Sachdev-Ye-Kitaev Non-Fermi-Liquid Correlations in Nanoscopic Quantum Transport

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SYK non Fermi Liquid Correlations in Nanoscopic Quantum Transport

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Electronic transport in nano-structures, such as long molecules or 2D exfoliated flakes, often goes through a nearly degenerate set of single-particle orbitals. Here we show that in such cases a conspiracy of the narrow band and strong *e*-*e* interactions may stabilize a non Fermi liquid phase in the universality class of the complex Sachdev-Ye-Kitaev (SYK) model. Focusing on signatures in quantum transport, we demonstrate the existence of anomalous power laws in the temperature dependent conductance, including algebraic scaling $T^{3/2}$ in the inelastic cotunneling channel, separated from the conventional Fermi liquid T^2 scaling via a quantum phase transition. The relatively robust conditions under which these results are obtained indicate that the SYK non Fermi liquid universality class might be not as exotic as previously thought.

Introduction: Electronic device miniaturization is now routinely operating at levels where quantum limits are reached. Examples where quantum effects are of key relevance and/or used as operational resources include single molecule transport[1–4], various realizations of qubits[5], and increasingly even commercial applications such as Q-dot display technology. Physically, such nanoscopic devices (henceforth summarily denoted as 'quantum dots') are frequently described[6] in terms of only few collective variables — their cumulative electric charge, a global superconducting order parameter, a collective spin, etc.

Starting from first principle many body representations, this 'universal Hamiltonian' approach[7, 8] is implemented via elimination of microscopic degrees of freedom which in turn rests on statistical arguments[7, 9, 10]. To illustrate the principle in the simplest physical setting, consider a small quantum system with $i = 1, \ldots, N$ electronic orbitals, assumed spinless for simplicity. This setting is described by the Hamiltonian $\hat{H} = \sum_i^N \epsilon_i c_i^\dagger c_i + \sum_{ijkl}^N \tilde{J}_{ijkl} c_i^\dagger c_j^\dagger c_k c_l$, where ϵ_i are the energies of the non-interacting orbitals, and $\tilde{J}_{ijkl}$ are the matrix elements of the particle interactions — generally strong in the case of nanoscopic device extensions. Systems of realistic complexity are typically non-integrable on the single particle level, implying effectively random matrix elements, $\tilde{J}_{ijkl}$. This randomness is usually taken as justification to discard all matrix elements except those with non-zero mean value. Specifically, focusing on contributions with $i = k$, $j = l$, or $i = l$, $j = k$, and assuming approximate equality of diagonal matrix elements on average, one is led to the representation

$$\hat{H} = \sum_i^N \epsilon_i c_i^\dagger c_i + \tfrac{1}{2} E_C\, \hat{n}^2 + \sum_{ijkl}^N J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l, \qquad (1)$$

where J_{ijkl} now excludes matrix elements with identical indices, $\hat{n} = \sum_i^N c_i^\dagger c_i$ is the total charge on the dot and the coefficient $E_C = e^2/C$ defines its effective electrostatic capacitance, C. The standard universal Hamiltonian approach [7, 8] defines $\hat{n}$ as the central collective variable, and ignores the contribution of the random sign matrix elements, J_{ijlk}, to the interaction energy.

In this paper, we caution that the neglect of the term $\hat{H}_{\rm SYK} \equiv \sum J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l$ may be less innocent then is commonly assumed. The point is that $\hat{H}_{\rm SYK}$ is a variant of the complex SYK Hamiltonian [11, 12][13], the latter being defined as an all–to–all interaction Hamiltonian with random matrix elements taken from a zero mean Gaussian distribution with $\langle J_{ijkl}^2 \rangle = J^2/N^3$. The pure SYK Hamiltonian [11, 12, 14–19] defines a universality class distinguished for a maximal level of entanglement, chaos, and non Fermi liquid (NFL) correlations, otherwise shown only by black holes (in 2D gravity the latter are related to SYK model via the holographic correspondence [20–22]). Correlations generated by arrays of SYK cells are increasingly believed[23–27] to be relevant in the physics of strongly correlated quantum matter, and it has been suggested that single copies of SYK-Hamiltonians might describe small sized samples of flat band materials[28]. In the following, we reason that even the low temperature physics of the much more generic class of systems described by the Hamiltonian above can be partially, or even fully governed by the SYK universality class. The latter is the case if the band width, W, of single particle orbitals, ϵ_i, is smaller than the interaction strength, $W < J$. For these values, strong quantum fluctuations generated by $\hat{H}_{\rm SYK}$ render the single particle contribution $\hat{H}_0 \equiv \sum_i \epsilon_i c_i^\dagger c_i$ irrelevant[29]. Conversely, for larger values the fluctuations themselves get suppressed by $\hat{H}_0$. However, even then the presence of $\hat{H}_{\rm SYK}$ shows in extended crossover windows in temperature where the dot shows NFL correlations. In the following we address both cases, focusing on signatures on low temperature transport.

The crucial feature of the SYK Hamiltonian is the presence of a weakly broken infinite dimensional conformal symmetry [12, 30, 31]. This symmetry breaking manifests itself in NFL correlations, and in the emergence of a set of Goldstone modes, which in the present context define a second set of low energy collective variables, $h(\tau)$, besides $\hat{n}(\tau)$. Depending on temperature, and the relative strength of interactions and the single-particle bandwidth, the conspiracy of these degrees of freedom can drive the system into a strongly correlated NFL phase of matter. In quantum transport, the presence of these regimes shows in non-monotonicity of the temperature dependent conductance, $g(T)$, and in power laws $g(T) \sim T^{\alpha}$ different from the T^2 of the Fermi liquid dot.

Symmetries: We start by identifying the symmetries of the system, which in turn determine its low-energy quantum fluctuations. This is best done in a coherent state representation, where the Hamiltonian is expressed via Grassmann valued time-dependent fields $(c, c^\dagger) \to (c(\tau), \bar{c}(\tau))$, depending on imaginary time $\tau \in [0, \beta]$. The system's action $S = \int d\tau (\bar{c}_i \partial_\tau c_i - H(c, \bar{c}))$ is then approximately invariant under the transformations[12, 30, 31]

$$c_i(\tau) \to e^{-i\phi(\tau)} \left[\dot{h}(\tau)\right]^{1/4} c_i(h(\tau)), \tag{2}$$

where the dot stands for the time derivative, and the U(1) phase ϕ is canonically conjugated to the charge operator, $\hat{n}$. The functions $h(\tau)$ are diffeomorphic reparameterizations of imaginary time and as such take values in the coset space $\mathrm{Diff}(S^1)/\mathrm{SL}(2, R)$, where $\mathrm{Diff}(S^1)$ is the set of smooth functions parameterizing the periodic interval of imaginary time and the factorization of $\mathrm{SL}(2, R)$ accounts for a few exact global symmetries of the action). The symmetries (2) are explicitly broken by both, the time derivative in the action, and the single particle contribution in Eq. (1). We first discuss the former and note that the action cost associated with temporal fluctuations of (ϕ, h) reads[12, 30, 31]

$$S_0[\phi, h] = \int d\tau \left[\tfrac{1}{2} E_C^{-1} \dot{\phi}^2 - m\{h, \tau\}\right], \tag{3}$$

where $\{h, \tau\} \equiv (h''/h')' - \frac{1}{2}(h''/h')^2$ is the Schwarzian derivative, the mass $m \propto N/J$ and we assume the following hierarchy of energy scales: $m^{-1} \ll E_c \ll J$[32].

The physical information relevant to our discussion below is contained in the fermion Green functions, $G_{\tau_1,\tau_2} = \langle c_i(\tau_1)\bar{c}_i(\tau_2)\rangle$. The transformations (2) affects the Green functions as $G \to G[\phi, h]$, where

$$G_{\tau_1,\tau_2}[\phi, h] = e^{-i\phi(\tau_1)} G_{\tau_1,\tau_2}[h]\, e^{i\phi(\tau_2)}; \tag{4}$$

$$G_{\tau_1,\tau_2}[h] = -\mathrm{sign}(\tau_1 - \tau_2) \left[\frac{\dot{h}(\tau_1)\dot{h}(\tau_2)}{(h(\tau_1) - h(\tau_2))^2}\right]^{1/4}.$$

Here, the terms in the numerator are consequences of Eq. (2), and the denominator reflects the 'engineering dimension' $\Delta = 1/4$ of fermions in a mean field approach to the SYK Hamiltonian[30, 33]. In the absence of reparameterizations, this leads to the NFL scaling $G_{\tau_1,\tau_2} \sim |\tau_1 - \tau_2|^{-1/2}$ signifying an interaction–dominated theory.

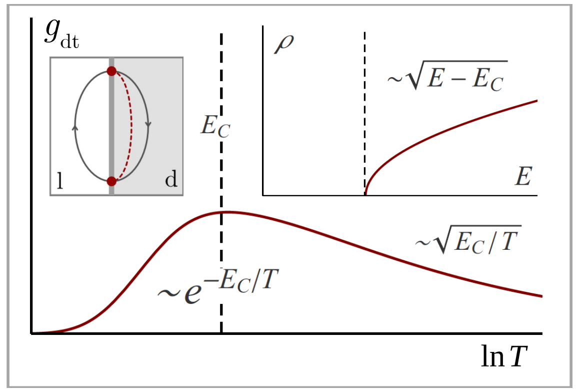

FIG. 1. Main Panel: temperature dependence of the direct tunneling contribution to the conductance. An exponential suppression at temperatures $T < E_C$ gives way to an $T^{-1/2}$ power law at larger temperatures. Left inset: diagram of the direct tunneling process; black lines are Green functions in the lead (l) and the dot (d), fat dots are tunneling vertexes at times $\tau_{1,2}$, respectively, and the red dashed line represents the charging correlation $D(\tau_1 - \tau_2)$, Eq. (5). Right inset: energy dependence of the average tunneling density of states on the dot at $T = 0$.

Isolated dot: For an isolated dot integration over the phase field with the action (9) generates the factor[34, 35]

$$D(\tau_1 - \tau_2) \equiv \left\langle e^{-i\phi(\tau_1)} e^{i\phi(\tau_2)} \right\rangle_\phi = e^{-E_C|\tau_1 - \tau_2|/2}. \tag{5}$$

Fourier transformation of $D(\tau)$ to the energy domain leads to a gap, E_C, in the excitation spectrum, and an exponential suppression of the single-particle density of states, the Coulomb blockade. The second factor $\langle G_{\tau_1,\tau_2}[h]\rangle_h$, which in the same from appears in the charge-neutral Majorana SYK model, has been studied extensively in Refs. [36, 37]. Here, the main observation is a crossover from temporal decay as $|\tau_1 - \tau_2|^{-1/2}$ for intermediate time scales $1 < J|\tau_1 - \tau_2| < N$, to $|\tau_1 - \tau_2|^{-3/2}$ in the fluctuation dominated long time regime, $N < J|\tau_1 - \tau_2|$. This change implies a crossover in the effective fermion dimension from $\Delta = 1/4$ to $\Delta = 3/4$. Fourier transformation of the long time power law reveals the presence of a soft zero-bias anomaly $\propto \sqrt{E - E_c}$ on top of the hard Coulomb blockade gap (see inset in Fig. 1.)

Tunneling conductance: We next consider the system connected to metallic leads via the tunneling Hamiltonian $H_T = \sum_{i,k} V_{ik} c_i^\dagger d_k + h.c.$. Here, d_k are annihilation operators in the normal leads and we assume the matrix elements V_{ik} to be effectively random with variance $\langle |V_{ik}|^2\rangle \equiv v^2$. To second order in perturbation theory, this generates the tunneling action

$$S_T[\phi, h] = -g_0 T \iint d^2\tau \frac{e^{-i\phi(\tau_2)} G_{\tau_2,\tau_1}[h] e^{i\phi(\tau_1)}}{\sin(\pi T(\tau_1 - \tau_2))}, \tag{6}$$

where $g_0 \propto \nu v^2 N/J$ is the bare dimensionless tunneling coupling of the lead-dot interface and ν is the

DoS in the leads. Equation (6) generalizes the celebrated Ambegaokar, Eckern and Schön (AES) [38] action for metallic quantum dots to system with NFL correlations. The difference amounts to the generalization $(\tau_1-\tau_2)^{-1}\to G_{\tau_1,\tau_2}[h]$, and this modified AES approach defines our starting point for the description of low temperature quantum transport.

In the following, we focus on the temperature dependent conductance, $g = g(T)$, as an observable diagnosing the presence of NFL correlations via anomalous power laws. The linear dc conductance through the dot is conveniently obtained by differentiation $g \sim \lim_{\omega\to 0}\omega^{-1}\delta^2 \ln Z[\mathcal{A}]/\delta\mathcal{A}_\omega\delta\mathcal{A}_{-\omega}|_{\mathcal{A}=0}$, of the generating function $Z[\mathcal{A}] = \int \mathcal{D}\phi\mathcal{D}h \exp(-S_0[\phi,h] - S_T[\phi+\mathcal{A},h])$ in a source vector potential $\mathcal{A}(\tau)$, minimally coupled to the tunneling action.

To the first order in the tunneling conductance $g_0 \ll 1$ [39] the calculation of $g(T)$ reduces to that of the Green function, $\langle G[\phi,h]\rangle_{\phi,h}$, of the isolated dot. In this approximation, the conductance $g \simeq g_{\rm dt}$ probes the *direct tunneling* processes amplitude converting a lead quasiparticle into a single-particle excitation of the dot (see the left inset in Fig. 1). With the time dependence of $\langle G_{\tau_1,\tau_2}[\phi,h]\rangle_{\phi,h}$, stated above, one then obtains (cf. Fig. 1)

$$g_{\rm dt}(T) = g_0 \begin{cases} e^{-E_C/T}; & T < E_C, \\ \sqrt{E_C/T}; & E_C < T < J, \end{cases} \tag{7}$$

where the first line is an immediate consequence of the Coulomb gap, Eq. (5). The second one follows from the smallness of fluctuations of both ϕ and h in the high temperature regime $T > E_C > J/N$, implying that the Green function $\langle G_{\tau_1,\tau_2}[\phi,h]\rangle_{\phi,h} \sim |\tau_1-\tau_2|^{-1/2}$ assumes its mean field NFL form[28, 40]. This leads to the non-monotonous temperature dependence of the direct tunneling conductance, as shown in Fig. 1.

Inelastic cotunneling: At low temperatures, $T < E_C$, the direct tunneling transport is taken over by a process of second order in g_0, which escapes exponential suppression. This transport channel, colloquially known as *inelastic cotunneling* [8, 41], $g_{\rm it}$, is a cooperative process where the tunneling of an electron onto the dot creates a virtual state, which relaxes after a short time, $\sim E_C^{-1}$, via the exit of *another* electron into the other lead. For a metallic dot, the corresponding contribution to the conductance reads $g_{\rm it}(T) = g_0^2 T^2/E_C^2$ [41], where the factor T^2 measures the phase volume available for particle-hole creation, and E_C^{-2} is the energy denominator picked up during the virtual excitation of the dot. A calculation outlined in the Supplemental Material shows [42] that for the NFL dot, this result changes to (cf. Fig. 2)

$$g_{\rm it}(T) = \frac{g_0^2}{E_C^2} \begin{cases} \sqrt{NJ}\,T^{3/2}; & T < J/N, \\ JT; & J/N < T < E_C, \end{cases} \tag{8}$$

where the power law at intermediate temperatures reflects the modified temporal exponent $(\tau^{-1})^2 \to (\tau^{-1/2})^2$

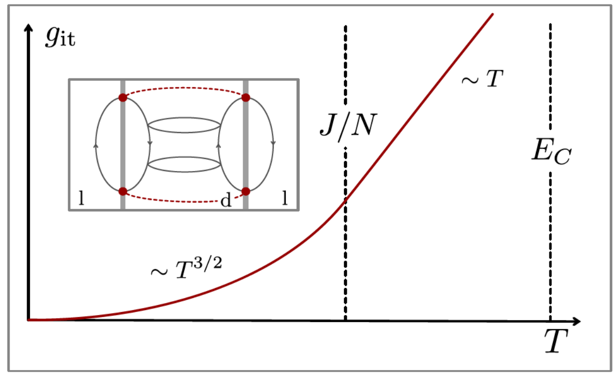

FIG. 2. Main Panel: schematic temperature dependence of the inelastic cotunneling contribution to the conductance. Inset: Inelastic cotunneling diagram, where a particle enters the dot from the left lead, while another particle exits to the right a short time $\sim E_C^{-1}$ after, and NFL 'particle-hole' excitation with the energy $\sim T$ is left behind on the dot.

for 'particle-hole' propagation as described by the NFL mean-field Green function. At lowest temperatures, $T < J/N$, strong reparameterization fluctuations result [36] in the 'particle-hole' propagator in the dot of the form $\langle (G_\tau[h])^2\rangle_h \propto \tau^{-3/2}$, leading to an unconventional fractional exponent $3/2$.

Equations (7) and (8) are our main results for the signatures of NFL correlations in quantum transport. Compared to a metallic system, the existence of the fluctuation scale J/N implies a higher amount of structure, i.e. different power laws, and regimes of non-monotonic temperature dependence. In the following, we ask how stable these findings are against perturbations, notably the presence of the free electron contribution to realistic model Hamiltonians.

Stability of the NFL phase: Turning to the role played by the single particle Hamiltonian, $\hat{H}_0$, we first consider the case of high temperatures $T > J/N$, where fluctuations of the conformal Goldstone modes are inessential. In the absence of $\hat{H}_0$, single particle excitations, then propagate via $G \sim (JT)^{-1/2}$, reflecting the NLF particle dimension, $\Delta = 1/4$. Second order perturbation theory in $\hat{H}_0$ leads to the energy shift $W^2 G \sim W^2/(JT)^{1/2}$, proportional to the square of the single particle bandwidth. This shift should be compared with the self consistent self energy due to interactions, $\sim J^2G^3 \sim J^2/(JT)^{3/2}$[23]. Equating these two scales, we find that for temperatures lower than $T \sim W^2/J$, $\hat{H}_0$ dominates and the dot turns into a FL subject to capacitive interactions. For larger temperatures, between W^2/J and the high energy scale of the SYK Hamiltonian, J, it is in a NFL regime. This window exists provided that $J > W$, which sets a zeroth order criterion for the observability of transport signatures such as such as the second line of (8).

Turning to the more subtle case of low temperatures, $T < J/N$, we reason that in this case there exists a critical value $W_c \propto J/N$ below which the low temperature physics is governed by strong NFL interactions. Conversely, larger band widths stabilize the FL dot. The Existence of this phase transition was first noted by Lunkin, Tikhonov and Feigelman[29]. Paraphrasing their arguments, large W suppresses reparametrization fluctuations, enforcing the mean-field NFL scaling dimension $\Delta = 1/4$. In this case, the single particle term $\sim \int d\tau \bar{c}_i c_i$ carries dimension $\tau^{1-2\Delta} = \tau^{1/2}$, and hence is relevant in a renormalization group (RG) sense. Conversely, if the single particle term is initially weak, fluctuations induce a dimensional crossover $\Delta \to 3/4$. In this case, the dimension of the single particle term is $\tau^{-1/2}$, indicating irrelevancy. The two scenarios must be separated by a critical value, W_c, which, however, this simple argument is not able to predict.

To better understand the transition, we develop its RG treatment along the lines of Ref. [27], where Majorana SYK arrays were considered. We begin by integrating over the fermion fields and averaging the action over a random distribution of ϵ_i to generate the term

$$S_W[h] = w \iint d^2\tau \, G_{\tau_1,\tau_2}[h] G_{\tau_2,\tau_1}[h], \tag{9}$$

where $w = NW^2/J$. Note that the number conservation of the single particle Hamiltonian implies that S_W depends on the field $h(\tau)$, but not on $\phi(\tau)$. Since the tunneling probe action (6) can be assumed arbitrarily weak we need to consider the two running constants m and w of the internal actions Eqs. (3) and (9), respectively. The renormalizability of this theory is safeguarded by its exact $\mathrm{SL}(2,R)$ symmetry, which constrains the form of relevant contributions to the action. In the supplementary material we show that the flow of the two couplings is governed by the equations

$$\frac{d\ln m}{dl} = -1 + \frac{1}{24} wm; \qquad \frac{d\ln w}{dl} = \frac{1}{2}. \tag{10}$$

These equations are best analyzed by defining $\lambda \equiv wm$, which leads to the closed equation $d\ln\lambda/dl = \lambda/24 - 1/2$. This indicates a critical value $\lambda_c = 12$, separating a NFL phase at smaller λ from FL phase at larger λ. In terms of the bare parameters $\lambda \propto (NW/J)^2$ and thus $W_c \propto J/N$.

Phase diagram: A summary of the regimes with different algebraic or exponential temperature dependence of the conductance is shown in Fig. 3. The two essential parameters organizing these regimes are the band width, W, and temperature, T. At zero temperature, a quantum critical point at $W_c \propto J/N$ separates a wide band width Fermi liquid phase renormalized by charging effects from a complementary NFL phase governed by the SYK Hamiltonian. The two domains are separated by a region where the temperature dependence of the conductance and of other observables is determined by the critical exponents of the phase transition. A detailed analysis of this regime is beyond the scope of the present paper.

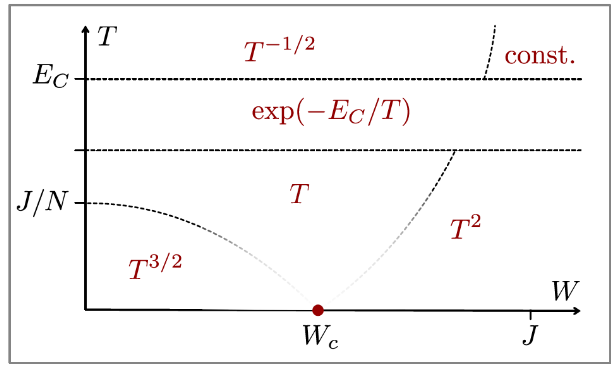

FIG. 3. Summary of the different regimes characterized by algebraic or exponential temperature dependence of the linear conductance through a quantum device with a narrow single particle band, W.

However, off criticality, we expect the formation of robust power laws, provided the dynamics on the single particle level is sufficiently chaotic, the number of involved orbitals is not too large ($W_c \propto N^{-1}$), and interactions are strong.

Summary and discussion: In this paper, we have drawn a bridge between the physics of low capacitance quantum devices and that of the SYK Hamiltonian. The observation that signatures of the SYK universality class might show in a wider class of systems rests on two principles: first, the conspiracy of interactions and chaoticity of single-particle wave functions makes the complex SYK Hamiltonian a natural contribution in the description of nanoscopic quantum systems. Second, at low energies, we then see the emergence of two soft modes, one, $\phi(\tau)$, representing soft fluctuations of the U(1) charge mode, and another, $h(\tau)$, that of conformal symmetry breaking. (The sole difference between the complex SYK system and the Majorana version, featuring in connection with holography, is the presence of the former mode.) As exemplified above, the respective effects of these two modes can be largely separated in the description of physical observables. Specifically, we have seen that low temperature quantum transport is strongly influenced by the infrared physics of the reparameterization mode. At the same time, we have also seen that the observability of these effects requires relatively narrow single-particle bands. Candidate systems where the effects discussed above might become observable include complex molecules[2], 'artificial atoms' based on semiconductor platforms, or exfoliated 2D materials. In view of the results discussed above it would be intriguing to search for signatures of NFL physics in such systems.

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