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Jesper Levinsen, Francesca Maria Marchetti, Jonathan Keeling, and Meera M. Parish

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Spectroscopic signatures of quantum many-body correlations in polariton microcavities

Jesper Levinsen,^{1,2} Francesca Maria Marchetti,³ Jonathan Keeling,⁴ and Meera M. Parish^{1,2}

¹*School of Physics and Astronomy, Monash University, Victoria 3800, Australia*

²*ARC Centre of Excellence in Future Low-Energy Electronics Technologies, Monash University, Victoria 3800, Australia*

³*Departamento de Física Teórica de la Materia Condensada & Condensed Matter Physics Center (IFIMAC), Universidad Autónoma de Madrid, Madrid 28049, Spain*

⁴*SUPA, School of Physics and Astronomy, University of St Andrews, St Andrews, KY16 9SS, United Kingdom*

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We theoretically investigate the many-body states of exciton-polaritons that can be observed by pump-probe spectroscopy in high-Q inorganic microcavities. Here, a weak-probe “spin-down” polariton is introduced into a coherent state of “spin-up” polaritons created by a strong pump. We show that the \downarrow impurities become dressed by excitations of the \uparrow medium, and form new polaronic quasiparticles that feature two-point and three-point many-body quantum correlations, which, in the low density regime, arise from coupling to the vacuum biexciton and triexciton states respectively. In particular, we find that these correlations generate additional branches and avoided crossings in the \downarrow optical transmission spectrum that have a characteristic dependence on the \uparrow -polariton density. Our results thus demonstrate a way to directly observe correlated many-body states in an exciton-polariton system that go beyond classical mean-field theories.

While the existence of Bose-Einstein statistics is fundamentally quantum, many of the properties of Bose-Einstein condensates can be understood from the phenomenology of nonlinear classical waves (see, e.g., Ref. [1]). In particular, the physics of a weakly interacting gas at low temperatures can generally be described by mean-field theories, involving coherent (i.e., semiclassical) states. Exceptions to this arise when the strength of interactions becomes comparable to the kinetic energy of the bosons. Here, one has correlated states and even quantum phase transitions, e.g., between superfluid and Mott insulating phases [2, 3]. For condensates comprised of short-lived bosonic particles such as magnons [4], photons [5], and exciton-polaritons (superpositions of excitons and cavity photons) [6], the possibility of realizing correlated states suffers a further restriction: the interaction energy scale must exceed the lifetime broadening of the system’s quasiparticles. For these reasons, observing quantum correlated many-body states with such quasiparticles remains a challenging goal.

In this Letter, we propose to engineer and probe quantum correlations in a many-body polariton system through *quantum impurity* physics. Here, a mobile impurity is dressed by excitations of a quantum-mechanical medium, thus forming a new quasiparticle or polaronic state [7, 8] that typically defies a mean-field description. Quantum impurity problems have been studied extensively with cold atoms, where one can explore both Bose [9–11] and Fermi [12–18] polarons (corresponding to bosonic and fermionic mediums, respectively). These studies have yielded insight into the formation dynamics of quasiparticles [17, 19, 20], and the impact of few-body bound states on the many-body system [21, 22]. Furthermore, in the solid-state context, the Fermi-polaron picture has recently led to a better understanding of ex-

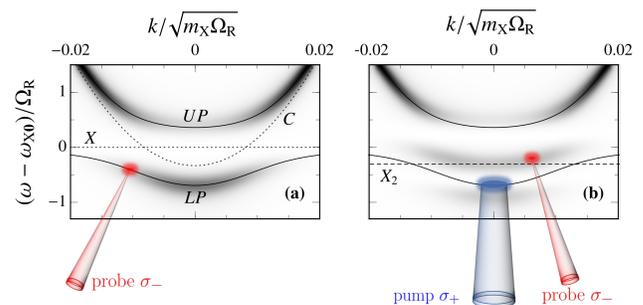


FIG. 1. Spectroscopic signature of a two-point many-body correlated state in the probe photon transmission $\mathcal{T}(\mathbf{k}, \omega)$ (see text and [26]) as a function of momentum and energy. (a) In the absence of pumping. The upper (UP) and lower (LP) polaritons are shown as solid lines, while the dotted lines correspond to the bare photon (C) and exciton (X) dispersions. (b) With a σ_+ pump resonant with the LP at zero momentum. Resonant coupling to a biexciton (X_2) at $\omega + \omega_{LP0} \simeq -E_B$ (dashed line) causes a splitting of the bare lower polariton into attractive and repulsive branches, as well as a blue-shift of the upper polariton. For this illustration, we take the σ_+ polariton density $n = m_X \Omega_R / 8\pi$, detuning $\delta = -\Omega_R / 3$, and $E_B = \Omega_R$.

citons immersed in an electron gas [23, 24], as well as the relation of this to the Fermi-edge singularity [25].

Here we will investigate correlated states of exciton-polaritons using the Bose polaron, which is naturally realized by macroscopically pumping a polariton state in a given circular polarisation (\uparrow), and then applying a weak probe of the opposite (\downarrow) species (Fig. 1). Indeed, experimental groups have already carried out polarization-resolved pump-probe spectroscopy in the transmission configuration [27, 28]. However, such measurements were interpreted in terms of a mean-field coupled-channel

model involving the vacuum biexciton state [29], which neglects the possibility of correlated polaronic states. In contrast, by considering a polaronic description of the state, our Letter shows the important role played by multi-point quantum correlations, how the character of the many-body polaronic state depends on density, and the role of multi-polariton continuum in influencing the transmission spectrum.

To model the quantum-impurity scenario, we go beyond mean-field theory and construct impurity \downarrow -polariton wave functions that include two- and three-point quantum many-body correlations. Such strong multi-point correlations can be continuously connected to the existence of multi-body bound states in vacuum, namely $\downarrow\uparrow$ biexcitons and $\downarrow\uparrow\uparrow$ triexcitons [30] (higher-order bound states have not been observed, as far as we are aware). We calculate the \downarrow linear transmission probe spectrum following resonant pumping of \uparrow lower polaritons, as illustrated in Fig. 1(b), and we expose how multi-point correlations emerge as additional splittings in the spectrum with increasing pump strength. There is thus the prospect of directly accessing polariton correlations from spectroscopic measurements performed in standard cryogenic experiments on GaAs-based structures [27, 28] — i.e., many-body *correlations* have measurable effects on transmission measurements, and do not require measurements of higher order *coherence* functions in order to be observed. Moreover, our theory requires few parameters which can be measured independently, and thus allows one to predict the pump-probe spectrum in other materials, such as transition metal dichalcogenides at room temperature [31, 32].

Model.— We consider a spin- \downarrow impurity excited by a σ_- probe immersed in a gas of spin- \uparrow lower polaritons excited by a σ_+ pump (see schematic in Fig. 1). The \downarrow probe is optical, but the coupling between \uparrow and \downarrow polarizations arises through the excitonic component. To capture the effect of the medium on this photonic component, it is natural to describe the impurity in terms of excitons ($\hat{b}_{\mathbf{k}}$), with dispersion $\omega_{X\mathbf{k}} = \frac{\mathbf{k}^2}{2m_X}$, and photons ($\hat{c}_{\mathbf{k}}$) with dispersion $\omega_{C\mathbf{k}} = \frac{\mathbf{k}^2}{2m_C} + \delta$. Here δ is the photon-exciton detuning (we take $\omega_{X\mathbf{0}} = 0$), m_X is exciton mass and m_C is the photon mass — in this Letter we always take $m_C/m_X \simeq 10^{-4}$. The photon-exciton coupling of strength Ω_R leads to the formation of lower (LP) and upper (UP) exciton-polaritons [33, 34] with dispersion:

$$\omega_{\text{LP,UP}\mathbf{k}} = \frac{1}{2} \left[\omega_{X\mathbf{k}} + \omega_{C\mathbf{k}} \mp \sqrt{(\omega_{C\mathbf{k}} - \omega_{X\mathbf{k}})^2 + \Omega_R^2} \right]. \quad (1)$$

We choose a pump that is resonant with the lower polaritons at zero momentum, yielding a macroscopically occupied single-particle $\mathbf{k} = 0$ state. Thus, we use the

following Hamiltonian [26] (setting \hbar and the area to 1):

$$\begin{aligned} \hat{H} = & \sum_{\mathbf{k}} \left[\omega_{X\mathbf{k}} \hat{b}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}} + \omega_{C\mathbf{k}} \hat{c}_{\mathbf{k}}^\dagger \hat{c}_{\mathbf{k}} + \frac{\Omega_R}{2} \left(\hat{b}_{\mathbf{k}}^\dagger \hat{c}_{\mathbf{k}} + \text{h.c.} \right) \right] \\ & + \sum_{\mathbf{k}} (\omega_{\text{LP}\mathbf{k}} - \omega_{\text{LP}\mathbf{0}}) \hat{L}_{\mathbf{k}}^\dagger \hat{L}_{\mathbf{k}} + \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} g_{\mathbf{k}\mathbf{k}'} \hat{L}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{q}-\mathbf{k}}^\dagger \hat{b}_{\mathbf{q}-\mathbf{k}'} \hat{L}_{\mathbf{k}'} \\ & + \sqrt{n} \sum_{\mathbf{k}, \mathbf{q}} g_{\mathbf{k}\mathbf{0}} \hat{b}_{\mathbf{q}-\mathbf{k}}^\dagger \hat{b}_{\mathbf{q}} \left(\hat{L}_{\mathbf{k}}^\dagger + \hat{L}_{-\mathbf{k}} \right), \end{aligned} \quad (2)$$

which is measured with respect to the energy of the Bose medium in the absence of excitations, $\omega_{\text{LP}\mathbf{0}}n$, where n is the medium density. Since only the \uparrow LP mode is occupied, we simplify our calculations by writing the medium in the polariton basis, with the finite-momentum LP creation operator $\hat{L}_{\mathbf{k}}^\dagger$ and excitation energy $\omega_{\text{LP}\mathbf{k}} - \omega_{\text{LP}\mathbf{0}}$. In order to extract the photon transmission, we however work with exciton and photon operators for the impurity. For simplicity, we have assumed that the polariton splitting and detuning are independent of polarization; however it is straightforward to generalize our results to polarization-dependent parameters.

We model the \uparrow - \downarrow interactions between excitons using a contact potential, which in momentum space is constant with strength g up to a momentum cutoff Λ . This is reasonable, since typical polariton wavelengths $\sim 1/\sqrt{m_C\Omega_R}$ greatly exceed the exciton Bohr radius that sets the exciton-exciton interaction length scale [35, 36]. The exciton-polariton coupling in Eq. (2) is given by $g_{\mathbf{k}\mathbf{k}'} = g \cos \theta_{\mathbf{k}} \cos \theta_{\mathbf{k}'}$, with the Hopfield factor [33]

$$\cos \theta_{\mathbf{k}} = \frac{1}{\sqrt{2}} \sqrt{1 + \frac{\omega_{C\mathbf{k}} - \omega_{X\mathbf{k}}}{\sqrt{(\omega_{C\mathbf{k}} - \omega_{X\mathbf{k}})^2 + \Omega_R^2}}}, \quad (3)$$

which corresponds to the exciton fraction in the LP state at a given momentum. As is standard in two-dimensional quantum gases (see, e.g., Ref. [37]), the coupling constant and cutoff are related to the biexciton binding energy E_B (which we define as positive) through the process of renormalization:

$$-\frac{1}{g} = \sum_{\mathbf{k} < \Lambda} \frac{1}{E_B + 2\omega_{X\mathbf{k}}} = \frac{m_X}{4\pi} \ln \left(\frac{\Lambda^2/m_X + E_B}{E_B} \right). \quad (4)$$

This treatment of the ultraviolet physics is justified as long as the biexciton size greatly exceeds that of the exciton, which is the case when the masses of the electron and hole making up the exciton are comparable [38]. We neglect interactions in the medium for simplicity — in [26] we show that adding interactions in the medium does not significantly change the results.

Probe photon transmission.— The transmission $\mathcal{T}(\mathbf{k}, \omega)$ of a photon at frequency ω and momentum \mathbf{k} is related to the photon retarded Green's function [39] via $\mathcal{T}(\mathbf{k}, \omega) = |G_C(\mathbf{k}, \omega)|^2$, where we ignore a constant prefactor that only depends on the loss rate through the

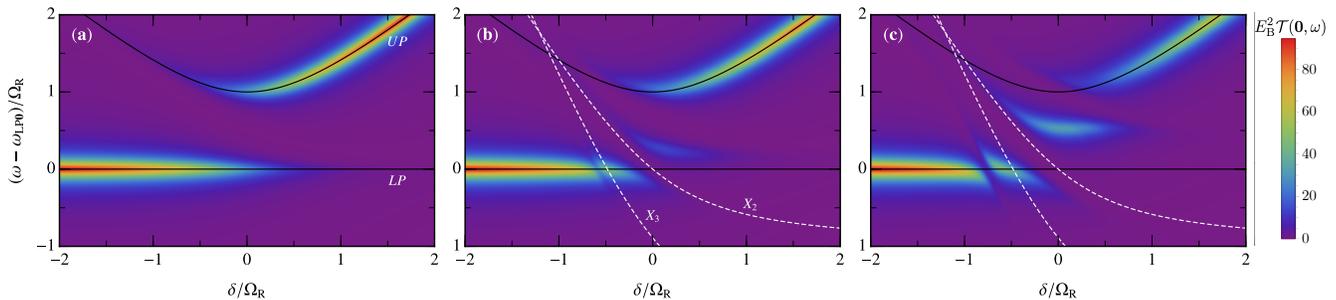


FIG. 2. Normal incidence pump-probe transmission $\mathcal{T}(\mathbf{0}, \omega)$ as a function of the photon-exciton detuning and the rescaled probe energy (relative to the LP energy) for increasing pump densities: (a) $n=0$, (b) $n = m_X \Omega_R / 16\pi$ and (c) $n = m_X \Omega_R / 4\pi$. In the experimentally realistic case of $\Omega_R = 3\text{meV}$, this corresponds to densities (b) $n = 3 \times 10^{10} \text{cm}^{-2}$ and (c) $n = 1.25 \times 10^{11} \text{cm}^{-2}$. In both cases we take $E_B = \Omega_R$, and a broadening $\Gamma = \Omega_R/10$. The lower and upper polariton energies in the absence of the medium are shown as black solid lines. The dashed white lines indicate the locations of the vacuum biexciton (X_2) and triexciton (X_3) resonances at $\omega + \omega_{\text{LP0}} = -E_B$ and $\omega + 2\omega_{\text{LP0}} = \varepsilon_T$, respectively, with triexciton energy $\varepsilon_T \simeq -2.4E_B$ [26].

mirrors. To evaluate this, we note that only the exciton component of the impurity interacts with the medium. Then, in the exciton-photon basis, the impurity Green's function has the form of a matrix,

$$\mathbf{G}(\mathbf{k}, \omega) = \begin{pmatrix} G_X^{(0)}(\mathbf{k}, \omega)^{-1} - \Sigma_X(\mathbf{k}, \omega) & -\Omega_R/2 \\ -\Omega_R/2 & G_C^{(0)}(\mathbf{k}, \omega)^{-1} \end{pmatrix}^{-1}, \quad (5)$$

where $G_C \equiv \mathbf{G}_{22}$. Here, the exciton and photon Green's functions in the absence of interactions are $G_{X,C}^{(0)}(\mathbf{k}, \omega) = 1/(\omega - \omega_{X,C\mathbf{k}} + i0)$, respectively, where the frequency poles are shifted infinitesimally into the lower complex plane. Importantly, Eq. (5) is an exact relation within the Hamiltonian (2), which highlights how any approximation to the probe transmission arises from the calculation of the exciton self-energy Σ_X .

In the following, we evaluate the photon Green's function by using the truncated basis method (TBM) [19]. Within this approximation, the Hilbert space of impurity wave functions is restricted to describe only a finite number of excitations of the medium. The Green's function can be found (as discussed below) by summing over all eigenstates in this basis. In the context of ultracold gases, such an approximation has been shown to successfully reproduce the experimentally observed spectral function of impurities immersed in a Bose-Einstein condensate [10], as well as the ground state [40, 41] and coherent quantum dynamics of impurities in a Fermi sea [17]. As such, the TBM is an appropriate approximation for the investigation of impurity physics, both in and out of equilibrium. Note further that the TBM in principle allows us to investigate other dynamical observables [19], such as higher-order coherence functions [42].

Impurity wave function. – To capture the signatures of strong two- and three-point correlations in the probe transmission, we introduce a variational wave function containing terms where the impurity is dressed by up to

two excitations of the medium:

$$|\Psi\rangle = \left(\gamma_0 \hat{c}_0^\dagger + \alpha_0 \hat{b}_0^\dagger + \sum_{\mathbf{k}} \alpha_{\mathbf{k}} \hat{b}_{-\mathbf{k}}^\dagger \hat{L}_{\mathbf{k}}^\dagger + \frac{1}{2} \sum_{\mathbf{k}_1 \mathbf{k}_2} \alpha_{\mathbf{k}_1 \mathbf{k}_2} \hat{b}_{-\mathbf{k}_1 - \mathbf{k}_2}^\dagger \hat{L}_{\mathbf{k}_1}^\dagger \hat{L}_{\mathbf{k}_2}^\dagger \right) |\Phi\rangle. \quad (6)$$

Here $|\Phi\rangle$ is the coherent state describing the medium in the absence of the impurity, and we consider a σ_- probe at normal incidence, where the total momentum imparted is zero. We take advantage of the fact that the large mass difference between photons and excitons acts to suppress terms in the wave function containing impurity photons at finite momentum — i.e., terms such as $\gamma_{\mathbf{k}} \hat{c}_{-\mathbf{k}}^\dagger \hat{L}_{\mathbf{k}}^\dagger$ and $\gamma_{\mathbf{k}_1 \mathbf{k}_2} \hat{c}_{-\mathbf{k}_1 - \mathbf{k}_2}^\dagger \hat{L}_{\mathbf{k}_1}^\dagger \hat{L}_{\mathbf{k}_2}^\dagger$ are far detuned in energy from the other terms in the wave function, and have thus been neglected. We then find the impurity spectrum by solving $\hat{H}|\Psi\rangle = E|\Psi\rangle$ within the truncated Hilbert space given by wave functions of the form (6). This procedure yields a set of coupled linear equations that we solve numerically [26].

Within the TBM, once all eigenvalues and vectors of the linear equations are known, the photon Green's function can be written as [26]:

$$G_C(\mathbf{0}, \omega) \simeq \sum_n \frac{|\gamma_0^{(n)}|^2}{\omega - E_n + i\Gamma}. \quad (7)$$

The sum runs over all eigenstates within the truncated Hilbert space, picking out the weight of the photon term from each. The factor $i\Gamma$ introduces broadening because of microcavity finite lifetime effects. For simplicity we take it to be independent of the state, which corresponds to considering equal exciton and photon lifetimes. This does not qualitatively affect the results of our work.

Results. – In Fig. 2 we show our calculated normal incidence pump-probe transmission as a function of the photon-exciton detuning and the probe frequency. In

the limit of vanishing pump power, Fig. 2(a), the probe transmission is given by the single-particle LP and UP branches as expected [33, 34], with the relative weights varying according to the photonic fraction of each branch. On increasing the pump strength, we observe first one and then two additional branches appearing with clear avoided crossings, as depicted in panels (b) and (c). This happens in the vicinity of where the LP and UP branches become resonant with either a biexciton (X_2) or a triexciton (X_3) state: Indeed, recalling that we set the $k = 0$ exciton energy to zero, the crossings between solid and dashed lines correspond to the *zero-density* resonance conditions $\omega^* + \omega_{\text{LP}0} = -E_B$ and $\omega^* + 2\omega_{\text{LP}0} = \varepsilon_T$, where ε_T is the vacuum triexciton energy, and $\omega^* \in \{\omega_{\text{LP}0}, \omega_{\text{UP}0}\}$. The resonant behavior results in an intriguing transmission spectrum, where both lower and upper polaritons split into red-shifted attractive and blue-shifted repulsive polaronic quasiparticle branches due to the X_3 and X_2 resonances. Furthermore, at sufficiently large densities, we see that the two LP repulsive branches smoothly evolve into the corresponding attractive and repulsive branches of the UP state.

It is important to distinguish the nature of the polaron state we describe here from the mean-field coupled-channel picture described elsewhere [27, 28], which, at low densities, produces a qualitatively similar spectrum. In the coupled-channel model, there is an anticrossing between the polariton branches and a pre-formed molecular state. By contrast, the X_2 splitting described in this Letter is a beyond-mean-field many-body effect due to two-point correlations which are enhanced by the biexciton resonance. Similarly, the appearance of additional branches at higher densities demonstrates the emergence of many-body three-point correlated states. Indeed, we see that the X_3 resonance position gets rapidly shifted from the vacuum triexciton energy when increasing the density, due to the influence of the continuum of unbound polariton states at high energies, see [26]. Note that our model is likely to overestimate the magnitude of the triexciton energy $|\varepsilon_T|$, since we have neglected the repulsion between \uparrow excitons. However, we can show that the triexciton remains bound even when there is an effective three-body repulsion (which mimics the \uparrow - \uparrow repulsion [22]), and the triexciton binding energy only weakly depends on this repulsion [26].

In order to quantify the density dependence of the two X_2 and X_3 resonances for the lower polariton, we evaluate in Fig. 3 the minimal splittings $\Delta\omega_{2,3}$ between repulsive and attractive branches and the corresponding detunings $\delta_{2,3}$ at which these anticrossings occur when the finite lifetime broadening Γ can be neglected [26]. In the low-density limit, one can formally show that the minimal splitting due to the X_2 resonance has the form $\Delta\omega_2 \sim \cos\theta_0 \sqrt{nE_B/m_X}$ [26]. This behavior is captured using two-point correlations only, and indeed we see in Fig. 3(a) that two-point correlations dominate even at

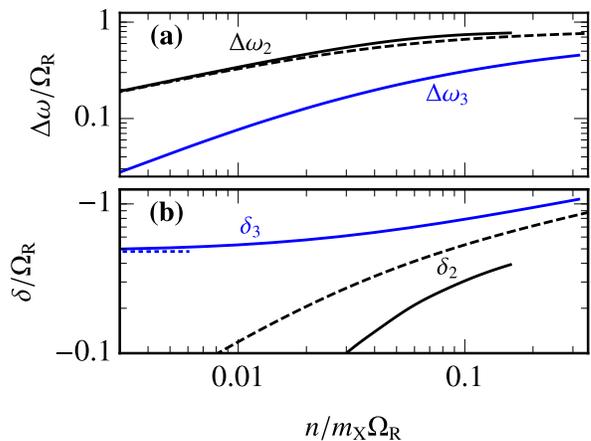


FIG. 3. (a) Minimal splitting between LP quasiparticle branches in the transmission spectrum, and (b) photon-exciton detuning at the minimal splitting. The splitting $\Delta\omega_3$ and detuning δ_3 for the lowest two branches — originating from the triexciton resonance — are shown as solid blue lines, where $\delta_3 \rightarrow -0.48\Omega_R$ (dotted line) in the limit $n \rightarrow 0$ due to the triexciton state. The splitting $\Delta\omega_2$ and detuning δ_2 for the biexciton resonance are shown as solid black lines. The dashed black lines depict the corresponding results calculated when including only two-point correlations (i.e., the Hilbert space with at most one excitation of the medium).

higher densities. However, the shift in the detuning δ_2 is a higher order density effect which can be affected by three-point correlations, as illustrated in Fig. 3(b). For the X_3 resonance, the splitting $\Delta\omega_3$ shown in Fig. 3(a) approaches a linear scaling with n as $n \rightarrow 0$. In this case, one can show that the energy shift of the attractive branch scales linearly with n at low densities, while the repulsive branch only shifts upwards once δ_3 moves away from the vacuum resonance position [26]. Note that, in the presence of broadening, a given splitting $\Delta\omega$ is only visible when $\Delta\omega \gtrsim \Gamma$.

Implications for experiments.— As previously mentioned, the pump-probe protocol employed in the experiments by Takemura *et al.* [27, 28] is similar to our impurity scenario. However, Ref. [28] had a large broadening Γ so that the splitting could not be resolved, while the experiment of Ref. [27] employed a broad pump that populated *both* \uparrow LP and UP branches. Nevertheless, if we take $\Omega_R = E_B = 3\text{meV}$, then the parameters chosen for Fig. 2(c) correspond to a density of $n = 1.25 \times 10^{11}\text{cm}^{-2}$, which approximately matches the parameters of Fig. 3 in Ref. [27]. Here, the splitting of the lower polariton close to the biexciton resonance was analyzed [27]. Qualitatively, our results for the attractive and repulsive energy shifts agree; however the measured energy shifts are somewhat smaller than what we find. This is likely to be due to the broad range of \uparrow states populated in Ref. [27], which will tend to wash out the effect of the resonances compared to when the bosonic medium is a macroscopi-

cally occupied single-particle state.

Conclusions and outlook.— In this Letter, we have shown how many-body correlations in the exciton-polariton system can be directly accessed using pump-probe spectroscopy. Such measurements are complementary to the sophisticated multi-dimensional optical spectroscopy techniques employed in, e.g., Ref. [43], which require multiple phase-stable optical pulses with controllable delays. Furthermore, depending on the material parameters, there is even the possibility of overlapping biexciton and triexciton resonances, where both two- and three-point correlations are enhanced [26]. Direct probes of many-body correlated states can also provide stringent bounds on the nature and spin structure of the polariton-polariton interaction. Such an approach is complementary to measurements of blueshift (which can be affected by reservoir excitons), and is crucial in the progress towards realizing truly quantum states. Recently there has been significant interest and some progress toward achieving anti-bunching in emission from fully confined photonic dots [44, 45]. However there is as yet little known about many-body correlated polariton states. Our results suggest that exploring impurity physics in polariton condensates provides a route to achieve this.

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