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Energy drag in particle-hole symmetric systems as a quantum quench

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Two conducting quantum systems coupled only via interactions can exhibit the phenomenon of Coulomb drag, in which a current passed through one layer can pull a current along in the other. However, in systems with particle-hole symmetry – for instance, the half-filled Hubbard model or graphene near the Dirac point – the Coulomb drag effect vanishes to leading order in the interaction. Its *thermal* analogue, whereby a thermal current in one layer pulls a thermal current in the other, does not vanish and is indeed the dominant form of drag in particle-hole symmetric systems. By studying a quantum quench, we show that thermal drag, unlike charge drag, displays a non-Fermi’s Golden Rule growth at short times due to a logarithmic scattering singularity generic to one dimension. Exploiting the integrability of the Hubbard model, we obtain the long-time limit of the quench for weak interactions. Finally, we comment on thermal drag effects in higher dimensional systems.

Since its inception [1], the Coulomb drag phenomenon – whereby a charge current in one layer pulls a reciprocal current in another through Coulomb interactions alone – has shed light on the special role of interaction effects in quantum transport [2]. Coulomb drag measurements have been instrumental in studying the microscopic structure of systems as diverse as double-quantum well structures [3, 4], excitons in electron-hole bilayers [5–8], quantum Hall states [9–13], Luttinger liquids [14, 15], spin currents in two-dimensional electron gases [16, 17], and bilayer graphene [18–24], among others. From the theoretical point of view, the Coulomb drag conductivity generally shows a rich dependence with temperature, with each regime dominated by different microscopic processes, and has been generalized in many directions [2]. Given the recent interest in the hydrodynamic behavior of electrons in solids [25–27], an analogy can also be made between the Coulomb drag and the shear viscosity, two processes leading to the equalization of currents in neighboring layers.

In light of this history, it stands to reason that the Coulomb drag effect between *thermal* currents, first studied to our knowledge in Ref. [28] – in which a thermal current in one layer may drag along a reciprocal thermal current in the other through Coulomb interactions – could elucidate the microscopic structure of quantum systems as well. In fact, in one particularly interesting class of quantum systems – those having particle-hole symmetry – Coulomb charge drag effects are known to vanish at leading order [21]. Momentum is transferred between the layers at this order, but it cannot result in a charge current [29]. This is not a straightforward effect of symmetry, which would lead to vanishing at all orders; rather the leading process in perturbation theory is independent of the sign of the scattering potential, as with the Born approximation, so that the currents induced by particle-particle and particle-hole scattering cancel. Such systems are prime candidates for the study of thermal drag, as thermal drag need not vanish under particle-hole symmetry, and we find it to be the domi-

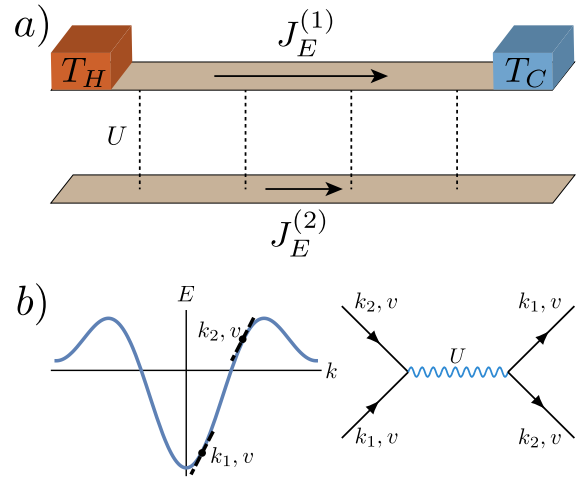


FIG. 1. (a) The thermal Coulomb drag geometry considered in this paper. A conducting quantum system’s top layer is held at a temperature gradient by connecting it to two reservoirs at temperatures $T_H > T_C$, causing a thermal current to flow; through quantum interactions U , a thermal current is dragged in the bottom layer. (b) The source of the divergent scattering process leading to the breakdown of the usual Fermi’s Golden Rule in one-dimensional systems, namely when all incoming and outgoing particles have the same velocity v but differ in energy.

nant form of drag in such systems. Examples include the Hubbard model at half filling, graphene near the Dirac point, and superconductors probed at low energy, among others.

In this Letter, we focus on thermal drag between particle-hole symmetric quantum systems, viewed through the lens of a quantum quench of the inter-layer interactions in a bilayer system. We find that thermal drag does indeed dominate drag physics in these systems and, in sharp contrast to charge drag, suffers from a scattering singularity generic to one-dimensional band structures. This singularity leads to a violation of the naïve Fermi’s Golden Rule, where the rate of change of the

thermal current is *logarithmic* in time rather than constant, in the thermodynamic limit. This implies that a simple scattering rate analysis is generally incorrect, and more sophisticated perturbation theory analysis must be used; in particular, the approximation of linearizing the spectrum cannot be used when dealing with thermal currents without some method of regulation.

A quench and a Kubo formula. To study the thermal drag, let us consider the paradigmatic one-dimensional Hubbard model,

$$H = -t \sum_{\langle ij \rangle, \sigma} c_{i, \sigma}^\dagger c_{j, \sigma} + U \sum_i (n_{i, \uparrow} - \frac{1}{2})(n_{i, \downarrow} - \frac{1}{2}) \quad (1)$$

where $\{c_{i, \sigma}^\dagger, c_{j, \sigma'}\} = \delta_{ij} \delta_{\sigma\sigma'}$. Let us view the two spin species as each forming separate quantum wires, with on-site interactions coupling them. We note that the limit of on-site interactions can be physically motivated as originating from a screened Coulomb potential with small screening length. Initialize one species, say spin-down, in a thermal state at temperature T with some small initial energy current, and initialize the other spin species in a thermal state with no energy current (with $U = 0$). Explicitly, since the free fermion chain may be diagonalized by a simple Fourier transform with energies $E_k = -2t \cos k$ and velocities $v_k = 2t \sin k$ (assuming periodic boundary conditions), such a state is given by

$$\langle n_k^\sigma \rangle = \frac{1}{1 + \exp(\beta(-2t \cos k - \mu))} - \delta_{\sigma\downarrow} \epsilon \sin(2k), \quad (2)$$

with ϵ a small parameter ensuring the validity of linear response. The charge and thermal current operators carried by the σ spin species are given respectively by $J^\sigma = L^{-1} \sum_k v_k n_k^\sigma$ and $J_E^\sigma = L^{-1} \sum_k E_k v_k n_k^\sigma$, hence this initial state has $\langle J_E^\sigma \rangle = \epsilon \delta_{\sigma\downarrow}$ and $\langle J^\sigma \rangle = 0$ (diagrammed in Fig. 1(a)). In this setup, the spin-down channel is the “drive” layer and the spin-up channel is the “response” layer in the usual terminology of Coulomb drag, with the caveat that the “drive” current is allowed to relax (which does not change the short-time dynamics). We note that, while somewhat unorthodox, this quench interpretation of the Coulomb drag problem is physically reasonable and allows for the use of techniques from scattering theory and integrability that would be inapplicable in an equilibrium description.

At time $t = 0$, let us quench on the interaction term U . We are interested in the change over time of the heat current in the spin-up channel. From the perspective of linear response, one would expect that an initial thermal current in the spin-down channel would drag along a thermal current in the spin-up channel, leading to the development of a temperature gradient for the spin-up species that is proportional to the initial energy current. This would give a thermal drag conductivity of

$$\kappa_D = \frac{J_E^{(1)}}{\nabla T^{(2)}} \quad (3)$$

where $J_E^{(1)}$ is taken at time $t = 0$, and here (1) refers to spin-up and (2) to spin-down. Now, generally speaking, there is no perturbing Hamiltonian for a temperature gradient, so there is no straightforward method of deriving a Kubo formula for thermal conductivities. One may argue, however, based on entropy production in the system, that there exists an effective perturbing Hamiltonian and from this derive a Kubo formula [30]. Adapting this method, we arrive at a Kubo formula for the thermal drag conductivity [28][31],

$$\kappa_{ab}^{\sigma\sigma'}(q, \omega) = \frac{1}{VT} \int_0^\infty dt e^{(i\omega - 0^+)t} \int_0^\beta d\lambda \langle J_{Q,b}^\sigma(-q, -i\lambda) J_{Q,a}^{\sigma'}(q, t) \rangle \quad (4)$$

with V the system size, σ and σ' layer indices, q the wavevector, J_Q the heat current, and a and b spatial indices (in the case of higher dimensional systems).

With this Kubo formula in hand, we can connect our quench picture to the thermal drag conductivity by the following argument: if the initial rate of change of the energy current in the spin-down species is some rate $\partial_t \langle J_E^\downarrow \rangle = \Gamma$, then by the fluctuation-dissipation theorem [32] we should expect that the two-point function is exponentially decaying with the same rate Γ . This would give $\kappa_D \sim \int_0^\infty dt e^{i\omega t} e^{-\Gamma t} = 1/(\Gamma - i\omega)$, which, identifying $\Gamma = 1/\tau$ with τ a scattering time, would reproduce the usual Drude relation. We caution that in this case, however, a naïve Drude analysis will fail due to the complicated behavior of the energy current post-quench, which we examine below.

To calculate Γ , we seek the quantity $\partial_t n_k^\uparrow$, under the perturbation of the Hubbard interaction. To lowest (second) order in U ,

$$\partial_t n_k^\uparrow = U^2 \sum_{k_2, k_3, k_4} S_{kk_2}^{k_3 k_4} \frac{\sin(t\Delta E)}{\Delta E} \delta(\Delta k), \quad (5)$$

where $S_{kk_2}^{k_3 k_4} = (1 - n_k^\uparrow)(1 - n_{k_2}^\downarrow)n_{k_3}^\uparrow n_{k_4}^\downarrow - n_k^\uparrow n_{k_2}^\downarrow(1 - n_{k_3}^\uparrow)(1 - n_{k_4}^\downarrow)$ is the net Fermi factor for the inward and outward scattering processes, $\Delta k = k + k_2 - k_3 - k_4$ and $\Delta E = E_k + E_{k_2} - E_{k_3} - E_{k_4}$. In the usual Fermi's Golden Rule, one takes the limit of large t , which sends $\sin(t\Delta E)/\Delta E \rightarrow \pi \delta(\Delta E)$ provided that the quantity being integrated against does not diverge at $\Delta E = 0$. This is the case for Coulomb drag of charge currents, which is well-behaved; however, this is not the case for the energy current, as we shall see, and we must deal with the divergence carefully.

Imposing momentum conservation, the energy current grows as

$$\partial_t J_E^\uparrow = \frac{2}{L} \sum_k \sin(2k) \partial_t n_k^\uparrow. \quad (6)$$

We can usefully rewrite this expression by moving the sum on k to an integral in energy space of a quantity $G(E)$, integrating against a kind of “density of

states" [33]. Focusing on half-filling $\mu = 0$, the function $G(E)$ contains the essential divergence of the response energy current, namely

$$G(E) \propto \int dk_1 dk_3 \sum_{\nu=1,2} \frac{F(k_1, k_{2,\nu}, k_3)}{|v(k_1 + k_{2,\nu} - k_3) - v(k_{2,\nu})|} \quad (7)$$

with $v(k) = \partial_k E(k) \propto \sin k$ the group velocity, the function F does not diverge, and ν indexes the solutions to $\Delta E - E = 0$. Clearly, the source of the divergence is the difference of velocities in the denominator, corresponding to a resonance of points in k space with different energies but the same velocity. Physically, this shows that the energy current operator diverges at small energies $\Delta E \approx 0$ which are directly probed by the $\text{sinc}(t\Delta E)$ term in perturbation theory, and it is because of this singular behavior that Fermi's Golden Rule breaks down.

There are two conditions under which the denominator diverges: the trivial case of $k_1 = k_3$, and the nontrivial second solution. In the first instance, one can readily see that the numerator also vanishes, and hence there is no divergence. For the second solution, which occurs here at $k_1 + k_2 - k_3 = \pi - k_2$ but must occur somewhere in a generic one-dimensional band structure, one finds that the numerator also vanishes for a charge current – and hence, it is well-behaved – while it does not for the energy current. The divergence is point-like, in the sense that for every incoming k there is a finite set of partners $\{k'\}$ with the same velocity. That there must be at least one partner is a consequence of the lattice, i.e. the periodicity of the band structure (see Figure 1(b)).

At small but finite E , we can regularize the denominator, ultimately leading to a logarithmic divergence. A careful accounting yields

$$g(E) = \epsilon \frac{4U^2}{(2\pi)^3} \int_{-\pi}^{\pi} dk \frac{f(k)}{|\sin k/2|} \log E \quad (8)$$

where $g(E) = (G(E) + G(-E))/2$ is the symmetric part of $G(E)$, $f(k) = -2 \sin^2(k) n(E_k) n(-E_k)$, and $n(E)$ is the Fermi-Dirac distribution. Finally, using $\int_{-\infty}^{\infty} dx \log(x) \text{sinc}(xt) = -\pi(\gamma + \log t)/t$, with γ the Euler-Mascheroni constant, and keeping only the dominant term in the large t limit, we arrive at the result

$$\frac{\partial_t J_E^\uparrow}{J_E^\uparrow(t=0)} = \alpha \log t + \mathcal{O}(1), \quad (9)$$

with

$$\alpha(T) = \frac{U^2}{\pi^2} \int_0^\pi dk |u(k)|^2 \frac{\sin^2 k \csc(k/2)}{1 + \cosh(2t\beta \sin(k/2))}, \quad (10)$$

where for generality, we have allowed for k -dependent interactions, $U(k) = Uu(k)$, and in the Hubbard model with onsite interactions $u(k) = 1$. We remark that this

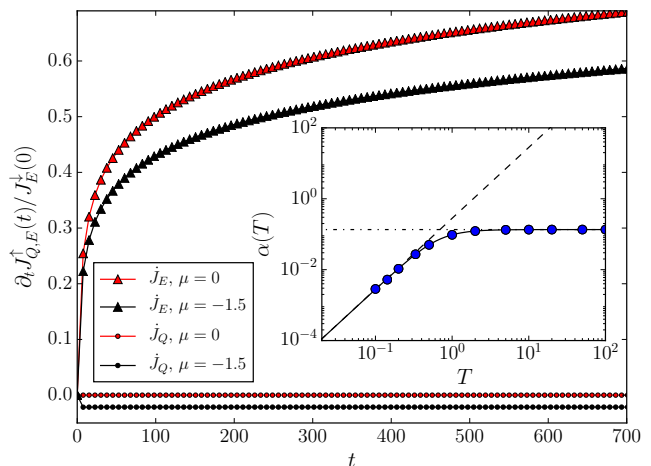


FIG. 2. The growth of the heat and energy currents in the bottom layer due to the Coulomb drag, to $\mathcal{O}(U^2)$ in perturbation theory. At half filling $\mu = 0$, no charge drag occurs due to particle-hole symmetry (red dots); this is no longer true away from half filling (black dots). In both cases, thermal drag is nonzero and the rate of change grows logarithmically in time as $\alpha(T) \log t$ (red and black triangles), rather than saturating to a constant as would be naïvely expected. Inset: the prefactor for this log growth $\alpha(T)$ as a function of temperature. Agreement with the analytical formula of Eq. 10 is excellent (solid line); the asymptotics are $\alpha(T) = 4U^2 T^2 \log 2/\pi^2$ for small T (dashed line) and $\alpha(T) = 4U^2/3\pi^2$ for large T (dotted line).

logarithmic behavior is quite general: we expect it for any lattice band structure in 1D, as such band structures must generically have points where $v(k) = v(k')$ but $E(k) \neq E(k')$. Further, other kinds of interactions only modify the prefactor of the log growth. This integral cannot be computed analytically, but for Hubbard, the low- and high-temperature limits are readily analyzed. First, at low temperatures, the denominator is a strongly peaked function about $k = 0$; expanding the numerator in Taylor series and performing the integration yields

$$\alpha(T) \underset{T \ll 1}{\approx} \frac{4U^2 \log 2}{\pi^2} T^2, \quad (11)$$

in units of Hubbard hopping $t = 1$ and $k_B = 1$. In the high temperature limit the denominator is approximately constant, yielding

$$\alpha(T) \underset{T \gg 1}{\approx} 4U^2/3\pi^2. \quad (12)$$

We have numerically checked this expression by exactly summing Eq. 5 on system sizes of $L > 3000$ and calculating $\partial_t J_E$ and $\partial_t J$. The results are shown in Fig. 2; the logarithmic growth of the energy current is clear both at half-filling ($\mu = 0$) and away from half-filling ($\mu = -1.5$). We recover the result that, as expected, there is no charge drag at half filling, confirming

that thermal drag dominates in this regime, while we do notice a drag thermopower effect away from half filling. Finally, the observed dependence on temperature of the prefactor of the log, obtained by fitting at various temperatures, is in excellent agreement with Eq. 10, which we integrate numerically and whose asymptotics we plot. This confirms that the processes considered in this section indeed dominate the thermal drag to an excellent approximation.

A few remarks are now in order. First, the breakdown of Fermi's Golden Rule for the energy current is generic to one-dimensional systems, as any band structure will display the same kind of divergence. Second, due to the divergence, the widespread technique of linearizing the spectrum [34] will fail badly in analyses of thermal drag. In this case, band curvature effects may be included directly in the field theory and treated perturbatively [35]. Third, the timescale for the validity of perturbation theory is parametrically reduced for thermal drag calculations: perturbation theory holds only up to a timescale $t_*^{-1} \sim U^2 \log U$. Finally, one may consider the effects of adding a small magnetic field: to lowest order, the field would simply shift the chemical potential in the two species in opposite directions [36], effectively breaking particle-hole symmetry. In that case, we no longer expect a vanishing charge drag. However, the logarithmic growth of the response heat current would remain, as it is present for any chemical potential, being a consequence of the band structure.

To access longer times, we make the approximation of a linear spectrum (Luttinger liquid) and regulate the breakdown of Fermi's Golden Rule [37]. Linearizing the spectrum produces a left- and a right-moving mode, described by wavevector $q_{L/R} = k \pm k_F$ with dispersion relation $E(q_{L/R}) = \mp v_F q_{L/R}$. We must then consider 8 possible scattering channels: two forward scattering channels, two Umklapp channels, and four backward scattering channels. For simplicity, we slightly modify the setup such that one spin species is kept at a temperature gradient with $k < 0$ at T_L and $k > 0$ at T_R , with the other species in the ground state ($T = 0$).

Analyzing these possible scattering channels, we find that, while the Umklapp and backscattering channels give a finite rate, the forward scattering channel leads to a divergence with system size, a one-dimensional incarnation of the well-known ‘‘collinear scattering singularity’’ in Dirac-dispersing systems [2, 38, 39]. This is due to the fact that, for the forward scattering channel, conservation of energy and momentum become the same constraint, leading to a delta function squared appearing under the scattering integral. This type of divergence was noted in Ref. [40] in the case of Coulomb drag for spinful Luttinger liquids. To recover a finite answer, it was proposed that one go past lowest order perturbation theory, inserting the RPA propagator in place of the bare propagator in the scattering integral (dubbed the ‘‘generalized Fermi's golden rule’’). In our case, it amounts to taking the incoming particles to have veloc-

ity v_F while the outgoing particles have velocity u , the Luttinger velocity, which is interaction dependent. Under this prescription, we find a heat current growth rate that is actually *first-order* in the interaction U ,

$$\partial_t J_E^\uparrow \sim U \frac{2\pi^4 \log 2}{3\hbar v_F} k_B^3 (T_R^3 - T_L^3), \quad (13)$$

due to the interaction-renormalized outgoing velocity cancelling a power of U . In sum, due to the unique divergences of heat drag as opposed to charge drag, we expect a logarithmic heat current growth rate at the shortest times that is second order in U , followed by a longer regime of heat current growth rate that is constant in time and first order in U . We emphasize that the charge drag in particle-hole symmetric systems vanishes to lowest order, and only enters at order U^3 (if at all); hence thermal drag is the dominant form of drag physics in this broad class of systems.

Long-time limit and higher dimensions. Generally speaking, the long-time limit of this quench is outside the realm of validity of perturbation theory, and therefore inaccessible. However, here we may exploit the integrability of the one-dimensional Hubbard model to make progress [41]. In particular, due to its integrability, the one-dimensional Hubbard model hosts a tower of conserved quantities, the number of which is extensive in system size. One such quantity, known as Q_3 , differs from the total energy current operator only by a term of order U ; that is,

$$J_E = t^2 \sum_{l,\sigma} i (c_{l+1,\sigma}^\dagger c_{l-1,\sigma} - c_{l-1,\sigma}^\dagger c_{l+1,\sigma}) - \frac{Ut}{2} \sum_{l,\sigma} (j_{l-1,\sigma} + j_{l,\sigma})(n_{l\bar{\sigma}} - 1/2),$$

which takes the same form as Q_3 except for a factor of 2 in the term proportional to U [42]. This implies that in the limit of small U , $J_E \approx Q_3$ and is hence conserved. (We note that even in the limit of stronger U , the overlap of J_E with Q_3 will be conserved, leaving some energy current in the final state.) Under the assumption of approach to a generalized Gibbs ensemble final state [43] with this same value of Q_3 , we expect that the energy current will be equally divided between the two wires. That is,

$$J_E^\uparrow(t \rightarrow \infty) = J_E^\downarrow(t \rightarrow \infty) = \frac{J_E^\downarrow(t=0)}{2}. \quad (14)$$

The conservation of the energy current is likely a special feature due to the integrability of the Hubbard model, but we remark that in this case it leads to an intriguing hydrodynamic transport of energy current reminiscent of the Dirac fluid [44].

Since the source of the divergent heat drag is related to special properties of scattering in 1D, we do not expect the same divergence to appear generically for higher

dimensional systems. As a check, we have considered the Hubbard model on the square lattice with nearest-neighbor hopping [45]. We have numerically explored this model for various values of the chemical potential and temperature on system sizes of up to $L_x = L_y = 100$. We find that the thermal drag indeed dominates near half-filling, and it does not appear to be divergent. We defer an exhaustive analysis of the two-dimensional case to future work.

Discussion. We have analyzed a thermal analogue of the Coulomb drag in interacting quantum systems with particle-hole symmetry via a quantum quench in the Hubbard model. We have found that, due to the vanishing of the charge Coulomb drag, the thermal drag effect dominates. In one dimension, its growth is drastically different than the charge drag due to the structure of the energy current operator: the short-time limit shows logarithmic non-Fermi's golden rule growth, followed by a longer regime of linear growth given by a generalized Golden rule, with the late-time limit in this case obtained from integrability arguments.

We expect these conclusions to apply to a broad range of experimentally realizable systems, including perhaps most prominently graphene near charge neutrality. It is an interesting question whether some components of the thermal Coulomb drag may be topologically quantized in certain systems, especially in light of recent experiments on the thermal Hall effect at nonchiral Hall edges [46]. We emphasize that, despite the vast literature on the charge Coulomb drag, the thermal drag effect is largely unexplored [47], and is ripe for further study.

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