Confinement in Nuclei and the Expanding Proton

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Confinement in Nuclei and the Expanding Proton

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High-precision knowledge of electromagnetic form factors of nuclei is an important current activity in nuclear and atomic physics. Such precision mandates that effects of the non-zero spatial extent of the constituent nucleons be treated carefully. A series of simple, Poincare-invariant, composite-proton models that respect the Ward-Takahashi identity and in which quarks are confined are used to study such effects. All of the models display a general theorem showing how medium modification of proton structure must occur. Combining this result with lattice QCD calculations leads to a conclusion that a bound proton must be larger than a free one.

Nucleons are composite particles made of quarks, gluons and quark-pairs bound by the confining forces of QCD. The composite nature means that nucleons bound in nuclei must be different than free ones. Many years of experiment and theory tells us the answer: the differences exist but are not very large. Early evidence was found in the EMC effect [1, 2] and also in kaon-nucleus scattering [3]. Recent reviews are Refs. [4–7]. The present manuscript presents a new approach to medium modification of the nucleon wave function that is related both to experiment and to lattice QCD calculations. The key result is that the proton gets bigger when it is bound in a nucleus.

The focus here is on elastic electron-nucleus scattering which has the simplifying feature that the initial and final nuclei are in the same quantum state. Elastic electromagnetic form factors of nuclei can be compared with *ab initio* nuclear structure calculations. For example, [8] measures isotope shifts in the radii of Ca isotopes to better than 1% accuracy. New muonic atom measurements [9] that determine the charge radii of light nuclei are now at about the 1% level. Furthermore, a planned Jefferson Laboratory experiment [10, 11] aims to measure the the difference between the charge radii of ^{3}He and ^{3}H to a precision of ± 0.02 fm. These high precision goals create a need to improve the treatment of the effects of the spatial extent of constituent nucleons.

This is because the nuclear electromagnetic form factor $\mathcal{F}_A(Q^2)$ has been approximated as:

$$\mathcal{F}_A(Q^2) = F_A(Q^2) G_E(Q^2), \tag{1}$$

where a spin-0 nucleus absorbs a space-like photon of four momentum q^μ, and $Q^2 = -q^2$, $G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$ is the proton Sachs electric form factor, where $F_{1,2}$ are Dirac and Pauli form factors (other charged particles are ignored here for simplicity). $F_A(Q^2)$ is the probability amplitude for a point proton to absorb momentum without changing the nuclear state and M is the proton mass. Eq. (1) is denoted as the *factorization approximation.*

The only derivation of Eq. (1) [12, 13] is based on non-relativistic classical physics. A quantum mechanical result is obtained by assuming that only free form factors, $F_{1,2}(Q^2)$ appear. The factorization approximation has been widely used even though it cannot be completely accurate because the struck protons are bound in nuclei. No examination has appeared in the literature.

Here I construct a diverse set of models of the free proton and then place that proton in the nucleus. Elastic electron-proton scattering is shown in Fig. 1a. In free space, $p^2 = p'^2 = (p+q)^2 = M^2$. The initial and final protons are on their mass shell. Suppose instead the proton is bound in the nucleus (see Fig. 1b). Interactions with nuclei involve evaluating Feynman graphs containing an integral over the four-momentum p of the initial nucleon that ranges over all possible values of p^2 from $-\infty$ to ∞, and the equality between square of the four-momentum and M^2 is not maintained. The nucleon form factors should depend on $\gamma \cdot p$ and $\gamma \cdot p'$ and functions (such as $(\gamma \cdot p)^2 = p^2$) thereof [14].

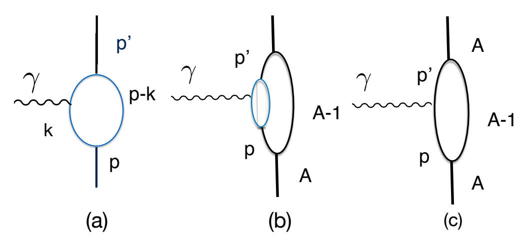

FIG. 1. Photon-nucleon electromagnetic interaction. (a) Photon hits quark in a free nucleon. (b) Photon hits quark in bound nucleon. c) Photon hits quark in a bare proton bound in the nucleus.

As a result medium modifications of nucleon structure may be determined by the virtuality, $V \equiv p^2 - M^2 = p'^2 - M^2$ (via Lorentz and time-reversal invariance) for elastic scattering on nuclei. The average value of the virtuality can be computed from the spectral function [15], but nuclear wave functions are not presented as a function of specific values of V. Therefore a first-order expansion in powers of V is used. The small nature of the binding energy might seem to justify neglecting differences between p^2, p'^2 and M^2. However, a better estimate can be obtained from the Schroedinger equation. For example, within the nuclear Hartree-Fock approximation a single particle wave function obeys the Schroedinger equation, with a dominant central binding potential $\mathcal{U}(\ll M)$. Therefore $\vec{p}^2/(2M) + \mathcal{U} = -B$, where $B > 0$ is the binding energy and $p^0 = M - B$. Then $p^2 - M^2 = (M-B)^2 - \vec{p}^2 - M^2 \approx 2M\mathcal{U}$. In the centers of typical nuclei $\mathcal{U}$ is about -50 MeV [16], so that $(p^2 - M^2)/M^2 \approx -0.1$, significantly different from zero, but small enough to be considered an expansion parameter.

The detailed study of the factorization approximation begins by evaluating the Feynman graphs of Figs. 1 a,b. The aim is to compute the dependence on the off-mass-shell invariants that appear in the nucleus. The calculations are done so that the Ward-Takahashi identity, which guarantees current conservation, is respected. For the present models, including the diagram of Fig. 1c along with that of Fig. 1b is necessary for this to occur. Furthermore, the models must embody confinement. These two aspects are dealt with below to arrive at the key result:

$$\Delta F_{1,2} = V \frac{\partial F_{1,2}}{\partial M^2}. \tag{2}$$

How can a property of the proton depend upon its mass, a value known to very high precision? The proton mass can be varied at will in different models. Eq. (2) is obtained when M^2 is associated with the four-momentum squared that appears in propagators of the Bethe-Salpeter equations determining the wave functions of the models used below. Moreover, results of fundamental lattice QCD calculations of nucleon properties depend implicitly on the proton mass via quark-mass dependence. In this paper, lattice QCD calculations are used only to provide input, not to test the idea of proton expansion itself.

Eq. (2) accounts for many effects that can be cast in the form of a modified form factor times the virtuality. It is very compact, sums a set of significant contributions, and arises naturally from using relativistic dynamics. It does not include the interactions between photons and charged mesons that are exchanged between two nucleons and the effects of non-nucleonic baryon components (such as the Δ-isobar) of the nuclear wave function.

Next I explain how Eq. (2) is derived. Five different models of the free proton are used:

- Quark-diquark, with spin 0 quarks and di-quarks with a scalar vertex function
- Quark-diquark with spin 1/2 quark, spin 0 di-quark with a scalar vertex function
- Quark-diquark with spin 1/2 quark, spin 1 di-quark with a vector vertex function (QED)
- Proton sometimes fluctuates into its neutron-π^+ component, pseudovector coupling.
- Proton sometimes fluctuates into a component consisting of a Δ-isobar and a pion, pseudovector coupling.

None of these models is realistic by itself, but each characterizes a significant aspect of proton structure.

Evaluating the Feynman graph of Fig. 1a for general off-shell kinematics renders it suitable for inclusion in Fig. 1b. The first-order approximation in V allows the separate study of each term that contributes to medium modifications. The models employed here share common features, so that the generality of Eq. (2) can be displayed by discussing only the salient aspects of the models. For each, the proton wave function involves a vertex function that converts a proton of momentum to a system of two constituents. One of the constituents, denoted by c is charged and interacts with the photon, and the other, denoted by d is a spectator. This notation is used for both quark-spectator models and pion-spectator models. In each model the three propagators provide a denominator of the form: $D \equiv (k^2 - m_c^2)((k+q)^2 - m_c^2)((p-k)^2 - m_d^2)$. These are combined with three Feynman parameters x, y, z respectively, such that $x + y + z = 1$, with a useful symmetry between x and y. The factor D can thus be re-written: $D \to (k^2 - \Delta)^3$, as

$$\Delta = xyQ^2 + m_c^2(x+y) + zm_d^2 - \frac{p^2 + p'^2}{2} z(1-z). \tag{3}$$

The on-mass-shell value of Δ, denoted as $\Delta_{\rm on}$ is obtained by replacing p^2 and p'^2 by M^2. By adding and subtracting the term $M^2 z(1-z)$ one obtains:

$$\Delta = (1 - V\frac{\partial}{\partial M^2})\Delta_{\rm on}, \tag{4}$$

The model-specifc scattering amplitudes depend on inverse powers of Δ (no terms involving $\log \Delta$ arise because Pauli-Villars regularization is used), so that one uses $1/\Delta \approx 1/\Delta_{\rm on}(1 + \frac{V}{\Delta_{\rm on}} \frac{\partial}{\partial M^2} \Delta_{\rm on})$ and the denominator terms are seen to give one set of contributions to Eq. (2).

The terms in the numerator take many forms including: $\not p$, $\not p'$, p^μ, $p'^\mu = (p+q)^\mu$, $2k^\mu$, $k \cdot k'$, $\not k \not k'$, $(k' \equiv k + q)$ where μ is the Lorentz-index of the photon-quark (or photon-pion) vertex. Let's start with the term $\not p$, which is re-written to first-order in V as follows:

$$\not p = M + \frac{p^2 - M^2}{\not p + M} \approx M + \frac{V}{2M} = (1 + V\frac{\partial}{\partial M^2})M, \tag{5}$$

and the pattern emerges. The same manipulations can be done for $\not{p}'$. Another term that enters is p^μ. Calculations are done in the Breit frame, with $\mu = 0$ or in the Drell-Yan frame with $\mu = +$. Then the identity $2p^\mu = \gamma^\mu \not{p} + \not{p}'\gamma^\mu + i\sigma^{\mu\nu} q_\nu$ is useful because the manipulations for $\not{p}$, $\not{p}'$ described above are applicable. The term involving $\sigma^{\mu\nu}$ contributes only to the on-mass-shell part of F_2.

The models involving struck pions contain a numerator term of the form $2(k^\mu + p^\mu z - q^\mu) \rightarrow 2p^\mu z$ because of parity and using either of the two mentioned frames. The model with an intermediate $\Delta-$isobar contains terms of the form $k \cdot k'$ and $\not{k}\not{k}'$. Upon applying the stated variable transformations, one finds

$$k \cdot k' \rightarrow k^2 + z^2 p \cdot p' + \frac{Q^2}{2} z(1-z). \tag{6}$$

The k^2 term is evaluated along with the denominators that are discussed above. The third term does not involve off-shell proton kinematics. The term $p \cdot p'$ may be re-written as $q \cdot p + p^2 = \frac{1}{2}(p'^2 + p^2 - q^2)$, and subtracting and adding $2M^2$ leads again to the result of Eq. (2). The manipulations needed to handle the term $\not{k}\not{k}'$ are essentially the same, upon using Eq. (5).

The net result is that Eq. (2) emerges from each term. The general argument is that for each of the terms that enter one may add and subtract the on-shell expression. To first-order in V : all terms in the difference between the on-and off-mass-shell expressions can be expressed as a derivative.

It is necessary to maintain the Ward-Takahashi (WT) identity [17], stating that the amplitude $\Gamma^\mu(p+q,p)$ for a photon of momentum q to be absorbed by a fermion of momentum p is related to the fermion-propagator $S(\not{p}) = \frac{1}{\not{p} - M_0 - \Sigma(\not{p})}$, via

$$q \cdot \Gamma(p+q,p) = S^{-1}(\not{p} + \not{q}) - S^{-1}(\not{p}), \tag{7}$$

where M_0 the bare mass and $\Sigma(p)$ the self-energy of the fermion. If this is respected, electron-nucleus interactions will satisfy current conservation. A similar identity is obtained for photon-pion interactions. Satisfying the WT identity is necessary for high-precision nuclear calculations to be valid.

If one evaluates the term of Fig. 1 (a), in which the photon-quark interaction is denoted as $\Gamma^{(q)}$, one finds:

$$q \cdot \Gamma^{(q)} = \Sigma(\not{p}) - \Sigma(\not{p}'), \tag{8}$$

and the right-hand-side vanishes for on-mass-shell kinematics ($p^2(p'^2) = M^2$). The graph of Fig. 1a is a reasonable model for free protons, but when the proton is bound to a nucleus (Fig. 1b) the WT identity is not respected. This problem is fixed by including the graph of Fig 1c. In that case one obtains $q \cdot \Gamma = (\not{p} + \not{q}) - \not{p} - (\Sigma((p+q)^2) - \Sigma(p^2))) = S^{-1}(p+q) - S^{-1}(p)$. The first two terms arise from Fig. 1c, and the next two from Fig. 1b.

The next step is to handle quark-confinement. Detailed evaluations of the Feynman graph of Fig. 1b fail dramatically to obey the factorization approximation, Eq. (1), if the quark propagator is that of a free quark. To see this, examine Eq. (3). For on-mass-shell kinematics the value of $\Delta > 0$ for all values of x, y and z provided the stability condition $M < m_c + m_d$ is obeyed. A similar stability condition holds for pion-baryon intermediate states. In evaluating the Feynman diagram of Fig. 1b, one integrates over all values of p^2, Δ can be negative. and the in-medium proton form factor is complex-valued. The free form factor is real-valued, so the factorization approximation Eq. (1) must break down. Moreover, the singularity associated with lack of confinement plays havoc in numerical integration, and the existence of such singularities in models is unphysical because nuclei are stable. Finally, the appearance of zeros in Δ means that an expansion of nucleon properties in terms of the virtuality cannot converge.

Negative values of Δ can also be understood by examining the proton self-energy, $\Sigma(p'^2)$ which involves the denominator $(k^2 - m_c^2)((p'-k)^2 - m_d^2) \rightarrow (k^2 + p'^2 u(1-u) - m_c^2(1-u) - m_d^2 u)^2$, where u is another Feynman parameter with $0 \leq u \leq 1$. This denominator has zeros for values of p' such that $p'^2 > (m_c + m_d)^2$, which is the condition required to knock a quark out of the proton. In Fig. 2, the final q and d can both be on the mass shell whenever $p'^2 > (m_c + m_d)^2$, which would not occur for systems respecting confinement.

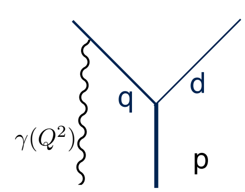

FIG. 2. (Color online) Photon of momentum $q^2 = -Q^2$ hits quark in a free proton of four-momentum p. Final quark and di-quark can both be on the mass shell.

Implementing the main feature (no singularities) of confinement must be included in the present models. This is done using quark (or di-quark) masses that occur in complex conjugate pairs, as summarized in the review [18] and used in Refs. [19]-[26]. Using a di-quark (spectator) propagator of the form

$$S_C(p) = \sum_{\lambda=-1,1} \frac{1}{p^2 + m_d^2 + i\lambda\epsilon}. \quad (9)$$

in Euclidean space removes the unphysical singularities. The previous analysis of the effects of virtuality has been applied using Eq. (9) to the models discussed above with the result Eq. (2). Furthermore, detailed Euclidean space calculations using the models described above have shown that the results of using such a propagator can be obtained in Minkowski space by simply using a complex di-quark mass and obtaining the form factors by taking the real part of the computed amplitude. The net result is that using complex-valued quark masses removes the unphysical singularities initially present in the simple models used here. This is necessary to justify expansions in terms of virtuality.

The first application of the key result, Eq. (2) is to study the proton charge radius [27] $r_E^2 \equiv -6G'_E(0)$. Eq. (2) leads to a change in r_E^2 given by $\delta r_E^2 = V \frac{\partial r_E^2}{\partial M^2}$ Evaluation requires knowing how the proton radius depends on its mass. This derivative is negative for the five presented models. The virtuality must have negative values, so the proton radius must expand when it is bound in a nucleus!

For the models presented here an off-mass-shell proton is equivalent to an on-mass-shell proton of a mass less than M. This is because off-shell effects of the energy denominator Δ of Eq. (3) (in which p^2 replaces the M^2 that appears as an eigenvalue of the Bethe-Salpeter eqaution) and the off-shell effects of the numerator end up looking like Eq. (2). Based on obtaining Eq. (2) in all of the models, I assume that it is a generally valid, first-order treatment of virtuality and discuss the necessary derivative in a broader framework.

In the MIT bag model [28] (with vanishing quark masses) the bag radius is inversely proportional to the mass of the nucleon, leading to $\frac{M^2}{r_E^2}\frac{\partial r_E^2}{\partial M^2} = -1$. The counterpoint is the non-relativistic quark model *e.g* Ref. [29], in which harmonic oscillator confinement is used with the size parameter: $b^2 \propto 1/m_q$. This leads to $\frac{M^2}{r_E^2}\frac{\partial r_E^2}{\partial M^2} = -1/2$. In Ref. [30] the dominant isovector contribution to the square of the nucleon radius is proportional to $\ln M/m_\pi$, where m_π is the pion mass. Using Eq.(4.2) therein leads to $\frac{M^2}{r_E^2}\frac{\partial r_E^2}{\partial m_\pi^2} = -0.6\frac{M^2}{m_\pi^2}$, potentially a very large effect. This idea has been developed further: *e.g.* [31]. In each of these more general models the proton mass increases with increasing pion mass so again the derivative $\frac{\partial r_E^2}{\partial M^2}$ is negative. Because $V < 0$ one again finds that the radius of a bound proton must be larger than that of a free one.

It is natural to turn to lattice QCD calculations of the proton radius because hadronic properties are computed as a function of quark masses (via the pion mass). One could expose the proton to an attractive, strong, external scalar field, ϕ, interacting with quarks, that is constant over the proton volume. An external vector field would not change the quark wave function [44–47]. The ϕ acts as a central nuclear potential giving bound nucleons their non-zero virtuality. One can calculate the proton radius using various values of ϕ. But a constant external scalar field acting on quarks is equivalent to shifting the quark masses. Thus one needs only to compute the radius as a function of the quark mass to evaluate the expansion. Lattice QCD results provide a more precise evaluation than the widely-used models [44–47] that require modeling of quark confinement.

Lattice QCD calculations of the proton charge radius have made significant recent progress [33–40], but various technical difficulties cause results to typically undershoot experiment by about 25%. The isovector radius is easier to calculate and dominant because the square of the proton charge radius is significantly larger than that of the neutron. Ref. [33, 34] computed the isovector radius for pion masses from 135 to 320 MeV using an analytic parametrization of the m_π^2 dependence. Using their formula gives $\frac{\partial r_E}{\partial m_\pi^2} \approx -2.6 \pm 0.3\,\mathrm{fm\,GeV}^2$. The nucleon mass is well-described as a function of the pion mass as $M \approx M_0 + 1.14\,\mathrm{GeV}^{-1} m_\pi^2$ [42] and this dependence in m_π^2 is expected [43]. Using the results [33, 34] one finds

$$\delta r_E = V \frac{\partial r_E}{\partial m_\pi^2}\frac{\partial m_\pi^2}{\partial M^2} = -\frac{V}{M^2}(1.1 \pm 0.1)\,\mathrm{fm}, \quad (10)$$

with the only stated uncertainty arising from $\frac{\partial r_E}{\partial m_\pi^2}$. Taking $V/M^2 = -0.1$ (the value at the nuclear center) leads to an increase of the proton radius by about 0.11 fm or about 12%. The sign is well-determined as the product of two numbers that are each strongly constrained to be negative. The magnitude is a reasonable estimate, improvable by future lattice calculations.

The expansion can be measured in six ways [7]. Our compact formulation encompasses all of the models cited therein. A 12% increase in the proton radius is a rather large effect, and one might wonder why it is has not already been seen. Most previous attempts use quasi-elastic scattering [48–51] in which In the final-state proton is essentially free. Thus any effect would be reduced by a factor of two, even before accounting for reductions caused when the reaction occurs near the nuclear surface. The current effect is not ruled out.

Our increase in radius represents a violation of the extensively-used factorization approximation Eq. (1).

Its importance can be understood by examining three-nucleon systems [10, 11]. The current experimental status is in Refs. [52, 53], and that of the theory is in [54, 55]. The standard procedure [56] for computing nuclear charge radii is to expand each of the terms of Eq. (1) to first-order in Q^2, yielding: $R_A^2 = R_{\rm pt}^2 + r_E^2$ (if the neutron contribution is neglected). The average virtuality for ^{3}He is reported [15] as $V/M^2 = -0.073$. Using this and Eq. (10), the computed shift in the proton radius is 0.08 fm. Using $R_{\rm pt} = 1.54$ fm [55] and changing r_E from 0.84 fm to 0.92 fm corresponds to a 2% increase in the computed ^{3}H charge radius. The 2% is comparable to present experimental uncertainties, but future experiments [9, 10] plan on achieving better than 1%. The increase of 0.08 fm is much larger than changes caused by meson exchange currents or variations in cut-offs of chiral perturbation theory [55]. This expansion is testable.

Ref. [11] has already achieved high-accuracy measurements of the Ca isotopes charge radii. It reports "unexpectedly large charge radii", based on discrepancies between their measurements and nuclear theory results. Confronting the present expansion idea with these data requires a new development in nuclear theory, namely the precision calculation of virtuality.

This paper treats nuclear medium effects on electromagnetic form factors. The calculations have many general features, so that one may speculate that Eq. (2) extends to other matrix elements of other one-body operators, $\mathcal{O}$ such that $\langle\mathcal{O}(p^2)\rangle \approx \langle\mathcal{O}(M^2)\rangle + (p^2-M^2)\frac{\partial}{\partial M^2}\langle\mathcal{O}(M^2)\rangle$. $\mathcal{O}$ could represent deep-inelastic scattering, and the present formulation could lead to an improved understanding of the EMC effect. The simplicity of this relation is very appealing. If valid, describing a wide variety of medium effects from the unified viewpoint of examining the dependence on virtuality would be possible.

Our calculations show that, for many models, a bound proton is larger than a free one. The necessary derivatives with respect to mass that appear in Eq. (2) may be computed using lattice QCD. Perhaps other proton properties can also be treated this way. A new approach to understanding nuclear modifications of nucleon properties that strengthens the connection between lattice QCD calculations and nuclear physics is provided here.

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