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# Quantum Criticality of Granular Sachdev-Ye-Kitaev Matter

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# Quantum criticality of granular SYK matter

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We consider granular quantum matter defined by Sachdev-Ye-Kitaev (SYK) dots coupled via random one-body hopping. Within the framework of Schwarzian field theory, we identify a zero temperature quantum phase transition between an insulating phase at weak and a metallic phase at strong hopping. The critical hopping strength scales inversely with the number of degrees of freedom on the dots. The increase of temperature out of either phase induces a crossover into a regime of strange metallic behavior.

*Introduction:* Despite decades of research, our understanding of strongly correlated (‘non-Fermi liquid’) quantum matter with metallic parent states remains incomplete. A universal feature of these materials is that seemingly incongruent phases of matter — superconducting, insulating, poorly conducting, metallic, etc. — coexist in close parametric proximity to each other[1]. The understanding of this diversity of competing phases, which finds its most prominent manifestations in the physics of the cuprates[2] or heavy fermion materials[3], requires universal blueprints of correlated fermion matter transcending the Landau quasiparticle paradigm. Recently, systems of coupled Sachdev-Ye-Kitaev (SYK)[4–16] quantum dots have gained popularity in this context. What makes these systems interesting is that a hallmark of many correlated fermion materials — crossover from a strange metal (SM) phase to a Fermi liquid (FL) upon lowering temperatures — is generated within a very simple mean field picture[6], which assumes the individual SYK cells to contain a thermodynamically large number  $N \rightarrow \infty$  of quantum particles. In this paper, we do not take this limit and explore what happens in ‘mesoscopic’ systems where  $N$  is large but finite. Our main finding is that the phase diagram becomes significantly more interesting and now features a zero temperature insulator–FL transition at a critical value of the inter-dot coupling inversely proportional to  $N$ . Extending the analysis to finite temperatures, we find an insulator/SM/FL phase separation as shown in (see Fig.1). Competitions of this type are seen in many contexts, indicating that the mesoscopic SYK network may capture essential ingredients for the phenomenological description of the correlated fermion matter.

The SYK model[4, 5] is a system of  $N$  Majorana fermions,  $\eta_i$ ,  $i = 1 \dots, N$ , subject to an all-to-all four fermion interaction  $H_{\text{SYK}} = (1/4!) \sum_{ijkl} J_{ijkl} \eta_i \eta_j \eta_k \eta_l$  with Gaussian distributed matrix elements  $J_{ijkl}$  of variance  $3!J^2/N^3$ . The system can be seen as a spatially local, zero dimensional paradigm of strongly interacting quantum matter: In the limit  $N \rightarrow \infty$ , the absence of a single particle term in the Hamiltonian implies that the fermion operators carry dimension  $[\text{time}]^{-1/4}$ , in marked

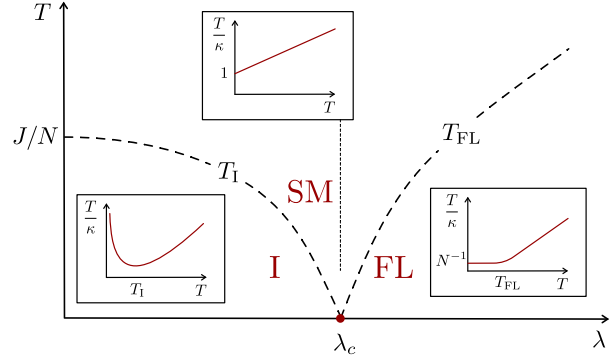


FIG. 1. Phase diagram of SYK array:  $T$  vs. dimensionless hopping strength  $\lambda = (NV/J)^2$ . At a critical value,  $\lambda_c = 8/Z$ , the system undergoes a zero temperature metal–insulator QPT. The two lines  $T_I(\lambda)$  and  $T_{FL}(\lambda)$  mark insulator (I) to strange metal (SM) and FL to SM crossovers, correspondingly. The insets show thermal resistivity  $T/\kappa(T)$  vs.  $T$  for  $\lambda < \lambda_c$ ,  $\lambda = \lambda_c$  and  $\lambda > \lambda_c$ .

distinction to the FL dimension  $-1/2$ . This motivates the extension to a  $d$ -dimensional array of nearest neighbor coupled non Fermi liquid cells. In view of the inherent randomness, it is natural to model the coupling by one-body operators  $H_T = (i/2) \sum_{\langle ab \rangle, ij} V_{ij}^{ab} \eta_i^a \eta_j^b$ , where  $a, b$  label the individual dots, and  $V_{ij}^{ab}$  are Gaussian distributed with variance  $V^2/N$ . Importantly, this coupling is a *relevant* perturbation of dimension  $[\int d\tau \eta \eta] = 1 - 2 \times 1/4 = +1/2$ . It implies a crossover from a non-FL ‘strange’ metal at high temperatures to a conventional, yet strongly renormalized, FL metal at low temperatures[6].

The above scenario makes reference to the engineering dimensions of the fermion operators and becomes valid in the thermodynamic limit. However, for finite  $N$ , very different behavior at low temperatures is expected. The non-FL nature of an isolated SYK dot manifests itself in an infinite dimensional ‘conformal’ symmetry[4, 17–21] under continuous reparameterizations of time. The above scaling dimension  $-1/4$  reflects the breaking of this symmetry at the large  $N$  mean-field level. However, as temperature is lowered below the energy scale  $J/N$ ,

strong Goldstone fluctuations associated to the conformal symmetry ensue, and effectively change the dimension of the fermion operator to  $-3/4$ [19, 20, 22, 23]. In this low energy regime, a single particle perturbation has dimension  $1 - 2 \times 3/4 = -1/2$  and now is RG *irrelevant*.

This dimensional crossover implies a competition between inter-dot couplings and intra-dot quantum fluctuations: depending on the bare strength of the coupling, Goldstone modes are either suppressed, or render the inter-dot coupling irrelevant. This implies the existence of a metal-insulator quantum phase transition (QPT) separating a phase of a strongly coupled FL from an insulating phase of essentially isolated dots. Below, we will explore this QPT within the framework of an effective low energy field theory describing granular SYK matter in terms of two coupling constants, representing intra-dot interaction and inter-dot coupling strength, respectively. We will demonstrate the renormalizability of the theory and from the flow of coupling constants (cf. Fig. 2 below) derive the manifestations of quantum criticality in two temperature scales marking an insulator/SM and FL/SM crossover at weak and strong coupling, respectively, cf. Fig. 1.

Before turning to the discussion of the model we note that reference [24] applied similar reasoning to predict a non-FL/FL phase transition for an isolated SYK dot subject to a one-body perturbation. We will comment on this result in relation to the I/FL transition in the array geometry after developing the proper theoretical framework. On general grounds we also expect similar physics in models of interacting *complex* fermions, the SY model[4–6]. However, the presence of U(1)-mode associated with particle number conservation in the SY system makes the theory more complicated. We here prefer to sidestep this complication and expose the relevant physics within the SYK framework, unmasked by the U(1) phase fluctuations [25]. In this system of electrically neutral Majorana fermions, thermal conductivity,  $\kappa(T)$ , is the main signature of transport, and from the Wiedemann-Franz law we infer that the ratio  $T/\kappa$  plays a role analogous to the electrical resistivity of complex fermion matter. We find that in the insulating phase it exhibits a minimum before diverging at small  $T$  as  $T/\kappa(T) \propto 1/T$  (cf. bottom left inset in Fig. 1). In the SM (FL) phase  $T/\kappa$  ratio exhibits  $T$ -linear (approximately  $T$ -independent) behavior, respectively.

*The model:* we consider a system described by the Hamiltonian[6]

$$H = \frac{1}{4!} \sum_a \sum_{ijkl} J_{ijkl}^a \eta_i^a \eta_j^a \eta_k^a \eta_l^a + \frac{i}{2} \sum_{(ab)} \sum_{ij} V_{ij}^{ab} \eta_i^a \eta_j^b, \quad (1)$$

where the mutually uncorrelated Gaussian distributed coefficients  $J_{ijkl}^a$  and  $V_{ij}^{ab}$  have been specified above. Following a standard procedure[17–21], the theory averaged over the coupling constant distributions is

described by an imaginary time functional  $Z = \int D(G, \Sigma) \exp(-S[G, \Sigma])$ , where  $G = \{G_{\tau_1, \tau_2}^a\}$  and  $\Sigma = \{\Sigma_{\tau_1, \tau_2}^a\}$  are time bi-local integration fields playing the role of the on-site SYK Green function and self-energy, respectively. The action  $S[G, \Sigma] \equiv \sum_a S_0[G^a, \Sigma^a] + \sum_{(ab)} S_T[G^a, G^b]$ , contains the ‘ $G\Sigma$ -action’,  $S_0$ , of the individual dots, and a tunneling action  $S_T[G^a, G^b] = \frac{1}{2} NV^2 \iint d\tau_1 d\tau_2 G_{\tau_1, \tau_2}^a G_{\tau_2, \tau_1}^b$  describing the nearest neighbor hopping. Here, we omit a replica structure[26] technically required to perform the averaging, but inessential in the present context.

While the explicit form of the  $G\Sigma$ -action[27] will not be needed, the following points are essential: (i) the action  $S_0$  possesses an exact  $SL(2, R)$ -invariance (see below) and approximate invariance under reparameterizations of time[4, 17–21],  $h : S^1 \rightarrow S^1, \tau \mapsto h(\tau)$ , where  $h$  is a diffeomorphism of the circle,  $S^1$ , defined by imaginary time with periodic boundary conditions onto itself. The infinite dimensional symmetry group  $\text{diff}(S^1)$  of these transformations is generated by a Virasoro algebra, hence the denotation ‘conformal’. (ii) The symmetry is subject to a weak explicit breaking by the time derivatives present in the action  $S_0$ . For low energies, the corresponding action cost is given by[4, 17–21, 28–30]  $S_0[h] = -m \int_0^\beta d\tau \{h, \tau\}$ , where  $\{h, \tau\} \equiv (\frac{h''}{h'})' - \frac{1}{2} (\frac{h''}{h'})^2$  is the Schwarzian derivative, and the proportionality  $m \propto N/J$  of the coupling constant indicates that quantum reparameterization fluctuations become stronger for small  $N$ . For temperature scales  $T < m^{-1}$  even large deviations,  $h$ , away from  $h(\tau) = \tau$  may have low action. This marks the entry into a low temperature regime dominated by strong reparameterization fluctuations. Finally, (iv) the mean-field Green function  $G_{\tau_1, \tau_2} = |\tau_1 - \tau_2|^{-1/2}$  (the square root dependence reflects the non-FL dimension of the fermions) transforms under reparameterizations as

$$G_{\tau_1, \tau_2} \rightarrow G_{\tau_1, \tau_2}[h] = \left( \frac{h'_1 h'_2}{[h_1 - h_2]^2} \right)^{1/4}, \quad (2)$$

where  $h_i \equiv h(\tau_i)$  and  $h'_i \equiv dh(\tau)/d\tau|_{\tau=\tau_i}$ . For an isolated dot, integration over the  $h$ -fluctuations effectively changes the Green function to  $\langle G_{\tau_1, \tau_2}[h] \rangle_h \stackrel{mT \ll 1}{\sim} m |\tau_1 - \tau_2|^{-3/2}$ , corresponding to a change of the fermion operator dimension to  $-3/4$ [19, 20, 22–24].

The effective low-energy *lattice Schwarzian theory* is formulated in terms of the reparameterizations  $h^a(\tau)$  on different dots. Its action  $S[h] = S_0[h] + S_T[h]$ , is defined through

$$S_0[h] = -m \sum_a \int d\tau \{h^a, \tau\}, \quad (3)$$

$$S_T[h] = -w \sum_{(ab)} \iint d\tau_1 d\tau_2 \left( \frac{h_1^a h_2^a}{[h_1^a - h_2^a]^2} \times \frac{h_1^b h_2^b}{[h_1^b - h_2^b]^2} \right)^{1/4},$$

where  $m$  and  $w$  are parameters with dimensions of [time] and [energy], and bare values  $m \propto N/J$  and  $w \propto NV^2/J$ .

A hallmark of the lattice Schwarzian action,  $S[h]$ , is its invariance under actions of  $\text{SL}(2, R)$ , where the group is represented via the Möbius transformations  $h(\tau) = \frac{\alpha\tau + \beta}{\gamma\tau + \delta}$  with  $\alpha\delta - \beta\gamma = 1$ . This shows that the  $h$ -transformations to be integrated cover the coset space  $\text{diff}(S^1)/\text{SL}(2, R)$ . The action itself is built from the two simplest  $\text{SL}(2, R)$  invariant blocks: local  $\{h, \tau\}$  and bi-local  $h'_1 h'_2 / [h_1 - h_2]^2$ . Maintained  $\text{SL}(2, R)$  symmetry imposes a stringent condition on the behavior of the theory under renormalization. A successive integration over  $h$ -transformations must leave the local and bi-local terms form invariant (multi-point terms may be generated but are irrelevant). The invariance condition thus implies that the renormalization results in a flow of the two couplings  $m$  and  $w$ .

*RG analysis:* we decompose fluctuations into 'fast' and 'slow' as  $h(\tau) = f(s(\tau)) \equiv (f \circ s)(\tau)$ , where  $f$  and  $s$  are fluctuations in the frequency range  $[\Lambda, J]$  and  $[0, \Lambda]$ , and  $\Lambda$  is a running cutoff energy[31]. We then integrate out the fast modes  $f(s)$ , and rescale time  $\tau \rightarrow \tau J/\Lambda$  to restore the UV cutoff  $\Lambda \rightarrow J$ . Consider first the case  $m^{-1} < \Lambda < J$ , where the reparameterization fluctuations are suppressed. The RG flow is then governed by the 'engineering' dimensions, resulting in:

$$\frac{d \ln m}{dl} = -1; \quad \frac{d \ln w}{dl} = +1, \quad (4)$$

where  $l = \ln(J/\Lambda)$ . For  $T > J/N$  this flow should be terminated when either  $\Lambda$  reaches  $T$ , or  $V(l) \sim \sqrt{w(l)}$  reaches the UV cutoff  $J$ . This defines the temperature scale  $T_{\text{FL}} = V^2/J$ , separating the high temperature SM and low temperature FL. In the SM phase  $w(T) = NV^2/T$  and  $T/\kappa(T) \propto J/w(T) \propto T/(NT_{\text{FL}})$  [6], while in FL the thermal resistivity saturates at  $T/\kappa(T) \propto 1/N$ .

We now turn to the regime of strong reparameterization fluctuations. When  $\Lambda$  reaches  $J/N$ ,  $m(l) = m(0)e^{-l}$  reaches the inverse UV cutoff  $m(l) \approx 1/J$ . To proceed with the further renormalization, we employ the Schwarzian chain rule

$$\{f \circ s, \tau\} = (s')^2 \{f, s\} + \{s, \tau\}, \quad (5)$$

to obtain the action:  $S_0[f \circ s] = S_0^{\text{fast}}[f, s] + S_0[s]$ , where the 'fast' Schwarzian action has a time-dependent mass  $m(s) \equiv ms^{a'}$ . At lowest order in  $w$  one needs to average the coupling action  $S_T[f \circ s]$  over the fast fluctuations. A straightforward application of the chain rule to the Green functions, Eq. (2), shows that

$$G_{\tau_1, \tau_2}[f \circ s] = G_{s_1, s_2}[f](s'_1 s'_2)^{1/4}, \quad (6)$$

so that  $\langle S_T[f \circ s] \rangle_f \propto \langle G_{s_1, s_2}[f^a] \rangle_{f^a} \times \langle G_{s_2, s_1}[f^b] \rangle_{f^b}$  splits into two fast averages. These expressions can be evaluated with the help of exact results [19, 22] for the 2-point propagator of the Schwarzian theory. Referring to the supplementary material for details [32], we note the

asymptotic expressions ( $s_{12} \equiv s_1 - s_2$ ):

$$\langle G_{s_1, s_2}[f] \rangle_f \simeq \begin{cases} |s_{12}|^{-1/2}, & s_{12} < m; \\ \sqrt{m(s_1)m(s_2)}|s_{12}|^{-3/2}, & m < s_{12} < \Lambda^{-1}; \\ m\Lambda|s_{12}|^{-1/2}, & \Lambda^{-1} < s_{12}. \end{cases} \quad (7)$$

This equation implies that the double time integral in the averaged tunneling action  $\langle S_T[f \circ s] \rangle_f \equiv S_{\text{int}} + S_{\text{long}}$  gets different contributions from intermediate ( $m < \tau_{12} < \Lambda^{-1}$ ) and long time differences ( $\tau_{12} > \Lambda^{-1}$ ). In processing the former, we use the general Taylor expansion ( $\tau = (\tau_1 + \tau_2)/2$ )

$$\left( \frac{s'_1 s'_2}{[s_1 - s_2]^2} \right)^\Delta \approx \frac{1}{[\tau_1 - \tau_2]^{2\Delta}} + \frac{\Delta}{6} \frac{\{s(\tau), \tau\}}{[\tau_1 - \tau_2]^{2\Delta-2}} + \dots, \quad (8)$$

with  $\Delta = 3/4$  to process the rational functions of the slow fields appearing upon substitution of Eqs. (5) and (7) into the action. Here, the second term indicates how the non-linear action of the tunneling term manages to feed back into the Schwarzian action under renormalization. Carrying out the details of the RG step (see supplementary material) and rescaling time to retain the value of the cutoff,  $\Lambda$ , we find that the integration over the intermediate time domain changes the coefficient of the local action as  $m \rightarrow m(l) \equiv e^{-l}(m + \frac{Z}{4}wm^2l)$ . The complementary integration over large time differences conserves the form of the tunneling action but changes the coupling constant as  $w \rightarrow w(l) = e^l w(m\Lambda)^2 = e^l w e^{-2l}$ .

From these results, RG equations are obtained by differentiation over  $l$  and putting  $l = 0$ . This leads to

$$\frac{d \ln m}{dl} = -1 + \frac{Z}{4}wm; \quad \frac{d \ln w}{dl} = +1 - 2. \quad (9)$$

The second equation reflects the aforementioned change of the dimension of  $w$  from  $+1$  to  $-1$ . While Eqs. (4) are applicable for  $mJ \gg 1$ , the new set of the RG equations (9) is derived in the opposite limit  $mJ \ll 1$ . (Indeed, this is the condition under which the exact expressions for the propagator[19, 22] can be reduced to the asymptotic expressions (7), see the supplementary material.)

*Analysis of the RG:* we first note that the limiting forms of the scaling equations, Eqs. (4) and (9), admit a closed representation in the dimensionless variable  $\lambda \equiv wm$ . In the regime  $mJ \gg 1$  one has  $d \ln \lambda / dl = 0$ , while for  $mJ \ll 1$ :

$$\frac{d \ln \lambda}{dl} = \left( \frac{Z}{4} \lambda - 2 \right). \quad (10)$$

This equation exhibits an unstable fixed point  $\lambda_c = \frac{8}{Z}$ , marking a transition between a FL phase at  $\lambda > \lambda_c$  and an insulating one at  $\lambda < \lambda_c$ . Since  $\lambda(0) \sim (NV/J)^2$ , one finds  $V_c \sim J/\sqrt{Z}N$ , inversely proportional to  $N$ , as stated in the introduction. Notice that according

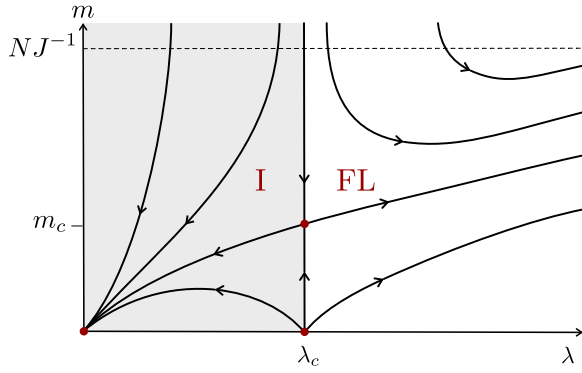


FIG. 2. RG flow in the plane of couplings ( $\lambda = mw, m$ ); here  $\lambda_c = 8/Z$  and  $m_c = \mathcal{O}(1)/J$ . The initial values are  $m(0) = N/J$  and  $\lambda(0) = (NV/J)^2$ .

to Eq. (9),  $d \ln m / d l|_{\lambda=\lambda_c} = +1$ , opposite to Eq. (4). The only way to reconcile the two limits is to have another fixed point at  $m_c \sim 1/J$ . The resulting two-parameter RG diagram in the plane  $(\lambda, m)$  is shown in Fig. 2. To first order in an expansion in  $w$ , but arbitrary  $m$ , this diagram may be derived from exact expressions for  $\langle G_{s_1, s_2}[f] \rangle_f$ , see supplementary material for details. In particular, the RG equation for  $w$  becomes  $d \ln w / d l = 2 - 4\Delta_\psi(m)$ , where  $\Delta_\psi(m)$  is the effective  $m$ -dependent scaling dimension of the fermion, see Fig. 3. The analysis of higher orders in  $S_T$  shows that the actual small parameter of the perturbative expansion is  $Z\lambda$ . Therefore, the fixed point is actually out of the perturbatively controlled regime and may not be used for quantitative evaluation of critical indices. However, second order calculations [33] show that RG flow keeps its qualitative form, Fig. 2.

The FL part of the RG diagram, Fig. 2, is well described by Eqs. (4) and the physics of the array is the one discussed in Ref. [6]. The only addition is that the crossover temperature  $T_{\text{FL}}(\lambda) \rightarrow 0$ , when  $\lambda \rightarrow \lambda_c$ , Fig. 1. This is due to the fact that for  $\lambda \approx \lambda_c$  the flow spends a long “time” in the vicinity of the  $(\lambda_c, m_c)$  fixed point, thus reaching progressively lower  $T$ . In the insu-

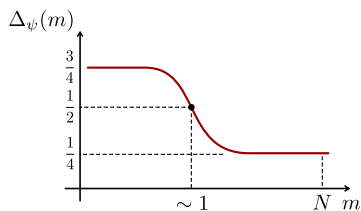


FIG. 3. The log-linear plot of the effective scaling dimension of fermion operators  $\Delta_\psi$ , as a function of the running scale  $mJ$ . For its exact definition in terms of 2-point function of the Schwarzian theory we refer to the supplementary material.

lating phase,  $\lambda \rightarrow 0$  and thus according to  $dw/dl = -w$  (Eq. (9)) and  $w \sim V^2$ ,  $V(T) \propto T^{1/2}$ . The diminishing of the inter-dot coupling at low temperatures implies that second order perturbation theory in  $V(T)$  may be applied to evaluate the thermal conductivity  $\kappa(T)$ . Therefore one finds  $\kappa(T)/T \propto |V(T)|^2 \propto T$  in the insulating phase.

To conclude, we have seen that the renormalization procedure indeed preserves the form of the lattice Schwarzian field theory. This stability follows from the conformal relations (5) and (8), but ultimately is required by the condition of maintained  $SL(2, R)$  symmetry. Our ability to deduce the entire RG flow (for  $Z\lambda \lesssim 1$ ) is owed to the knowledge of the reparameterization averaged Green function  $\langle G[f] \rangle_f$  for any  $m$ , which in turn follows from mapping of the local Schwarzian action to Liouville quantum mechanics [19]. We finally note that the RG procedure introduced in this Letter may likewise be applied to an isolated SYK dot subject to a random one-body perturbation [24]. The most important difference is that the action  $S_T$  is now subject to only one, and not two different reparameterization modes. This leads to a set of RG equations [34], different from the present ones in that strength of the one-body term,  $w$ , remains always relevant. At the same time, there is a transition in the scaling of  $m$ , separating a FL phase ( $m \gg 1/J$ ) from a phase of strong quantum fluctuations ( $m \rightarrow 0$ ), in line with the prediction of Ref. [24].

**Summary** — In this work we have shown that, regardless of dimensionality and geometric structure, an array of SYK dots coupled by one-body hopping exhibits a zero temperature metal-insulator transition. This phenomenon is rooted in the conformal invariance of the non-FL states supported by the individual SYK dots. The presence of this symmetry in turn is a direct consequence of an asymptotically strong dot-local interaction and may transcend the specific model employed here. A mutually suppressive competition between conformal fluctuations on the dots and the conformal symmetry breaking tunneling operators implies the presence of a transition between an insulating and a metallic phase, and a crossover into a strange metal regime at finite temperatures. Read in this way, the main message of our study is that phenomenology present in many strongly correlated materials, may follow from a rather basic principle. Although, the underlying Schwarzian lattice theory will not be able to describe the specific physics of realistic quantum materials, it will be intriguing to find out if the universality class of its phase transition can encompass strong correlations phenomena beyond those discussed here.

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