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Phys. Rev. Lett. **124**, 073602 — Published 19 February 2020

DOI: [10.1103/PhysRevLett.124.073602](https://doi.org/10.1103/PhysRevLett.124.073602)

Squeezed light induced symmetry breaking superradiant phase transition

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(Dated: January 27, 2020)

We theoretically investigate the quantum phase transition in the collective systems of qubits in a high quality cavity, where the cavity field is squeezed via the optical parametric amplification process. We show that the squeezed light induced symmetry breaking can result in quantum phase transition without the ultrastrong coupling requirement. Using the standard mean field theory, we derive the condition of the quantum phase transition. Surprisingly, we show that there exist a tricritical point where the first- and second-order phase transitions meet. With specific atom-cavity coupling strengths, both the first- and second-order phase transition can be controlled by the nonlinear gain coefficient, which is sensitive to the pump field. These features also lead to an optical switching from the normal phase to the superradiant phase by just increasing the pump field intensity. The signature of these phase transitions can be observed by detecting the phase space Wigner function distribution with different profiles controlled by the squeezed light intensity. Such superradiant phase transition can be implemented in various quantum systems, including atoms, quantum dots and ions in optical cavities as well as the circuit quantum electrodynamics system.

Quantum phase transitions (QPTs) are phase transitions (PT) between two robust stable states caused by a non-thermal parameter that occur at zero temperature ($T = 0$ K) [1]. In general, there exist two categories of QPTs: the first-order phase transition and the second-order phase transition. The second-order phase transition is so called the continuous phase transition. In the second-order phase transition, the quantum fluctuations must diverge at a critical point so that the two kinds of phases with different symmetries can be matched. However, in the first-order phase transitions, these two stable phases can coexist in a critical hysteresis regime, where the quantum fluctuations exhibit a jump across the phase transition boundary. In seventies, a second-order phase transition, from normal to superradiant phase, was proposed in the Dicke model with the ultrastrong coupling, where the QPTs are driven by the quantum fluctuations and take place for a certain critical qubit-cavity coupling strength g_c [2–5]. If the coupling strength $g < g_c$, the cavity field is in the vacuum and the atoms are in their ground states. Such state is recognized as the normal phase (NP) and the system exhibits an U(1) symmetry. However, in the case of $g > g_c$, the cavity mode is intensely populated with two degenerate ground states and the atoms are excited simultaneously. Therefore, the U(1) symmetry has to be broken and a macroscopic average photon number can be detected in this state, which is widely known as the symmetry broken superradiant phase (SP). Though these studies did not require the external drive, many subsequent papers introduced external drive to study the phase transitions [6, 7]. Further it

was realized that the dissipation can significantly change the character of the phase transition [8–13]. Many theoretical generalizations were considered [14–19]. These included a number of studies on superradiant lattices. An experimental study on Dicke phase transition was reported using the BEC systems [20] which led to new theoretical work [21–23]. Recently phase transition was reported using few superconducting qubits driven by a coherent field [24].

Note that the original work on Dicke phase transition included the counter rotating terms which become significant in the limit of ultrastrong coupling [25]. Such a coupling is now becoming possible using circuit QED and hence considerable work on phase transitions got revived due to advances in circuit QED [26–31]. The ultrastrong coupling is especially needed for realizing first-order phase transitions. Besides, dissipation is something which can not be avoided in experiments and thus should be accounted for in studying the phase transitions.

In this letter, we present a new approach to the study of the phase transitions. We do not require ultrastrong coupling, we use the Tavis-Cummings model, include dissipation. We introduce a new element in the cavity - we add parametric medium in the cavity and thus break the inherent U(1) symmetry of the rotating wave TC model. The parametric term in the Hamiltonian $G(a^2 + a^{\dagger 2})$ break the symmetry of system, leading to the occurrence of the normal-superradiant PT without the requirement of the ultrastrong coupling. Using the standard mean field approximation and the quantum fluctuation analysis, we find that the first- and second-order PT coexist in our system and they meet at a critical point. We show that the signature of these superradiant PTs can be observed by detecting the phase-space Wigner function distribution, whose profile changes with the nonlinear gain

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coefficient G . Moreover, we show that the PT in our system can be optically controlled, as opposed to many traditional proposals for realizing PT by changing the parametric coupling strength. Using these advantages in our system, an optical switching from normal phase to superradiant PT can be accomplished by adjusting the pump field intensity to increase the nonlinear gain coefficient. We show that the OPA process will enhance the atom-cavity coupling, leading to the splitting of the normal phase and the implementation of the first-order phase transition, which can't be observed without atoms. Moreover, with the increase of the driving strength, the system can enter the strong coupling regime and even the ultrastrong coupling regime which is challenging in other models.

FIG. 1. The setup of the cavity-QED system with N identical two-level qubits. The cavity mode (red) with angular frequency ω_{cav} is generated by a strong pump field (green) with angular frequency $2\omega_{\text{sq}}$ via the $\chi^{(2)}$ nonlinearity. The qubits with resonant frequency ω_A interact with the cavity mode with qubit-cavity coupling strength λ . The ground (excited) state of the qubit is labeled as $|g\rangle$ ($|e\rangle$). Here, γ and κ are the qubit and cavity decay rates, respectively.

As depicted in Fig. 1, the system under consideration is a typical Tavis-Cummings model. N identical two-level qubits with resonant frequency ω_A are trapped inside a single-mode cavity (the mode frequency is ω_{cav}). In our system, the cavity mode is generated through a crystal with second-order nonlinearity [32–35], which is widely known as the OPA process. As shown in Fig. 1, a strong pump field (green) with angular frequency $2\omega_{\text{sq}}$ drives the nonlinear crystal coherently. This coupling is proportional to the second order susceptibility of the medium and the amplitude of the pump field. Since we work in the region below parametric oscillation threshold, we assume that the pump field is not depleted. Then, the generated intracavity field interacts with the qubits with the same coupling strength λ [36]. Assuming $a^\dagger$ and a are the creation and annihilation operators of the cavity mode, respectively, the Hamiltonian of the system is given by

$$H = \hbar\Delta_C a^\dagger a + \hbar\Delta_A S_z + \frac{\hbar\lambda}{\sqrt{N}}(S_+ a + S_- a^\dagger) + \hbar G(a^2 + a^{\dagger 2}), \quad (1)$$

where $\Delta_C = \omega_{\text{cav}} - \omega_{\text{sq}}$ and $\Delta_A = \omega_A - \omega_{\text{sq}}$ are the detunings of the cavity and qubits, respectively. The operators $S_\pm = S_x \pm iS_y$ and $S_\alpha = \sum_{j=1}^N \sigma_\alpha^j/2$ ($\alpha = x, y, z$) are the collective spin operators constituted from the individual Pauli spin operator σ_α^j which are used to describe the j -th two-level qubit systems. It is noted that the last term on the right-hand side of Eq. (1) represents the nonlinear term by the OPA process with a gain coefficient G , which result in a squeezed cavity mode [32, 33]. In the absence of the pump field (i.e., $G = 0$), such a system has a continuous $U(1)$ -symmetry for the transformation $a \rightarrow ae^{i\phi}$ and $S_+ \rightarrow S_+ e^{-i\phi}$ so that the superradiant phase transition will not take place [8]. However, in the presence of the pump field (i.e., $G \neq 0$), the $U(1)$ -symmetry is broken and the superradiant phase transition can be observed, leading to an optical switching from normal phase to superradiant phase by increasing the pump field intensity.

We note that in the absence of the atoms ($\lambda = 0$), the model (1) leads to a second-order phase transition if $G > \sqrt{\kappa^2 + \Delta_C^2}/2$ (see Refs [32], p. 140). In order to study the phase transitions which arise due to atom-field coupling we avoid the above regime of the parameters, i.e., we work in the regime $G < \sqrt{\kappa^2 + \Delta_C^2}/2$.

In general, the dynamical evolution of the TC model is described by the master equation for density matrix ρ of the system, which reads

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \mathcal{L}_{\text{cav}}\rho + \mathcal{L}_A\rho, \quad (2)$$

where $\mathcal{L}_{\text{cav}}\rho = \kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)$ and $\mathcal{L}_A\rho = \gamma(2S_- \rho S_+ - S_+ S_- \rho - \rho S_+ S_-)/N$ are the dissipations of the cavity and qubits, respectively. In our system, we assume that the decay rate of the qubit γ is much smaller than the cavity decay rate κ . Although the decay of qubits may break the phase transition by softening the quantum fluctuations, we set $\gamma = 0$ in the following analytical derivations for mathematical simplicity and the conservation of spin is considered, which plays an important role. It is also noted that the dissipation of cavity can also cause the symmetry breaking and alter the phase diagram of the system, which is considered in the following derivations [8].

To study the phase transition induced by the OPA generated squeezed field, we begin with the Heisenberg equations of motion for operators, which reads

$$\frac{da}{dt} = -i\Delta_C a - 2iGa^\dagger - i\frac{\lambda}{\sqrt{N}}S_x - \frac{\lambda}{\sqrt{N}}S_y - \kappa a, \quad (3a)$$

$$\frac{dS_x}{dt} = -\Delta_A S_y + i\frac{\lambda}{\sqrt{N}}S_z(a - a^\dagger), \quad (3b)$$

$$\frac{dS_y}{dt} = \Delta_A S_x - \frac{\lambda}{\sqrt{N}}S_z(a + a^\dagger), \quad (3c)$$

$$\frac{dS_z}{dt} = \frac{\lambda}{\sqrt{N}}S_y(a + a^\dagger) - i\frac{\lambda}{\sqrt{N}}S_x(a - a^\dagger). \quad (3d)$$

The parametric term in Eq. (1) is known to produce squeezed light in the cavity. In principle one might think

of replacing this with the injecting of broadband squeezed vacuum. However, in this case, the mean field equations for $\langle a \rangle$, $\langle a^\dagger \rangle$, $\langle \vec{S} \rangle$ are same as in the absence of the broadband squeezed vacuum. Thus, phase transitions can not be studied using the traditional mean-field approach, requiring separate investigation.

Generally, Eq. (3) can be linearized by the mean-field approximation by expanding an arbitrary operator $\mathcal{O}$ as the form of $\mathcal{O} = \langle \mathcal{O} \rangle + \delta\mathcal{O}$, which is widely used for describing many-body system. Here, $\langle \mathcal{O} \rangle$ is the mean value of the operator $\mathcal{O}$, and $\delta\mathcal{O}$ represents the quantum fluctuation on top of the mean value. For mathematical simplicity, one can rewrite the Eqs. (3) by using the following definitions $\langle S_x \rangle = NX$, $\langle S_y \rangle = NY$, $\langle S_z \rangle = NZ$ and $\langle a \rangle = \sqrt{N}\alpha$, where $\alpha = \alpha_{\text{Re}} + i\alpha_{\text{Im}}$.

Under the steady-state approximation and the spin-conservation law $X^2 + Y^2 + Z^2 = 1/4$, the Eqs. (3) can be solved analytically (assuming $\Delta_C = \Delta_A \equiv \Delta > 0$), yielding $X = 2\lambda Z\alpha_{\text{Re}}/\Delta$ and $Y = -2\lambda Z\alpha_{\text{Im}}/\Delta$. To obtain non-trivial solutions of α_{Re} and α_{Im} , the constraint on parameter Z is given by

$$Z_{\pm} = \frac{\Delta}{2\lambda^2}(-\Delta \pm \sqrt{4G^2 - \kappa^2}). \quad (4)$$

Note that the Eq. (4) is only valid for $\Delta \neq 0$. If $\Delta = 0$, then one can obtain $Z = 0$ and $Y = (2G \mp \sqrt{4G^2 - \kappa^2})X/\kappa$ with $X^2 = (2 \pm \sqrt{4G^2 - \kappa^2}/G)/16$. Since the qubits is resonant with the cavity mode, these non-trivial solutions are unstable in this case. Thus, we focus on the case of $\Delta \neq 0$ in this work.

Obviously, $2G \geq \kappa$ is required to make $Z_{\pm}$ be real. This condition implies that there exist only trivial solutions (normal phase) for α_{Re} and α_{Im} if $G < \kappa/2$. When $G > \kappa/2$, the superradiant phase may take place, leading to a set of non-trivial solutions. We also note that for $G = 0$ and $\kappa \rightarrow 0$, the condition reduces to the well known condition for the undriven TC model for phase transition $\lambda > |\Delta|$. This phase transition has been experimentally studied by Feng et al [24]. To obtain stable physical solutions for α_{Re} and α_{Im} , we carry out standard stability analysis [37]. It is found that only the solutions associated with Z_+ are stable in our system. Using the spin conservation condition, we get

$$\alpha_{\text{Re}} = \pm \sqrt{p/q} \left[q + \frac{q\kappa^2\Delta^2}{(2\lambda^2 Z + \Delta^2 - 2G\Delta)^2} \right], \quad (5a)$$

$$\alpha_{\text{Im}} = \frac{\kappa\Delta}{2\lambda^2 Z + \Delta^2 - 2G\Delta} \alpha_{\text{Re}}, \quad (5b)$$

where $p = 1/4 - Z^2$ and $q = 4\lambda^2 Z^2/\Delta^2$. It is worth noting that the solutions are the opposite of one another, reflecting the Z_2 symmetry breaking of system.

In Fig. 2, to show the key physics, we plot α_{Re} [panel (a)] and α_{Im} [panel (b)] as functions of the normalized coupling strength λ/Δ and the nonlinear gain coefficient G/κ , respectively. Here, we choose $\Delta = 10\kappa$.

FIG. 2. Panels (a) and (b) describe the real and imaginary parts of the expectation value $\langle a \rangle$, respectively, as functions of the normalized qubit-cavity coupling strength λ/Δ and the nonlinear gain coefficient G/κ , respectively. The white areas represent $\alpha_{\text{Re}} = \alpha_{\text{Im}} = 0$, corresponding to the normal phase regimes, while the colored areas represent the superradiant phase regimes. In panel (b), the horizontal dash-dotted line ($G = \kappa/2$) and the dashed curve ($G = \sqrt{\kappa^2 + (\Delta^2 - \lambda^2)^2/\Delta^2}/2$) indicate the boundaries from normal phase to superradiant phase.

The white areas denote the normal phase regimes, corresponding to $\alpha_{\text{Re}} = \alpha_{\text{Im}} = 0$. The areas with $\alpha_{\text{Re}} \neq 0$ and $\alpha_{\text{Im}} \neq 0$ represent the superradiant phase regimes. Obviously, there exist two boundaries for this phase transition indicated in panel (b). One boundary condition satisfies $G = \kappa/2$ (horizontal dash-dotted line), representing that the phase transition occurs only if $G > \kappa/2$. The other boundary condition satisfies $G = \sqrt{\kappa^2 + (\Delta^2 - \lambda^2)^2/\Delta^2}/2$ (dashed curve) obtained by setting $Z_+ = -1/2$. In the left-side area of this boundary condition, one can obtain $|Z_+| > 1/2$ so that the system is in the normal phase. In the right-side area, stable solutions of α_{Re} and α_{Im} can be obtained, leading to the superradiant phase regime [41].

To show more characteristics of the superradiant phase transition, we must examine the quantum fluctuations of the operators a and $a^\dagger$ on top of the stable mean-field values. Defining the fluctuation operators $\delta a = \sqrt{N}\delta\alpha$ and $\delta S_\beta = N\delta\beta$ ($\beta = X, Y, Z$), the quantum fluctuations can be evaluated by linearizing the quantum Langevin equations given in the supplementary material. This procedure is equivalent to using the Holstein Primakoff (HP) transformation [42]. For the normal phase, the variance of cavity photon number which is discussed in detail in the supporting material is given by

$$\langle \delta a^\dagger \delta a \rangle \approx \frac{2G^2[(2\Delta^2 - \lambda^2) + \Delta^2(\kappa^2 - 4G^2 + \lambda^2)]}{(\kappa^2 - 4G^2 + 4\Delta^2)[(\Delta^2 - \lambda^2) + \Delta^2(\kappa^2 - 4G^2)]}.$$

However, the explicit expression of the variance of cavity mode quadrature for the superradiant phase is too lengthy to be given.

In Fig. 3(a), we show the expected value of the photon number fluctuations $\langle \delta a^\dagger \delta a \rangle$ evaluated based on the mean-field solutions. For clarity, $\ln(\langle \delta a^\dagger \delta a \rangle + 1)$ is plotted as functions of the normalized coupling strength λ/Δ and the nonlinear gain G/κ , respectively. Here, the blue and

FIG. 3. (a) The photon number fluctuations calculated on top of the stable mean-field solutions versus the normalized coupling strength λ/Δ and the nonlinear gain G/κ . Blue and red refer to fluctuations on top of the normal phase and the superradiant phase, respectively. In panel (a), each regime is marked by the number of stable physical solutions. Panels (b) and (c) show the photon number fluctuations at (a) $\lambda = 0.9\Delta$ and (b) $\lambda = 1.4\Delta$, respectively.

red correspond to the fluctuations of the normal phase and the superradiant phase, respectively. Since the normal phase splitting induced by the nonlinear term at $\lambda = \Delta$, it is clear to see that there exist three regimes for which the photon fluctuations vanish, marked by the number of coexisting solutions. To show these features clearly, we choose $\lambda = 0.9\Delta$ and 1.4Δ , respectively. From regime I to regime II ($\lambda = 0.9\Delta$), the photon fluctuations diverge continuously across both sides of the transition, known as the continuous second-order transition which morph into two discontinuous first-order transitions [see panel (b)]. From regime I to regime III ($\lambda = 1.4\Delta$), however, one can observe discontinuous first-order transition between the normal phase and the superradiant phase accompanying with the coexistence regime [see panel (c)].

FIG. 4. Optical switching from normal phase to superradiant phase by increasing the nonlinear gain G . The red solid and blue dashed curves denote $|\alpha_{Re}|$ and $|\alpha_{Im}|$, respectively. The coupling strengths are chosen as $\lambda = 0.9\Delta$ and $\lambda = 1.4\Delta$ in panel (a) and (b), respectively.

In sight of these features demonstrated above, optical switching from the normal phase to the superradiant phase can be accomplished by increasing the nonlinear gain G . In Fig. 4, we demonstrate two kinds of opti-

cal switching operations by taking $\lambda = 0.9\Delta$ (a) and $\lambda = 1.4\Delta$ (b), respectively. One is the switching from normal phase to superradiant phase without coexisting solutions [see panel (a)]. The other is the switching from normal phase to superradiant phase with coexisting solutions [see panel (b)]. As shown in Fig. 4(a), the values of α_{Re} and α_{Im} change continuously at $G > \kappa$, representing an expected second-order transition. However, there exist an abrupt change for the values of α_{Re} and α_{Im} at $G = \kappa/2$, which is a witness for the first-order phase transition.

FIG. 5. Mean photon number $\langle a^\dagger a \rangle$ versus the nonlinear gain G for (a) $\lambda = 4.5$ MHz and (b) $\lambda = 7$ MHz, respectively. The inserted figures are the density plot of the Wigner functions in regime I, II, and III.

It is known that the fluctuations calculated within the framework of HP transformation or the linearized Langevin equations diverge near the critical point. However, a more rigorous approach regularizes these divergences (see for example the laser phase transition [43]). It is worth examining these within the framework of the full master equation. While, now it is not possible to examine these for large number of atoms since we are limited by the computational capabilities. Though we are limited this still has the advantage that experiments are more likely to be done with finite number of qubits and thus it is important to have better results for fluctuations. Here, we consider eight two-level qubits, such as ^{87}Rb atoms, are trapped in a single-sided optical Fabry-Perot cavity with cavity damping rate $\kappa = 0.5$ MHz. The cavity mode frequency is assumed to be the same as the atomic resonance frequency, and the pump field drives the nonlinear crystal with the detuning $\Delta = 5$ MHz. The accuracy of our numerical calculations is limited by the finite size of the Hilbert space spanned by 50 photon states. We must point out that the number of atoms is enough to exhibit most of the properties of the phase transition process [44]. In our numerical simulation, we also consider the atomic decay by taking $\gamma = 1$ kHz, which is valid for the Rydberg states and metastable states coupled by two-photon process. As shown in Fig. 5, the mean photon number grows continuously for $\lambda = 4.5$ MHz, while an abrupt but continuous increase in the mean photon number can be observed for $\lambda = 7$ MHz.

Although the mean photon number changes as expected, the superradiant phase transition can't be assessed by just measuring the mean photon number. Thus, we calculate the Wigner function to show the phase characteristics of the cavity field. For small nonlinear gain, the system is in the normal phase regime and the Wigner function is that of the vacuum state, which has Gaussian distribution (regime I). When the system enters into the superradiant phase regime, the Wigner function varies significantly. For $\lambda = 4.5$ MHz, there exist two peaks in the Wigner function distribution and the shapes of these two peaks are compressed in the X-direction [45]. This double-peaked Wigner function demonstrate the squeezed superradiant phase (regime II) clearly. For $\lambda = 7$ MHz, there exist three peaks in the Wigner function, which show the mixture of the normal phase and the superradiant phase (regime III) [46].

In summary, we have studied the quantum phase transitions induced by an OPA generated squeezed field in a high Q cavity with N qubits. Using the standard mean field approximation and quantum fluctuation analysis, we linearize the equations for operators and find the critical point for realizing the superradiant phase transition. Increasing the nonlinear gain, i.e., the pump field intensity, an optical switching from the normal phase to superradiant phase can be accomplished. By analyzing the quantum fluctuation and detecting the phase space Wigner function distribution, we show that the normal phase undergoes a splitting with the increase of the pump field. As a result, not only the first-order phase transition but also the second-order phase transition can be achieved if a specific qubits-cavity coupling strength is chosen. For the second-order phase transition, we show that the system is changed from the normal phase with classical mode to the superradiant phase with two squeezed modes continuously. For the first-order phase transition, however, a sudden change from the normal phase to the superradiant phase with coexisting modes can be accomplished in this system. These results demonstrated in this paper can be used to optically control the phase transitions and the feasibility of our scheme has direct implications for various quantum systems, including the atoms, quantum dots and ions cavity QED systems as well as the circuit QED system.

We acknowledge discussions with Dr. Ricardo Gutierrez Jauregui and Dr. Jie Li. CJZ and YPY thank the support from the National Key Basic Research Special Foundation (Grant No.2016YFA0302800); the Shanghai Science and Technology Committee (Grants No.18JC1410900) and the National Nature Science Foundation (Grant No.11774262). GSA thanks the support of Air Force Office of scientific Research (Award No. FA-9550-18-1-0141).

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- [41] It is noticed that the nonlinear gain strengths are quite feasible in experiments [see Ref. [33]]. Assuming that the nonlinear coefficient of the crystal is $d = 18$ pm/V. The crystal is 12 mm long, and two external mirrors are separated by 63 mm. The transverse radius of the pump field is 30 mm. Then, the nonlinear gain coefficient $G = (\epsilon d/\hbar)V_{\text{int}}E_p|E_{\text{cav}}|^2 \approx \kappa/2 = 0.25$ MHz with the pump power 5 mW. In our model, the coefficient G increases to 2κ , corresponding to a pump power 80 mW.
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- [44] As shown in Ref. [24], about 4 qubits are almost enough to realize many features of phase transition. In order to make our work convincing, we present the numerical simulations with 8 atoms.
- [45] The amount of squeezing can be calculated by the formula $-10\log_{10}(2\min(\Delta X^2, \Delta Y^2))$ with quadratures ΔX^2 and ΔY^2 of operators $X = (a + a^\dagger)/\sqrt{2}$ and $Y = i(a^\dagger - a)/\sqrt{2}$. For the driving strength of 0–1, the amount of cavity field squeezing almost linearly changes from 0 to 1.5 dB.
- [46] The multipeak structure of the wigner function should be very relevant for experiments with few qubits although this might get blurred at macroscopic state.