E^{τ}_m of H_t, and τ the duration of the driving protocol. This relation has been used to experimentally determine the work distribution in closed quantum systems such as NMR, trapped ion, and cold atom systems [40–42], for which $\Delta U = W$.

However, in open quantum systems, the total energy change cannot in general be uniquely separated into heat and work [43]. We consider Markovian open quantum systems described by a master equation for the density operator ρ_t of the form [44],

$$\frac{d\rho_t}{dt} = -\frac{i}{\hbar}[H_t, \rho_t] + \mathcal{L}\rho_t, \quad (2)$$

where $\mathcal{L}$ is a Lindblad dissipator. In this case, the first law has been written in the usual form, $\Delta\bar{U} = \bar{Q} + \bar{W}$, (the bar denotes the ensemble average) with [23–25],

$$\bar{Q} = \int_0^{\tau} dt\, \mathrm{tr}\left[\frac{d\rho_t}{dt} H_t\right], \quad \bar{W} = \int_0^{\tau} dt\, \mathrm{tr}\left[\rho_t \frac{dH_t}{dt}\right]. \quad (3)$$

As in classical thermodynamics, $\bar{Q}$ is the energy supplied to the system by the environment and $\bar{W}$ the work done by external driving. The above definition of quantum work has been originally introduced by Pusz and Woronowicz in a C^*-algebraic context [45] and recently applied to individual discrete quantum jumps [26–29].

In our experiment, we examine how quantum heat and work can be consistently identified for systems whose environment consists of a continuously coupled quantum limited detector, an effectively zero temperature reservoir [1]. The ability to track quantum state trajectories enables energy changes to be decomposed separately into heat and work components [32, 33]. The starting point of our analysis is that the quantum state evolution consists of both a unitary part and, because of the continuous monitoring, an additional nonunitary component: the former is again identified as work, the latter as heat, in analogy to macroscopic thermodynamics [32, 33, 46]. Specifically, for an infinitesimal time interval dt, a change of the conditional density operator for a single trajectory may be written as $d\tilde{\rho}_t = \delta\mathbb{W}[\tilde{\rho}_t]dt + \delta\mathbb{Q}[\tilde{\rho}_t]dt$, where $\delta\mathbb{W}[\tilde{\rho}_t]$ and $\delta\mathbb{Q}[\tilde{\rho}_t]$ are superoperators associated with the respective unitary and nonunitary dynamics [32]. The tilde here marks quantities that are evaluated in different realizations of the experiment, as opposed to quantities averaged over the possible trajectories. The first law along a single quantum trajectory $\tilde{\rho}_t$ then reads $d\tilde{U} = \delta\tilde{W} + \delta\tilde{Q}$, with $\delta\tilde{W} = \mathrm{tr}[\tilde{\rho}_{t-dt} dH_t]$ and $\delta\tilde{Q} = \mathrm{tr}[H_t d\tilde{\rho}_t]$ [47]. When integrated over time, the first law reads,

$$\Delta\tilde{U} = \int_0^{\tau} \frac{d\tilde{U}}{dt} dt = \int_0^{\tau} \frac{\delta\tilde{W}}{dt} dt + \int_0^{\tau} \frac{\delta\tilde{Q}}{dt} dt, \quad (4)$$

for each quantum trajectory. Equation (4) is a quantum extension of the first law of stochastic thermodynamics. It relates the average change of energy $\Delta\tilde{U}$ with the path-dependent heat $\tilde{Q}$ and work $\tilde{W}$. Similarly, we may distinguish quantum heat and work contributions to changes of the transition probabilities [32],

$$d\tilde{P}_{m,n} = \delta\tilde{P}^W_{m,n} + \delta\tilde{P}^Q_{m,n}, \quad (5)$$

along single quantum trajectories [47].

The consistency of the decompositions (4) and (5) may be established in three independent ways: *(i)* the total energy change along a trajectory, $\Delta\tilde{U} = \sum d\tilde{U}$, and the total transition probability, $\tilde{P}_{nm} = \sum d\tilde{P}_{nm}$, may be compared to the TPM approach [39], *(ii)* the stochastic heat and work contributions (4) may be compared to the mean quantities (3) after averaging over stochastic and quantum fluctuations, and finally, *(iii)* the work (4) along a trajectory may be directly compared to the TPM result (1) in the unitary limit when heat vanishes. In that case, $\Delta\tilde{U} = \Delta U = E^{\tau}_m - E^0_n = W$ [47].

Experimental set-up. The qubit is realized by the near-resonant interaction of a transmon circuit [48] and a three dimensional aluminum cavity [49] capacitively coupled to a 50 Ω transmission line. Resonant coupling between the circuit and cavity results in an effective qubit which is described by the Hamiltonian, $H_{\mathrm{q}} = -\hbar\omega_{\mathrm{q}}\sigma_z/2$, and depicted in Fig. 1a. The radiative interaction between the qubit and transmission line is given by the interaction Hamiltonian, $H_{\mathrm{int}} = \hbar\gamma(a\sigma_+ + a^{\dagger}\sigma_-)$, where γ is the coupling rate between the electromagnetic field mode corresponding to a ($a^{\dagger}$), the annihilation (creation) operator, and the qubit state transitions denoted by σ_+ (σ_-), the raising (lowering) ladder operator for the qubit. By virtue of this interaction Hamiltonian, a homodyne measurement along an arbitrary quadrature of the quantized electromagnetic field of the transmission line, $ae^{-i\varphi} + a^{\dagger}e^{+i\varphi}$, results in weak measurement along the corresponding dipole of the qubit, $\sigma_+e^{-i\varphi} + \sigma_-e^{+i\varphi}$ [50]. In order to perform work on the qubit, we introduce a classical time-dependent field described by $H_{\mathrm{R}} = \hbar\Omega_{\mathrm{R}}\sigma_y\cos(\omega_{\mathrm{q}}t + \varphi)$, where ω_{q} is the resonance frequency of the qubit and Ω_{R} is the Rabi drive frequency.

Homodyne monitoring is performed with a Josephson parametric amplifier [51, 52] operated in phase-sensitive mode. We adjust the homodyne detection quadrature such that the homodyne signal dV_t obtained over the time interval $(t, t + dt)$ provides an indirect signature [53] of the real part of $\sigma_- = (\sigma_x + i\sigma_y)/2$. The detector signal is given by $dV_t = \sqrt{\eta}\gamma\langle\sigma_x\rangle dt + \sqrt{\gamma}dX_t$, where η is the quantum efficiency of the homodyne detection, γ is the

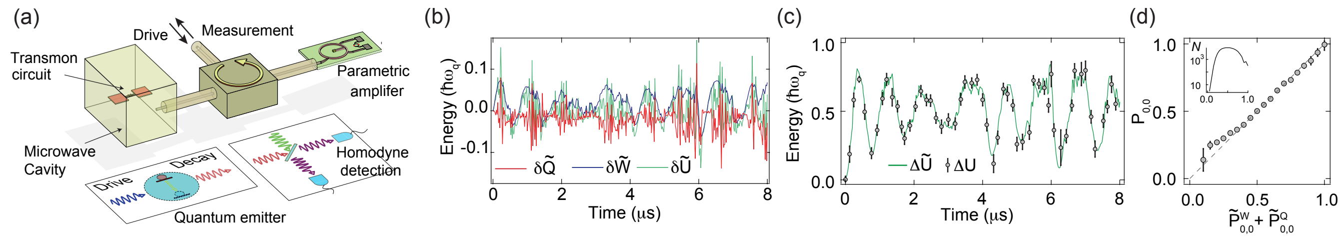

FIG. 1. Evaluating heat and work along single quantum trajectories. (a), Schematic of the qubit system, drive, and homodyne detection. (b), Work (blue), heat (red) and energy (green) along a single trajectory. The discrete timestep resolution is $\delta t = 20$ ns, the smallest compatible with the detection bandwidth (c), The total energy along a single quantum trajectory (green) compared to the total energy as determined from an ensemble of projective measurements at each time point (circles). The error bars indicate the standard error of the mean. (d), Projective measurements binned and averaged according to the sum of the work and heat contributions $\tilde{P}^W_{0,0} + \tilde{P}^Q_{0,0}$. The error bars indicate the standard error of the mean based on the number of occurrences (N) for each value of $\tilde{P}^W_{0,0} + \tilde{P}^Q_{0,0}$ (inset).

radiative decay rate, and dX_t is a zero-mean Gaussian random variable with variance dt.

The qubit evolution, given both driven evolution $H_{\rm R}$ and homodyne measurement results dV_t, is described in the rotating frame by the stochastic master equation [54],

$$d\tilde{\rho}_t = -\frac{i}{\hbar}[H_R, \tilde{\rho}_t]\, dt + \gamma\mathcal{D}[\sigma_-]\tilde{\rho}_t\, dt + \sqrt{\eta\gamma}\mathcal{H}[\sigma_- dX_t]\tilde{\rho}_t, \quad (6)$$

where $\mathcal{D}[\sigma_-]\tilde{\rho} = \sigma_-\tilde{\rho}\sigma_+ - \frac{1}{2}(\sigma_+\sigma_-\tilde{\rho} + \tilde{\rho}\sigma_+\sigma_-)$ and $\mathcal{H}[O]\tilde{\rho} = O\tilde{\rho} + \tilde{\rho}O^\dagger - \mathrm{tr}[(O + O^\dagger)\tilde{\rho}]\tilde{\rho}$ are the dissipation and jump superoperators, respectively. By taking the ensemble average, Eq. (6) reduces to a master equation of the form (2) with dissipator $\mathcal{L}\rho_t = \gamma\mathcal{D}[\sigma_-]\rho_t$, which describes the coupling to a zero-temperature reservoir [1].

We next introduce the experimental protocols to determine the instantaneous heat and work contributions. We identify the work contribution $\delta\mathbb{W}[\tilde{\rho}_t]$ with the first (unitary) term in Eq. (6), while the heat contribution $\delta\mathbb{Q}[\tilde{\rho}_t]$ is associated with the latter two (nonunitary) terms. Experimentally, we use Eq. (6) to track $\tilde{\rho}$ from a known initial state; at each time step, $d\tilde{\rho}_t$ is decomposed into $\delta\mathbb{W}[\tilde{\rho}_t]$ and $\delta\mathbb{Q}[\tilde{\rho}_t]$[47]. Although the system could, in general, exchange energy with the detector in the form of heat or work, the homodyne measurement in our experiment only induces a zero-mean stochastic back-action, which guarantees no extra work is done by the detection process.

Having access to the stochastic heat and work contributions from an individual quantum trajectory, we now verify the first law in the form of Eqs. (4) and (5). For this, we initialize the qubit in the eigenstate n, and then drive the qubit while collecting the homodyne measurement signal. Figure 1b shows the path-dependent heat and work contributions, $\delta\tilde{Q}$ and $\delta\tilde{W}$, and the corresponding changes in internal energy $d\tilde{U}$ for a single trajectory originating in $n = 0$. After time τ, we utilize the Jaynes-Cummings nonlinearity readout technique [55] to projectively measure the qubit in state m and then repeat the experiment several times. Using individual heat and work trajectories we now address the consistency of these decompositions in three independent ways.

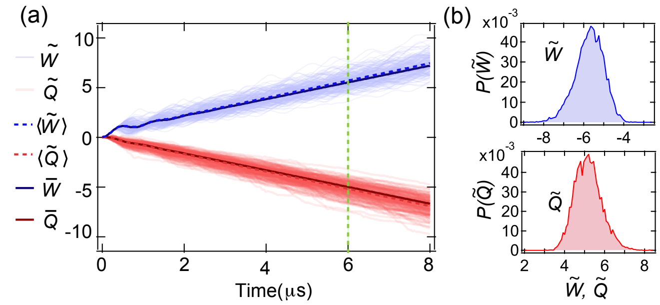

FIG. 2. Comparison of stochastic and average heat and work quantities. (a), Individual heat and work trajectories $\tilde{Q}$, $\tilde{W}$ are displayed as transparent red and blue traces. The mean of these individual trajectories $\langle\tilde{Q}\rangle$, and $\langle\tilde{W}\rangle$ are displayed as dashed lines which are in good agreement with the mean values from the master equation, $\bar{Q}$ and $\bar{W}$, Eq. (3), solid lines. (b), Distributions of $\tilde{Q}$ and $\tilde{W}$ at evolution time $\tau = 6$ μs.

(i) Total energy change—In order to establish the consistency with the TPM scheme [39], we first show in Fig. 1c the path-dependent total energy variation $\Delta\tilde{U} = \sum \delta\tilde{U}$ for a single trajectory and the path-independent total energy change $\Delta U = (\hbar\omega_q)P^\tau_{1,0}$ obtained via projective measurements performed at various intermediate times [47]. We find that the path-independent energy changes are in excellent agreement with the energy changes along a single trajectory. In Fig. 1d we further compare the path-independent transition probability $P_{0,0}$ to the sum of the path-dependent work and heat contributions, $\tilde{P}^W_{0,0} + \tilde{P}^Q_{0,0}$, for experiments of variable duration $\tau = [0, 8]$ μs. We again observe very good agreement.

(ii) Correspondence with master equation definitions—Figure 2 displays the time evolution of the heat $\tilde{Q}$ and work $\tilde{W}$ along single trajectories, as well as their respective mean values. The ensemble average of the individual work $\langle\tilde{W}\rangle$ and heat $\langle\tilde{Q}\rangle$ trajectories agrees well with the the averaged values, $\bar{Q}$ and $\bar{W}$, Eq. (3), thus recovering the expression by Pusz and Woronowicz [45] at the level of unraveled quantum trajectories. In addition, the individual trajectories allows examination of the heat and work distributions (Fig. 2b) at each timestep.

(iii) The unitary limit—We finally show correspondence of the quantum trajectory work $\tilde{W}$ and the TPM work, $W = E_m^\tau - E_n^0$, for a single realization by experimentally isolating the system with a quantum feedback loop [1]. The essence of feedback is to compensate for the effect of the detector $\delta\mathbb{Q}[\tilde{\rho}_t]$ by adjusting the Hamiltonian at each timestep, thus making the system effectively closed. The dynamics of the system is then simply described by unitary evolution where only the work $\delta\mathbb{W}[\tilde{\rho}_t]$ contributes to changes in the state. In order to implement feedback, we adapt the phase-locked loop protocol introduced in Ref. [7]. This is achieved by multiplying the homodyne measurement signal with a reference oscillator of the form $A[\sin(\Omega_{\rm R}t+\phi)+B]$ yielding a feedback control, $\Omega_{\rm F} = \sqrt{\eta}(\cos(\Omega t+\phi)-1)dV_t/dt$, that modulates the Rabi frequency of the qubit drive. The detector heat exchange is eliminated by applying additional work, $\delta\mathbb{W}_F[\tilde{\rho}_t] = (i/\hbar)[\hbar\Omega_{\rm F}\sigma_y\cos(\omega_q t+\phi), \rho_t]$.

Figure 3a shows the instantaneous feedback work, $\delta\tilde{W}_{\rm F} = \hbar\omega_{\rm q}{\rm tr}\,[\Pi_{m=1}\delta\mathbb{W}_{\rm F}[\tilde{\rho}_t]]\,dt$ (with Π_m the projector onto eigenstate m), together with the corresponding instantaneous heat, $\delta\tilde{Q} = \hbar\omega_{\rm q}{\rm tr}\,[\Pi_{m=1}\delta\mathbb{Q}[\tilde{\rho}_t]]\,dt$, along a trajectory for a quantum efficiency of 35%. We observe that the feedback partially cancels the heat at each point in time. The anti-correlation between the instantaneous feedback and heat contributions depicted in Figure 3b confirms that the feedback loop compensates for exchanged heat at each timestep. In addition, by averaging the heat and work contributions to the transition probability over many iterations of the experiment (Fig. 3c), we clearly see how feedback works toward canceling the heat on average. Similarly, at the level of single trajectories, the total transition probability may be written as $\tilde{P}^\tau_{m,n} = \tilde{P}^W_{m,n} + \tilde{P}^Q_{m,n} + \tilde{P}^F_{m,n}$, with the work contribution from feedback $\tilde{P}^F_{m,n}$. Figure 3d shows the transition probabilities $\tilde{P}^W_{0,0}$ versus $\tilde{P}^Q_{0,0} + \tilde{P}^F_{0,0}$. By comparing the transition probabilities with and without feedback, we observe a significantly reduced heat contribution.

In the presence of the quantum feedback loop we can decompose the instantaneous work along trajectories into work imparted by the feedback and work associated with the driving protocol, $\delta\tilde{W}$. In the absence of the feedback loop, the quantum dynamics of the qubit are given by work $\delta\mathbb{W}[\tilde{\rho}_t]$ and heat $\delta\mathbb{Q}[\tilde{\rho}_t]$ superoperators; the heat changes the state, causing the observed $\delta\tilde{W}$ to differ from

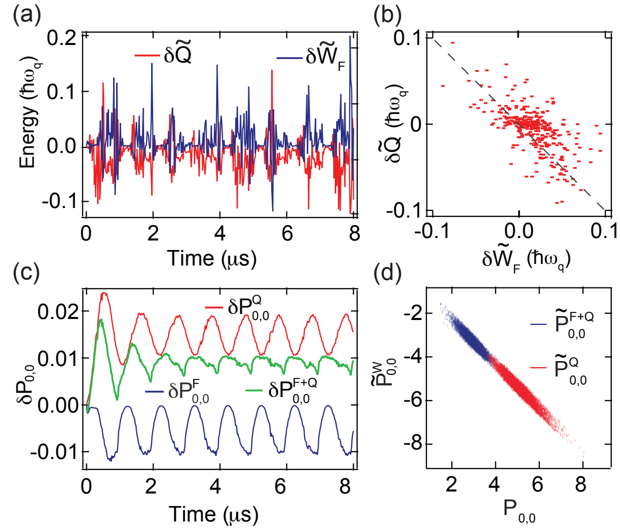

FIG. 3. Quantum feedback loop. (a), Instantaneous heat and feedback work along a single trajectory. The feedback work has been time shifted by 20 ns to account for the time delay in the feedback circuit. The anti-correlation ($r = -0.68$) of heat and feedback work is evident in the scatter plot (b). (c), Average of the instantaneous contribution of heat and feedback to the transition probability for 10^4 experimental iterations. (d), Parametric plot of $\tilde{P}^W_{0,0}$ versus $\tilde{P}^Q_{0,0}$ (red) and $\tilde{P}^{F+Q}_{0,0}$ (blue) showing how the feedback cancels the heat, narrowing and shifting the distribution toward zero for $\tau = 6\ \mu$s.

the case of closed unitary evolution, $\delta\tilde{W}_{\rm u}$. With the feedback loop, the heat contribution is compensated at each timestep causing the instantaneous work $\delta\tilde{W}$ to match the expected unitary work $\delta\tilde{W}_{\rm u}$. Figure 4 displays $\delta\tilde{W}$ for a single quantum trajectory in the presence of feedback (blue) and for a different trajectory in the absence of feedback (red) compared to the expected unitary work $\delta\tilde{W}_{\rm u}$ (green). Figure 4b,c show that in the presence of feedback the work is more closely correlated with the unitary work, with the correlation only limited by the efficiency of the feedback loop [47]. In the limit of unit quantum efficiency and null loop delay, a feedback loop could exactly compensate for the exchanged heat [47].

Conclusions. The field of quantum thermodynamics strives to understand heat and work at the level of single energy quanta where coherence and measurement backaction play a leading role in the energy dynamics. Our study has explored how individual quantum coherent trajectories can be used to identify heat and work exchanged with a detector. In contrast to classical detectors, heat exchanged with (zero-temperature) quantum detectors is not negligible; this heat thus needs to be included in the energy balance in addition to heat flows to (finite-temperature) heat baths. Our findings are therefore crucial for future experimental and theoretical studies in quantum thermodynamics [56] at the single trajectory

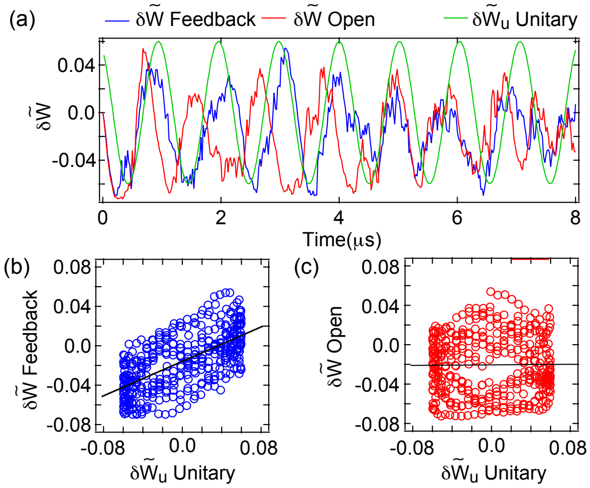

FIG. 4. Work along trajectories with and without feedback. (a), the instantaneous work $\delta\tilde{W}$ along a single trajectory in the presence of feedback (blue) and the open loop configuration (no feedback) (red) is compared to the calculated instantaneous work expected for pure unitary evolution (green). (b, c), The correlation between the instantaneous work $\delta\tilde{W}$ and the work for a unitary evolution along a quantum trajectory. The linear regression fit (black lines) show correlation (slope 0.44, estimated error 0.03) when the feedback loop is employed, and no correlation in the open loop configuration (slope 0.007, estimated error 0.04).

level.

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