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Quantifying the mesoscopic nature of the Einstein-Podolsky-Rosen nonlocality

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Evidence for Bell's nonlocality is so far mainly restricted to microscopic systems, where the elements of reality that are negated predetermine results of measurements to within one spin unit. Any observed nonlocal effect (or lack of classical predetermination) is then limited to no more than the difference of a single photon or electron being detected or not (at a given detector). In this paper, we analyze experiments that report Einstein-Podolsky-Rosen (EPR) steering form of nonlocality for mesoscopic photonic or Bose-Einstein condensate (BEC) systems. Using an EPR steering parameter, we show how the EPR nonlocalities involved can be quantified for four-mode states, to give evidence of nonlocal effects corresponding to a two-mode number difference of 10^5 photons, or of several tens of atoms (at a given site). We also show how the variance criterion of Duan-Giedke-Cirac and Zoller for EPR entanglement can be used to determine a lower bound on the number of particles in a pure two-mode EPR entangled or steerable state, and apply to experiments.

Einstein-Podolsky-Rosen (EPR) presented in 1935 a seemingly compelling argument that quantum mechanics was incomplete [1]. In their gedanken experiment, properties of a system B can be predicted ultra-precisely, by the measurements of a distant observer, popularly called Alice. EPR assumed no "spooky action-at-a-distance", to argue that Alice's measurement is noninvasive, and therefiore that Alice's prediction represents a predetermined property (an "element of reality") of system B. Further, they showed that the set of all such predetermined properties could not be consistent with any local quantum state description for B, and thus concluded that quantum mechanics was an incomplete theory. The assumptions made in EPR's argument are collectively known as local realism (LR). Bell's theorem negated these premises, by showing LR could be falsified [2].

Understanding whether and how local realism fails macroscopically remains an open question in physics. Loophole-free experiments confirming Bell's theorem are so far limited to microscopic systems e.g. system B is a single photon or electron [3, 4]. In these cases, the predetermined properties that EPR called elements of reality give predictions to within a single spin unit. The failure of LR that is inferred from the experiments is therefore a microscopic effect only, in the sense that this pertains only to predictions specified to an accuracy of one spin unit, for a microscopic particle. Similar accuracies are required in almost all of the experiments predicted to violate LR for multi-particle systems [5, 6].

By contrast, EPR's experiment (called an "EPR-steering" experiment [7–11]) has been investigated experimentally for mesoscopic optical fields [12–24], atomic ensembles [25–32], and, recently, for mesoscopic mechanical oscillators [33–41]. In many of these experiments, not only are the systems sizeable, but the outcomes are over a larger range, corresponding to several or many spin units. There is thus the possibility to test for a mesoscopic EPR nonlocal effect, where the predetermined "elements of reality" that are falsified give predictions with an indeterminacy of several spin units. One may then ask how much "spooky action-at-a-distance" is occurring in terms of spin units? How to do the quantification is not obvious, however. It is not simply the size of the entangled system, nor the range of outcomes. Previous measures inform us how many atoms are mutually entangled [42, 43], or what fraction of particle pairs behave locally versus nonlocally [44], but these need not imply large differences in the actual *outcomes* of observables due to nonlocal effects.

The situation is clear if the physical quantities measured by observers Bob and Alice (at different locations) have two mesoscopically-distinct outcomes + and −, e.g. N particles in an *up* position versus N particles in a *down* position ($N \gg 1$) [45, 46]. One may then extend EPR's premises, to define δ-scopic local realism (δLR), which asserts [47, 48]: (1) any measurement by Alice cannot instantly induce a change $\delta = 2N$ to the outcome of measurement at Bob's location; and (2) if the outcome + or − for Bob's system can be predicted with certainty by Alice, then Bob's system is always predetermined to be in a state that gives *either* the result + or −. The failure of such premises then implies a "spooky action" effect of size $\delta = 2N$. However, examples of EPR systems with just two outcomes ± separated by δ are limited.

In this paper, we present a practical approach to test EPR premises based on δ-scopic local realism (δLR). We consider a version of EPR's argument based on the premise of δLR, where the separation of outcomes + and − is quantified by δ, but where there is a continuous range of outcomes. Our analysis leads to a criterion sufficient to demonstrate EPR δ-scopic nonlocality, that there is inconsistency between the completeness of quantum mechanics and δLR. The size of δ quantifies in the δLR assumption the upper bound on the amount of change that can occur to Bob's system, due to Alice's measurements. Failure of δLR where δ is large implies large nonlocal effects. We analyze EPR experiments which have a mesoscopic, continuous range of outcomes for Alice and Bob's measurements, to present preliminary evidence for quantifiable mesoscopic EPR nonlocalities.

Quantifying the EPR Paradox: The EPR argument can be generalised to pairs of measurements $\{X_A, P_A\}$ and $\{X_B, P_B\}$ on two spatially separated systems A and B. We consider X_B, P_B to be scaled noncommuting observables which satisfy $[X_B, P_B] = 2$, so that the Heisenberg uncertainty relation is $\Delta X_B \Delta P_B \geq 1$. To demonstrate the paradox, one measures the variances $V_B(X_B|X_A)$and $V_B(P_B|P_A)$ of the respective conditional distributions $P(X_B|X_A)$ and $P(P_B|P_A)$ [9]. Here $P(X_B|X_A)$ is the probability for result X_B, given a measurement X_A. The average conditional variance $(\Delta_{inf}X_B)^2 = \sum_{X_A} P(X_A)V_B(X_B|X_A)$ determines the accuracy of inference of the results for X_B, based on the measurements at A. The $\Delta_{inf}P_B$ is defined similarly. Using EPR's logic, these inference variances define the average indeterminacy of the two respective elements of reality, for X_B and P_B. If

$$\varepsilon \equiv \Delta_{inf}X_B \Delta_{inf}P_B < 1 \tag{1}$$

then an EPR paradox arises, since the simultaneous predetermination for X_B and P_B is more accurate than allowed by the uncertainty principle [9, 13]. The condition (1) is a condition for "*EPR steering* of system B" [8, 10].

We now construct a quantified version of the EPR argument, by relaxing EPR's premises. The assumptions of δ_X-scopic local realism (δ_XLR) are: (1) A measurement made at A might disturb the system B, so that the outcome for a simultaneous measurement X on B can be altered, but the change to the outcome cannot be greater (in magnitude) than δ_X. (2) If the value for a physical quantity X is predictable, without disturbing the system by more than δ_X, then the value of that physical quantity is a predetermined property of the system (the "element of reality" for X), the predetermined value being given to within $\pm\delta_X$ of the predicted value. We will refer to δ_X as the degree of "*nonlocal indeterminacy*", with respect to the EPR observable X.

The assumption of δLR changes the condition for an EPR paradox, making it more difficult to demonstrate the paradox. Applying δ_XLR, the indeterminacy in the predictions for X_B associated with the element of reality has increased, but by a limited amount only. We show in the Supplemental materials that the maximum value of this indeterminacy becomes [51]

$$(\Delta_{inf,\delta_X}X_B)^2 = (\Delta_{inf}X_B)^2 + \delta_X^2 + 2\delta_X \sum_{X_A,X_B} P(X_A, X_B)|X_B - \langle X_B|X_A\rangle|, \tag{2}$$

where $P(X_A, X_B)$ is the joint probability. Defining $\Delta_{inf,\delta_P}P_B$ in a similar manner, the experimental realisation of

$$\varepsilon_\delta \equiv \Delta_{inf,\delta_X}X_B \Delta_{inf,\delta_P}P_B < 1 \tag{3}$$

will therefore imply an inconsistency between the premise of δLR and the completeness of quantum mechanics. The calculation of ε_δ is straightforward, once the distributions $P(X_A, X_B)$ and $P(P_A, P_B)$ are known. When $\delta = 0$, Eq. (3) reduces to the standard EPR condition (1). The inequality is progressively more difficult to satisfy, as δ increases.

Gaussian δ-scopic EPR nonlocality: We consider EPR experiments based on field modes at locations A, B. $X_{A/B}$, $P_{A/B}$ are defined according to $a = (X_A + iP_A)/2$ and $b = (X_B + iP_B)/2$, where a, b are the annihilation operators of each mode. The δ-scopic EPR inequality reduces to Eq. (3). A widely-used source of EPR-correlated fields is the parametric amplifier, the ideal output of which is the two-mode squeezed state [9, 13]. Here, the conditionals $P(X_B|X_A)$ and $P(P_B|P_A)$ are Gaussian. Moreover, a Gaussian profile is maintained in non-ideal situations where losses and thermal noise are present [13, 49, 50].

Assuming Gaussianity, the prediction of ε_δ given measured values of $\Delta_{inf}X_B$ and $\Delta_{inf}P_B$ is straightforward. Using Eq. (2) and that for a Gaussian distribution $\langle|X_B - \mu_X|\rangle = \Delta_{inf}X_B\sqrt{2/\pi}$ (where μ_X is the mean of $P(X_B|X_A)$), we find $\varepsilon_\delta = \sigma^2 + \delta^2 + 2\delta\sigma\sqrt{2/\pi}$ [51]. For the sake of simplicity, we have taken $\sigma = \Delta_{inf}X_B = \Delta_{inf}P_B$ and $\delta = \delta_X = \delta_P$. We see that $\varepsilon < \left[-\delta\sqrt{2/\pi} + \sqrt{2\delta^2/\pi - (\delta^2 - 1)}\right]^2$ will be sufficient to imply the δ-scopic EPR nonlocality.

Extensive data has been reported for continuous variable EPR experiments [12–27] (see Fig. 1). Gaussian distributions are predicted in almost all cases plotted (including the data indicated by (g)) as has been verified experimentally [13, 50]. For rigorous testing, a full construction of the distributions with space-like separated measurement events is required [2, 4]. With this proviso, we note that the recently achieved values of the EPR parameter $\varepsilon \sim 0.176$ [20] will imply a δ-scopic EPR nonlocality, with $\delta \sim 0.633$.

To determine the significance of the value of δ, one needs to resort to the details of the individual experiments. The nonlocal indeterminacy δ is given relative to the quantum noise level, which for the optical experiments is usually considered microscopic. On the other

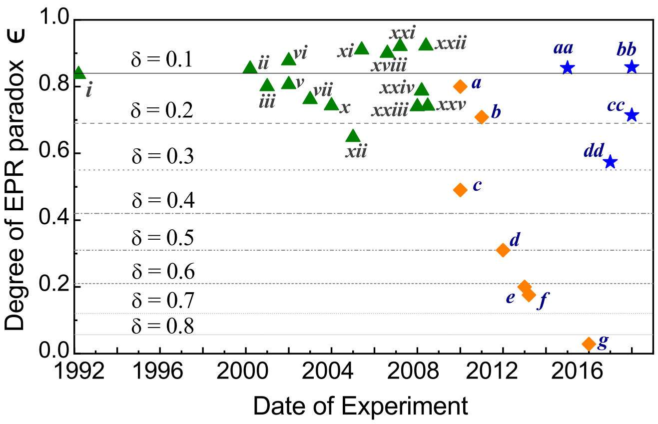

Figure 1. The δ-scopic EPR nonlocality is realised ($\epsilon_\delta < 1$) when ϵ is below the line shown, for the given δ. Data $i - xxv$ are for the experiments referenced in Fig. 9 of Ref. [13], while data a ([15]), b ([16]), c ([22]), d ([23]), e ([19]), f ([20]), g ([24]), aa ([25]), bb ([26]), cc ([27]), dd ([32]) are later experiments. All results are for spatially separated optical fields, except those given by blue stars which are for mesoscopic groups of cooled atoms (aa, bb, cc) or for hybrid systems (dd). The cold atom groups have no (as in aa) or small spatial separations of $\sim 10\mu$m (bb, cc).

hand, entanglement has now been detected between two mechanical oscillators [33, 34, 38], and between an oscillator and a field [35]. Entanglement however does not imply the EPR steering condition (1). It has been proposed to detect the EPR condition (1) for these cases [33, 36, 40, 41], where X_B, P_B refer to the quadratures of the phonon modes of the oscillator. Eq. (3) enables a quantification of the EPR nonlocality that would be observed in such an experiment. Here, δ is quantifiable at the Planck scale [60], and corresponds to a nonlocal indeterminacy with respect to mechanical motion.

EPR nonlocality using Schwinger spins: For some experiments, the quantum noise level and hence δ may correspond to a large number of photons. This is understood by considering the Heisenberg relation $\Delta J^Z \Delta J^Y \geq \left|\langle J^X\rangle\right|/2$ for spin systems, where measurements are made of the spin components $J^{X,Y,Z}$. For high spins, $\left|\langle J^X\rangle\right|$ can be large.

Indeed, EPR states exist for which $\left|\langle J^X\rangle\right|$ is a scalable large number. In these cases, the EPR observables are two-mode Schwinger spins, defined as $J_A^X = (a_+^\dagger a_- + a_+ a_-^\dagger)/2$, $J_A^Y = (a_+^\dagger a_- - a_+ a_-^\dagger)/2i$, $J_A^Z = (a_+^\dagger a_+ - a_-^\dagger a_-)/2$, and $J_B^X = (b_+^\dagger b_- + b_+ b_-^\dagger)/2$, $J_B^Y = (b_+^\dagger b_- - b_+ b_-^\dagger)/2i$, $J_B^Z = (b_+^\dagger b_+ - b_-^\dagger b_-)/2$, where $a_\pm$, $b_\pm$ are annihilation operators for four modes [47]. The four modes are created from spatially separated modes a, b prepared in an EPR state $|\psi\rangle$. Each mode a b interferes (via a beam splitter) with an intense "local oscillator" field (denoted by annihilation mode operators b_{LO}, a_{LO}). This creates a macroscopic photonic state $|\psi\rangle_M$ involving four fields $a_\pm = (a \pm a_{LO})/\sqrt{2}$, $b_\pm = (b \pm b_{LO})/\sqrt{2}$ at sites A and B respectively. The fields at each site pass through second polarising beam splitters set at respective angles θ_A and θ_B. The number of particles in each arm is detected, as a large number, and the difference gives a measure of J^Z, J^Y or J^Z depending on the choice of θ_A, θ_B. Based on the Heisenberg uncertainty relation, the EPR criterion is

$$\Delta_{inf,\delta_J}(J_B^Z)\Delta_{inf,\delta_J}(J_B^Y) < |\langle J_B^X\rangle|/2, \tag{4}$$

which normalises to Eq. (3) on defining $X_B/P_B = J_B^{Z/Y}/\sqrt{|\langle J_B^X\rangle|/2}$ and $\delta = \delta_J/\sqrt{|\langle J_B^X\rangle|/2}$. Here, $|\langle J_B^X\rangle| = |\langle b_{LO}^\dagger b_{LO} - b^\dagger b\rangle/2|$ which becomes $\langle b_{LO}^\dagger b_{LO}\rangle/2$ since $\langle b^\dagger b\rangle/\langle b_{LO}^\dagger b_{LO}\rangle$ is small. The intensity of the local oscillator is macroscopic and $\delta_J \sim \delta\sqrt{\langle b_{LO}^\dagger b_{LO}\rangle/4}$ (the nonlocal indeterminacy in the values of J_X^B, J_Y^B) can therefore also be large.

EPR nonlocality for Schwinger spins has been realised in the experiments of Bowen et al [14], where $a_\pm$ ($b_\pm$) correspond to two orthogonal horizontally ("H" or "x") and vertically ("V" or "y") polarised field modes at A (B). From the description of their experiment [14, 51], $|\langle J_B^X\rangle| \sim 10^{11}$ photons, implying a δ_J of order 10^5 photons. The relative value $\delta_J/|\langle J_B^X\rangle|$ is however small.

Analogy to Schrödinger cat: The Schwinger-spin experiment provides a simple parallel to Schrödinger's cat gedanken experiment [45, 47]. In the original cat paradox, a macroscopic superposition is created by the process of measurement, which couples the microscopic system (prepared in a superposition state) to a measurement apparatus. In the experiment, the microscopic EPR state $|\psi\rangle$ is indeed coupled to a macroscopic system (the local oscillator fields) at each site, and a four-mode amplified state $|\psi\rangle_M$ is produced that enables a macroscopic readout of $X_{A/B}$ and $P_{A/B}$ of the original fields.

The many-particle state $|\psi\rangle_M$ is created prior to the measurements $J_A^{\theta_A}$, $J_B^{\theta_B}$ and it is this feature that enables the demonstration of mesoscopic nonlocality. The $|\psi\rangle_M$ is a superposition of states with definite outcomes for $J_B^{\theta_B}$, where those outcomes are given by $J_B^{\theta_B} = EX_B/2$, or $EP_B/2$ (here $E^2 = \langle b_{LO}^\dagger b_{LO}\rangle$) depending on θ_B. The superposition $|\psi\rangle_M$ comprises many states that have a large range of continuous outcomes for $J_B^{\theta_B}$, rather than just two distinct states as in Schrodinger's case. In the Supplemental Materials we prove that the observation of the δ_J-scopic EPR nonlocality can only arise if $|\psi\rangle_M$ comprises at least two states that differ in outcome for $J_B^{\theta_B}$ by at least δ_J [51]. Such states (when δ_J is large) give a nonzero mesoscopic quantum coherence, and signify a generalised Schrodinger-cat paradox (refer Refs. [56–59]). These states, for which EPR nonlocality is also demonstrated, are well nested *within* the overall superposition state however. For an experimental value $\epsilon \sim 0.42$ (where $\delta \sim 0.4$), their separation is typically

$\delta_J = 0.4E/2$, whereas the state $|\psi\rangle_M$ predicts a Gaussian distribution for $J_B^{\theta_B}$, with $\Delta J_B^{\theta_B} \sim 1.3E/2$.

The δ-scopic EPR nonlocality manifests without the significant decoherence that normally prevents formation of Schrödinger cat states, because the separation δ_J between states of the superposition is not amplified relative to the quantum noise level. The EPR steering parameter ε_δ is unchanged, consistent with the requirement that entanglement cannot be created by local entangling transformations, such as produced by beam splitters [61].

EPR nonlocality between distinct atom groups: The experiments of Refs. [28–31] investigate EPR entanglement between two spatially separated macroscopic atomic ensembles, A and B. In their experiment, $J_{A/B}^{X,Y,Z}$ are the collective spins of each ensemble, defined relative to two atomic levels. The observation of the condition $D \equiv \{[\Delta(X_A - X_B)]^2 + \Delta[(P_A + P_B)]^2\}/4 < 1$ implies entanglement between subsystems A and B [62]. For spins, this entanglement condition becomes [63]

$$D \equiv \frac{[\Delta(J_A^Z + J_B^Z)] + [\Delta(J_A^Y + J_B^Y)]^2}{|\langle J_A^X\rangle| + |\langle J_B^X\rangle|} < 1. \qquad (5)$$

Measurements give $D \sim 0.8$ for thermal atomic ensembles [28, 29]. The value of $D \sim 0.5$ would imply an EPR steering nonlocality (4) [13, 64, 65]. For a rigorous demonstration of EPR nonlocality, it is however necessary to measure the EPR observables independently and locally, so that information is gained simultaneously about each of $J_A^{\theta_A}$ and $J_B^{\theta_B}$.

EPR steering correlations meeting the condition (1) have however recently been observed for matter-wave fields created with Bose-Einstein condensates (BEC) [25–27, 66, 67]. In the experiments [25, 26, 67], twin atom-beam states are generated by a parametric interaction [67]. Atoms are created in pairs, one into each group A and B that correspond to different spins. The atom field quadratures $X_{A/B}$, $P_{A/B}$ are measured by an atomic homodyne method, where the local oscillators are a different, larger group consisting of $E = \langle a_{LO}^\dagger a_{LO}\rangle \sim \langle b_{LO}^\dagger b_{LO}\rangle > 10^2$ atoms [67]. Experiments of Piese et al observe EPR correlations (1) between mesoscopic atomic groups A and B with $\varepsilon \sim 0.85$ (with no spatial separation) [25]. Recently, Kunkel et al observe an EPR steering with $\varepsilon \sim 0.71$ between atom groups spatially separated by $\sim 10\mu m$ [26]. Assuming results are unchanged if the experiment were reconfigured along the lines of the Schwinger spin experiments, and that the distributions are approximately gaussian, this suggests nonlocalities with $\delta_J > E\delta > 20$ atoms.

Using a different method of generation, Fadel et al observe EPR steering correlations ($\varepsilon \sim 0.74$) between the Schwinger spins J_A^Z, J_B^Z (and J_A^Y, J_B^Y) of two atomic groups separated by $\sim 4\mu m$ [27]. Here, group A (B) consists of two BEC components, $a_\pm$ ($b_\pm$). Their measurement of spins (J^X or J^Y) is achieved with a variable pulse rotation θ, in analogy to the polarising beam splitter of Bowen et al. (although without independent selection of the two measurement angles). The BEC experiments thus reveal quantifiable nonlocal indeterminacies in the *atom* number differences $J_B^{\theta_B}$ at the given site. The nonlocality being tested here is whether an action at the site A can create a change in the number of atoms between the groups b_+ and b_-, at site B.

Quantification of number of bosons in the two-mode entangled state: The large values of δ_J arise for four-mode states. However, the *two-mode* EPR state $|\psi\rangle$ can itself be constrained to have a certain degree of "largeness". For any *entangled* pure state $|\psi\rangle$, the mean total number of bosons is $\bar{n} = \langle\psi|a^\dagger a + b^\dagger b|\psi\rangle$. The value of D places a lower bound on $\bar{n}$. Using the identity $|\langle ab\rangle| \leq \sqrt{(\langle a^\dagger a\rangle + 1)\langle b^\dagger b\rangle}$ [68], we find $D \geq D_{\bar{n}}^{(l)}$ [51], where $D_{\bar{n}}^{(l)} = 1 + \bar{n} - \bar{n}\sqrt{1 + 2/\bar{n}}$ decreases with $\bar{n}$, and is achieved for the two-mode squeezed state for which $D = (1-x)/(1+x)$. In an experiment, the two-mode system is generally not a pure state. The measured $\langle a^\dagger a + b^\dagger b\rangle$ does not then reflect the mean number of bosons in an entangled state, because there may be components of the mixture that are not entangled. However, the observation of $D < D_{\bar{n}}^{(l)}$ certifies that a pure entangled state $|\psi\rangle$ with $\langle a^\dagger a + b^\dagger b\rangle > \bar{n}$ must be a component of the mixed state (refer [51]). Similarly, by expanding all pure states in the basis of number states $|i\rangle_a|j\rangle_b$, we prove in the Supplemental Materials that the value D places a lower bound on the minimum number of bosons

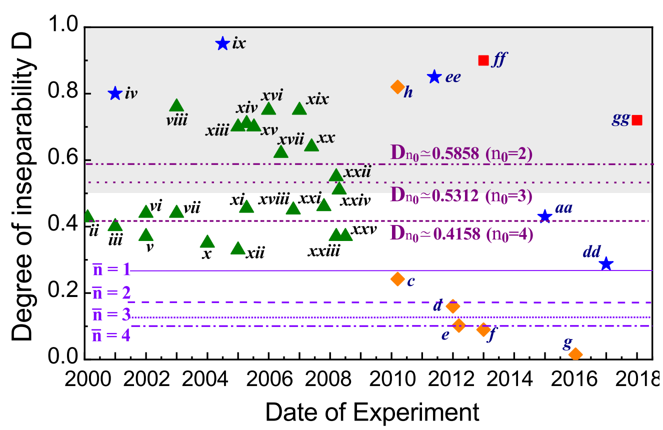

Figure 2. The entanglement parameter D is plotted versus date for a sample of experiments. Data ($ii - xxv$) are the values reported for atomic (stars) and optics (triangles) experiments respectively, as listed in Fig. 9 of Ref. [13]. Data c ([22]), d ([23]), e ([19]), f ([20]), g([24]), h ([18]) are for optical experiments (diamonds), data aa ([25]), dd ([32]), ee ([29]) are for atomic or hybrid experiments (stars), and ff ([35]), gg ([38]) are for mechanical oscillator experiments (squares). $D < D_{\bar{n}}^{(l)}$ implies the EPR state $|\psi\rangle$ has a mean number of bosons greater than $\bar{n}$. The $D_{\bar{n}}^{(l)}$ are plotted for $\bar{n} = 1, 2, 3, 4$. $D < D_{n_0}$ requires states of more than n_0 bosons. Plotted is D_{n_0} for $n_0 = 2, 3, 4$. For $n_0 = 10$, $D_{n_0} \simeq 0.2228$.

$n_0 = i + j$ contributing a nonzero term to the expansion. If the bosons are atoms, the state $|i\rangle_a|j\rangle_b$ has an entanglement depth of $n_0 = i + j$, meaning all n_0 atoms are mutually entangled [42, 43, 51].

Experimental values of D are plotted in Fig. 2. The values $D < 0.2228$ confirm two-mode optical EPR states $|\psi\rangle$ involving more than 10 photons ($n_0 > 10$). These states are different to states constructed from photon pairs, for which $n_0 \leq 2$. Where $D < 0.5$, the two modes of the state $|\psi\rangle$ are both EPR steerable [69]. Measurements by Piese et al [25] observe $D < 0.43$, implying two-way EPR steerable states $|\psi\rangle$ with more than 3 atoms (if spatial separation could be achieved) [51].

Conclusion: We have given evidence for a mesoscopic EPR nonlocality that "delocalises" $\delta_J \sim 10^5$ photons between two polarisation modes at a given site. This represents a tenth of the full range of outcomes (defined as that within 3 standard deviations of the mean) for the polarisation photon-number difference at the site. Recent experiments with Bose-Einstein condensates show similar EPR nonlocalities involving four atomic modes. This motivates new experiments where it may be feasible to demonstrate an EPR nonlocality "delocalising" $\delta_J \sim 10$ atoms across two highly-occupied atomic modes at a given site. The criteria presented in this paper may also have a practical application. The curves of Fig. 1 can be used to detect a genuine EPR effect, even when a causal effect is present. If the maximum disturbance due to the causal effect can be quantified (to be δ_C say), then an EPR nonlocality can be deduced if $\epsilon_\delta < 1$ where $\delta > \delta_C$. An example of such a causal effect ("cross-talk") is given in Ref. [27].

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