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Hyperbolic Sound Propagation over Nonlocal Acoustic Metasurfaces

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Hyperbolic metasurfaces, supporting extreme anisotropy of the surface impedance tensor, have been recently explored in nanophotonic systems for robust diffraction-less propagation over a surface, offering interesting opportunities for sub-diffraction imaging and enhanced Purcell emission. In acoustics, due to the longitudinal nature of sound transport in fluids, these phenomena are forbidden by symmetry, requiring the acoustic surface impedance to be inherently isotropic. Here we show that nonlocalities produced by strong coupling between neighboring impedance elements enable extreme anisotropic responses for sound traveling over a surface, supporting negative phase and energy velocities, as well as hyperbolic propagation for acoustic surface waves.
The recent progress in metamaterials research has enabled many exciting opportunities for wave manipulation and its applications, resulting in the discovery of novel devices, such as cloaks [1]-[3], hyperlenses [4]-[7], and hyperbolic metamaterials [8]-[14], among several others. These concepts were firstly proposed in electromagnetics [1]-[14], and later extended to acoustics [15]-[18] due to the analogies between the two wave phenomena. In the design and realization of these metamaterial devices, a key requirement is the implementation of extremely anisotropic material properties. Due to the vector nature of electromagnetic waves, both permittivity and permeability can be anisotropic. However, the longitudinal nature of sound waves in fluid, and the associated symmetry constraints make the opportunities more limited in acoustics. Although anisotropy is not a common property of fluids, the realization of anisotropic mass density has been explored in recent years [19]-[21], and its implementation has been applied to the design of various acoustic metamaterials, especially in the framework of transformation acoustics [22]-[24]. Although anisotropic stiffness is possible [25],[26], there is no implementation of anisotropic bulk modulus for fluid acoustics. The reason for this missed opportunity lies in the symmetries obeyed by the equations governing sound propagation in fluids: the momentum equation $\nabla p + \rho_{\text{eff}} \frac{\partial \vec{v}}{\partial t} = 0$ and the mass conservation equation $\frac{\partial p}{\partial t} + B_{\text{eff}} \nabla \cdot \vec{v} = 0$ [27]. Here $p$, $\vec{v}$, $\rho_{\text{eff}}$ and $B_{\text{eff}}$ represent the acoustic pressure, particle velocity, effective density, and bulk modulus, respectively. Since both $\nabla p$ and $\frac{\partial \vec{v}}{\partial t}$ are vectors, a tensorial effective density in the momentum equation can be expected, yielding $\nabla p + \tilde{\rho}_{\text{eff}} \cdot \frac{\partial \vec{v}}{\partial t} = 0$, which simply implies that the three components of the equation are generally coupled together, and the response changes for different velocity directions. The mass conservation equation, however, is scalar, hence the effective bulk modulus cannot take a tensorial form. The fact that only the mass density can be anisotropic in acoustic metamaterials has not prevented the application of various
electromagnetic metamaterial concepts to acoustics. However, when exploring sound interactions with metasurfaces, this missing element imposes severe restrictions.

Metasurfaces are the planarized version of metamaterials, yielding unique opportunities for wave manipulation along a surface. Translating bulk metamaterial concepts to metasurfaces allows minimizing the effect of loss, and enables easier access to field enhancement [28]-[29]. Hyperbolic metasurfaces in photonics have recently become an important class of such devices, enabling the unusual propagation properties of hyperbolic metamaterials, such as the unbounded local density of states and sub-diffraction imaging and canalization, over a planarized surface [30]-[32]. Like in the case of hyperbolic metamaterials, hyperbolic metasurfaces require an extreme anisotropic response for the two field polarizations along the surface, namely a surface impedance tensor with oppositely signed entries in the diagonal elements [30].

While bulk hyperbolic metamaterials have been realized in acoustics relying on mass anisotropy [18], so far there has been no concept nor implementation of acoustic hyperbolic metasurfaces. The reason lies in the symmetry constraints stemming from the longitudinal nature of sound waves outlined above: the acoustic metasurface properties are described by the impedance relation \( p = Z_s v_n \), where \( Z_s \) and \( v_n \) indicate surface impedance and particle velocity normal to the surface, respectively. The scalar nature of acoustic pressure and normal velocity restricts the acoustic surface impedance to be inherently scalar and isotropic, and hence prevents the realization of hyperbolic metasurfaces for sound, which would inherently require extreme anisotropy in the surface plane. While the discovery of metamaterials and metasurfaces have extensively enriched the portfolio of material and impedance properties for acoustic waves, such as negative bulk modulus [33] and density-near-zero propagation [34], the nature of acoustic wave propagation appears to forbid the realization of anisotropic bulk modulus and surface
impedances, challenging the implementation of hyperbolic propagation over a surface.

**Anisotropic impedance stemming from nonlocality.** The introduction of nonlocal responses over an acoustic metasurface allows relaxing the mentioned symmetry constraints, and enables the implementation of acoustic anisotropic metasurfaces, opening a path towards extreme anisotropic responses in the surface impedance [35]. In particular, we consider strong coupling and energy tunneling between neighboring unit cells over a surface, which introduces a new relation governing acoustic wave propagation over a surface. Different levels of anisotropy can be achieved in this regime by simply modulating coupling and energy tunneling in different directions, including extreme values that open to the possibility of hyperbolic sound transport over a surface. These same features also allow surface wave propagation with negative energy velocity. Different from previously reported bianisotropic metasurfaces using Willis resonant coupling [36], here anisotropy is induced through nonlocality, enabling a direct extension of electromagnetic anisotropy to acoustics.

Figure 1(a) sketches the usual definition of acoustic surface impedance, in which acoustic microstructures vibrate locally and separately from each other [37]. These microstructures, which may be formed by Helmholtz resonators [38], acoustic maze-like structures [39] or impedance tubes [40], have been extensively applied to the design of acoustic metasurfaces, achieving several functionalities, such as anomalous reflection, focusing, beamforming, enhanced absorption and more [41],[42]. All these microstructures, no matter how complex, can be mathematically described by their local impedance, defined through the impedance relation \( p = Zv_n \). This relation only considers the local response of these microstructures and discards coupling between unit cells. Strictly speaking, this formula is only valid when the microstructures are deeply subwavelength, and with ideal rigid boundaries. However, it works
well also in modeling realistic scenarios, and the reported theoretical and experimental results in various metasurface geometries using this approach match each other very well [37]-[42], even for underwater metasurfaces in which the structure cannot be treated as rigid due to the high impedance of water [43]. This is mainly because the coupling between neighboring cells is typically negligible. An interesting way to overcome the scalar nature of the surface impedance and break symmetry consists in enabling and controlling the strong coupling between neighboring cells.

Figure 1(b) depicts the proposed nonlocal metasurface, in which extra paths are opened to connect adjacent microstructures so that acoustic energy can tunnel from one unit cell to the next, transverse to the surface. In this scenario, the impedance relation can be described as [44]

\[ p = Z_s y_x + Z_s \frac{\partial (Y_x y_x)}{\partial x} + Z_s \frac{\partial (Y_y y_y)}{\partial y}, \]  

(1)

where \( Z_s \) is the local acoustic impedance, which describes the individual microstructures in blue in Fig. 1(b). \( Y_x \) and \( Y_y \) are the coupling coefficients, which quantify the coupling strength between units in the \( x \)- and \( y \)-directions, as indicated by the red and green channels, respectively. Eq. (1) is generally applicable, but, if we limit ourselves to coupling through the mentioned channels, the coupling coefficients satisfy \( Y_x = \frac{i \omega \rho_0 \Delta x}{(Z_x \Delta y)} \) and \( Y_y = \frac{i \omega \rho_0 \Delta y}{(Z_y \Delta x)} \), respectively, where \( Z_x \) and \( Z_y \) are the acoustic impedance of the microstructures in the \( x \)- and \( y \)-directions [44] and \( \Delta x \), \( \Delta y \) are the corresponding periods. By tailoring the coupling coefficients, we can mold strong transverse responses in the metasurface, which have been neglected in previous metasurface implementations for sound, opening the path towards nonlocal and anisotropic responses over an acoustic surface.
**Negative-index airborne surface acoustic waves.** Eq. (1) opens a new degree of freedom to tailor acoustic metasurfaces. In the following, we limit ourselves to homogeneous scenarios, i.e., \( \partial Y_x/\partial x = 0 \) and \( \partial Y_y/\partial y = 0 \). In this case, Eq. (1) can be simplified as

\[
p = Z_s v_x + Z_s Y_x \frac{\partial v_x}{\partial x} + Z_s Y_y \frac{\partial v_y}{\partial y}.
\]  

(2)

Consider an airborne acoustic surface wave traveling along the metasurface in the \( x \)-direction, with pressure \( p = p_0 e^{-i\omega t} e^{ik_x x} \) and normal velocity \( v_x = -v_z = \gamma p_0 e^{-i\omega t} e^{ik_x x}/(i\omega \rho_0) \). For a traditional impedance surface, i.e., \( Y_x = 0 \), the impedance satisfies \( Z_s = p/v_x = i\omega \rho_0/\gamma \). Only when \( \gamma > 0 \), the surface acoustic wave can be excited, thus the impedance must be capacitive, i.e., \( Z_s/i = \omega \rho_0/\gamma > 0 \), to support surface wave propagation. However, if we consider nonlocal coupling, i.e. \( Y_x \neq 0 \), according to Eq. (2), we find the new dispersion relation \( Z_s/i = \omega \rho_0/(\gamma - Y_x k_x^2) \). For \( Y_x > \gamma/k_x^2 \), then \( Z_s/i < 0 \), indicating that an inductive surface can now support surface acoustic waves based on nonlocal interactions.

Figure 2(a) presents three unit cells of the proposed nonlocal anisotropic metasurface, in which a coupling path is opened only in one direction. The left part, indicated by the blue color in each unit, is a local impedance tube, widely used in conventional acoustic metasurface designs [40]. Here, we intentionally design the impedance tube to support an inductive response at the frequency of interest, by controlling its length. The right part, indicated by green color in each unit, is an additional path connecting adjacent local impedance tubes, which enables energy tunneling from one tube to the next, yielding strong nonlocality. Figure 2(b) presents the acoustic pressure distribution for a line source excitation along the \( y \)-direction, which launches a wave
traveling along $x$ (the direction of the coupling paths, $Y_x \neq 0$, $Y_y = 0$). As predicted by our theory, a surface acoustic wave travels along the metasurface in the $x$-direction, despite the inductive nature of the surface elements. To make sure that the surface wave is indeed supported by the metasurface nonlocality, we remove the coupling paths and analyze the corresponding pressure distribution [Figure 2(d)]. The result clearly shows that no surface wave can be excited over a local inductive metasurface, verifying that the surface acoustic wave excitation in Fig. 2(b) is due to the metasurface nonlocality. In order to verify the anisotropic properties of the metasurface, we rotate each unit cell by 90 degrees, so that the coupling paths are oriented along $y$ (i.e., $Y_x = 0$, $Y_y \neq 0$). Now the coupling paths have no effect on waves propagating along $x$, and hence surface acoustic waves are not expected for this excitation, as verified in Fig. 2(e).

The grey arrows in each panel of Fig. 2 indicate the phase velocity direction. In Fig. 2(b), we find that the phase velocity points towards the source, suggesting propagation with a negative phase index. Negative index propagation is common in bulk metamaterials, supporting an energy flow away from the source, while the phase velocity points towards it [45]. However, negative index propagation over metasurfaces is more challenging: Fig. 2(c) presents the $x$-component of the Poynting vector along the surface for the design shown in Fig. 2(b), indicating that, interestingly, the surface wave energy flow above the metasurface also points towards the source. Intuitively, this phenomenon appears to violate causality and energy conservation. To gain physical understanding into this phenomenon, we analytically calculate the power flow of the surface wave in the $x$-direction above the metasurface

$$P_{surf} = \frac{1}{2} \int_0^\infty \text{Re} \left( p v_x^* \Delta y \right) dz = \frac{|p_0|^2 k_x}{4 \gamma \omega \rho_0} \Delta y.$$  

(3)
Equation (3) indicates that $P_{surf}$ and $k_x$ have the same sign, i.e., the direction of power flow and phase velocity should be the same, confirming that a negative phase velocity corresponds to a negative energy velocity flowing of the surface wave towards the source, as shown in Fig. 2(b) and 2(c). This paradox can be resolved by considering nonlocality: since the metasurface is nonlocal, additional power can flow through the coupling paths, which contributes to the total energy flow [44]:

$$P_{tunnel} = \frac{1}{2} \text{Re}(pU_x^*) = -\frac{|p_0|^2 k_x Y_x}{2\omega \rho_0} \Delta y .$$

The total power flow is then the sum of the power traveling above the metasurface and the tunneling power

$$P_{tot} = P_{surf} + P_{tunnel} = \frac{|p_0|^2 k_x Y_x}{4\omega \rho_0} \Delta y \frac{1-2\gamma Y_x}{\gamma} .$$

Eq. (5) represents the power flow expression when considering nonlocality. From Eq. (5), we find that, when the term $(1-2\gamma Y_x)$ is positive, (i.e. $Y_x < 1/(2\gamma)$), $P_{tot}$ and $k_x$ have the same sign, indicating positive index propagation. When the term $(1-2\gamma Y_x)$ is negative, (i.e., $Y_x > 1/(2\gamma)$), however, $P_{tot}$ and $k_x$ have opposite signs. Since $P_{tot}$ indicates the total power radiated from the source, which is positive, $k_x$ is actually expected to be negative, requiring backward surface wave propagation, and yielding an effective negative index propagation driven by nonlocality. According to Eq. (3), the power flow of the surface wave above the metasurface $P_{surf}$ is also negative. This phenomenon is enabled by the strong energy flow, with opposite sign, sustained in the coupling channels responsible for the strong nonlocality. Figure 2 corresponds to the
situations in which the surface wave is backward.

**Hyperbolic sound propagation over a surface.** The anisotropic surface wave shown in Fig. 2(b) and 2(d) enables extreme wave propagation features. By introducing an additional tube to the geometry shown in Fig. 2(a), as indicated by the red color, we open coupling paths in both $x$- and $y$-directions. The resulting metasurface geometry, shown in Fig. 3(a), is excited by a point source located in the middle of the sample. A strong surface wave is excited and localized on the metasurface, yielding a hyperbolic profile of the surface wavefronts, with energy localized along with specific directions, shown in Fig. 3(b). The wave, with propagation properties consistent with hyperbolic surface wave propagation in nanophotonics [31], is confined on the surface, and rapidly decays as we move out of the plane. In order to verify the propagation properties of the surface wave $p = \rho_0 e^{-r_y \epsilon} e^{i(k_x x + k_y y)}$, we replace this expression into Eq. (2), and obtain

$$\omega \rho_0 = \sqrt{k_x^2 + k_y^2 - k_0^2}. \quad (6)$$

The asymptotes for $k_x \rightarrow \infty$ and $k_y \rightarrow \infty$ are $k_x = \sqrt{-Y_x / Y_y k_x}$. Thus, in order to achieve hyperbolic surface wave propagation the signs of $Y_x$ and $Y_y$ must be opposite, yielding in principle an unbounded local density of states over the entire frequency range over these two parameters can be maintained oppositely signed.

By modulating the cross-sectional area of the coupling tube (red color) in the $y$-direction, we can modulate the value of $Y_y$, and hence impart different hyperbolic profiles to the dispersion diagram. Figure 3(c) presents the pressure profile for a unit with a smaller cross-sectional area, and hence smaller $Y_y$ compared with Fig. 3(b). Figures 4(a) and (b) present the isofrequency contours of the metasurface shown in Fig. 3(b) and (c), respectively, which confirm a hyperbolic
profile with varying asymptotes. In these plots, the inner region corresponds to higher frequencies, while the outer region corresponds to low frequencies. Hence, the group velocity $\tilde{v}_g = \nabla_k \omega$ in the $x$-direction is opposite to the phase velocity, as indicated by the red arrows. Negative index propagation can also be verified in the Supplementary Animation [44]. According to the asymptote expression $k_y = \sqrt{-Y_x/Y_y} k_x$, for smaller $Y_y$, the asymptote gets closer to the $y$-direction. However, compared with Fig. 3(b), surface wave propagation in Fig. 3(c) is more focused in the $x$-direction. This is because the phase velocity direction is different from the group velocity in hyperbolic surfaces, as shown in the isofrequency contour in Fig. 4. In the extreme case $k_x \to \infty$, $k_y \to \infty$, the group velocity is perpendicular to the phase velocity. Hence, an asymptote closer to the $y$-direction implies more energy flowing towards $x$. This is indeed verified in Fig. 3(d), in which we consider the extreme scenario $Y_y \to 0$, i.e., no coupling in the $y$-direction. In this case, the asymptote $k_y = \sqrt{-Y_x/Y_y} k_x$ becomes parallel to the $y$-axes and energy propagates in the $x$-direction, corresponding to the canalization regime [46]-[47], in which surface waves propagate without diffraction along the surface in specific directions, with exciting opportunities for imaging. The corresponding isofrequency contour is shown in Fig. 4(c), confirming the canalization properties of the surface.

In conclusion, in this work we have shown that strong nonlocal responses enabled by tunneling channels connecting neighboring unit cells in acoustic metasurfaces can overcome the inherent isotropy of the acoustic surface impedance imposed by symmetry constraints in sound propagation, and enable extreme acoustic anisotropy driven by nonlocality over a surface. This new degree of freedom opens several opportunities for surface acoustic waves, e.g., wave propagation above inductive surfaces, hyperbolic surface wave transport, negative phase, and
energy velocities of surface waves. These findings can be an important tool for the design of new surface acoustic wave devices and acoustic metasurfaces. This work has been supported by the National Science Foundation and the Simons Foundation.

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Fig. 1. (a) Traditional design of acoustic local metasurface, whose units vibrate locally and separate from each other. (b) Anisotropic metasurface with coupling effects between units in different directions.
Fig. 2. (a) Three unit cells of the nonlocal anisotropic metasurface. (b) Pressure distribution for a $y$-directional line source exciting acoustic waves traveling along the $x$-direction, with $x$-direction coupling paths. (c) $x$-direction Poynting vector distribution. On the surface, the energy flows towards the source. (d) Pressure distribution for a $y$-directional line source exciting above an inductive metasurface with only local impedance tubes. No surface wave can be excited. (e) Pressure distribution for a $y$-directional line source exciting acoustic waves traveling along $x$ with $y$-direction coupling paths.
Fig. 3 (a) Hyperbolic acoustic metasurface based on nonlocality. (b and c) Pressure distribution of the hyperbolic metasurface designed with coupling tubes with different cross-sectional area in $y$-directions. (d) Pressure distribution of hyperbolic metasurface working in the canalization region by completely removing the coupling tube in the $y$-direction.
Fig. 4. (a, b, c) Dispersion relation for the metasurface shown in Fig. 3 (b,c,d), respectively. The dispersion relation refers to the modes supported by the structure, which include the fields traveling over the surface and those that tunnel underneath. The dispersion relation confirms hyperbolic propagation with varying asymptotes, controlled by nonlocality.