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Controlling Quantum Transport via Dissipation Engineering

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Inspired by the microscopic control over dissipative processes in quantum optics and cold atoms, we develop an open-system framework to study dissipative control of transport in strongly interacting fermionic systems, relevant for both solid state and cold atom experiments. We show how subgap currents exhibiting Multiple Andreev Reflections – the stimulated transport of electrons in the presence of Cooper-pairs – can be controlled via engineering of superconducting leads or superfluid atomic gases. Our approach incorporates dissipation within the channel, which is naturally occurring and can be engineered in cold gas experiments. This opens opportunities for engineering many phenomena with transport in strongly interacting systems. As examples, we consider particle loss and dephasing, and note different behaviour for currents with different microscopic origin. We also show how to induce nonreciprocal electron and Cooper-pair currents.

Introduction. Understanding and controlling the out-of-equilibrium dynamics of strongly interacting many-body systems constitutes one of the key forefronts in quantum physics across a variety of subfields in experiment and theory. In this context, opportunities to achieve their control via dissipation mechanisms have arisen [1, 2], as is applied for few-body systems in quantum optics [3, 4]. This is especially true in cold-atom platforms, where large separations between frequency scales allows well-controlled theoretical models and implementations of dissipative processes, as realized for laser cooling and trapping [5]. The longer timescales of cold atom experiments also allow dynamics to be tracked and potentially controlled time-dependently [6, 7]. Out-of-equilibrium transport dynamics remain a ubiquitous paradigm in the solid state [8], and recent developments in cold atom systems have also made it possible to engineer quantised transport of atoms between reservoirs, as well as quantum point contacts and waveguides [9–12]. Here we explore the emerging new opportunity of using dissipation engineering to achieve control of quantum transport properties, that are relevant for both cold-atom and solid-state platforms.

We study transport in a system of strongly interacting fermions coupled to weakly interacting reservoirs, as can be reaslied with cold atoms using optical tweezers connecting larger superfluids, or with solid-state devices using quantum dots (QD) coupled to superconducting leads (S). In a traditional S-QD-S tunelling junction, subgap transport is known to be suppressed for weak electron tunneling as compared to the gap of the attached leads [14]. Here we demonstrate that subgap transport can be recovered even in the regime of weak tunnelling. This is done via reservoir engineering that allows for independent control of Cooper-pair and single-electron channels. Such channel separation can be accomplished in the solid state by adding two large-gap superconductors to a traditional S-QD-S junction, producing a four terminal structure, or in cold atoms considering driving from a molecular Bose-Einstein condensate [13].

Subgap currents in this context are produced by Multiple Andreev Reflections (MARs) [14–17], i.e., stimulated transport of electrons via exchange of Cooper-pairs. MARs have been observed in the solid state [18–21] and cold atoms [11], and their signatures can be used to reveal topological phase transitions related to Majorana bound states formation [22]. We show how to engineer well-resolved MAR peaks under weak electron tunneling, and show how these behave in the presence of dissipation in the channel - providing a diagnostic tool for the microscopic nature of the current. We also show that for asymmetric coupling, the reciprocity of the engineered system is broken, yielding electron and Cooper pair currents dependent on the bias direction. This represents a genuinely new way of generating nonreciprocal transport of electrons and Cooper-pairs.

We investigate the transport properties of the junction with an open system approach, while most of the theoretical works rely on Keldish non-equilibrium Green functions or scattering techniques. These approaches are able to treat the tunnelling rate γ between the QD and the leads non-perturbatively, but usually treat the Coulomb interaction U between the QD electrons perturbatively or within a mean-field treatment [23–26]. In contrast, open system approaches such as input-output theories [27–30] or master equations [31–33] work well in the opposite regime: for arbitrary interaction U but weak tunnelling rate γ, implying that MARs have been left beyond their scope. In our framework, the large gap superconducting leads behave effectively as time dependent coherent drives of Cooper-Pairs on the QD (analogous to laser fields in quantum optics). This dynamical model is naturally cast as a dissipative Floquet system, for which we derive a Floquet-Born-Markov master equation [34–37] capturing MARs up to arbitrary order. Our open-system framework provides an opportunity to study the effects of controlled or uncontrolled dissipation acting on the QD. We thus analyze the response of the currents to fermion losses and dephasing, and show in particular robustness of the currents against dephasing. We use in the following natural units in which $\hbar = k_B = e = 1$, where $-e$ is the electron charge.

Model. To represent the separate control of Cooper pair driving, we consider a four-terminal QD connected to two pairs of left (L) and right (R) superconducting leads by tunnel junctions, as depicted in Fig. 1. In each pair, we consider one lead in the single-particle mean-field description with a moderate energy gap Δ_ℓ ($\ell = L, R$), and one described only by its condensed fraction of Cooper-pairs, assuming that the gap is so large that single-particle excitations are irrelevant. A bias voltage $V = V_L - V_R$ is generated between the pairs of superconductors, where V_L and V_R are the voltages of each side. The QD Hamiltonian reads

$$H_{\rm QD} = \sum_{s=\downarrow,\uparrow} \omega c_s^\dagger c_s + U c_\uparrow^\dagger c_\uparrow c_\downarrow^\dagger c_\downarrow, \tag{1}$$

and describes electrons of spin s, energy ω, and Coulomb interaction U. The QD is an effective 4-level system spanned by the non-occupied, single occupied, and double-occupied states $\{|0\rangle, |\downarrow\rangle, |\uparrow\rangle, |\downarrow\uparrow\rangle\}$. The coupling of the QD to the large-gap superconducting leads (red superconductors in Fig. 1) gives rise to a pairing of the QD electrons, i.e., the proximity effect [17], and results in an effective time-dependent QD Hamiltonian of the form

$$H_{\rm QD}^{\rm eff}(t) = H_{\rm QD} + \sum_{\ell=L,R} \left(g_\ell e^{2iV_\ell t} c_\downarrow c_\uparrow + \text{h.c.} \right), \tag{2}$$

where g_ℓ is the Cooper-pair tunnelling amplitude between the QD and the large-gap superconducting $\ell = L, R$. Hence, the coupling of the large-gap superconductors with the QD takes the form of a driving of the transition between the non-occupied and double-occupied states $|0\rangle$ and $|\downarrow\uparrow\rangle$ of the QD.

We obtain the dissipative dynamics of the QD by coupling the Hamiltonian (2) to the superconductors with moderate gaps Δ_ℓ (blue superconductors in Fig. 1) under an open system approach, by deriving a Floquet-Born-Markov master equation [34–37] for the QD. The leads, considered in a mean-field single-particle description, act as baths of Bogoliubov quasiparticles of density of states $D_\ell(E) \propto \Theta(|E| - \Delta_\ell)|E|/(\sqrt{E^2 - \Delta_\ell^2})$. The tunnelling of electrons between the leads and the QD is described by a standard tunnelling Hamiltonian of the form $H_{\rm int} = \sum_\ell \kappa_\ell \sum_{ks} (b_{\ell k s}^\dagger c_s + \text{h.c.})$, where κ_ℓ is the electron tunnelling amplitude and $b_{\ell k s}$ the annihilation operator of an electron of spin s and momentum k in the moderate-gap lead ℓ. The derivation of the master equation, in second-order in $H_{\rm int}$, results in a single-particle tunnelling rate $\gamma_\ell \propto \kappa_\ell^2$ (the typical line widths of the QD levels) considered as the smallest parameter. Note that while treating perturbatively the single-particle coupling, the master equation describes the QD Coulomb interaction U exactly, as in [32, 33]. See Supplemental Material for details of the derivation [38].

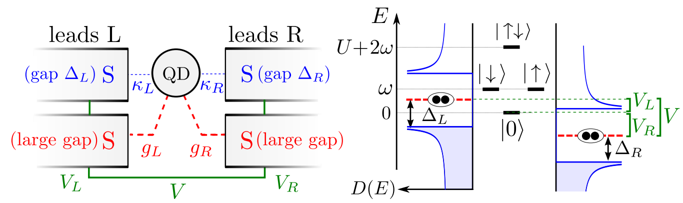

FIG. 1. (left) Sketch of the four-terminal QD tunnelling junction with Cooper-pair and electron tunnelling of amplitudes g_ℓ and κ_ℓ ($\ell = L, R$). (right) Corresponding energy diagram. The moderate-gap superconducting leads are characterized by Bogoliubov quasiparticles of density of states $D(E)$ (blue). The large-gap superconducting leads are only characterized by their Cooper-pair condensates (red). The applied bias voltage V_ℓ is the same for the moderate and large gap leads.

Engineering of transport. From the solutions of the master equation, we calculate the particle current in the leads as a function of the applied bias voltages, taken as opposite from each other for the sake of simplicity ($V_L = -V_R = V/2$). Figure 2 shows the particle current I in both the moderate and large-gap right leads as a function of V. We consider the electron-hole symmetric case $\omega = -U/2$, and vanishing or not Cooper-pair tunnelling $g \equiv g_\ell$, taken here real and identical for left and right leads. We also consider identical single-particle tunnelling rates $\gamma \equiv \gamma_\ell$ between the QD and the moderate-gap superconducting leads.

When $g = 0$, the large-gap superconducting leads are disconnected from the QD, and our system simply consists in a conventional S-QD-S tunnelling junction [32, 33]. Only one peak of current is observed (see panel A in Fig. 2), whose the shape is related to the superconductor density of states $D_\ell(E) \propto \Theta(|E| - \Delta_\ell)|E|/(\sqrt{E^2 - \Delta_\ell^2})$. The peak appears when $E_i > \omega > E_f$, where E_i is the energy of the highest occupied state of the left lead and E_f is the energy of the lowest non-occupied state of the right lead (see energy diagram I in Fig. 2). For high bias, the particle current tends to the value 2γ of normal leads. For low bias ($\gamma \ll V < 4\Delta_\ell$), i.e. in the subgap region (where no resonance between left-lead occupied and right-lead non-occupied states exist), no current is observed as a result of the weak coupling approximation. Indeed, for $\gamma \ll \Delta$, Andreev reflection at the interface with the moderate-gap superconductor is negligible.

Connecting the large-gap superconducting leads to the QD (i.e., setting $g \neq 0$) allows Andreev reflections to occur. Under such process, an electron (hole) is reflected as a hole (electron) producing the emission (absorption) of a Cooper-pair in the large-gap superconducting leads (see panel B in Fig. 2). After some reflections, electrons of the QD acquire enough energy to tunnel into the moderate-gap superconducting lead. This produces well-resolved single-particle subgap currents more and more pronounced as g increases. These processes are represented in our Floquet-Born-Markov formalism by decay channels corresponding to QD transition (quasi)energies shifted by multiple of Cooper-pair energies (see Supplemental Material). The subgap currents are located at $V = 2(|\omega| + \Delta)/(2n + 1)$ where $n = 1, 2, \ldots$ denotes the n^{th} MAR (see energy diagrams II and III corresponding respectively to the first and second Andreev reflections). This can be obtained from the condition $E_i + nV = \omega = E_f - nV$, in which n denotes the number of Cooper-pairs transfer from the left to the right lead. Note however that in general the bias voltage at which a MAR peak appears is a function of both the QD charging energy U and ω (see Supplemental Material). Hence, while the tunnelling between the QD and the moderate-gap leads is always sequential due to the weak-coupling regime (one electron per time), it can be assisted by transfer of an arbitrary number of Cooper-pairs between the large-gap superconductors, thanks to the stronger tunnelling amplitude g. This represents reservoir engineering of subgap single-particle currents. This is our first important result. Interestingly, the Cooper-pair current in the right large-gap superconducting lead is negative outside the subgap region. We attribute this phenomenon to a supercurrent (i.e., Cooper-pair current) reversal, due to the modification of parity of the QD when the voltage exceed the value delimiting the subgap border, as can be seen in the panel C in Fig. 2 [14, 43]. Note that the sign and amplitude of the supercurrent are dependent of the phases of the superconductors (not shown).

Effects of particle loss and dephasing. In the previous section, we showed that dissipation induced by reservoir engineering can be used to control subgap transport. Here we examine the robustness of the produced subgap currents against the presence of incoherent processes, that are inherent in real experimental setups. We incorporate these effects into our master equation through an additional dissipator of the form $\mathcal{D}_I(\rho) = \gamma_I \left(2L\rho L^\dagger - \left\{L^\dagger L, \rho\right\}\right)$, where γ_I is the rate of the incoherent process and L the corresponding Lindblad operator (see Supplemental Material). For cold atoms experiments, the dissipation in the channel is often in this Markovian form as can be derived from first principles [2].

We first consider the effects of particle loss (i.e., $\gamma_I \equiv \gamma_{\rm loss}$, $L = c_s$) acting on the QD. This occurs naturally in the cold-atom platforms through background gas collisions, and could be engineered using electron beams [44] or light scattering quantum gas microscopes with single-site resolution [45–47] (analogous to x-ray scattering in the solid state). In Fig. 3 (panels A-D), we show the

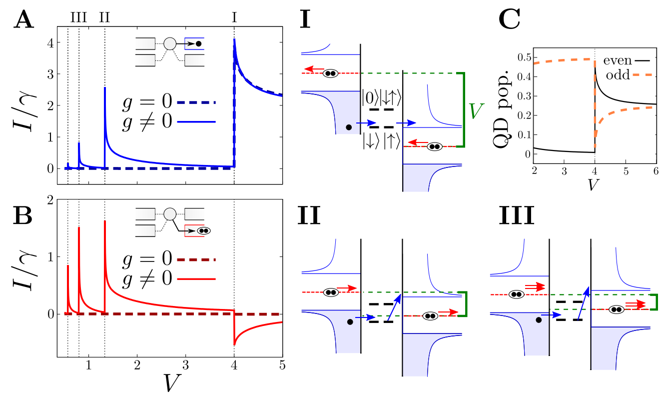

FIG. 2. Particle current I (in units of γ) in the right moderate-gap lead (**A**) and large-gap lead (**B**) as a function of the bias voltage V for $g = g_\ell = 0$ (dashed line) and 0.5 (solid line). Other parameters are $U = 2$, $\omega = -1$, $\gamma = 10^{-2}$ and $T_\ell = 0$, in units chosen so that $\Delta_\ell \equiv \Delta = 1$. Subgap currents via MARs appear for non-vanishing g. (**I-III**) Energy diagrams corresponding to the standard resonant tunnelling (**I**) and the first and second MARs (**II** and **III**). **C**: Steady states QD averaged populations for states with even ($|0\rangle, |\downarrow\uparrow\rangle$, black line) and odd ($|\downarrow\rangle, |\uparrow\rangle$, dashed orange line) number of electrons.

particle currents in all the leads as a function of the bias voltage for increasing loss rates $\gamma_{\rm loss}$. The presence of losses results in competing effects. On the one hand, the additional decay channel tends to empty the QD faster. This results in an increase (decrease) of the currents of electrons entering (reaching) the moderate-gap superconducting leads. On the other hand, pushing the QD towards the non-occupied state $|0\rangle$ increases the effects of the driving (since the driving only affects the QD in the non-occupied or double-occupied states), which favors MARs and thus raises subgap currents. Hence, while source currents (panels A and C) only increase due to electrons losses, drain currents (panels B and D) are subjected to these competing effects, exhibiting amplitude increase or decrease depending on the voltage bias.

We then consider the effects of dephasing (i.e., $\gamma_I \equiv \gamma_{\rm deph}$, $L = c_s^\dagger c_s$) acting on the QD, which occurs naturally through coupling to additional degrees of freedom in the solid state, and can be engineered in cold atoms through light scattering or noise [2, 48–51]. We show that dephasing acting on the QD affects identically the source and drain Cooper-pair currents, whereas leaves unchanged the electron currents. Figure 3 (panels E) shows the current of Cooper-pair leaving the QD to reach the right large-gap superconductor for different dephasing rate $\gamma_{\rm deph}$. Our results show that increasing the dephasing rate reduces the size of the subgap peaks (see panel F). This can be understood as a consequence of the blurring of the QD energy levels caused by the dephasing. Hence, dephasing tend to destroy

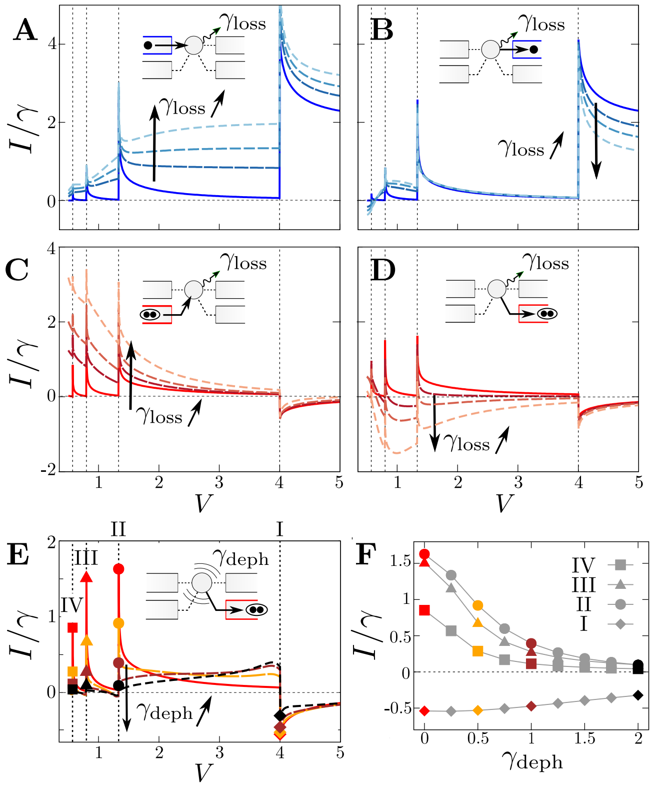

FIG. 3. Current-voltage characteristics under the effects of electron loss (**A-D**) and dephasing (**E-F**) acting on the QD. Currents of electrons entering (**A**) and leaving (**B**) the QD and of Cooper-pairs entering (**C**) and leaving (**D**) the QD as a function of the bias voltage V for different electron loss rate $\gamma_{\rm loss}$ (from solid to dashed lines, $\gamma_{\rm loss} = 0,\ 0.5\gamma,\ \gamma,\ 2\gamma$). Other parameters are $U = 2$, $\omega = -1$, $\gamma = 10^{-2}$, $T_\ell = 0$ and $g_\ell = 0.5$, in units of $\Delta_\ell \equiv \Delta$. Current of Cooper-pairs in the right large-gap superconducting lead as a function of V for $\gamma_{\rm deph} = 0,\ 0.5{,}1$ and 2 (**E**); and as a function of $\gamma_{\rm deph}$ for $V = 2(|\omega| + \Delta)/(2n+1)$ with $n = 0,\ 1,\ 2$ and 3 corresponding to the peaks I, II, III and IV (**F**). Other parameters as above.

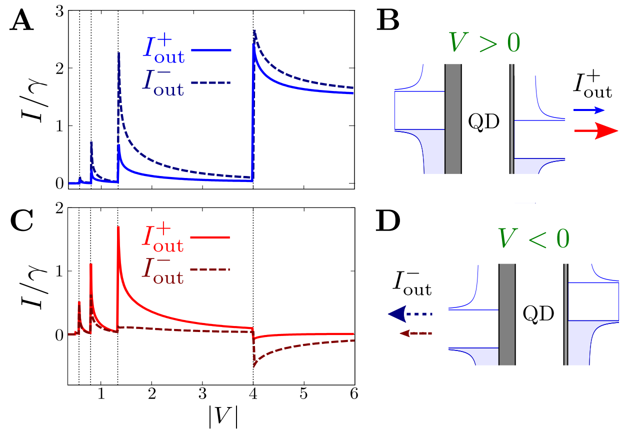

FIG. 4. Current-voltage characteristics in the moderate-gap (**A**) and large-gap (**C**) superconducting leads for asymmetric single-particle tunnelling rate $\gamma_L = 3\gamma_R = 1.5\ 10^{-2}$. In both plots, the solid (dashed) lines curves correspond to the current for positive (negative) bias voltage V, as depicted in the diagram **B** (**D**) on the right. Other parameters are $U = 2$, $\omega = -1$, $\gamma = 10^{-2}$, $T_\ell = 0$ and $g_\ell = 0.5$, in units of $\Delta_\ell \equiv \Delta$.

Cooper-pair subgap currents, but does not affect the single particle currents. This suggests these latter are robust against phonon/photon scattering in condensed matter/cold atomic systems.

Nonreciprocal subgap transport. Finally, we show how to generate nonreciprocal subgap transport. Bias-direction-dependent properties is generally a desired feature of nanoscale devices, and are known to result from the presence of asymmetry and nonlinearity. Nonreciprocal transport at the quantum level has been investigated in spin [52–60] and QD systems [61–65]. This includes the paradigmatic Pauli Blockade effects in a double-QD junction, where a nonreciprocal electron current has been observed for asymmetric QD energy levels [62]. For a single QD, the required asymmetry can be provided by different left and right tunnelling rates. In a S-QD-S junction, in the intermediate coupling regime ($\gamma_\ell \sim \Delta_\ell$), non reciprocal conductance has been observed and explained as originating from asymmetric Kondo resonance at the contact with the leads [66]. Here we show that for asymmetric weak single-particle tunnelling rates $\gamma_L \neq \gamma_R$, the reciprocity of the transport properties can be broken as soon as the Cooper-pair tunnelling amplitudes g_ℓ is non-zero. In Figure 4, we plot the current-voltage characteristics for the moderate (panel A) and large gap (panel C) superconducting leads for positive and negative bias voltage (see diagrams B and D) for $\gamma_L = 3\gamma_R$. While the total current (the sum of electron and Cooper-pair currents) is still reciprocal (not shown), its electron and Cooper-pair contributions become dependent of the bias direction, as can be clearly seen in Fig. 4. In particular, the current of electrons (Cooper-pair) is larger (smaller) for negative (positive) bias. We interpret this phenomenon as a Cooper-pair-assisted nonreprocical transport, since it occurs only for non-zero g_ℓ. Indeed, for $g_\ell = 0$, the electron current is reciprocal (not shown). We believe it is a genuinely new way of breaking reciprocity, since while keeping reciprocal the total current, its electron and Cooper-pair contributions – which could be measured independently in our four-terminal scheme – become asymmetric.

Conclusion. We developed a quantum-optics-inspired framework to study the dynamics of strongly interacting fermions in tunnelling junctions under the influence of dissipation and driving, relevant for both solid-state and cold-atom platforms. For concreteness, we studied the dynamics of a QD coupled to superconducting leads in a four-terminal configuration, where two large-gap superconducting leads are added to a traditional S-QD-S tunnelling junction. We demonstrate the possibility of controlling subgap transport via dissipation engineering. We showed that the added leads generate subgap transport based on MARs despite weak electron tunnelling, and studied the effects of electron loss and dephasing acting on the QD. Finally, we showed that the Cooper-pair driving provided by the added leads is a new way of breaking the reciprocity of the junction, generating nonreciprocal electron and Cooper-pair subgap currents based on MARs.

Our results could be investigated in both solid-state and cold-atom experiments. They could be generalized to multi-QD tunnelling junction, and to include the presence of measurement and feedback loop to control the transport dynamics of fermions in tunnelling junctions [67]. More possible outlooks include reservoir engineering of (Floquet)-Majorana fermions [13, 68, 69], or studies of the interplay between dissipation and driving in thermodynamics problems such as thermoelectric effects [70] or quantum heat engines [71] involving superconductors.

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