

This is the accepted manuscript made available via CHORUS. The article has been published as:

Constrained Invasion Percolation Model: Growth via Leath Bursts and the Origin of Seismic b-Value

John B. Rundle, Ronaldo Orteç, Joachim Königslied, and Donald L. Turcotte

Phys. Rev. Lett. **124**, 068501 — Published 13 February 2020

DOI: [10.1103/PhysRevLett.124.068501](https://doi.org/10.1103/PhysRevLett.124.068501)

Constrained Invasion Percolation Model: Growth via Leath Bursts and the origin of Seismic b-Value

John B. Rundle,^{1,2,3,*} Ronaldo Orteç,^{1,†} Joachim Königslied,^{1,‡} and Donald L. Turcotte^{2,§}

¹*Department of Physics, One Shields Ave.,
University of California, Davis, CA 95616, United States*

²*Department of Geology, One Shields Ave.,
University of California, Davis, CA 95616, United States*

³*Santa Fe Institute, Santa Fe, NM 87501, United States*

(Dated: December 23, 2019)

We analyze a new model for growing networks, the constrained Leath invasion percolation (CLIP) model. Cluster dynamics are characterized by bursts in space and time. The model quantitatively reproduces the observed frequency-magnitude scaling of earthquakes in the limit that the occupation probability approaches the critical bond percolation probability in $d=2$. The model may have application to other systems characterized by burst dynamics.

I. INTRODUCTION

Many driven physical processes in nature do not occur at constant rates, but rather have a burst-like character in space and time, clustering in space and time. Examples include earthquake seismicity[1], price changes price in financial markets[2], avalanche dynamics and forest fires[3], and transcriptional bursts in genomic systems[4]. In turn, many of these systems and their associated models have been mapped onto percolation models, which is a simple model for clustering[5]. An example of this type of mapping for financial markets is described in ref.[6]. An example for earthquake systems is discussed in ref.[7].

Here we discuss the invasion percolation model[8] that was originally developed to describe

*Electronic address: rundle@ucdavis.edu

†Electronic address: raortez@ucdavis.edu

‡Electronic address: joachim.koenigslied@gmail.com

§Electronic address: dlturcotte@ucdavis.edu

fluid injection into a porous medium, then apply it specifically to the problem of earthquake dynamics and statistics. Invasion percolation (IP) is a variation on the standard models of site and bond percolation[5], and is a type of connected graph-theoretic model wherein the nodes and edges can represent many types of quantities.

Similar to the Leath method[9] in site percolation, one starts with a central seed site and grows the cluster outward. However, in the the IP model, bonds connected to existing cluster sites are opened in order of lowest probability or bond strength first, then next-lowest, and so forth. Eventually the cluster grows to "infinity" (or a pre-defined maximum size). One of the characteristics of the classical IP model is that there is only one time scale, the time scale on which bonds are progressively opened.

To summarize our results: We propose a new model for burst-like dynamics, the constrained Leath invasion percolation (CLIP) model. We show that this model is loopless similar to the model in ref.[10] Interpreting the percolation sites as units of energy release, we show that the model reproduces the observed natural scaling of earthquakes with the correct scaling exponent in the limit that the occupation probability equals the critical bond percolation probability in $d = 2$, $p_{occ} = 0.5$. Comparing these results to observed scaling of earthquakes in several geological regimes, we find good quantitative agreement.

II. BURSTS

One of the characteristics of the IP model is the existence of bursts. Once a strong bond is opened, fluid may enter a region where weaker bonds may exist[3, 11, 12]. Burst sites are defined relative to an (arbitrarily) defined burst threshold strength, usually taken to be very near the critical bond probability value p_{bc} . A burst is defined to include all bonds that are opened sequentially, where the bond strength is less than an arbitrarily defined burst threshold strength. As a practical example, ref.[13] associated invasion percolation bursts with resistance jumps observed in laboratory studies of mercury injection into a porous medium.

A burst begins when an opened bond strength is smaller than the arbitrarily defined threshold and ends when an opened bond strength is greater than the threshold. Because the bursts are defined in this way, the particular dynamics used to grow the cluster will determine whether the individual bursts are spatially connected as well as temporally sequential. In

other words, a sequence of opened bonds may not imply that a burst is a spatially connected object.

There is no concept of time independent from the sequence of opened bonds in the classical IP model. However, with respect to earthquakes in nature, there are multiple time scales. Bursts of activity may occur episodically in time, separated by a period of repose as the system "recharges" for the next seismicity burst[1]. For a more general model, we now modify the classical IP model to allow for bursts that are both spatially and temporally localized, and in which time increments have a meaning independent of the temporal development of the bursts. Or to state in an alternate way, in our modification of the model, we allow for multiple time scales.

III. CONSTRAINED LEATH INVASION PERCOLATION

In the CLIP model, we combine the idea of growing clusters via the Leath algorithm, with the constraint that each site can only be connected to the origin by means of a single pathway of bonds. We start with the "injection site" as the origin, although models with multiple injection sites could be constructed. In this model, we occupy sites in the growing cluster via Leath events or bursts. We also apply the constraint that a site can only be connected by a single path of bonds to the origin, so that multiple connection paths are not allowed, similar to constraints imposed by the method in ref. [14]

Here we consider two time scales, an "injection scale" or long time scale on which the injections occur, and a "burst scale" or short ("instantaneous") time scale on which the bond-opening events occur. The cluster begins with growth from the origin on the first long time step.

On a square lattice in $d = 2$, the four nearest neighbor sites to the origin are first identified. As in the Leath algorithm[9], each of these four sites are tested by generating a uniformly distributed random number on $(0,1)$. If the random number is less than an occupation probability p_{occ} , the site is occupied and the bond between the origin and that site is opened. Testing and opening of the bonds is assumed to occur on the short time scale. This step is regarded as the first burst.

Once the testing is completed on the four nearest neighbors to the origin on the first long time step, the model proceeds to the second long time step, during which the second burst

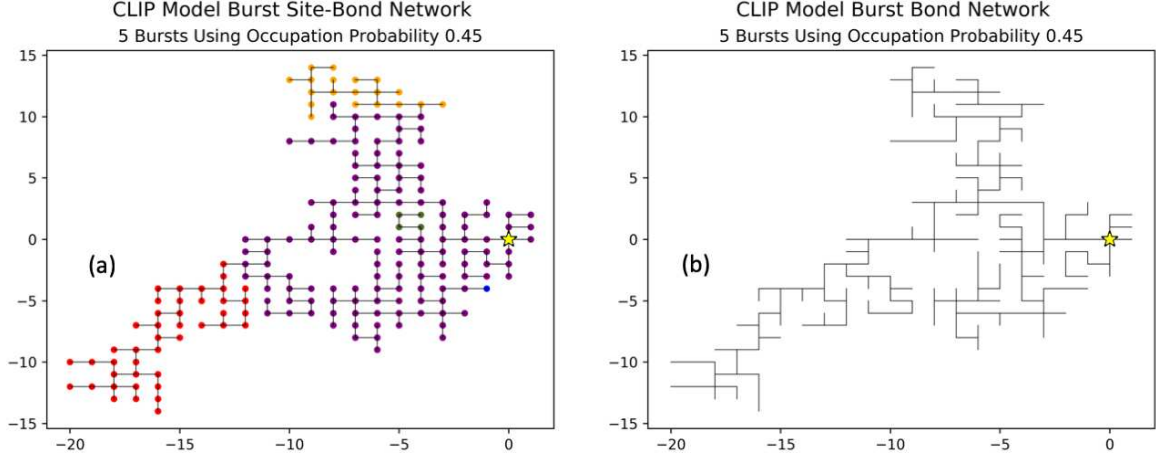


FIG. 1: Example of 5 bursts with associated open bonds. Bursts are shown as separate colors. Yellow star denotes the initial seed. (a) Bursts sites are color-coded, connecting bonds are shown as dark lines. (b) Open bonds only shown as lines.

occurs. Two tests are carried out. The first test demands that the random number be less than p_{occ} , and the second test demands that an occupied site can only be connected to the origin by a single path of bonds.

The process is repeated for all later generations of bursts. The cluster grows by a series of Leath bursts, constrained by the requirement that a site is only connected to the origin once. Because the model is fundamentally constrained by bond pathways, we expect that the critical value of occupation probability should be 0.5, the value for bond percolation in $d = 2$. This expectation is borne out by simulations. Once the a burst is completed, one of the sites in the cluster is then chosen at random as the next growth site and its neighbors are identified and tested by comparing random numbers to p_{occ} and the process repeats.

An example of such a cluster composed of 5 bursts is shown in Figure 1(a) for the value $p_{occ} = 0.45$. In Figure 1(b) it can be seen that there are no open regions in the cluster network that are totally surrounded by opened bonds and thus isolated. In the conceptual physical model, all pre-existing fluid therefore has the possibility of "draining" out of the medium as bonds are opened. It can be seen that the bursts, which occur sequentially over the long time scale characteristic of fluid injection at the origin, are spatially connected. Each burst is assumed to develop over the short burst time scale.

In Figure 2 we show the number-size non-normalized probability density functions $f(S)$

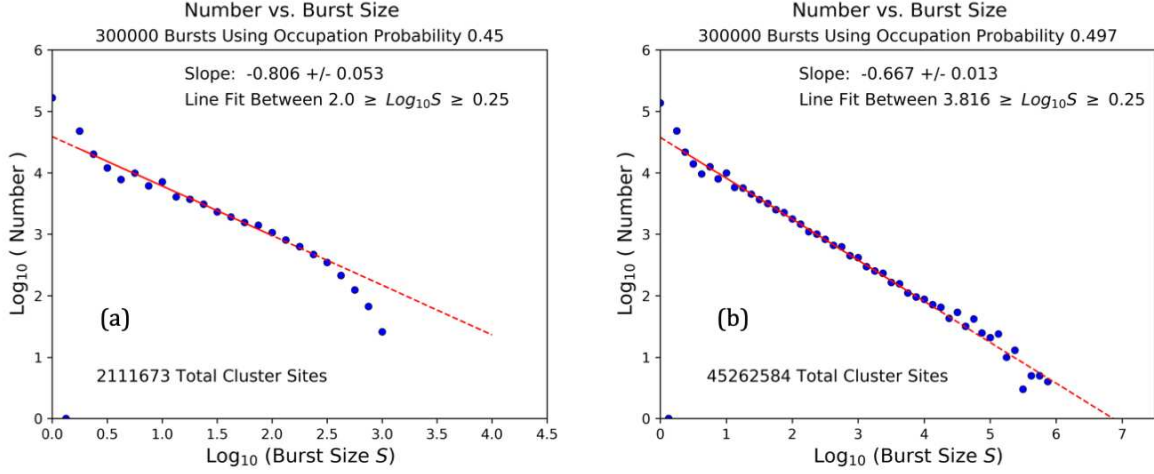


FIG. 2: Non-normalized probability density functions for number of bursts vs. burst magnitude size (number of sites in a burst) on log-log axes for two values of occupation probability. Line slopes as shown. (a) Left, $p_{occ} = 0.45$. (b) Right, $p_{occ} = 0.497$.

for bursts in models with two different values of occupation probability, $p_{occ} = 0.45$ (a) and $p_{occ} = 0.497$ (b). Figure 2(a,b) are plotted on Log-Log axes, so that scaling, or power law functions will appear as a straight line. Here number is the number of bursts and burst size S is the number of occupied sites in the burst. Note that this plot bins the data prior to fitting the scaling line, but the data are computed to machine precision. Quoted errors in data fits, which are calculated by least squares here and in Figure 3, arise from the fit shown.

Both Figures 2(a,b) are statistics for calculations with 300,000 bursts. Figure 2(a) has a shorter scaling region, whose best fit scaling line between $2.0 \leq \text{Log}_{10}(S) \leq 2.5$ has a slope of -0.806 ± 0.053 . Figure 2(b), nearer to the critical occupation probability of $p_{bc} = 0.5$, has a longer scaling region. The best fitting scaling line between $3.816 \leq \text{Log}_{10}(S) \leq 2.5$ is $-0.667 \pm 0.013 \approx -2/3$. This latter slope in Figure 2(b) has a significance that will become apparent shortly. We note that problems in fitting earthquake data have been discussed extensively in [15, 16].

The fact that the slope of the scaling line should be more negative in Figure 2(a) than Figure 2(b) is clear. Both plots have 300,000 bursts, and as a $p_{occ} \rightarrow p_{bc} = 0.5$, bursts will tend to grow larger once they are initiated. This fact is borne out by the data in Figure 2, as (a) has 2.11 million sites in the growing cluster, whereas (b) has 45.26 million sites

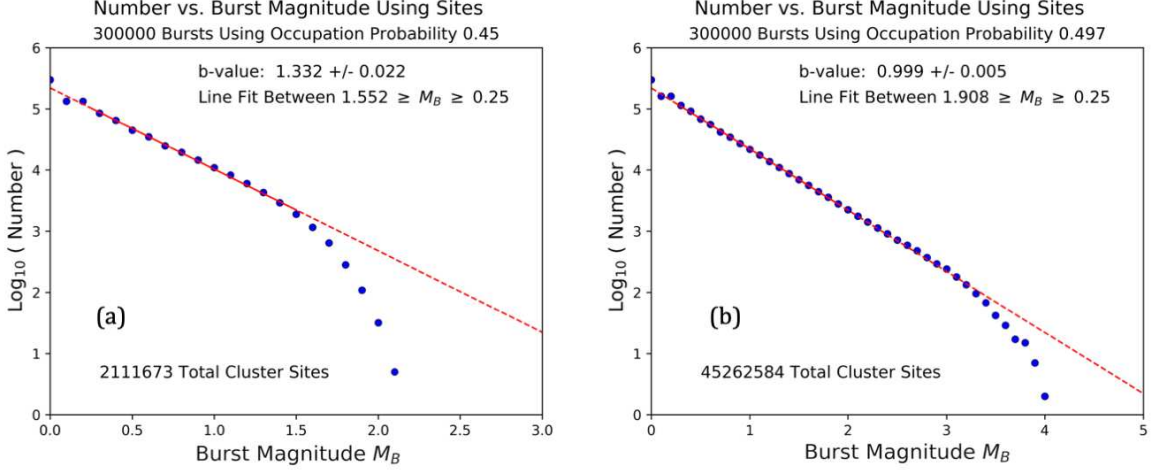


FIG. 3: Non-normalized survivor (exceedance) distributions for number of bursts vs. burst magnitude (defined in the text) on log-log axes for two values of occupation probability. b-values as shown. (a) Left, $p_{occ} = 0.45$. (b) Right, $p_{occ} = 0.497$.

in the growing cluster. In Figure 2(b), there are proportionally many more large clusters relative to the number of small clusters than in Figure 2(a). As a result, the magnitude of the scaling line slope in Figure 2(a) should be larger than the magnitude of the scaling line slope in Figure 2(b).

For naturally occurring earthquakes, the standard in the literature is to plot the Gutenberg-Richter frequency-magnitude (or number-magnitude) relation as a non-normalized survivor distribution or exceedance distribution for earthquakes greater than a magnitude M as in Figure 3(a,b). Earthquake magnitude is typically defined based on the energy release in earthquakes, a quantity that is characterized by the seismic moment W [17]. More specifically, the standard definition of moment magnitude M_w in SI units is:

$$1.5M_w = \text{Log}_{10}(W) - 9.0 \quad (1)$$

where:

$$W = \mu U A \quad (2)$$

Here μ is the elastic shear stiffness, U is the displacement on the earthquake fault, and A is the slipped area on the fault.

We now convert the simulation data shown as the non-normalized probability density functions in Figure 2(a,b), to non-normalized survivor (exceedance) distributions as shown

in Figure 3(a,b). Here there is no need to bin the data. In addition, rather than assuming that each site in a burst represents an element of burst area, let us assign each site to represent an element of seismic moment, consistent with the idea of CLIP as a general graph-theoretical model.

Analogous to equation (1), we define the burst magnitude M_B by the relation:

$$1.5M_B = \text{Log}_{10}(S) \quad (3)$$

where again S is the burst size, or number of occupied sites in the burst. The results are shown in Figure 3(a,b). Here, the slope of the scaling line on the survivor distribution plot is typically called the Gutenberg-Richter[1] b-value. Earthquake number- magnitude scaling relations are empirically found to be approximately described by the equation:

$$\text{Log}_{10}(N) = a - bM_W \quad (4)$$

where a and b are constants. Examples are shown below.

Similar to the results of Figure 2, it can be seen that the b-value of 1.332 ± 0.022 in Figure 3(a) is larger than the b-value of 0.999 ± 0.005 in Figure 3(b). Again, this is because the number of overall sites in the cluster is larger for larger p_{occ} having the same number of bursts. In Figure 3(b), which was a model for which $p_{occ} = 0.497$ (near $p_{bc} = 0.5$), the b-value is very close to $b = 1.0$. The data in Figure 3 were fit between $1.552 \geq M_B \geq 0.25$ for Figure 3(a), and between $1.908 \geq M_B \geq 0.25$ for Figure 3(b). We note that to verify the b-values, we also computed them by the Maximum Likelihood method and found very similar results [18–20]

More generally, for a sequence of values of p_{occ} , we find the results shown in Figure 4. Error bars for the b-values are shown as well (68% confidence). The short dashed extension to the red line represents the extrapolation of the data to the critical value of probability $p_{bc} = 0.5$. It is found that the extrapolated b-value is $b \rightarrow 1.002 \pm 0.006$ as $p_{occ} \rightarrow p_{bc} = 0.5$. As we discuss below, this b-value is characteristic of values seen in observed earthquake seismicity.

To show why the limiting value of b in Figure 4 approaches $b = 1$, we write the exceedance distribution $N(> M_B)$ for M_B in terms of the probability density function $f_B(M_B)$ as:

$$N(> M_B) = \int_{M_B}^{\infty} f_B(M'_B) dM'_B \quad (5)$$

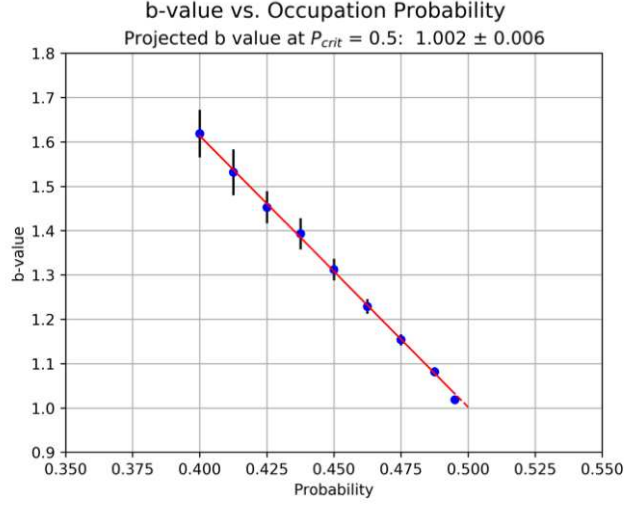


FIG. 4: b-value vs. occupation probability p_{occ} for CLIP bursts. Data were fit for the range of values $0.45 \leq p_{occ} \leq 0.497$. Extrapolation to the critical value $p_{occ} = p_{bc} = 0.5$ indicates that the b-value at criticality for the Number-Magnitude relation is expected to be $b = 1.002 \pm 0.006$.

As discussed previously, we see from Figure 2(b) that as $p_{occ} \rightarrow p_{bc} = 0.5$, the probability density function $f(S)$ asymptotically approaches a power law:

$$f(S) \rightarrow cS^{-x} \quad \text{as } p_{occ} \rightarrow p_{bc} = 0.5 \quad (6)$$

where c is a constant and $x \rightarrow 2/3$.

Combining (3) and (6), we find the probability density function $f_B(M_B)$ is:

$$f_B(M_B) = f(S(M_B)) = c[(10^{1.5M_B})]^{-x} \rightarrow c10^{-M_B} \quad \text{as } p_{occ} \rightarrow p_{bc} = 0.5 \quad (7)$$

Substituting (7) into (5) we finally find that:

$$N(> M_B) = \int_{M_B}^{\infty} f_B(M'_B) dM'_B = c \log_{10} 10^{-M_B} \quad (8)$$

From definition (4) and equation (8), we therefore see that b and $a \rightarrow \log_{10}(c \log 10)$ as the occupation probability $p_{occ} \rightarrow p_{bc} = 0.5$.

IV. EARTHQUAKE DATA

To compare with observed earthquake data, we show the b -value data for multiple sites and geologic regimes in Table 1, for the seismicity data in circular regions. Data are from

Table 1. Data for b-values in various locations and regimes (T = Tectonic; I/F = Injection/Fracking; V = Volcanic)

Location (Circle Radius)	Regime	Time Interval	Line Fit Range	b-value
Global	T	1950-2019	$6.0 \leq M_w \leq 8.0$	1.06 ± 0.02
Tokyo, Japan (1000 km)	T	1950-2019	$6.0 \leq M_w \leq 7.75$	0.99 ± 0.01
Port Moresby, Papua New Guinea (1000 km)	T	1950-2019	$6.0 \leq M_w \leq 7.5$	1.06 ± 0.02
Manila, Philippines (1000 km)	T	1950-2019	$6.0 \leq M_w \leq 7.5$	0.92 ± 0.02
Taipei, Taiwan (1000 km)	T	1950-2019	$6.0 \leq M_w \leq 7.0$	1.08 ± 0.02
Santiago, Chile (1000 km)	T	1950-2019	$6.0 \leq M_w \leq 7.5$	0.95 ± 0.02
Los Angeles, CA, USA (300 km)	T	1990-2019	$1.5 \leq M_w \leq 5.0$	0.98 ± 0.01
Prague, OK, USA (200 km)	I/F	1990-2019	$3.0 \leq M_w \leq 5.0$	1.53 ± 0.02
The Geysers, CA, USA (100 km)	I/F	1990-2019	$1.25 \leq M_w \leq 5.0$	1.14 ± 0.01
Groningen, Netherlands (200 km)	I/F	1990-2019	$2.75 \leq M_w \leq 5.0$	1.26 ± 0.02
Kilauea Summit, HI, USA (200 km)	V	1990-2019	$2.75 \leq M_w \leq 5.0$	1.1 ± 0.02
Mt. Pinatubo, Philippines (200 km)	V	1990-2019	$4.75 \leq M_w \leq 6.0$	1.09 ± 0.03
Mt. Etna, Sicily (200 km)	V	1990-2019	$4.75 \leq M_w \leq 6.0$	1.09 ± 0.08

the US Geological Survey. Tectonic regimes (T) are generally characterized by b-values close to $b \sim 1$, consistent with nearly critical behavior $p_{occ} \sim 0.5$. In these locations, very large earthquakes (large "bursts") are possible and often observed. In nature, values $b < 1$ are unusual, and are often found to be due to observational problems of detecting small earthquakes[1]. Another factor may be uneven coverage of seismometers, such as in areas that combine land, where the coverage is usually good, and oceanic areas, where the coverage is often less reliable.

Volcanic regimes (V) are generally characterized by somewhat higher non-critical probabilities and b-values near $b \sim 1.1$, since larger earthquakes are not often observed on the smaller fault systems present in volcanic edifices. Injection and fracking locations (I/F) have yet smaller fault systems, and consequently smaller earthquakes, with higher b-values near $b \sim 1.2 - 1.5$. But see also [21, 22] for a discussion.

V. DISCUSSION

The CLIP model extends the original IP model. In earthquakes, activity is often observed to begin in a location after a period of quiescence, then progresses in series of burst-like events

to cluster in space and time [23–31]. These bursts include foreshock-mainshock-aftershock sequences, as well as swarms [32].

Previous papers have developed simple models for earthquakes based on percolation[5] and slider blocks[33, 34]. In the mean field versions of these models, the frequency-size exponent $\tau - 1$ is generally found to have the value $\tau - 1 = 1.5$ [34], to be compared to Figure 2.. Interpreting a connected site as an element of moment release as we have assumed in this paper, one would find $b = 1.5$, compared to the observed value near $b \sim 1$.

On the other hand, using a slider block model with damage, ref.[7] found that models could be developed in which $b \sim 1.0$. In both of these other models, τ is a constant irrespective of model parameters. As another example, the SOC model of [35] is characterized by an area-scaling exponent of $\tau - 1 \sim -1$, so it too would have $b \sim 1$. However, for all these models, the area-scaling exponents are constant, and therefore the b -value is constant.

The CLIP model, on the other hand, has a variable b -value. As the CLIP occupation probability approaches the critical value $p_{occ} \rightarrow p_{bc} = 0.5$, larger bursts become progressively easier to generate, leading to a lower b -value that approaches the observed value in the limit.

We note that other scaling laws characteristic of earthquakes can be obtained from the CLIP model. For example, we find that the fractal dimension of the clusters $D_f = 1.89 \pm 0.021$, in good agreement with the observationally measured value of $D_f = 1.9$ [36].

As described in many previous publications of long standing, the mobilization of pore fluids is thought to be intimately connected to the physics of earthquakes [37–45], providing possible justification for the CLIP model. The model will also allow earthquake seismicity data to be interpreted in terms of current values of burst probabilities p_{occ} [46, 47].

VI. ACKNOWLEDGEMENTS AND DATA

The research conducted by JBR and RO has been supported in part by a grant from the US Department of Energy to the University of California, Davis, DoE grant number DE- SC0017324. We are also grateful for thoughtful reviews by Professor I. Main and an anonymous reviewer. Data for the analysis was obtained from the USGS global earthquake catalog, available at: <https://earthquake.usgs.gov/earthquakes/search/>

-
- [1] C. H. Scholz, *The Mechanics of Earthquakes and Faulting* (Cambridge University Press, Cambridge, 2018).
 - [2] R. N. Mantegna and H. E. Stanley, *An Introduction to Econophysics, Correlations and Complexity in Finance* (Cambridge University Press, Cambridge, 2004).
 - [3] M. Paczuski, S. Maslov, and P. Bak, Phys. Rev. E **53**, 414 (1996), URL <https://link.aps.org/doi/10.1103/PhysRevE.53.414>.
 - [4] K. B. Halpern, S. Tanami, S. Landen, M. Chapal, L. Szlak, A. Hutzler, A. Nizhberg, and S. Itzkovitz, Molecular Cell **58**, 147 (2015), ISSN 1097-2765, URL <http://www.sciencedirect.com/science/article/pii/S1097276515000507>.
 - [5] D. Stauffer and A. Aharony, *Introduction to percolation theory* (Taylor & Francis, 2018).
 - [6] D. Stauffer and D. Sornette, Physica A: Statistical Mechanics and its Applications **271**, 496 (1999).
 - [7] C. Serino, K. Tiampo, and W. Klein, Physical review letters **106**, 108501 (2011).
 - [8] D. Wilkinson and J. F. Willemsen, Journal of Physics A: Mathematical and General **16**, 3365 (1983).
 - [9] P. Leath, Physical Review B **14**, 5046 (1976).
 - [10] J. Q. Norris, D. L. Turcotte, and J. B. Rundle, Physical Review E **89**, 022119 (2014).
 - [11] S. Roux and E. Guyon, Journal of Physics A: Mathematical and General **22**, 3693 (1989).
 - [12] S. Maslov, Physical review letters **74**, 562 (1995).
 - [13] J.-N. Roux and D. Wilkinson, Phys. Rev. A **37**, 3921 (1988), URL <https://link.aps.org/doi/10.1103/PhysRevA.37.3921>.
 - [14] C. P. Stark, Nature **352**, 423 (1991).
 - [15] J. Greenhough and I. Main, Geophysical Research Letters **35** (2008).
 - [16] A. F. Bell, M. Naylor, and I. G. Main, Geophysical Research Letters **40**, 2585 (2013).
 - [17] T. C. Hanks and H. Kanamori, Journal of Geophysical Research: Solid Earth **84**, 2348 (1979).
 - [18] K. Aki, Bull. Earthq. Res. Inst., Tokyo Univ. **43**, 237 (1965).
 - [19] T. Utsu, Geophys. Bull. Hokkaido Univ. **13**, 99 (1965).
 - [20] Y. Shi and B. A. Bolt, Bulletin of the Seismological Society of America **72**, 1677 (1982).
 - [21] N. S. Roberts, A. F. Bell, and I. G. Main, Journal of Volcanology and Geothermal Research

- 308**, 127 (2015).
- [22] N. S. Roberts, A. F. Bell, and I. G. Main, *Geophysical Research Letters* **43**, 4288 (2016).
 - [23] J. Gardner and L. Knopoff, *Bulletin of the seismological society of America* **64**, 1363 (1974).
 - [24] P. Reasenber, *Journal of Geophysical Research: Solid Earth* **90**, 5479 (1985).
 - [25] D. Marsan and O. Lengline, *Science* **319**, 1076 (2008).
 - [26] I. Zaliapin and Y. Ben-Zion, *Journal of Geophysical Research: Solid Earth* **118**, 2865 (2013).
 - [27] I. Zaliapin and Y. Ben-Zion, *Bulletin of the Seismological Society of America* **106**, 846 (2016).
 - [28] I. Zaliapin and Y. Ben-Zion, *Geophysical Journal International* **207**, 608 (2016).
 - [29] I. Zaliapin, A. Gabrielov, V. Keilis-Borok, and H. Wong, *Physical review letters* **101**, 018501 (2008).
 - [30] A. Peresan and S. Gentili, *Physics of the Earth and Planetary Interiors* **274**, 87 (2018).
 - [31] B. Luen and P. B. Stark, *Geophysical journal international* **189**, 691 (2012).
 - [32] C. S. Weaver and D. P. Hill, *Pure and Applied Geophysics* **117**, 51 (1978).
 - [33] J. B. Rundle and W. Klein, *Journal of statistical Physics* **72**, 405 (1993).
 - [34] W. Klein, M. Anghel, C. Ferguson, J. Rundle, and J. S. Martins, *Geophysical Monograph - American Geophysical Union* **120**, 43 (2000).
 - [35] P. Bak, C. Tang, and K. Wisenfeld, *Phys. Rev. A* **38**, 364 (1988).
 - [36] M. Sahimi, M. C. Robertson, and C. G. Sammis, *Physical review letters* **70**, 2186 (1993).
 - [37] A. Nur and J. R. Booker, *Science* **175**, 885 (1972).
 - [38] R. Sibson, *Nature Physical Science* **243**, 66 (1973).
 - [39] R. H. Sibson, *Earthquake prediction: an international review* **4**, 593 (1981).
 - [40] R. Sibson, *Tectonophysics* **211**, 283 (1992).
 - [41] R. Sibson, J. M. M. Moore, and A. Rankin, *Journal of the Geological Society* **131**, 653 (1975).
 - [42] R. M. Wood, *Geological Society, London, Special Publications* **78**, 85 (1994).
 - [43] G. Peltzer, P. Rosen, F. Rogez, and K. Hudnut, *Science* **273**, 1202 (1996).
 - [44] M. M. Scuderi and C. Collettini, *Scientific reports* **6**, 24852 (2016).
 - [45] M. Scuderi, C. Collettini, and C. Marone, *Earth and Planetary Science Letters* **477**, 84 (2017).
 - [46] R. Orte, and et al., *To Be Published* (2019).
 - [47] J. Konig and et al., *To be Published* (2019).